



The photonic nanowire: an emerging platform for highly efficient single-photon sources for quantum information applications

Gregersen, Niels; Munsch, Mathieu; Malik, Nitin S.; Bleuse, Joël; Dupuy, Emmanuel; Delga, Adrien; Mørk, Jesper; Gérard, Jean-Michel; Claudon, Julien

Published in:

Proceedings of SPIE, the International Society for Optical Engineering

Link to article, DOI:

[10.1117/12.2020622](https://doi.org/10.1117/12.2020622)

Publication date:

2013

[Link back to DTU Orbit](#)

Citation (APA):

Gregersen, N., Munsch, M., Malik, N. S., Bleuse, J., Dupuy, E., Delga, A., ... Claudon, J. (2013). The photonic nanowire: an emerging platform for highly efficient single-photon sources for quantum information applications. Proceedings of SPIE, the International Society for Optical Engineering, 8749, [87490W]. <https://doi.org/10.1117/12.2020622>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

The photonic nanowire: An emerging platform for highly efficient single-photon sources for quantum information applications.

Niels Gregersen ^{*a}, Mathieu Munsch^b, Nitin S. Malik^b, Joël Bleuse^b, Emmanuel Dupuy^b, Adrien Delga^b, Jesper Mørk^a, Jean-Michel Gérard^b and Julien Claudon^b

^aDTU Fotonik, Department of Photonics Engineering, Technical University of Denmark, Building 343, DK-2800 Kongens Lyngby, Denmark

^bCEA-CNRS-UJF group "Nanophysique et Semiconducteurs," CEA, INAC, SP2M, F-38054 Grenoble, France

ABSTRACT

Efficient coupling between a localized quantum emitter and a well defined optical channel represents a powerful route to realize single-photon sources and spin-photon interfaces. The tailored fiber-like photonic nanowire embedding a single quantum dot has recently demonstrated an appealing potential. However, the device requires a delicate, sharp needle-like taper with performance sensitive to minute geometrical details. To overcome this limitation we demonstrate the photonic trumpet, exploiting an opposite tapering strategy. The trumpet features a strongly Gaussian far-field emission. A first implementation of this strategy has led to an ultra-bright single-photon source with a first-lens external efficiency of 0.75 ± 0.1 and a predicted coupling to a Gaussian beam of 0.61 ± 0.08 .

Keywords: Single-photon source, photonic nanowire, inverted conical taper, photonic trumpet, spin-photon interface, efficiency, Gaussian beam.

1. INTRODUCTION

Optimizing the coupling between a localized quantum emitter [1] and a well defined optical channel represents a powerful route to realize the bright sources of the non-classical light states required in quantum communication [2] and information processing applications. [3],[4] Reversibly, the efficient absorption of a photon impinging on the emitter is key to realize strong non-linear interactions between single photons required for the spin-photon interface, the node of future quantum networks. [5] The semiconductor quantum dot (QD) embedded in a engineered semiconductor microstructure appears as a promising platform for both the single-photon source (SPS) and the single-photon absorber. The self-assembled InAs QD features a high radiative yield without bleaching or blinking, it has a narrow spectral linewidth at cryogenic temperatures and may be either optically or electrically pumped. For practical applications, it is desirable to trigger the photon using an electrical pulse, and this can be achieved by integrating the QD into a *p-i-n* structure to realize a single-photon LED.

For the quantum communication and quantum information applications, a near-unity efficiency ε , defined as the number of detected photons by the collection optics per trigger, is desired. [6] The fabrication of self-assembled QDs allowing for a nearly perfect conversion of excitons to emitted photons is well-established, however to obtain a high efficiency the photonic environment must be engineered such that all the emitted light couples to the collection optics.

