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# Finite Element Implementation of a Structurally-Motivated Constitutive Relation for the Human Abdominal Aortic Wall with and without Aneurysms

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**Abstract**— The structural integrity of the abdominal aorta is maintained by elastin, collagen, and vascular smooth muscle cells. Changes with age in the structure can lead to development of aneurysms. This paper presents initial work to capture these changes in a finite element model (FEM) of a structurally-motivated anisotropic constitutive relation for the “four fiber family” arterial model. First a 2D implementation is used for benchmarking the FEM implementation to fitted biaxial stress-strain data obtained experimentally from four different groups of persons; 19-29 years, 30-60 years, 61-79 years and abdominal aortic aneurysm (AAA) patients. Next the constitutive model is implemented in an anisotropic 3D FEM formulation for future simulation of intact aortic geometries. The 2D simulations of the biaxial test experiment show good agreement with experimental data with a standard deviation below 0.5% in all cases. The maximum axial and hoop stress in the group of AAA patients was 94.9 kPa ( $\pm 0.283$  kPa) and 94.3 kPa ( $\pm 0.224$  kPa) at maximum stretch ratios of 1.043 and 1.037, respectively. In the 3D simulations, the maximum stress is also found to occur in the AAA patient group, with the highest stress in the circumferential direction (275 kPa). Comparison with an already published isotropic model indicates that the latter underestimates the peak stress significantly. Based on these results it is concluded that the four fiber family model has been successfully implemented into a 3D anisotropic finite element model and that this model can provide more accurate insight into the stress conditions in aortic aneurysms.

**Keywords**— Biomechanics, aortic aneurysms, four fiber family model, anisotropic finite element analysis.

## I. INTRODUCTION

The wall of the normal human aorta is a layered structure consisting of three layers; the intima, the media and the adventitia. The primary structural components of the aortic wall are the elastic fibers (elastin and associated microfibrils), collagen fibers and vascular smooth muscle cells (vSMC). With age the structure of the aortic wall changes, it becomes stiffer, and more vulnerable to damage leading to diseases like atherosclerosis and aneurysms. So, it is interesting to construct a simulation model to capture these structural changes and gain more insight into the pathology of these diseases from a biomechanical point of view. The current challenge is to determine the arterial wall stress and

strength accurately. This presents some difficulty, because arterial tissue is anisotropic and nonlinear in the stress-stretch relationship, displays pseudo-elastic behavior, and changes material properties with age due to structural change and remodeling. The aim of this work is to implement the structurally-motivated phenomenological “four fiber family” model introduced by Baek et al. [1] for simulation of biomechanical properties in the human aorta with and without aneurysms. As a first step, a 2D finite element model (FEM) implementation is presented and used as a benchmark to numerically reproduce the stress-strain relations obtained in biaxial stress-strain experiments [2,3]. Next, the four fiber family model is implemented in a 3D anisotropic FEM and its ability to reveal detailed stress-strain information in arterial tissue is compared to that of an already published isotropic model [8].

## II. MATERIALS AND METHODS

### A. Constitutive framework

It is assumed that the aortic wall is a constrained mixture of four locally parallel families of collagen fibers (axial, circumferential, symmetric diagonal) embedded in an amorphous isotropic matrix dominated by elastic fibers. The biomechanical properties of a normal abdominal aorta and an aneurysm are described using the general formulation of the Cauchy stress (true stress) [4]

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2\mathbf{F} \frac{\partial W}{\partial \mathbf{C}} \mathbf{F}^T, \quad (1)$$

where  $\boldsymbol{\sigma}$  [Pa] is the Cauchy stress tensor,  $p$  is a Lagrange multiplier,  $\mathbf{I}$  is the identity tensor,  $\mathbf{F}$  is the deformation gradient tensor,  $W$  [Pa] is the SEF and  $\mathbf{C} = \mathbf{F}^T \mathbf{F}$  is the right Cauchy-Green tensor. In order for the SEF to be as general as possible the model accounts for compressibility by splitting the SEF in a purely volumetric elastic response,  $W_{vol}(J)$ , and a purely isochoric elastic response,  $W_{iso}(\mathbf{C}, \mathbf{M}^{(k)})$  [5],

