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# Loss Allocation in a Distribution System with Distributed Generation Units

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**Abstract** — In Denmark, a large part of the electricity is produced by wind turbines and combined heat and power plants (CHPs). Most of them are connected to the network through distribution systems. This paper presents a new algorithm for allocation of the losses in a distribution system with distributed generation. The algorithm is based on a reduced impedance matrix of the network and current injections from loads and production units. With the algorithm, the effect of the covariance between production and consumption can be evaluated. To verify the theoretical results, a model of the distribution system in Brønderslev in Northern Jutland, including measurement data, has been studied.

**Index Terms** — Distributed generation, wind power, loss allocation

## 1. INTRODUCTION

Since the mid eighties, a large number of wind turbines and distributed combined heat and power plants have been connected to the Danish power system. Especially in the Western part, comprising Jutland and Funen, the penetration is high compared to the load demand. In some periods the wind power alone can even cover the entire load demand.

Traditionally, the distributed generation (DG) units have to some extent been regarded as passive negative loads with the main purpose of producing energy and not disturbing the operation of the distribution systems.

Since the mid nineties, the Danish electrical power system, like most European power systems, has been going through a liberalization process, where services such as production, transmission, distribution, power balancing, ancillary services etc. are being unbundled. When evaluating the economy of DG, more aspects than the annual energy production must be taken into account. Dependent on the coincidence with the load demand and the location, the DG units can for example help reducing the power system losses in cases where they supply local consumers and work as peak shaving in high load periods.

The installation of DG in a distribution system affects the total system losses. In systems where the penetration of DG is low, the DG units are located close to load centers and there is a large coincidence between load and production, the DG units can contribute to reduction of the total system losses. On the other hand, if the power from the DG units connected to the MV or LV network has to be exported to the transmission system, because the local production exceeds the local demand, the total system losses will be higher than if the power were produced at a large power plant connected directly to the transmission system.

## 2. LOSS ALLOCATION

Different loss allocation methods have been developed to quantify the influence of different participants on the total

system losses. In a liberalized market, this knowledge can be used to avoid cross subsidizing in the transmission and distribution fees of consumers and producers [1], to generate incentives of the participants to change the consumption or production in periods with congestion [2;3] or to estimate the value of distributed generation in an area [4]. In systems where the investments and operation are partially or fully centrally controlled, the allocation of losses can be used to optimize the operation and investments and to minimize the losses. One of the problems about separating the cause of losses is their non linear nature.

In literature, the following main approaches of loss allocation based on deterministic methods are found [1;5-7]: Pro Rata procedures where the losses are allocated to producers and consumers proportionally to the delivered or consumed energy, Marginal Loss Allocation procedures where the losses are allocated according to the change in losses corresponding to a small change in production or consumption and Proportional Sharing procedures, also referred to as Tracing [6], where the losses are allocated according to the total power flows in the system generated by the participants. Further, the Z-Bus allocation method has been proposed in [8] where the losses are allocated based on the current flows in the system rather than the power flows.

### 2.1. Allocation based on current injections

A new method for loss allocation based on current injections rather than bus voltages or power flows is proposed here. The difference between this method and the method presented in [8] is that here, a single slack bus is assumed. This is considered reasonable in distribution systems which typically only has one infeed from the transmission system.

#### 2.1.1. Mathematical formulation

The theory is based on the standard system impedance matrix as described for example in [9]. For the investigations, the network busses have been divided into three types: Fixed voltage busses, fixed current busses, and busses without sources. The relation between the voltage and the current in the fixed voltage and fixed current busses can be expressed with the full impedance matrix in (1). The rows and columns corresponding to busses without sources have been removed from the matrix.

$$\begin{bmatrix} \overline{\mathbf{V}}_v \\ \overline{\mathbf{V}}_i \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \cdot \begin{bmatrix} \overline{\mathbf{I}}_v \\ \overline{\mathbf{I}}_i \end{bmatrix} \quad (1)$$

The voltage at the busses with fixed current injections can be expressed as (2) where  $\underline{\mathbf{Z}}_i$ , defined in (3), is a reduced impedance matrix for the busses with fixed current injection

when the busses with fixed voltage have been short circuited.  $\underline{\mathbf{K}}_{21}$ , defined in (4), represents the relation between the voltage at the busses with fixed current infeed and the busses with fixed voltage. Analogously, the current injections at the constant voltage busses can be calculated using (5).

$$\overline{\mathbf{V}}_I = \underline{\mathbf{Z}}_I \cdot \overline{\mathbf{I}}_I + \underline{\mathbf{K}}_{21} \overline{\mathbf{V}}_V \quad (2)$$

$$\underline{\mathbf{Z}}_I = \underline{\mathbf{Z}}_{22} - \underline{\mathbf{Z}}_{21} \cdot \underline{\mathbf{Z}}_{11}^{-1} \underline{\mathbf{Z}}_{12} \quad (3)$$

$$\underline{\mathbf{K}}_{21} = \underline{\mathbf{Z}}_{21} \cdot \underline{\mathbf{Z}}_{11}^{-1} \quad (4)$$

$$\overline{\mathbf{I}}_V = \underline{\mathbf{Z}}_{11}^{-1} \overline{\mathbf{V}}_V - \underline{\mathbf{K}}_{12} \cdot \overline{\mathbf{I}}_I \quad (5)$$

$$\underline{\mathbf{K}}_{12} = \underline{\mathbf{Z}}_{11}^{-1} \underline{\mathbf{Z}}_{12} \quad (6)$$