For an InAs QD embedded in GaAs, the large refractive index of GaAs limits the photon extraction efficiency to a few percent. This extraction efficiency can be improved by placing the emitter inside a high quality (Q) microcavity [7] and taking advantage of cavity quantum electrodynamics. The relevant figures of merit are the β factor describing the spontaneous emission rate into the fundamental cavity mode over the emission total rate and the probability γ describing the ratio of the power collected by the collection optics and that of the fundamental mode. [6] Assuming a perfect

* ngre@fotonik.dtu.dk; phone +45 45253789; fax +45 45936581; www.fotonik.dtu.dk

radiative yield and that contributions from other modes to the detected power are negligible, the total efficiency ε is then given by the product $\beta\gamma$. In high-Q microcavities, a β close to unity is achieved using the Purcell effect, and an efficiency ε of 0.79 was recently reported [8] for a micropillar SPS. However, this microcavity approach requires a careful spectral and spatial alignment between the emitter and the cavity mode leading to a complicated in-situ fabrication technique. [9] The purity of the single-photon emission from the high-efficiency cavity-based sources demonstrated so far has not been ideal with typical values of $g^{(2)}(\tau=0) \geq 0.15$ at saturation. [8],[10] These values are most likely due to the presence of numerous QDs and cavity feeding effects. [11]-[13] Furthermore, these resonant systems only operate over the limited bandwidth of the cavity resonance.

As alternative we have explored the photonic nanowire geometry, [14]-[21] well adapted to efficient collection of light. This structure features no cavity and relies on a geometrical screening of radiation modes [14] rather than resonant effects to ensure a strong coupling between the quantum dot and the optical waveguide mode. This means that spectral alignment between the emitter line and a narrow cavity line is not required, which represents a huge practical advantage in the fabrication. The combination of a top conical tapering [15] and a metal mirror [16] allows for control of the far-field radiation pattern. This has led to the proposal of optically [17] and electrically [18] pumped designs with predicted efficiencies of ~ 0.9 and to the experimental demonstration of an optically pumped device fabricated using a top-down approach with a measured efficiency ε of 0.72. [19] Furthermore, for a given dot density, the smaller area of the nanowire QD layer compared to that of the micropillar means that fewer dots are present, and very pure photon emission with a measured $g^{(2)}(\tau=0)$ as low as 0.008 [19] has been obtained. However, the device requires a delicate, sharp needle-like taper for efficient out coupling of light, and the performance of this taper is sensitive to minute geometrical details on the order of 1° . While such sharp tapers have been realized using a bottom-up fabrication approach, [20] the measured efficiency of 0.42 was limited by the difficulty of implementing a metal mirror in this approach.

In this work, we demonstrate a new photonic nanowire design, the photonic ‘‘trumpet,’’ [20] exploiting an opposite tapering strategy based on an inverted conical taper to ensure efficient in- and out-coupling. The trumpet allows for opening angles above 10° , thus greatly relaxing fabrication tolerances. Moreover, the trumpet features a Gaussian far-field emission, a strong asset for most applications, and is fully compatible with electrical contacting. We report on the first implementation of this strategy and demonstrate an ultra-bright single-photon source with a first-lens external efficiency ε of 0.75 ± 0.1 and a predicted coupling ε_G to a Gaussian beam of 0.61 ± 0.08 .

This paper is organized as follows. In section 2 we present the general characteristics of the photonic nanowire SPS designs. The simplified model used to analyze the SPS efficiency and the simulation method is described in section 3. The performance of the photonic trumpet and the experimental results are given in section 4, followed by a conclusion in section 5.

2. THE PHOTONIC NANOWIRE GEOMETRY

The two photonic nanowire SPS designs considered in this work are illustrated in Fig. 1. The photonic nanowire is simply a GaAs cylinder placed on a substrate. Single photons are emitted by an InAs QD and they couple to the fundamental mode of the nanowire with a probability which can be optimized by choosing the correct nanowire radius. At an emission wavelength of $\lambda = 950$ nm, a β of 0.96 is obtained [21] for an optimum radius R_{nw} of 120 nm. This high β is the result of a geometrical screening effect dampening [14] the electric field strength of radiation modes inside the nanowire. For even smaller radii, the guided mode is no longer supported and the same screening effect results in strong inhibition of the spontaneous emission [14] realized without the presence of a bandgap.

