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# Measurement based analysis of active and reactive power losses in a distribution network with wind farms and CHPs

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**Key Words**  
Wind power  
Loss allocation  
Regression

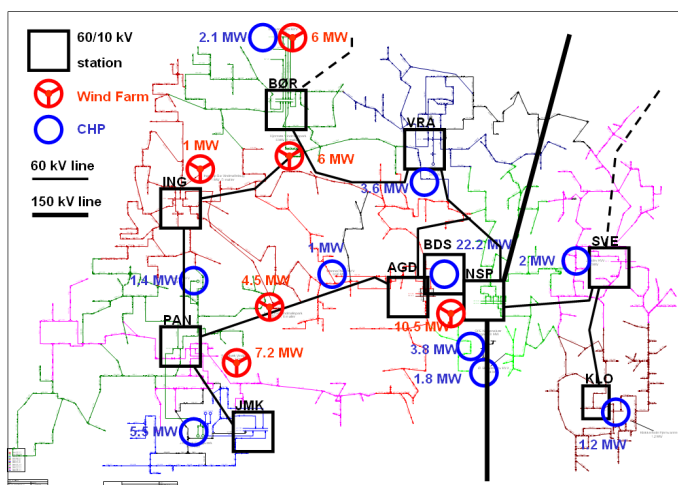
*The paper presents an investigation of the active and reactive power losses in a distribution network with wind turbines and combined heat and power plants. The investigation is based on 15 min average power measurements and load flow calculations in the power system simulation tool PowerFactory®. Based on the measurements and simulations, a regressive model for calculation and allocation of active and reactive power losses has been derived. The influence of the covariance between load and production on the system losses is investigated separately.*

## 1 Introduction

In Denmark, a large part of the electricity is produced by wind turbines and combined heat and power plants (CHPs). Most of them are connected to the network through distribution systems. Because the production units, the distribution systems and the transmission system are owned by different companies, rules regulating the conditions and fees for transmission of power apply. In Western Denmark, the TSO has made a regulation specifying bands for the exchange of reactive power between the 60 kV and the 150 kV networks. Further, the TSO is obligated to compensate the distribution network operator for the losses on the 10 kV radials fed by wind turbines. In order to make fair regulations, to optimize the operation of the distribution systems and to be able to utilize wind forecasts for planning, it is important to know, how the decentralized production affects the active and reactive power losses in the system. In a liberalized market, this knowledge can be used to avoid cross subsidizing in the transmission and distribution fees of consumers and producers [1], to generate incentives of the participants to change the consumption or production in periods with congestion [2;3] or to estimate the value of distributed generation in an area [4]. In systems where the investments and operation are partially or fully centrally controlled, the allocation of losses can be used to optimize the operation and investments and to minimize the losses. In periods with high load demand, the production from the decentralized production units will cover consumption in the vicinity. This can contribute to the reduction of the system losses. On the other hand, in periods where the decentralized production exceeds the local demand, it will cause the losses to increase. It is therefore clear that the influence of distributed generation on the system losses is determined by the location of the production units, the amount of distributed generation relative to the load demand and the simultaneity between demand and production. The aim of this paper is to describe these three effects in a simple way.

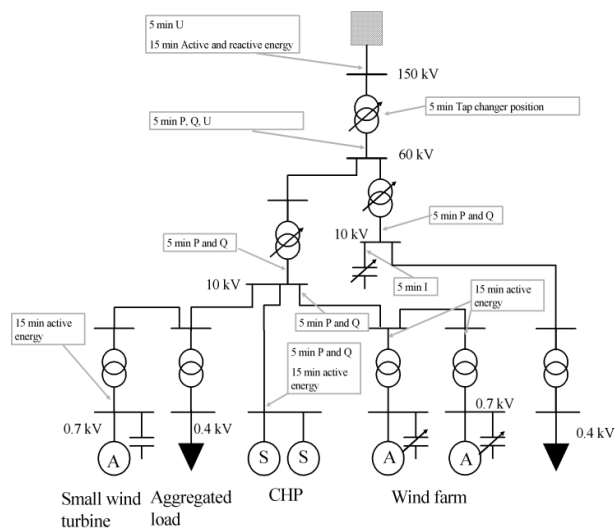
## 2 Test system

As case study, the distribution network in Brønderslev in Western Denmark has been investigated. A model of the 60 kV and 10 kV networks, including 65 wind turbines (total 40 MW), 29 synchronous generators (total 50 MW) and 1792 bus bars has been implemented in PowerFactory. The load demand is between 15 and 45 MW, which means that power is often exported to the transmission system. *Figure 1* shows an overview of the 60 and 10 kV network.



**Figure 1:** The 60 and 10 kV grid

Ten months of 15 min measurement data have been obtained with the SCADA system. The data, containing for example active and reactive power flows through the 60 / 10 kV transformers, voltage measurement on the 150 kV infeed and production data from the wind turbines and CHPs, has been inserted as time scales in the model. The active and reactive loads and losses have been estimated by performing a series of load flows. *Figure 2* shows the principal structure of the network and the location of the different measurements points.



