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Lower Bound for the Radiation Q of Electrically Small Magnetic Dipole Antennas With Solid Magnetodielectric Core

Oleksiy S. Kim and Olav Breinbjerg

Abstract—A new lower bound for the radiation Q of electrically small spherical magnetic dipole antennas with solid magnetodielectric core is derived in closed form using the exact theory. The new bound approaches the Chu lower bound from above as the antenna electrical size decreases. For $ka < 0.863$, the new bound is lower than the bounds for spherical magnetic as well as electric dipole antennas composed of impressed electric currents in free space.

Index Terms—Chu lower bound, electrically small antennas, fundamental limitations, magnetic dipole, quality factor.

I. INTRODUCTION

An ideal electrically small magnetic dipole antenna composed of an impressed electric current on the surface of an imaginary sphere exhibits a radiation Q three times the Chu lower bound [1], [2]. When the surface current encloses a magnetic core the internal stored magnetic energy reduces, and the Chu lower bound [3] can be reached for a vanishingly small antenna. However, the behavior of a finite-size magnetic dipole antenna is more complicated due to the internal resonances in the magnetic core. Exact analytical expressions derived in [4] show that the minimum achievable Q is a function of the antenna electrical size. This function represents the lower bound for the radiation Q of a magnetic dipole antenna with a magnetodielectric core. In this communication, taking account of the rigorous theory presented in [4], we derive an analytical expression for this lower bound and define its range of validity.

II. LOWER BOUND FOR THE QUALITY FACTOR

We consider an impressed electric current on the surface of a material spherical core with relative permeability μ_r and permittivity ε_r. In a spherical coordinate system $(r\theta\phi)$ the surface current density radiating the TE_{10} spherical mode (magnetic dipole mode) is

$$\mathbf{J} = \hat{\mathbf{a}}_\phi J_0 \sin\theta \tag{1}$$

where $\hat{\mathbf{a}}_\phi$ is the azimuthal unit vector and J_0 is the amplitude (A/m). Closed-form expressions for the stored magnetic W_H and electric W_E energies as well as for the radiated power P_{rad} are derived in [4]. From those, the radiation quality factor Q can be expressed in compact form, subject to $W_H \geq W_E$, as

$$Q(ka, \mu_r, \varepsilon_r) = \left\{ 1 + \frac{1}{\mu_r} \Lambda(k_s a) \right\} Q_{Chu}(ka) \tag{2}$$

where k is free-space wave number, a is the radius of the antenna, $k_s = \sqrt{\varepsilon_r \mu_r} k$ is the wave number in the core material, the function $\Lambda(k_s a)$

$$\Lambda(k_s a) = \frac{1 + (k_s a) j_0(k_s a) y_0(k_s a)}{2 j_1^2(k_s a)} - 1 \tag{3}$$
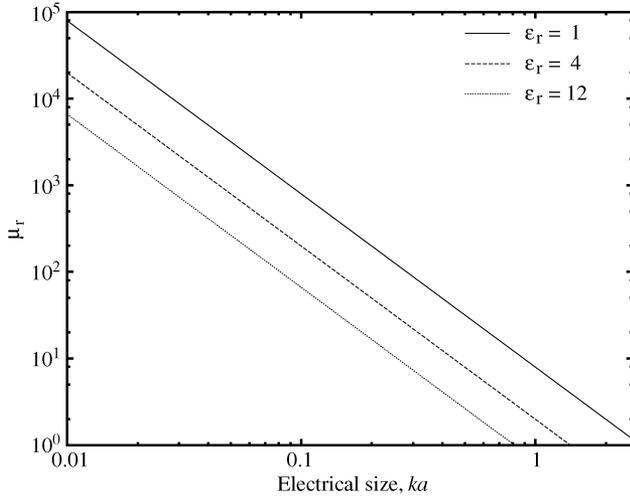
Fig. 1. Optimum permeability (8) as a function of the antenna electrical size ka for $n = 1$ and $\varepsilon_r = 1, 4, 12$.