Due to symmetry, half of the photons in the fundamental mode will be propagating towards the substrate and it is necessary to implement a bottom metal mirror [16] reflecting these photons back towards the top. We include a thin dielectric layer of thickness ~ 10 nm to suppress coupling to surface plasmons resulting in a reflection coefficient $R_{11} = |r_{11}|^2$ of 0.91.

Using the mirror, we ensure that the emitted photons in the fundamental mode propagate towards the top of the nanowire, however for a nanowire radius R_{nw} of 120 nm, the fundamental mode waist is narrow leading to a wide far-field radiation pattern and a poor coupling to the objective lens. [22] To improve this coupling it is thus necessary to expand the mode waist such that the resulting output beam divergence is reduced. [15] The designs presented in Fig. 1. represent two different strategies for achieving this mode expansion.

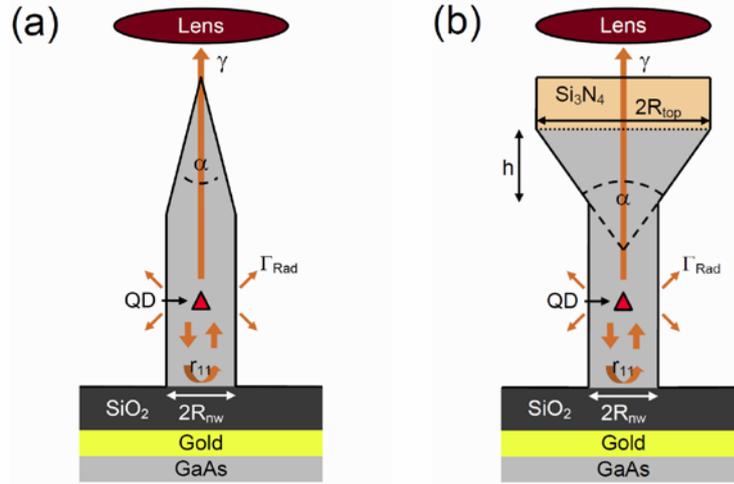


Fig. 1. Optically pumped photonic nanowire designs based on a regular tapering (a) and the inverted “trumpet” taper (b).

The design in Fig. 1(a) features a regular conical tapering of opening angle α . As the fundamental mode propagates forward, the radius of the nanowire decreases. For R_{nw} below ~ 100 nm, the nanowire can no longer support a guided mode, and the fundamental mode expands rapidly. For sufficient small values of α , the taper thus allows for an adiabatic expansion of the fundamental mode, [15] resulting in a large mode waist and a narrow far field profile. Furthermore, the mode is adiabatically expanded into the surrounding air leading to a strong reduction of the reflection at the semiconductor-air interface. The adiabatic expansion primarily takes part in the lower part of the taper, and thus a truncated taper with truncation radius of $R_r = 60$ nm gives the same performance as the perfectly sharp tip.

The top part of the photonic nanowire is engineered to suppress reflection. There is thus no cavity meaning that the emitted photons escape immediately and are thus much less subject to scattering from fabrication imperfections than they are in e.g. the high-Q micropillar cavity. A scanning electron micrograph (SEM) of a full device based on the regular conical tapering is illustrated in Fig. 2(a). Indeed, this device features a measured efficiency of 0.72 combined with a very clean single photon emission with $g^{(2)}(\tau = 0)$ of 0.008. [19]

However, the design in Fig. 1(a) suffers from several weaknesses. The fabrication of the regular conical tapering with opening angles $\leq 5^\circ$ is quite challenging using our top-down approach. [19] Furthermore, due to the large part of the mode extending into the surrounding air, the implementation of a top metal contact which does not perturb the optical field profile is not immediate.

For this reason, we have investigated a photonic nanowire “trumpet” design implementing an inverted conical tapering



Fig. 2. Scanning electron micrographs of photonic nanowires implementing a regular conical tapering (a) and an inverted conical “trumpet” tapering (b).