**Figure 2:** Principal structure of the system. The grey boxes indicate measurements

The time dependent consumption and the losses have been estimated based on the power balance of each feeder. The procedure has been described in [5].

### 3 Calculation and allocation of losses

The technical losses in an electrical system can be divided into load dependent losses and load independent losses. [6;7]. The load independent losses, also referred to as shunt losses, given by  $|V|^2 \cdot Y^*$ , where  $Y$  is the shunt admittance, describe the losses in the system which are independent of the loading when disregarding the change in voltage level. For a system with  $N$  busses this can be expressed as (3.1)

$$\mathbf{S}_{\text{loss-shunt}} = \sum_{k=1}^N [V_k^2 \cdot \mathbf{Y}_{k,k\text{-shunt}}^*] \quad (3.1)$$

The load dependent losses can roughly be described as series losses which are given by  $|I|^2 \cdot \mathbf{Z}$ , where  $\mathbf{Z}$  is the series impedance of cables, overhead lines, transformers etc. Given a situation where two participants are sharing the same line, the series losses can be expressed as in (3.2).

$$\mathbf{S}_{\text{loss-series}} = \frac{(\mathbf{S}_1 + \mathbf{S}_2) \cdot (\mathbf{S}_1^* + \mathbf{S}_2^*)}{\mathbf{V} \cdot \mathbf{V}^*} \cdot \mathbf{Z}_{\text{series}} = \frac{[\mathbf{S}_1 \mathbf{S}_1^* + \mathbf{S}_2 \mathbf{S}_2^* + 2 \cdot \text{real}(\mathbf{S}_1 \mathbf{S}_2^*)]}{\mathbf{V} \cdot \mathbf{V}^*} \cdot \mathbf{Z}_{\text{series}} \quad (3.2)$$

One of the problems about separating the cause of losses is the non linear nature of (3.2). The two first terms which represent the square of the two current contributions can easily be allocated to the two participants. The last term, however, is a cross term which is dependent on the magnitude of both current contributions and the angle between them. This means that the contribution of one participant on the losses depends on the behavior of the neighboring participants.

In literature, three main approaches of loss allocation based on deterministic methods are found [1;8-10]: Pro Rata procedures where the losses are allocated to producers and consumers proportionally to the delivered or consumed energy, Marginal Loss allocation procedures where the losses are allocated according to the change in losses corresponding to a small change in production or consumption, and Proportional Sharing procedures, also referred to as tracing [9], where the losses are allocated according to the total power flows in the system generated by the participants.

### 3.1 Linear model of the system losses

The aim of this paper is to derive a relatively simple way of describing the interaction between distributed generation and consumption in an area with a linear regression analysis. For that purpose, the simplified expression for estimation of the system losses in (3.3) is used. The first term represents the no-load losses in the system, and will be used as the constant term in the regression analysis. The second term describes the load dependent losses, where  $\bar{P}$  is a vector of active power injections of aggregated participants. Assuming a voltage of 1 p.u. with an angle of 0 deg. in the entire system and purely active power injections,  $\bar{P}$  would be equal to the current injections. The sign is considered positive for generation units.  $\underline{\mathbf{Z}}$  is an equivalent “impedance matrix”, which will be identified by the regression analysis.

$$\hat{\mathbf{S}}_{\text{loss}} = \mathbf{S}_{\text{loss-no-load}} + \bar{P}^T \underline{\mathbf{Z}} \cdot \bar{P} \quad (3.3)$$

Based on (3.3), the least square regression problem can be expressed as (3.4). The theory behind the linear regression is described in [11-13]. For simplicity, only the three following categories have been used: The total active power consumption, the total active power production from the CHPs and the total active power production from the wind turbines.

$$\hat{\mathbf{S}}_{\text{loss}} = \bar{\mathbf{X}} \cdot \bar{\mathbf{B}} = \begin{bmatrix} 1 \\ P_{\text{load}} P_{\text{load}} \\ P_{\text{load}} P_{\text{CHP}} \\ P_{\text{load}} P_{\text{Wind}} \\ P_{\text{CHP}} P_{\text{CHP}} \\ P_{\text{CHP}} P_{\text{wind}} \\ P_{\text{wind}} P_{\text{wind}} \end{bmatrix}^T \cdot \begin{bmatrix} S_{\text{loss-no-load}} \\ \mathbf{Z}_{11} \\ \mathbf{Z}_{12} + \mathbf{Z}_{21} \\ \mathbf{Z}_{13} + \mathbf{Z}_{31} \\ \mathbf{Z}_{22} \\ \mathbf{Z}_{23} + \mathbf{Z}_{32} \\ \mathbf{Z}_{33} \end{bmatrix} \quad (3.4)$$

### 3.2 Separation in mean values and covariances

When  $\mathbf{S}_{loss\_no\_load}$  and  $\mathbf{Z}$  have been estimated according to (3.4), the expected mean value of the losses can be calculated from the estimated covariance and mean values of  $\bar{P}$  according to (3.5).  $\bar{\mathbf{1}}$  is a unity column vector with three elements, and  $\bullet$  denotes the element wise matrix product.