$$W(\mathbf{C}, \mathbf{M}^{(k)}) = W_{vol}(J) + W_{iso}(\mathbf{C}, \mathbf{M}^{(k)}), \quad (2)$$

where  $J = \det(\mathbf{F})$  is the deformed-to-undeformed volume ratio and  $\mathbf{M}^{(k)}$  being a unit vector describing the direction of orientation of the collagen fiber families. Here the aortic tissue is assumed to be incompressible. To infer incompressibility the so-called penalty method is used in the finite element implementation. Here the tissue is modeled as slightly compressible by applying a very high bulk modulus in the volumetric elastic response, which has the simple form

$$W_{vol}(J) = \frac{\kappa}{2}(J - 1)^2, \quad (3)$$

where  $\kappa$  [Pa] is the bulk modulus [4]. The isochoric response is modeled by the four fiber family constitutive relation [1]

$$W_{iso}(\mathbf{C}, \mathbf{M}^{(k)}) = \frac{c}{2}(I_C - 3) + \sum_{k=1}^4 \frac{c_1^{(k)}}{4c_2^{(k)}} \left\{ \exp\left(c_2^{(k)}(IV_C^{(k)} - 1)^2\right) - 1 \right\} \quad (4)$$

where  $c$ ,  $c_1^{(k)}$ ,  $c_2^{(k)}$  are material parameters,  $I_C$  is the first invariant of  $\mathbf{C}$  and  $IV_C^{(k)} = \mathbf{M}^{(k)}\mathbf{C}\mathbf{M}^{(k)}$  is the fourth invariant of  $\mathbf{C}$ . This model has proven useful by providing increased insight into differences in the mechanical behavior due to structural abnormalities in the arterial wall [6]. For detailed information on the material parameters used in this study we refer to [6]. In brief, the determination of material properties is based on biaxial testing of tissue slabs from four different age/patient groups; 19-29 years, 30-60 years, 61-79 years, and AAA patients. Within each group the mean value of each material parameter is used.

#### B. Simulation of biaxial and inflation-extension test of arteries

Biaxial tension test of arteries is a well-known method for deducing the biomechanical properties of arteries [7]. Here we have simulated the biaxial testing of both normal abdominal aorta and pathological AAA tissue described by Vande Geest et al. [2,3] using COMSOL Multiphysics v4.1 (COMSOL AB, Stockholm, Sweden). In the simulation a tension value of 120 N/m is applied to the tissue corresponding to the circumferential tension per unit axial length in a thin-walled cylindrical tube pressurized to 113 mmHg, and the resulting stretch ratios and Cauchy stress components in the tissue sample are calculated. The inflation-extension test is also a commonly used experiment for determination of arterial properties, since the normal geometrical configuration of the artery is preserved [4]. Here a uniform internal pressure,  $P_i = 15$  [kPa] is applied corresponding to 113 mmHg, which results in a radial force on the

interior wall of a circular axis-symmetric cylinder. The cylinder has a radius of 1 cm and a length of 5 cm.

#### C. Analysis of simulated experiments

The implementation of the four fiber family model involves programming equations (1) – (4) into the finite element program. As the 2D variant of eq. (1) was used by Ferruzzi et al [6] to estimate the parameters of the constitutive model from the biaxial test data, the same equation can serve as a reference for benchmarking a 2D FEM implementation. In this perspective, stress-strain relations computed with a correct FEM implementation should be superimposed on the stress-strain relations calculated by hand using eq. (1). After benchmarking the implementation against biaxial test data the model is tested for predictability in the 3D case by comparing the anisotropic model to the isotropic model proposed by Raghavan and Vorp [8]. In this paper a negative Cauchy stress is interpreted as a compressive stress, and a positive stress is interpreted as a tensile stress. In addition, the unloaded configuration of the tissue is assumed to be stress free.