[20] with opening angle α . The design is illustrated in Fig. 1(b). The bottom part of the design is identical to that in Fig. 1(a), however the top part features an inverted conical tapering and an anti-reflection coating. Here, the fundamental mode is adiabatically expanded not outside the nanowire as it is the case for the regular tapering, but inside the photonic nanowire. Since the mode stays inside the nanowire, the reflection at the top facet semiconductor-air interface must be suppressed, and to this aim we introduce a Si_3N_4 anti-reflection coating with an index $n_{AR} = \text{sqrt}(n_{GaAs})$ and thickness $\lambda / (4n_{AR})$.

This work focuses on the out coupling performance of the new trumpet design, and in the following we thus study two figures of merit, namely the total transmission γ to a 0.75 numerical aperture lens as well as the transmission γ_G to a Gaussian beam profile.

3. MODELING TECHNIQUE

The QD is treated as a radially oriented point dipole placed in the center of the rotationally symmetric nanowire, and all calculations are performed at a wavelength of $\lambda = 950$ nm. We employ the eigenmode expansion technique [23] combined with improved perfectly matching layers [24] for the numerical simulations. The tapers are modeled using a staircase approximation. [25]

In an exact calculation of the SPS efficiency ε we should place the point dipole inside the full structure and compute the corresponding far field emission. Though rigorous, this approach does not give a clear physical insight into the physical mechanisms governing the efficiency. For this reason we instead employ a single-mode model based on an element-splitting approach. This simplified model, the validity of which was demonstrated in Ref. [17], allows us to analyze and optimize the various elements separately. Assuming an emitter positioned in an antinode of the electric field, the total efficiency ε in the element-splitting approach is described by

$$\varepsilon = \frac{\beta\gamma(1+|r_{11}|)^2}{2(1+\beta|r_{11}|)}, \quad (1)$$

where r_{11} is the modal reflectivity of the metal mirror and γ is the transmission through the conical tapers to the lens, as illustrated in Fig. 1. In this model the efficiency is given by the product $\beta\gamma$ and a correction taking into account the reflectivity of the metal mirror. Equation (1) is a transparent expression for the total efficiency ε . The element-splitting approach thus allows us to analyze the four building blocks separately and directly study the influence of the characteristic parameters β , r_{11} , and γ on the SPS efficiency ε .

4. PERFORMANCE AND RESULTS

The predicted transmissions γ and γ_G for the two taper strategies are given in Fig. 3 as function of taper opening angle α . The total transmission γ into a 0.75 numerical aperture lens (black thin lines) as well as the transmission γ_G to a Gaussian beam (red full lines) are presented. For the regular design, we observe in Fig. 3(a) that an opening angle below 5° is required to achieve a total transmission γ above 0.85. An ultra-sharp needle with an opening angle of 2° has been realized using a bottom-up fabrication approach, and for such a small angle a transmission above 0.99 is obtained. However, we observe that even for $\alpha < 5^\circ$, the computed transmission γ_G to a Gaussian profile remains below 0.5. The regular conical tapering strategy is thus unsuitable for achieving a near-unity coupling to a Gaussian profile.

The performance of trumpet design is shown in Fig. 3(b). In this calculation we have fixed the taper height h to $12 \mu\text{m}$, and the top taper radius R_{top} thus has a near-linear dependence on α . For small values of α , the mode waist at the top of the inverted taper is small leading to a wide output beam profile and a low transmission. However, we observe that in the interval $6^\circ < \alpha < 15^\circ$ the total transmission γ is above 0.96. The inverted trumpet design thus allows for significantly larger opening angles than the regular design and is much less sensitive to small variations in angle. Furthermore, we observe that the transmission γ_G to a Gaussian profile reaches a predicted maximum of 0.97 for an opening angle of $\sim 12^\circ$. For larger opening angles, the adiabatic expansion of the fundamental mode is compromised and the transmission γ_G drops.

These results show that the trumpet design approach is less demanding in terms of side wall angle and thus represents an attractive alternative well adapted to our top-down fabrication technique. Furthermore, the trumpet design allows for near-unity coupling to a Gaussian profile, an important asset for the realization of spin-photon interfaces.