$j_n(x)$ and $y_n(x)$ are the n-order spherical Bessel functions of the first and second kind, respectively, and Q_{Chu} is the Chu lower bound [3], [5]

$$Q_{Chu}(ka) = \frac{1}{(ka)^3} + \frac{1}{ka}. \qquad (4)$$

In the limiting case of a vanishingly small antenna, expression (2) reduces to the well-known formula by Wheeler [1]

$$\lim_{ka \to 0} Q(ka, \mu_r, \varepsilon_r) = Q(ka, \mu_r) = \left(1 + \frac{2}{\mu_r}\right) \frac{1}{(ka)^3}. \qquad (5)$$

For a finite-size antenna, the function $\Lambda(k_s a)$ exhibits singularities at zeroes of the Bessel function $j_1(k_s a)$, and, consequently, so does the quality factor Q in (2). The singularities originate from the internal TE_{10}-mode resonances in the spherical core, when the external fields, and thus the radiated power, are zero. To find the minimum Q between the resonances and the corresponding optimal permeability of the core, we have to solve the following equation

$$\frac{\partial Q(ka, \mu_r, \varepsilon_r)}{\partial \mu_r} = 0. \qquad (6)$$

Substituting (2) into (6) and letting $u = k_s a$, we obtain

$$\frac{u}{2} \frac{\partial \Lambda(u)}{\partial u} - \Lambda(u) \equiv \frac{F_1(u)}{F_2(u)} = 0. \qquad (7)$$

This equation has an infinite number of solutions—zeroes $u_n (n = 1, 2, 3, \ldots)$ of the oscillating function $F_1(u)$. The corresponding optimal permeability is then found as

$$\mu_r(ka, \varepsilon_r) = \frac{u_n^2}{\varepsilon_r (ka)^2} \qquad (8)$$

and illustrated in Fig. 1.

Obviously, for each solution u_n there is a corresponding quality factor $Q_n(ka)$, which can be obtained from (2) taking into account (8) as

$$Q_n(ka, \varepsilon_r) = \left\{1 + (ka)^2 \varepsilon_r \frac{\Lambda(u_n)}{u_n^2}\right\} Q_{Chu}(ka)$$
$$\equiv \left\{1 + (ka)^2 \varepsilon_r \lambda_n\right\} Q_{Chu}(ka). \qquad (9)$$

TABLE I
ZEROS u_n OF THE FUNCTION $F_1(u)$ IN (7) AND THE CORRESPONDING CONSTANTS λ_n IN (9)

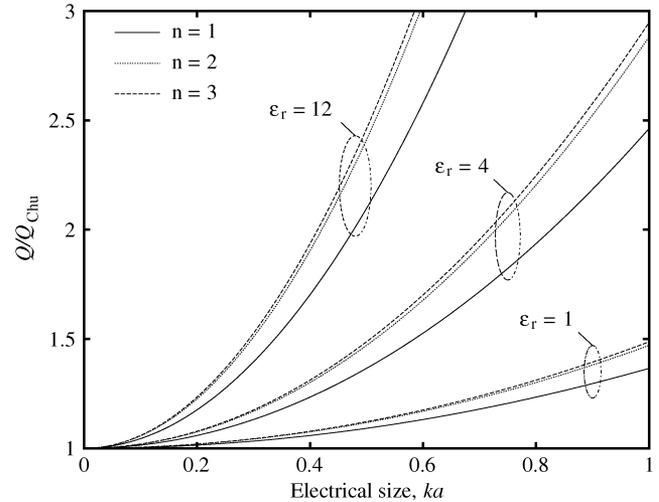| n	1	2	3	4
u_n	2.8161	6.1915	9.3684	12.5252
λ_n	0.3658	0.4707	0.4872	0.4928

Fig. 2. Normalized quality factor Q_n/Q_{Chu} (9) corresponding to the optimum permeability (8) for $n = 1, 2, 3$ and $\varepsilon_r = 1, 4, 12$.