### III. RESULTS

#### A. Simulation of biaxial test

Comparison of the numerical simulation of the biaxial test and the analytical solution for the Cauchy stress components is shown in Fig. 3. The superimposition of the FEM results on the hand calculated curves confirms a correct FEM implementation of the model. The maximum stress values are seen in the circumferential direction (hoop stress) ranging from 85-175 kPa (638-1313 mmHg) compared to 85-165 kPa (638-1238 mmHg) for the axial direction. The maximum standard deviation was below 0.5% for both the hoop and axial stress;  $\pm 0.224$  kPa and 0.283 kPa respectively for the AAA patient group, which has the highest standard deviation compared to the other groups (not shown). In general the aortic tissue becomes less compliant with age, and AAA tissue is significantly stiffer than normal abdominal aortic tissue. However, using the mean values of the material parameters indicate that the biomechanical properties of the normal AA for the groups 30-60 years and 61-79 years are similar. The tissue from the group 61-79 years is less compliant in the axial direction compared to the group of 30-60 year-olds. But in the circumferential direction the difference is minimal. This clearly shows that the anisotropy of the aortic tissue is captured by the constitutive relation, since the stretch ratios in the two directions are different from each other for all four groups of test subjects.

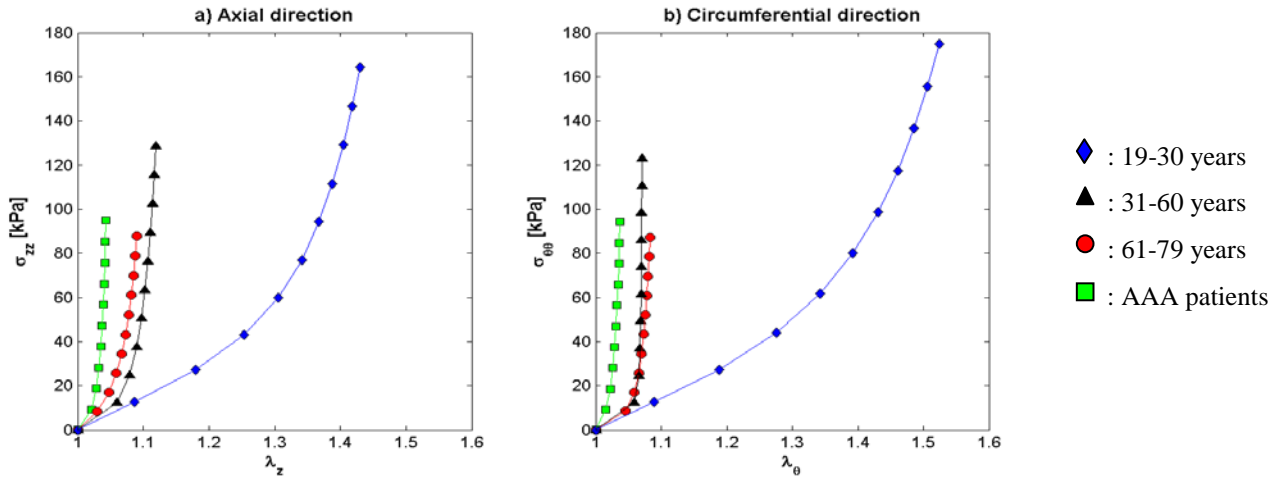
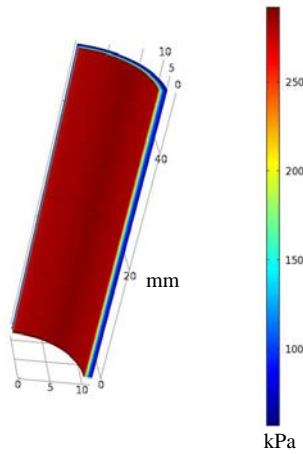


Fig 1 Stress-stretch plot comparing the known analytical solution for biaxial loading. (a) shows the axial Cauchy stress ( $\sigma_{zz}$ ) as a function of axial stretch ratio ( $\lambda_z$ ) for all four patient groups. (b) shows the Cauchy hoop stress ( $\sigma_{\theta\theta}$ ) as a function of circumferential stretch ratio ( $\lambda_\theta$ ) for all four patient groups. The solid lines are the solutions of the experimental fit, and the symbols indicate the numerical solution.