For the loss allocation, all production units and loads are considered as fixed current injections, and a single slack bus with a fixed voltage magnitude and angle is assumed. The total system losses can be expressed as the sum of all power injections in the system (7).

$$S_{\text{loss}} = \mathbf{I}_{\text{SL}}^* \mathbf{V}_{\text{SL}} + \overline{\mathbf{I}}_I^H \cdot \overline{\mathbf{V}}_I \quad (7)$$

The current of the slack bus and the voltage of the load and generation busses can be eliminated using (2) and (5). After some manipulation and assuming that the reduced impedance matrix is symmetric, i.e. no phase shifting transformers are present, the losses in (7) can be reformulated as (8).

$$S_{\text{loss}} = \underbrace{\mathbf{V}_{\text{SL}} (\underline{\mathbf{Z}}_{11}^{-1})^* \mathbf{V}_{\text{SL}}^*}_{\text{No-load losses}} + \underbrace{\overline{\mathbf{I}}_I^H \underline{\mathbf{Z}}_I \overline{\mathbf{I}}_I}_{\text{Load dependent losses}} + \underbrace{j \cdot 2 \cdot \overline{\mathbf{I}}_I^H \Im (\underline{\mathbf{K}}_{21}) \cdot \mathbf{V}_{\text{SL}}}_{\text{Cross effect}} \quad (8)$$

The expression consists of three terms. The first term describes the no-load losses which are dependent only on the voltage at the slack bus. This includes shunt losses in transformers and series losses related to reactive power flows in the shunt elements. The second term represents the losses which are related to the square of the current infeeds. The last term represents a cross coupling between the two first terms. The last term describes the change in losses related to supplying the shunt elements from different busses. For example, one can think of a transformer with a large magnetizing current, located far away from the slack point. If a part of the magnetizing current is supplied at a connection point close to the transformer, it will contribute to reduction of the overall losses. This effect is not covered by the load dependent quadratic term. If the shunt impedances in the system are large compared to the series impedances, it can be seen that  $\underline{\mathbf{K}}_{21}$  will be close to unity and the last term in (8) will be relatively small.

The most interesting term is the term describing the load dependent losses, because this term describes the effect of the power flows in the system. The second term in (8) only gives a scalar value. To separate the contributions from the individual participants and the cross couplings between them, the load dependent term can be reformulated as in (9). The factor in the square brackets is an  $N$  by  $N$  matrix where  $N$  is the number of current injections.

The real part of the diagonal elements will always be positive. This means that any traffic of active and reactive current in the system will cause active power losses. The real

part of the off-diagonal elements can either be positive or negative, dependent on the loading of the network.

$$S_{\text{loss-series}} = \overline{\mathbf{I}}_I^H \underline{\mathbf{Z}}_I \overline{\mathbf{I}}_I = \overline{\mathbf{I}}_I^T \cdot \left[ \left( \overline{\mathbf{I}}_I \cdot \overline{\mathbf{I}}_I^H \right)^* [\bullet] \underline{\mathbf{Z}}_I \right] \cdot \overline{\mathbf{I}}_I \quad (9)$$

### 2.1.2. Allocation based on covariance and mean flows

The reduced impedance matrix,  $\underline{\mathbf{Z}}_I$ , shows how the different cross products of the currents affect the losses. The contribution of the cross products to the mean losses is dependent on the simultaneity between activity of the different producers and consumers. A measure of the simultaneity is given by the covariance matrix. The covariance matrix of a random vector,  $\overline{\mathbf{F}}$ , is defined as (10), where  $E$  denotes the expected values [10]. The generalization of the theory to include complex random vectors is discussed in [11].

$$\text{cov}[\overline{\mathbf{F}}] = E \left[ (\overline{\mathbf{F}} - E[\overline{\mathbf{F}}]) \cdot (\overline{\mathbf{F}} - E[\overline{\mathbf{F}}])^H \right] \quad (10)$$

Rearranging (10), the expected value of the outer product of the vector with itself can be expressed as (11).

$$E[\overline{\mathbf{F}} \cdot \overline{\mathbf{F}}^H] = \text{cov}[\overline{\mathbf{F}}] + E[\overline{\mathbf{F}}] \cdot E[\overline{\mathbf{F}}]^H \quad (11)$$

If the current vector is treated as a vector of complex stochastic variables with a mean value and a variance, the expected value or the mean value of the losses can be formulated as (12) by combining (9) and (11).

$$E[S_{\text{loss-series}}] = \overline{\mathbf{I}}_I^T \cdot \left[ \underbrace{\text{cov}[\overline{\mathbf{I}}_I]}_{\text{Contr. from variance}} + \underbrace{E[\overline{\mathbf{I}}_I] \cdot E[\overline{\mathbf{I}}_I]^H}_{\text{Contr. from mean flows}} \right] [\bullet] \underline{\mathbf{Z}}_I \cdot \overline{\mathbf{I}}_I \quad (12)$$

Equation (12) shows that the effect of the mean values of the current infeeds on the losses can be separated from the effect of the covariance between the current infeeds. The cross term of (8) could be considered a part of the losses related to the mean power flows, because if the voltage at the slack point is relatively constant, this term depends on the mean currents.