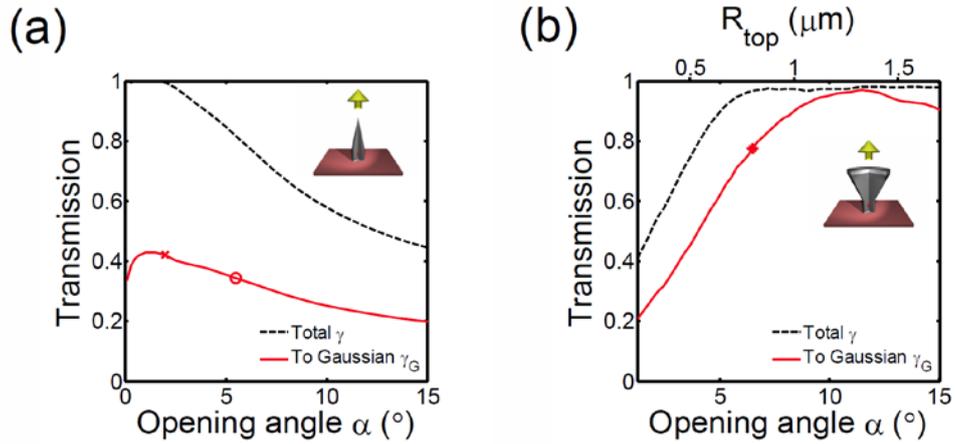


Fig. 3. Computed total transmission (black dashed lines) and transmission to a Gaussian profile (red full lines) for the perfectly sharp regular (a) and the inverted “trumpet” (b) taperings for a lens with numerical aperture of 0.75 and a taper height $h = 12 \mu\text{m}$. Experimental realizations: Needle taper X (Ref. [20]) and O (Ref. [19]) and photonic trumpet * (This work).

We have fabricated photonic nanowire SPSs based on the inverted trumpet design, [20] and a representative device is illustrated in Fig. 2(b). The trumpet is $12 \mu\text{m}$ high and its top facet features a radius $R_{top} = 775 \text{ nm}$. The facet is covered by a 115 nm thick Si_3N_4 antireflection coating. The bottom part of the trumpet features a radius R_{nw} in the $100 - 120 \text{ nm}$ range and the taper angle is $\alpha = 6.5^\circ$. The structure is supported by a gold-silica planar mirror and embeds a few ($\sim 5 - 10$) InAs self-assembled QDs, located 110 nm above the mirror. The sample has been processed out of a planar sample grown by molecular beam epitaxy using a top-down approach.

The first lens external efficiency ε of the source was determined through careful calibration of the setup, conducted using a laser tuned to the QD emission wavelength as a reference. The following values have been corrected from residual multiphoton events. When driven to saturation an efficiency $\varepsilon = 0.75 \pm 0.1$ is measured. The computed transmission γ for this device is 0.96. Insertion of $\beta = 0.96$ and $|r_{11}|^2 = 0.91$ into (1) leads to $\varepsilon = 0.95 \gamma$, and thus a predicted efficiency of 0.9. We attribute the slight discrepancy with the measured efficiency to non-ideal off-axis positioning of the QD.

The external coupling efficiency to a Gaussian beam can be derived from $\varepsilon / \varepsilon_G = \gamma / \gamma_G$. Using the computed values $\gamma = 0.96$ and $\gamma_G = 0.78$ and the measured $\varepsilon = 0.75$, we obtain $\varepsilon_G = 0.61 \pm 0.08$. This figure of merit represents a major improvement over the state-of-the-art ($\varepsilon_G = 0.24$ for QD-oxide aperture micropillar cavity, 0.32 for a regular photonic nanowire).