(a) Hoop stress in the normal AA



(b) Distribution of Cauchy stress within the aortic wall

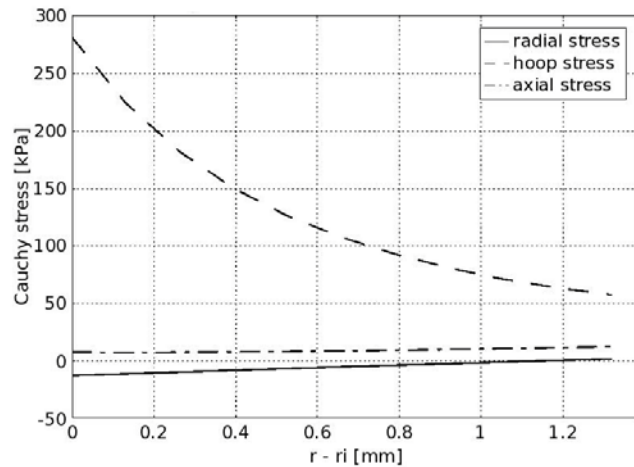


Fig 2 mm related inflation-extension experiment showing the amount of hoop stress within the aortic wall for the AAA patient group. (b) shows the stress distribution within the aneurismal wall for the AAA patient group. The dashed line is the hoop stress, the dash-dot line is the axial stress, and the solid line is the radial stress.

### B. Simulation of inflation-extension test

The four fiber family constitutive relation was implemented in an anisotropic 3D FE model and applied to a circular, axis-symmetric cylinder. To exploit symmetry only one quarter of the cylinder is simulated. The simulation result for the hoop stress is shown in Fig 2a for the AAA patient group. A maximum hoop stress of 275 kPa is seen at the innermost part of the cylinder, and 60 kPa at the external

part of the cylinder. With the new 3D model it is possible to investigate the anisotropic nature of the stress distribution within the aortic wall for the AAA patient group (see Fig 2b). The largest stress component is the hoop stress. The axial stress is almost constant varying from 8-10 kPa, and the radial (outward) stress is 15 kPa at the inner wall, corresponding to 113 mmHg, and zero at the external part of the wall. Comparing the results of the anisotropic model to the isotropic model for AAA tissue suggested by Raghavan and Vorp [8] (results not shown) the isotropic model underestimates the magnitude of the stress components within the

wall (peak hoop stress is 70 kPa) and the change in stress distribution within the wall is more uniform within the aortic wall.

#### IV. DISCUSSION AND CONCLUSION

In this paper a finite element implementation of the four fiber family constitutive relation in COMSOL Multiphysics is presented. When the condition of plane stress is secured in the biaxial test protocol, the axial and circumferential Cauchy stress components can be deduced analytically. The analytical model represented by eq. (1) serves a dual purpose. First, it can be used to fit parameters of a strain energy density function to experimental data and thereby provide a constitutive model for the stress-strain relationship. Secondly, the analytical model can serve as a reference for benchmarking numerical models such as finite element models. The former application was used by Ferruzzi et al [6] to develop a constitutive anisotropic model of arterial tissue from biaxial test data obtained by Vande Geest et al. [2]. The latter application was used successfully in this paper to benchmark an implementation of the four fiber model in the finite element program COMSOL Multiphysics. The use of mean values of the material parameters in finite element models is common [10]. But here the mean values indicate that there is not a significant difference between the groups of 31-60 years and 61-79 years in the biomechanical properties. This is surprising due to a significant difference in mean age (43 and 70 years respectively). The number of subjects in each group is the same with similar distribution among the sexes. This raises the question whether the division in the current age groups is suitable. An alternative could be to subdivide the group of 31-60 year-olds into smaller intervals of five or ten years, since it seems that the most significant change in arterial structure takes place in this period. Another possibility is to use the median of the material parameters, since this would eliminate the effect of outliers in the different patient groups. This exploration of the parameter space, together with extension of the mechanical tests to include inflation-extension tests of both normal abdominal aortic and aneurismal tissue, could improve the current model. In addition, with these improvements it might also be possible to obtain more complete knowledge about when the critical damage to the aortic tissue is most likely to occur. The model considered here is purely passive and does not account for the contribution from activation of vascular smooth muscle cells. The reasons for not including the active part are two-fold. There is lack information on the change in smooth muscle activity in normal AA. Secondly,

AAA contains limited amounts of smooth muscle cells, [4,6].

Extending this finite element implementation to patient-specific model geometries with matching patient-specific blood flow will give the clinician a very powerful tool for detailed evaluation of AAAs.

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