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Neutron to proton mass difference, parton distribution functions and baryon resonances from dynamics on the Lie group $u(3)$

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Abstract
We present a hamiltonian structure on the Lie group $u(3)$ to describe the baryon spectrum. The ground state is identified with the proton. From this single fit we calculate approximately the relative neutron to proton mass shift to within half a percent of the experimental value. From the same fit we calculate the nucleon and delta resonance spectrum. For specific spin eigenfunctions we calculate the delta to nucleon mass ratio to within one percent.

We derive parton distribution functions. The distributions are generated by projecting the proton state to space via the exterior derivative. This projection has shown to yield parton distribution functions that compares rather well with those of the proton valence quark distributions already in a first order approximation.

The allospatial hypothesis
It is the hypothesis of the present work, that the eigenstates of the above space and assume the following Hamiltonian

$$H = \sum_{i=1}^{3} a_i \sum_{j=1}^{3} \sum_{k=1}^{3} \psi_{ij}(x) \frac{1}{2m_i} \left( \frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial x_k^2} - \frac{\partial^2}{\partial x_{ij}^2} \right) \phi_{ij}(x) + \psi_{ij}(x) \frac{1}{2m_i} \left( \frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial x_k^2} - \frac{\partial^2}{\partial x_{ij}^2} \right) \phi_{ij}(x)$$

where $\psi_{ij}(x)$ and $\phi_{ij}(x)$ are the components of the spinor and the wave function, respectively.

The theory unfolded
The Laplacian in (1) contains off-diagonal derivatives which are represented by the off-diagonal Goldstone matrixes. We choose those three to represent spin and group them into $H_{12}, H_{23}, H_{31}$. This interpretation is supported by their commutation relations as body fixed angular momentum. The relation between space and allospatial is like the relation in number theory between the space-like and the other body fixed coordinate systems for the description of the orientational degrees of freedom. The remaining three are grouped into $H_{11}, H_{22}, H_{33}$, which is related to hypercharge and isospin. They act upon the algebra by commuting into the subspace of $\mathfrak{su}(3)$. The fully parameterised Laplacian in polar decomposition reads

$$\Delta = \frac{1}{2m_i} \left( \frac{\partial^2}{\partial \theta_i^2} + \frac{\partial^2}{\partial \phi_i^2} \right)$$

The constant term is interpreted as a curvature potential and the ellipsoidal term is analogous to the centrifugal term in the usual treatment of the radial wave function for the hydrogen atom.

$$\Delta \approx \frac{1}{2m_i} \left( \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta_i^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi_i^2} \right)$$

With the periodic potential in (2) our complete allospatial equation reads with $E = \hbar^2 / 8$, and $\lambda = 1$ with $E = \hbar^2 / 2$. The third term is given for $\theta_1 = \theta_2 = \theta_3 = \phi_1 = \phi_2 = \phi_3 = 0$.

And a similar factorisation of $\psi_{ij}(x) \phi_{ij}(x) \psi_{ij}(x) \phi_{ij}(x)$ gives for $\theta_1 (0, 0, 0, 0, 0, 0)$, with $(0, 0, 0, \theta_1, \theta_2, \theta_3)$:

$$- \Delta \approx \frac{1}{2} \psi_{ij}(x) \frac{1}{2} \psi_{ij}(x) \approx \frac{1}{2} \psi_{ij}(x) \frac{1}{2} \psi_{ij}(x)$$

The figure shows parameter eigenstates with periodicity $2\pi$ to the left and $2\pi$ to the right. We can couple a dreemituding period doubling in level two with an additional period doubling in level one. We interpret these coupled period doubling as representing the transformation from a neutral state (i.e. the reaction) to a charge state (e.g. the proton).

$$n \rightarrow p$$

if it is similar to spin rotation functions

$$\psi_r (x) \psi_r (x) \approx \frac{1}{2} \psi_r (x) \frac{1}{2} \psi_r (x)$$

Conclusions
Approximate energy levels for baryonic states are found by combinations of three parameters: eigenvalues of the three torus angles. These eigenvalues can be limited to have the same periodicity as the proton. However, a coupled period doubling can decrease the total energy.

Periodic potential and reduced zone scheme
The period doubling is projected into the Bloch wave number choices for the neutron (left) and the proton (right).

Parton distributions
We project from a state constructed from trigonometric functions to mimick the period doubling implied in the decay to the proton state.

References

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Appendix A: list of people have been helpful