The real part of the term containing the mean values is difficult to change. The mean value of the production or consumption over longer period is given by the actual energy demand. The mean value of the reactive power can be changed, e.g. by installing or removing a capacitor or changing the power factor of a synchronous machine. The diagonal elements of the covariance matrix describe the variation of the consumption or production of each connection point. The off-diagonal elements describe the simultaneity of the variations of different current sources. Traditionally the information contained in the covariance matrix has been represented with a coincidence factor or Velander's coefficients [12]. The simultaneity between different loads and productions is caused by several effects with different time periods including hourly, daily, weekly and seasonal variations. An estimate of the mean values and covariances therefore only describes the behavior within the period where the measurements were taken.

The element wise product of the covariance matrix and the reduced impedance matrix can give an indication of where there is potential for power savings, e.g. by changing the production pattern of a CHP to better match the load pattern of a group of consumers in the vicinity. The impedance matrix may not be constant during the entire period under

consideration. For example, the position of the under load tap changers of the transformers changes from time to time. If the changes are relatively small, a mean value of the impedance matrix can be used. Alternatively, analyses can be performed separately for time periods with different network configurations.

2.1.3. Allocation based on linear regression

In cases where large sets of measurement data are available, but the network parameters are not exactly modeled, regression methods can be used to separate the causes of the active and reactive power losses from each other and to anticipate future losses based on prognoses. In [13] the causes of the Reactive power exchange between a distribution system and a transmission system have been allocated to the wind turbines, CHPs and consumers using a linear regression analysis. This approach has also been used in [14] to determine the impact of wind turbines on the reactive power losses in the distribution transformers of a system. [15;16] propose a cluster wise linear regression method based on fuzzy logic to anticipate and allocate active power losses in a distribution system.

The idea of the linear regression analysis is to represent the losses as a linear combination of a number of input variables. Generally, the linear regression problem can be specified as (13) [17].  $\hat{\mathbf{y}}$  is a column vector with one sample of the estimated quantity per entry,  $\mathbf{X}$  is a matrix with a row for each observation and a column for each input parameter,  $\bar{\mathbf{B}}$  is a column vector with one coefficient per input parameter, and  $\bar{\mathbf{I}}$  is an identity column vector with the same size as  $\hat{\mathbf{y}}$ .

$$\hat{\mathbf{y}} = \mathbf{b}_0 \bar{\mathbf{I}} + \mathbf{b}_1 \bar{\mathbf{x}}_1 + \mathbf{b}_2 \bar{\mathbf{x}}_2 + \dots + \mathbf{b}_k \bar{\mathbf{x}}_k = \mathbf{X} \cdot \bar{\mathbf{B}} \quad (13)$$

There are standard algorithms for determining the coefficient vector which leads to the smallest quadratic deviation between the measured and the estimated output. However, it is important to know the basic structure of the problem to select a set of input parameters which provide sufficient but not redundant information.

Reformulating (8) leads to the expression in (14). If there are no tap changing transformers and switchable capacitor batteries in the system, the model in (14) gives a complete description of the system. This means that if  $\mathbf{s}_{\text{loss}}$  and  $\bar{\mathbf{X}}$  are exactly known, for a large number of samples, and the input variables are not linearly dependent or constant, a regression analysis will give the  $\bar{\mathbf{B}}$ -vector in (14).

$$\hat{\mathbf{S}}_{\text{loss}} = \bar{\mathbf{X}} \cdot \bar{\mathbf{B}} = \begin{bmatrix} \mathbf{V}_{\text{SL}} \cdot \mathbf{V}_{\text{SL}}^* \\ \bar{\mathbf{I}}_1^* \bar{\mathbf{I}}_1 \\ \bar{\mathbf{I}}_1^* \bar{\mathbf{I}}_2 + \bar{\mathbf{I}}_2^* \bar{\mathbf{I}}_1 \\ \vdots \\ \bar{\mathbf{I}}_N^* \bar{\mathbf{I}}_N \\ 2j \cdot \bar{\mathbf{I}}_1^* \mathbf{V}_{\text{SL}} \\ \vdots \\ 2j \cdot \bar{\mathbf{I}}_N^* \mathbf{V}_{\text{SL}} \end{bmatrix}^T \cdot \begin{bmatrix} (\mathbf{Z}_{11}^{-1})^* \\ \mathbf{Z}_{1,1} \\ \mathbf{Z}_{1,2} \\ \vdots \\ \mathbf{Z}_{1,N,N} \\ \Im(\mathbf{K}_{21})_{1,1} \\ \vdots \\ \Im(\mathbf{K}_{21})_{N,1} \end{bmatrix} \quad (14)$$

If the current infeeds are not known, they can be estimated using a load flow algorithm, or by assuming that the voltage has a magnitude of 1 p.u. and an angle of 0 in the entire system which is equivalent to inserting the conjugate of the complex power contributions.