5. CONCLUSION

In conclusion, photonic trumpets offer a unique combination of broad operation bandwidth, high extraction efficiency and Gaussian far-field emission. The realization of an on-demand, ultra bright single-photon source represents a first implementation of this approach. Regarding advanced quantum light sources, the circular top facet is very convenient to implement a top electrode, [10],[18] which is desirable to provide an electrical charge injection in the QD, [26] or to tune its fine spectral properties with an electric field. [27] We thus anticipate photonic trumpets will constitute a robust and generic platform for a wide range of solid-state quantum optics applications.

ACKNOWLEDGMENTS

We acknowledge support from the Danish Research Council for Technology and Production (Contract No. 10-080752) and from the NATEC Center financed by VILLUM FONDEN.

REFERENCES

- [1] Shields, A. J., "Semiconductor quantum light sources," *Nat. Photonics* **1**(4), 215-223 (2007).
- [2] Gisin, N., and Thew, R., "Quantum communication," *Nat. Photonics* **1**(3), 165-171 (2007).
- [3] O'Brien, J. L., "Optical Quantum Computing," *Science* **318**, 1567-1570 (2007).
- [4] Knill, E., Laflamme, R., and Milburn, G. J., "A scheme for efficient quantum computation with linear optics," *Nature* **409**(6816), 46-52 (2001).
- [5] Kimble, H. J., "The quantum internet," *Nature* **453**, 1023-1030 (2008).
- [6] Barnes, W. L., Björk, G., Gérard, J. M., Jonsson, P., Wasey, J. A. E., Worthing, P. T., and Zwiller, V., "Solid-state single photon sources: light collection strategies," *Eur. Phys. J. D.* **18**(2), 197-210 (2002).
- [7] Gérard, J. M., Sermage, B., Gayral, B., Legrand, B., Costard, E., and Thierry-Mieg, V., "Enhanced Spontaneous Emission by Quantum Boxes in a Monolithic Optical Microcavity," *Phys. Rev. Lett.* **81**(5), 1110-1113 (1998).
- [8] Gazzano, O., Michaelis de Vasconcellos, S., Arnold, C., Nowak, A., Galopin, E., Sagnes, I., Lanco, L., Lemaître, A., and Senellart, P., "Bright solid-state sources of indistinguishable single photons," *Nat. Comm.* **4**, 1425 (2013).
- [9] Dousse, A., Lanco, L., Suffczynski, J., Semenova, E., Miard, A., Lemaître, A., Sagnes, I., Roblin, C., Bloch, J., and Senellart, P., "Controlled Light-Matter Coupling for a Single Quantum Dot Embedded in a Pillar Microcavity Using Far-Field Optical Lithography," *Phys. Rev. Lett.* **101**, 267404 (2008).
- [10] Reitzenstein, S., Heindel, T., Kistner, C., Albert, F., Braun, T., Hopfmann, C., Mrowinski, P., Lermer, M., Schneider, C., Höfling, S., Kamp, M. and Forchel, A., "Electrically Driven Quantum Dot Micropillar Light Sources," *IEEE J. Sel. Top. Quantum Electron.* **17**(6), 1670-1680 (2011).
- [11] Hennessy, K., Badolato, A., Winger, M., Gerace, D., Atature, M., Gulde, S., Falt, S., Hu, E. L., and Imamoglu, A., "Quantum nature of a strongly coupled single quantum dot-cavity system," *Nature* **445**(7130), 896-899 (2007).
- [12] Winger, M., Volz, T., Tarel, G., Portolan, S., Badolato, A., Hennessy, K. J., Hu, E. L., Beveratos, A., Finley, J., Savona V., and Imamoglu, A., "Explanation of Photon Correlations in the Far-Off-Resonance Optical Emission from a Quantum-Dot-Cavity System," *Phys. Rev. Lett.* **103**(20), 207403 (2009).