The first four values of u_n and the corresponding constants λ_n are given in Table I. From Table I as well as from Fig. 2, which illustrates the expression (9), it is clear, that the first value u_1 yields the lowest λ_n, and, consequently, the lowest $Q_n(ka, \varepsilon_r)$. Since the $Q_n(ka, \varepsilon_r)$ also reduces as the permittivity ε_r decreases, the optimal core must be pure magnetic with relative permeability

$$\mu_r(ka) = \frac{u_1^2}{(ka)^2}. \qquad (10)$$

Finally, the lower bound for magnetic dipole antennas can be written from (9) as

$$Q_{LB}(ka) = Q_1(ka, \varepsilon_r = 1)$$
$$= \left\{1 + (ka)^2 \lambda_1\right\} Q_{Chu}(ka)$$
$$= \frac{1}{(ka)^3} + \frac{\lambda_1 + 1}{ka} + (ka)\lambda_1. \qquad (11)$$

The conclusion is that for a given antenna electrical size ka there is an optimum magnetic core permeability (10) that ensures the lowest possible Q as given in (11). This Q is thus the lower bound for magnetic dipole antennas with solid magnetodielectric core.

As illustrated in Fig. 3, the new bound (11) is higher than the Chu lower bound, but approaches it as $ka \to 0$. At the same time, it shows that with a properly selected magnetic core an electrically small magnetic dipole antenna exhibits a Q that is lower not only as compared to the same antenna with air core, but also as compared to an air-core electric dipole antenna[1] of the same size. We note, however, that in a region $0.863 < ka < 1.325$—although, in this region an antenna can hardly be considered electrically small—an air-core electric dipole antenna performs slightly better.

[1]Here, and throughout the communication, we assume that air-core magnetic and electric dipole antennas are composed of electric current distributions on an imaginary spherical surface in free space, that is, no other materials or objects, neither inside, nor on the surface, nor outside of the antenna, are present.
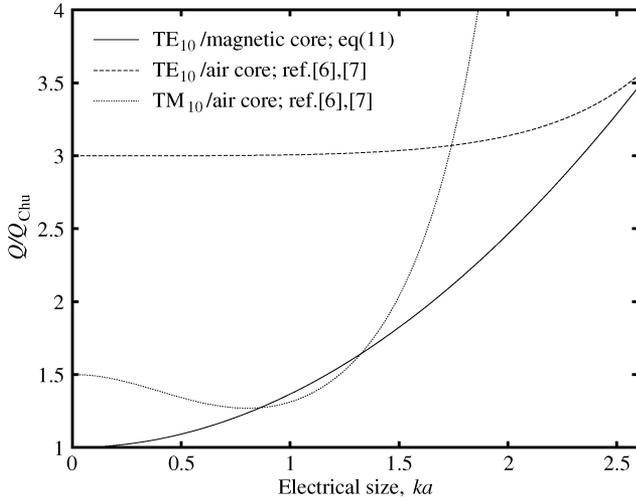
Fig. 3. The lower bound for magnetic dipole antennas with magnetodielectric core is compared to the bounds for air-core electric and magnetic dipole antennas [6], [7]. The bounds are normalized to the Chu lower bound Q_{Chu}.

Fig. 4. Values (ka, μ_{r}) corresponding to the resonances in a magnetic dipole antenna with a pure magnetic core ($\varepsilon_{\mathrm{r}} = 1$). For the parameters in the shaded region $W_{\mathrm{H}} < W_{\mathrm{E}}$. Optimum permeability (10) is also plotted.