One problem with the regression analysis is that many of the current injections are highly correlated with each other. For example wind farms, located close to each other. This problem is denoted multicollinearity and can lead to a large variance in the estimated coefficients when analyzing different sets of samples. The problem of multicollinearity can partly be overcome by applying a Ridge Regression or a Principal Component Regression, which can reduce the variance of the estimated coefficients at the cost of a bias in the estimated output vector [10;18;19].

2.1.4. Aggregation of current sources

In a real distribution system, there is usually a very large number of customers and production units. In the BOE case in Chapter 3, there are for example 721 aggregated loads, 65 induction machines and 29 synchronous machines in the model. With 815 current sources, (14) would require 333336 elements in the input vector, which would not be realistic. Further, there are not measurements of each of the 400 V loads in the system. Therefore, it is advantageous to group some of the sources together and assume that they behave as one lumped source, connected to one virtual node. The grouping of similar components also reduces the problem of multicollinearity.

Assuming that the current injections can be expressed as a linear combination of a reduced number of aggregated currents like in (15), the load dependent losses can be calculated exactly using (16) and (17).

$$\bar{\mathbf{I}}_1 = \mathbf{K}_1 \cdot \bar{\mathbf{I}}_{\text{red}} \quad (15)$$

$$\mathbf{S}_{\text{loss-series}} = \bar{\mathbf{I}}_{\text{red}}^H \cdot \mathbf{K}_1^H \cdot \mathbf{Z}_1 \cdot \mathbf{K}_1 \cdot \bar{\mathbf{I}}_{\text{red}} \quad (16)$$

$$\mathbf{S}_{\text{loss-cross}} = j \cdot 2 \cdot \bar{\mathbf{I}}_{\text{red}}^H \mathbf{K}_1^H \cdot \Im(\mathbf{K}_{21}) \cdot \mathbf{V}_{\text{SL}} \quad (17)$$

When the reduced current vector is inserted in (14), the estimated  $\bar{\mathbf{B}}$ -vector will contain elements from  $\mathbf{K}_1^H \cdot \mathbf{Z}_1 \cdot \mathbf{K}_1$  and  $\mathbf{K}_1^H \Im(\mathbf{K}_{21})$ .

$\mathbf{K}_1$  is a transformation matrix with a number of rows corresponding to the number of busses in the system and a number of columns corresponding to the number of aggregated currents.

One approach is to define an aggregated current for each feeder based on the sum of all loads of the feeder and another aggregated current based on the sum of all production of the feeder. Often, only the total power of the loads of a feeder is known. To estimate an aggregated current for the feeder, an aggregated voltage must be assumed. As a first approach, the voltages at the connection points of the feeders can be used as basis for calculating the current at the virtual nodes.

The method of loss allocation based on aggregated loads and consumers has been used in the investigation of the losses in the BOE network, presented in [20].

2.1.5. The influence from reactive power flows

The current dependent losses can be split into contributions from the active power transfer, the reactive power transfer and a cross effect between the two. Equation (18) shows the separation of the current injections into a part corresponding to the real power injections and a part corresponding to the reactive power injections. The changes in voltage caused by the current injections have not been considered.

$$\bar{I}_1 = \bar{I}_P + \bar{I}_Q = (\bar{S}[\cdot]/\bar{V})^* = (\bar{P}[\cdot]/\bar{V})^* + (j\bar{Q}[\cdot]/\bar{V})^* \tag{18}$$

$$\bar{I}_P = (\bar{P}[\cdot]/\bar{V})^* \text{ and } \bar{I}_Q = (j\bar{Q}[\cdot]/\bar{V})^* \tag{19}$$

Inserting (18) in (9) yields (20)

$$S_{\text{loss-series}} = \bar{I}^T \cdot \left[ \underbrace{\bar{I}_P \cdot \bar{I}_P^H}_{\text{Contr. from P}} + \underbrace{\bar{I}_Q \cdot \bar{I}_Q^H}_{\text{Contr. from Q}} + \underbrace{(\bar{I}_Q \cdot \bar{I}_P^H + \bar{I}_P \cdot \bar{I}_Q^H)}_{\text{Cross effect}} \right] \cdot [\bullet] Z_1 \cdot \bar{I} \tag{20}$$

The three terms in (20) represent the contribution from the active power injections, the reactive power injections and the cross effect. Since the transfer of active power is usually regarded as the main objective, the cross effect could be considered a part of the losses, allocated to the reactive power.

3. CASE STUDY: BRØNDERSLEV OG OPLANDS ELFORSYNING

As case study, the distribution network in Brønderslev in Western Denmark has been investigated. A model of the 60 kV and 10 kV networks, including 65 wind turbines (total 40 MW), 29 synchronous generators (total 50 MW) and totally 1792 nodes has been implemented in PowerFactory®. The load demand is between 15 and 45 MW, which means that power is often exported to the transmission system. Fig. 1 shows an overview of the 60 and 10 kV network.