- [13] Hohenester, U., "Cavity quantum electrodynamics with semiconductor quantum dots: Role of phonon-assisted cavity feeding," *Phys. Rev. B* **81**(15), 155303 (2010).
- [14] Bleuse, J., Claudon, J., Creasey, M., Malik, N. S. and Gérard, J. M., "Inhibition, Enhancement and Control of Spontaneous Emission in Photonic Nanowires," *Phys. Rev. Lett.* **106**(10), 103601 (2011).
- [15] Gregersen, N., Nielsen, T. R., Claudon, J., Gérard, J. M., and Mørk, J., "Controlling the emission profile of a nanowire with a conical taper," *Opt. Lett.* **33**(15), 1693-1695 (2008).
- [16] Friedler, I., Lalanne, P., Hugonin, J. P., Claudon, J., Gérard, J. M., Beveratos, A., and Robert-Philip, I., "Efficient photonic mirrors for semiconductor nanowires," *Opt. Lett.* **33**(22), 2635-2637 (2008).
- [17] Friedler, I., Sauvan, C., Hugonin, J. P., Lalanne, P., Claudon, J., and Gérard, J. M., "Solid-state single photon sources: the nanowire antenna," *Opt. Express* **17**(4), 2095-2110 (2009).
- [18] Gregersen, N., Nielsen, T. R., Mørk, J., Claudon, J., and Gérard, J. M., "Designs for high-efficiency electrically pumped photonic nanowire single-photon sources," *Opt. Express* **18**(20), 21204-21218 (2010).
- [19] Claudon, J., Bleuse, J., Malik, N. S., Bazin, M., Jaffrennou, P., Gregersen, N., Sauvan, C., Lalanne, P. and Gérard, J. M., "A highly efficient single-photon source based on a quantum dot in a photonic nanowire," *Nat. Photonics* **4**(3), 174-177 (2010).
- [20] Reimer, M. E., Bulgarini, G., Akopian, N., Hocevar, M., Bavinck, M. B., Verheijen, M. A., Bakkers, E. P., Kouwenhoven, L. P., and Zwiller, V., "Bright single-photon sources in bottom-up tailored nanowires," *Nat. Comm.* **3**, 737 (2012).
- [21] Munsch, M., Malik, N. S., Dupuy, E., Delga, A., Bleuse, J., Gérard, J. M., Claudon, J., Gregersen, N. and Mørk, J., "Dielectric GaAs-antenna ensuring an efficient broadband coupling between an InAs quantum-dot light emitter and a Gaussian optical beam," to be published in *Phys. Rev. Lett.*
- [22] Nowicki-Bringuier, Y.-R., Hahner, R., Claudon, J., Lecamp, G., Lalanne, P., and Gérard, J. M., "A novel high-efficiency single-mode single photon source," *Ann. Phys. Fr.* **32**, 151-154 (2007).
- [23] Maslov, A. V., and Ning, C. Z., "Far-field emission of a semiconductor nanowire laser," *Opt. Lett.* **29**(6), 572-574 (2004).
- [24] Bienstman, P., and Baets, R., "Optical modelling of photonic crystals and VCSELs using eigenmode expansion and perfectly matched layers," *Opt. Quantum Electron.* **33**, 327-341 (2001).

- [25] Gregersen, N., and Mørk, J., "An Improved Perfectly Matched Layer for the Eigenmode Expansion Technique," *Opt. Quantum Electron.* **40**, 957-966 (2008).
- [26] Gregersen, N., Nielsen, T. R., Tromborg, B., and Mørk, J., "Quality factors of nonideal micro pillars," *Appl. Phys. Lett.* **91**(1), 011116 (2007).
- [27] Yuan, Z., Kardynal, B. E., Stevenson, R. M., Shields, A. J., Lobo, C. J., Cooper, K., Beattie, N. S., Ritchie, D. A., Pepper, M., "Electrically Driven Single-Photon Source" *Science* **295**, 102-105 (2002).
- [28] Finley, J. J., Sabathil, M., Vogl, P., Abstreiter, G., Oulton, R., Tartakovskii, A. I., Mowbray, D. J., Skolnick, M. S., Liew, S. L., Cullis, A. G. and Hopkinson, M., "Quantum-confined Stark shifts of charged exciton complexes in quantum dots," *Phys. Rev. B* **70**, 201308(R) (2004).