III. RANGE OF VALIDITY

All expressions given in Section II are exact and valid as long as the stored electric energy does not exceed the stored magnetic energy. Since these take turns in being dominant as the resonances are crossed, to define the range of validity for our expressions, we first need to find values of $(ka, \mu_{\mathrm{r}}, \varepsilon_{\mathrm{r}})$, for which

$$W_{\mathrm{H}}(ka, \mu_{\mathrm{r}}, \varepsilon_{\mathrm{r}}) = W_{\mathrm{E}}(ka, \mu_{\mathrm{r}}, \varepsilon_{\mathrm{r}}). \tag{12}$$

The first set of values corresponds to the total resonance

$$W_{\mathrm{H}}^{-}(ka, \mu_{\mathrm{r}}, \varepsilon_{\mathrm{r}}) + W_{\mathrm{H}}^{+}(ka, \mu_{\mathrm{r}}, \varepsilon_{\mathrm{r}})$$
$$= W_{\mathrm{E}}^{-}(ka, \mu_{\mathrm{r}}, \varepsilon_{\mathrm{r}}) + W_{\mathrm{E}}^{+}(ka, \mu_{\mathrm{r}}, \varepsilon_{\mathrm{r}}) \tag{13}$$

where $W_{\mathrm{H}}^{\pm}$ and $W_{\mathrm{E}}^{\pm}$ are, respectively, the magnetic and electric energies stored inside and outside the antenna core. The solution of (13) is not straightforward and has to be found numerically.

The second set of values is associated with the internal TE_{10}-mode resonances in the spherical core of the antenna, when in the absence of the external fields the internal magnetic W_{H}^{-} and electric W_{E}^{-} stored energies are balanced

$$W_{\mathrm{H}}^{-}(ka, \mu_{\mathrm{r}}, \varepsilon_{\mathrm{r}}) = W_{\mathrm{E}}^{-}(ka, \mu_{\mathrm{r}}, \varepsilon_{\mathrm{r}}). \tag{14}$$

In contrast to (13), the solution of (14) is known—$k_s a = v_n$, where v_n are zeroes of the spherical Bessel function $j_1(v)$.

Since the bound (11) implies a pure magnetic core ($\varepsilon_{\mathrm{r}} = 1$), we can plot the result in a (ka, μ_{r}) space, as shown in Fig. 4. Except for the shaded region, between the straight line representing the internal resonance (dependence $k_s a = v_1$) and a curve denoting the total resonance, the stored magnetic energy dominates over the stored electric energy. Thus, the optimum parameters (10), also shown in Fig. 4, corresponding to the lower bound (11) are located in the valid region $W_{\mathrm{H}} > W_{\mathrm{E}}$, which extends from the electrically small regime, $ka \ll 1$, up to $ka \approx 2.6$, at which (10) yields a total resonance. Consequently, the lower bound (11) is applicable in the range of antenna electrical sizes $0 < ka < 2.6$.
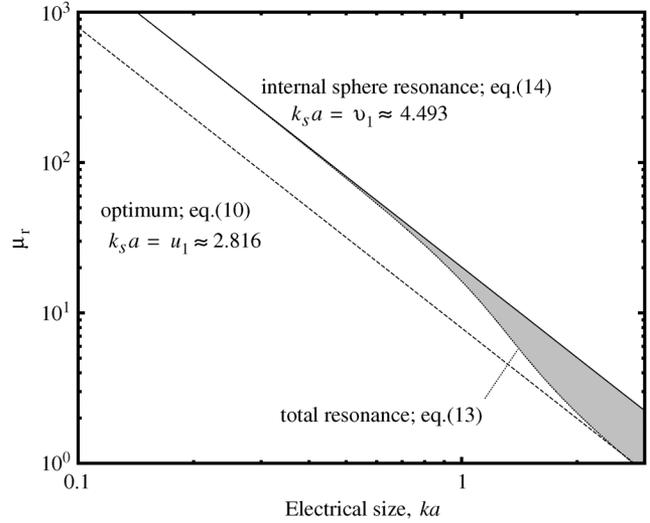
IV. CONCLUSION

The lower bound for the radiation Q of magnetic dipole antennas with solid magnetodielectric core is derived explicitly. The bound can be reached by a spherical magnetic dipole antenna with a pure magnetic core, whose permeability is properly selected according to the antenna electrical size. Such an antenna exhibits a lower Q than the air-core magnetic and electric dipole antennas of the same size in the range of $ka < 0.863$ as well as for $1.325 < ka < 2.6$. The later value sets the upper limit for the range of validity of the presented bound.

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