Ten months of 15 min measurement data have been obtained with the SCADA system. The data, containing for example active and reactive power flows through the 60 / 10 kV transformers, voltage measurement on the 150 kV infeed and production data from the wind turbines and CHPs, has been inserted as time scales in the model. The active and reactive loads and losses have been estimated by performing a series of load flows. The time dependent consumption and the losses have been estimated based on the power balance of each feeder. The procedure has been described in [14].

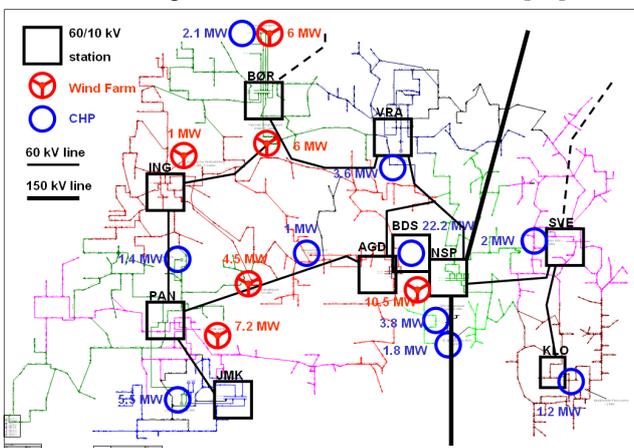


Fig. 1 The 60 and 10 kV network

3.1. Loss allocation

The losses of the system have been analyzed according to the methods described in Chapter 2. The aim of the analyses is twofold. Firstly, they are supposed to provide an overview of the losses in the distribution system. The following questions should be considered:

1. How large are the total losses compared to the load and production?
2. Where in the system are the losses dissipated?
3. What are the losses caused by the integration of DG?
4. What are the losses caused by the transfer of reactive power?
5. What are the potential savings in losses if the simultaneity between load and production is increased?

Secondly, the analyses will serve as a validation of the loss allocation methods presented in Chapter 2.

The analyses are based on measurements obtained in the period April 6<sup>th</sup> 2006 to February 6<sup>th</sup> 2007. During the period, a few days of data are missing due to communication problems in the SCADA system. The estimated mean values of losses etc. have not been corrected for the difference in load and production pattern between the missing two months and the rest of the year.

Fig. 2 shows an overview of the mean active power losses, divided into the components causing the losses. The total mean-losses make approximately 1.27 MW, from which 72 % is dissipated at 10 kV level and below. It should be noted that the real system also comprises a large number of 0.4 kV lines which have not been modeled. The shunt losses of the transformers which are practically independent of the loading, make 49 % of the total active power losses. The mean load dependent losses of the 150/60 kV transformers only amount to 5 kW.

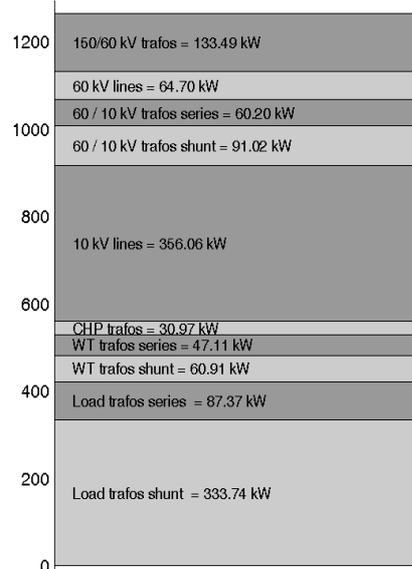


Fig. 2 Mean active power losses

The network comprises ten 60 / 10 kV stations. Three of the stations comprise two transformers which are not operated in parallel. Table 1 shows the mean load and production from CHPs and wind turbines of the feeders under each of the transformers.

	A Load	B Wind	C CHP	D Sum
AGD 2	-2.17	0	0.1	<b>-2.07</b>
AGD 1	-1.62	0.13	0.4	<b>-1.09</b>
BDS 2	0.00	0.00	7.72	<b>7.72</b>
BDS 1	-2.96	3.10	1.97	<b>2.11</b>
BØR	-3.09	3.26	0.62	<b>0.79</b>
ING	-2.28	1.37	0.60	<b>-0.31</b>
JMK	-2.69	0.03	2.51	<b>-0.16</b>
KLO	-1.34	0.03	0.68	<b>-0.63</b>
NSP	-3.10	0.05	0.00	<b>-3.05</b>
PAN 1	-2.00	1.66	0.00	<b>-0.34</b>
PAN 2	-2.29	0.22	0.52	<b>-1.55</b>
SVE	-1.39	0.05	0.81	<b>-0.53</b>
VRÅ	-3.73	0.04	0.56	<b>-3.13</b>
<b>Sum</b>	<b>-28.66</b>	<b>9.93</b>	<b>16.48</b>	<b>-2.25</b>

Table 1: Mean values of active power contributions of each feeder [MW]

To study the impact of the distributed generation, an allocation of the system losses is performed as described in Chapter 2. The following combination of the methods has been used:

Firstly, the losses of the 60 kV network, the 150 / 60 kV transformers and the 60 / 10 kV transformers are allocated to the individual feeders, based on the impedance matrix of that part of the network. For comparison, the allocation is made both using the marginal loss allocation method and the statistical method based on current injections.

Secondly, the losses at 10 kV level and below for each feeder are allocated to the four categories; loads, wind turbines, CHPs and shunt losses. The allocation is performed using the regression method using the apparent power as input and neglecting the cross effects.

Finally, the losses at 60 kV and above are allocated to the loads, wind turbines, CHPs and shunt losses of the individual feeders. The approach is that the load or generation of each category minus the low voltage losses allocated to the specific category are converted to an equivalent current injection on the 10 kV side of the 60 / 10 kV transformers, and the same approach as in step one is used.

Table 2 shows the allocation of the losses at 60 kV level and above, including the 60/10 kV transformers to the individual feeders. Column A contains the contribution from the mean power flows of the feeders. Column B shows the

	A Contribution from mean currents	B Contribution from the covariance	C Contribution from self-impedances	D Total allocated losses	E Marginal loss allocation
AGD 2	1.54	-0.04	2.29	1.50	1.44
AGD 1	1.07	0.32	1.29	1.38	1.45
BDS 2	11.92	23.56	37.98	35.47	35.00
BDS 1	1.90	5.93	6.80	7.83	7.76
BØR	-0.73	18.12	11.74	17.39	17.24
ING	1.08	10.53	5.10	11.61	11.90
JMK	0.52	7.28	5.69	7.80	8.02
KLO	1.11	0.80	1.47	1.90	1.96
NSP	3.85	0.44	4.39	4.29	4.34
PAN 1	1.17	10.36	5.70	11.53	11.41
PAN 2	5.34	4.35	5.46	9.69	9.93
SVE	0.58	0.98	1.12	1.56	1.58
VRÅ	11.21	1.44	11.52	12.65	12.54
<b>Sum</b>	<b>40.56</b>	<b>84.06</b>	<b>100.55</b>	<b>124.61</b>	<b>124.56</b>

Table 2: The load dependent losses of the 60 kV network, and the 150/60 kV and 60/10 kV transformers. The numbers represent the losses in kW

influence from the covariance. Column C shows the diagonal elements of the loss matrices. These losses correspond to the load dependent losses if only one of the feeders were connected. Column D is the sum of column A and B, representing the total losses allocated to each of the feeders. Column E represents the losses allocated to the different feeders using the sensitivity coefficients. The sum of the rows in column D is approximately equal to the sum of the rows in column E. The allocation to the individual feeders, however, deviates up to 5 %. The fundamental difference between the two methods is that the current injection method assumes a constant current where as the sensitivity method assumes a constant power infeed. This means that the sensitivity analysis takes the change in bus voltages and thereby changes in current injections caused by the change of a single power injection into account. The advantage of the current injection method is that it makes it possible to separate the influence from the mean values and the covariances. A comparison of column A and column B in Table 2 shows that the feeders with a high wind penetration like BDS 1, BØR, ING and PAN 1 have higher losses related to the covariance than those related to the mean value. The same applies for feeders with high penetration of CHP production like BDS 2 and JMK. For feeders with a relatively low penetration of distributed generation like AGD 2, NSP and VRÅ, the highest contribution comes from the mean value. For AGD 2, the contribution from the variance is even negative, because it is located close to the large CHP in Brønderslev.

Table 3 shows the division of the losses at 60 kV and above in a part caused by the active power flows and a part caused by the reactive power flows. Column A and C have been calculated using the current injection algorithm, and column B and D have been calculated using the sensitivity algorithm. The total sum of losses allocated to the reactive power flows is similar for the two methods, but for the individual busses, the two methods give diverging results for the reactive power contribution in column C and D.

The problem with the current injection method here is that it assumes that the part of the current which is perpendicular to the bus voltage is related to the reactive power flow. The current injections, however, change the bus voltages.

With both methods, it is found that the reactive power injections only cause approximately 5 % of the load dependent losses at the 60 kV level and above.

	A Active power – current injection	B Active power – marginal allocation	C Reactive power – current injection	D Active power – marginal allocation
AGD 2	1.38	1.36	0.13	0.08
AGD 1	1.28	1.27	0.10	0.18
BDS 2	33.95	33.91	1.52	1.08
BDS 1	7.73	7.67	0.10	0.09
BØR	16.69	16.61	0.70	0.63
ING	10.85	10.87	0.76	1.03
JMK	7.55	7.53	0.25	0.50
KLO	1.61	1.61	0.29	0.35
NSP	3.78	3.77	0.51	0.57
PAN 1	11.01	10.94	0.52	0.47
PAN 2	9.33	9.43	0.36	0.50
SVE	1.50	1.50	0.07	0.08
VRÅ	11.84	11.94	0.81	0.60
<b>Sum</b>	<b>118.48</b>	<b>118.41</b>	<b>6.13</b>	<b>6.15</b>

Table 3: Separation of the losses on 60 kV level and above in contributions from active and reactive power flows kW

Table 4 shows the allocation of all the losses in the model to load, wind, CHP and constant shunt losses. The losses at 10 kV and below have been allocated using the linear regression method and the rest of the losses have been allocated using the current injection method. It can be seen that the no-load losses make approximately half the losses. In AGD 2 which has very little production from CHP units, the losses allocated to that category is negative and for JMK which has very little wind production, the losses allocated to wind power are negative. This means that these units actually contribute to reduction of the losses. Although the mean production from the CHPs is 65 % larger than from the wind turbines, the total losses allocated to the wind turbines are larger than the losses allocated to the CHPs. There are three main reasons for that. Firstly, all the wind turbines comprise a step up transformer, where as most of the larger CHPs are connected directly to the 10 kV network (or the transformer is not modeled). Secondly, nearly half the CHP production comes from the Brønderslev KVV which is located only half a kilometer from the substation, BDS 1. Thirdly, the ratio between the mean value and the standard deviation of the production is higher for the CHPs than for the wind turbines. The high correlation between the CHP production and the load demand is not assumed to have a large impact on the losses at 10 kV level and below, since most of the large CHPs have their own radials.

	A Shunt losses	B Load	C Wind	D CHP	E Sum
AGD 2	19.30	14.49	0	-0.11	<b>33.69</b>
AGD 1	32.67	16.48	0.89	5.74	<b>55.78</b>
BDS 2	0.05	0.00	0	47.48	<b>47.53</b>
BDS 1	53.53	11.62	68.62	13.78	<b>147.54</b>
BØR	33.63	35.07	49.90	1.11	<b>119.70</b>
ING	57.71	28.92	41.70	1.57	<b>129.89</b>
JMK	25.27	27.82	-0.12	16.05	<b>69.02</b>
KLO	27.44	9.17	0.21	5.79	<b>42.62</b>
NSP	48.96	29.85	0.10	0	<b>78.91</b>
PAN 1	29.98	36.51	29.92	0	<b>96.41</b>
PAN 2	31.52	52.82	3.46	2.94	<b>90.74</b>
SVE	28.61	8.41	0.31	10.31	<b>47.64</b>
VRÅ	35.84	48.32	0.19	3.29	<b>87.64</b>
60 kV	221.25	0	0	0	<b>221.25</b>
<b>Sum</b>	<b>645.76</b>	<b>319.5</b>	<b>195.18</b>	<b>107.94</b>	<b>1268.3</b>

Table 4: Allocation of all the losses in kW

To put the losses that have been allocated to the different categories into context, they have been presented in Table 5 as percentages of the total power flows and of the total system losses. Only the load dependent losses are allocated to participants with the loss allocation methods used above. Therefore, the no-load losses of the transformers related to the loads, wind turbines and CHPs are added to the losses allocated to the respective categories.

	A Allocated losses [kW]	B Mean Volume [MW]	C % of Volume	D % of all losses
Load	319.5	28.7	1.1	25.2
10/0.4 trafo no load	333.7	28.7	1.2	26.3
Load total	<b>653.2</b>	<b>28.7</b>	<b>2.3</b>	<b>51.5</b>
Wind	195.2	9.9	2.0	15.4
Wind trafo no load	60.9	9.9	0.6	4.8
Wind total	<b>256.1</b>	<b>9.9</b>	<b>2.6</b>	<b>20.2</b>
CHP	107.9	16.5	0.7	8.5
CHP trafo nl	11.7	16.5	0.1	0.9
CHP total	<b>119.7</b>	<b>16.5</b>	<b>0.7</b>	<b>9.4</b>
Rest	<b>239.4</b>			<b>18.9</b>
Total	<b>1268.4</b>	<b>55.1</b>	<b>2.3</b>	<b>100.0</b>

Table 5: The allocated losses relative to the total flows

#### 4. CONCLUSION

The paper has described, how the losses in a distribution system can be allocated to load and distributed generation units. The marginal loss allocation method and the current injection method have been used in the case study to allocate the losses. For the 60 kV system, the results from current injection method have been compared to results from the sensitivity analysis, and the two algorithms show identical results. The advantages of the sensitivity analysis are firstly that the algorithm is a part of most power system simulation tools. In PowerFactory® the calculation of loss sensitivities, however, requires an invocation of the sensitivity tool for each bus under consideration. This can be automated, but it extends the total simulation time. Secondly, the interpretation is well suited for e.g. incentive generating price signals, since it directly gives the price of a small change in production / consumption. The advantages of the current injection method are firstly that it is based on the reduced impedance matrix, which contains the short circuit impedances. It is possible to make a rough estimate of the cost of transferring power from one place to another just by looking at the reduced impedance matrix. Like the sensitivity analysis, the algorithm requires a load flow calculation per measurement sample to determine the current infeeds. PowerFactory® does not directly support the export of the impedance matrix. It is, however, possible that it could get implemented in a future version of the tool.

The linear regression method is a simple way of getting an overview of the losses at 10 kV and below. It is, however, not possible to separate the losses related to components in the same feeder with similar load or production time profiles due to the multicollinearity problem.

For the 60 kV system and above, there is a clear synergy effect between production and load. These losses, however, only account for 10 % of the total system losses. For the 10

kV system, it is concluded that the cross effects between load and production make a relatively small part of the total system losses, because the larger wind farms and CHPs are connected to the 60 / 10 kV stations through their own radials. Based on the regression analysis of feeders with only a few smaller wind turbines and CHPs, it is, however, concluded that some of the smaller units do contribute to lowering the losses. The reactive power transfer through the 60/10 kV transformers and above only generates 5 % of the load dependent losses.

APPENDIX

*Nomenclature*

<b>Symbol</b>	<b>Definition</b>
.	Matrix product
[./]	Element wise vector or matrix division – equivalent to ./ in Matlab®
[.•]	Element wise vector or matrix product equivalent to .* in Matlab®
<b>F</b>	Complex quantity
<b>F*</b>	Complex conjugate
$\Re(\mathbf{F})$	Real part of a complex quantity
$\Im(\mathbf{F})$	Imaginary part of a complex quantity
$\bar{F}$	Column Vector
$F$	Matrix
$F^T$	Transposed vector or matrix
$F^H = F^{T*}$	Conjugate transposed vector or matrix
$[F]_{i,j}$	Row <i>i</i> , column <i>j</i> of the matrix
$[F]_i$	Element <i>i</i> of the vector
$E(\mathbf{F})$	Estimate of mean value of a stochastic variable
$\text{cov}(\bar{\mathbf{F}})$	Covariance matrix

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REFERENCES

- [1] Costa, P. M. and Matos, M. A., "Loss allocation in distribution networks with embedded generation," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 384-389, 2004.
- [2] Cardell, J. B. Improved Marginal Loss Calculations During Hours of Transmission Congestion. 2005. 38th Annual Hawaii International Conference on System Sciences.
- [3] Herter, K., "Residential implementation of critical-peak pricing of electricity," *Energy Policy*, vol. 35 pp. 2121-2130, 2007.
- [4] Hoff, T. E., Perez, R., Braun, G., Kuhn, M., and Norris, B. The Value of Distributed Photovoltaics to Austin Energy and the City of Austin. 2006. Austin Energy.
- [5] Conejo, A. J., Arroyo, J. M., Alguacil, N., and Guijarro, A. L., "Transmission loss allocation: a comparison of different practical algorithms," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 571-576, 2002.
- [6] Al-Rajhi, A. N. and Bialek J.W. Marginal and Tracing Pricing of Transmission an Empirical Comparison. 2002. 14th Power Systems Computation Conference.
- [7] Mutale, J., Strbac, G., Curcic, S., and Jenkins, N., "Allocation of losses in distribution systems with embedded generation," *IEE Proceedings-Generation Transmission and Distribution*, vol. 147, no. 1, pp. 7-14, 2000.
- [8] Conejo, A. J., Galiana, F. D., and Kockar, I., "Z-bus loss allocation," *Power Systems, IEEE Transactions on*, vol. 16, no. 1, pp. 105-110, 2001.
- [9] Glover, J. D. and Sarma, M. S., *Power System Analysis and Design* Brooks/Cole, 2002.
- [10] Johnson, R. A. and Wichern, D. W., *Applied multivariate statistical analysis*, 5. ed. Prentice-Hall, Inc. Upper Saddle River, NJ, USA, 2002.
- [11] Svensson, L. and Lundberg, M. Estimating complex covariance matrices. Thirty-Eighth Asilomar Conference on Signals, Systems and Computers , 2151-2154. 2004.
- [12] Lakervi, E. and Holmes, E. J., *Electricity distribution network design* Peter Peregrinus Ltd, 1995.
- [13] Østergaard, J. and Jensen, K. K. Ny Mvar-ordning. 506. 1-9-2004. DEFU.
- [14] Lund, T., Nielsen, J. E., Hylle, P., Sørensen, P., Nielsen, A. H., and Sørensen, G., "Reactive Power Balance in a Distribution Network with Wind Farms and CHPs," *International Journal of Distributed Energy Resources*, vol. 3, no. 2, pp. 113-138, 2007.
- [15] Hong, Y. Y., Chao, Z. T., and Yang, M. S., "A fuzzy multiple linear regression based loss formula in electric distribution systems," *Fuzzy Sets and Systems*, vol. 142, no. 2, pp. 293-306, 2004.
- [16] Hong, Y. Y. and Chao, Z. T., "Development of energy loss formula for distribution systems using FCN algorithm and cluster-wise fuzzy regression," *Power Delivery, IEEE Transactions on*, vol. 17, no. 3, pp. 794-799, 2002.
- [17] Mendenhall, W., Beaver, B. M., and Beaver, R. J., *Introduction to Probability and Statistics* Thomson Brooks/Cole, 2003.
- [18] Draper, N. R. and Smith, H., *Applied Regression Analysis*, 2. ed. John Wiley & Sons New York, 1981.
- [19] Jobson, J. D., *Applied Multivariate Data Analysis* Springer-Verlag, 1991.
- [20] Lund, T. Measurement based analysis of active and reactive power losses in a distribution network with wind farms and CHPs. 2007. European Wind Energy Conference & Exhibition, Milan, EWEA.