$$\hat{\mathbf{E}}[\mathbf{S}_{loss}] = \mathbf{S}_{loss-no-load} + \bar{\mathbf{1}}^T \cdot \left[ \left( \hat{\mathbf{E}}[\bar{P}] \cdot \hat{\mathbf{E}}[\bar{P}]^T \right) \bullet \mathbf{Z} + \hat{\mathbf{cov}}[\bar{P}] \bullet \mathbf{Z} \right] \cdot \bar{\mathbf{1}} \quad (3.5)$$

The first term in (3.5) represents the losses which are independent of the loading of the system. This would correspond to the losses, if all load and generation were switched off, when disregarding the influence from the tap changers of the transformers. The second term describes the contribution of the mean values of the load and production. If all the consumers, CHPs and wind turbines had a constant power consumption/production, the losses would correspond to the sum of the two first terms in (3.5). The last term is the increase in losses due to the variance and covariance in consumption/production.

## 4 Results

The data for the period between April 2006 and February 2007 has been partitioned in two data sets, for comparison and verification of the model. The first data set corresponds to every second week of the period, and the second data set contains the remaining weeks. The system losses for the distribution system have been calculated for both periods. The losses, the load and production for the first period have been inserted in (3.4), and an equivalent  $\mathbf{Z}$ -matrix has been derived with the function `regress()` in Matlab®. Table 1 shows the  $\mathbf{Z}$ -matrix identified from data set 1, and Table 2 shows the same for data set 2. Since the input in (3.4) is active power rather than current,  $\mathbf{Z}$  is strictly speaking not an impedance matrix, and the unit is not

	$\mathbf{P}_{load}$	$\mathbf{P}_{CHP}$	$\mathbf{P}_{Wind}$
$\mathbf{P}_{load}$	0.49 + 1.6i	0.073 + 0.49i	0.22 + 1.6i
$\mathbf{P}_{CHP}$	0.073 + 0.49i	0.14 + 1.7i	0.16 + 1.1i
$\mathbf{P}_{Wind}$	0.22 + 1.6i	0.16 + 1.1i	1.1 + 4.0i

**Table 1:** Estimated  $\mathbf{Z}$ -matrix from data set 1. Unit: [kVA/MVA<sup>2</sup>]

Ohms.

The diagonal elements are a measure of the weighted impedance between all the units of each category to the infeed from the transmission system. The off diagonal elements are a measure of the mutual impedance between the different categories. A high mutual impedance between two categories indicate large savings in losses, if power can be transferred directly from one category to another. There is a deviation of several percents between the identified matrices of the two data sets. Especially the cross coupling between the load and the CHP production is sensitive to changes in the data set. The reason is that the load and the CHP production are intentionally correlated, since the CHPs mainly operate in high load periods, when the prices are high. Table 3 shows the cross correlation matrix of the categories. It can be seen that the correlation

	$\mathbf{P}_{load}$	$\mathbf{P}_{CHP}$	$\mathbf{P}_{Wind}$
$\mathbf{P}_{load}$	0.49 + 1.6i	0.067 + 0.53i	0.22 + 1.7i
$\mathbf{P}_{CHP}$	0.067 + 0.53i	0.13 + 1.7i	0.14 + 1.2i
$\mathbf{P}_{Wind}$	0.22 + 1.7i	0.14 + 1.2i	1.1 + 4.0i

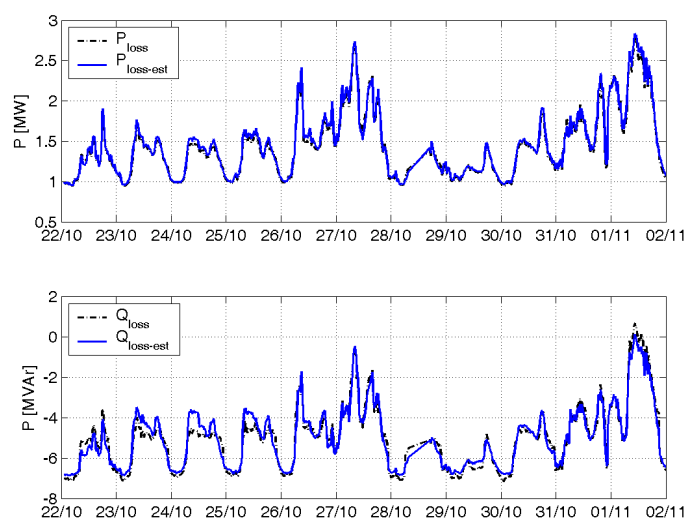
**Table 2:** Estimated  $\mathbf{Z}$ -matrix from data set 2. Unit: [kVA/MVA<sup>2</sup>]

coefficient between the load and the CHP production is 75 % for the period under consideration. That makes it difficult to separate this cross effect from the diagonal elements. The problem of multicollinearity can partly be overcome by applying a Ridge Regression or a Principal Component Regression [11-13].

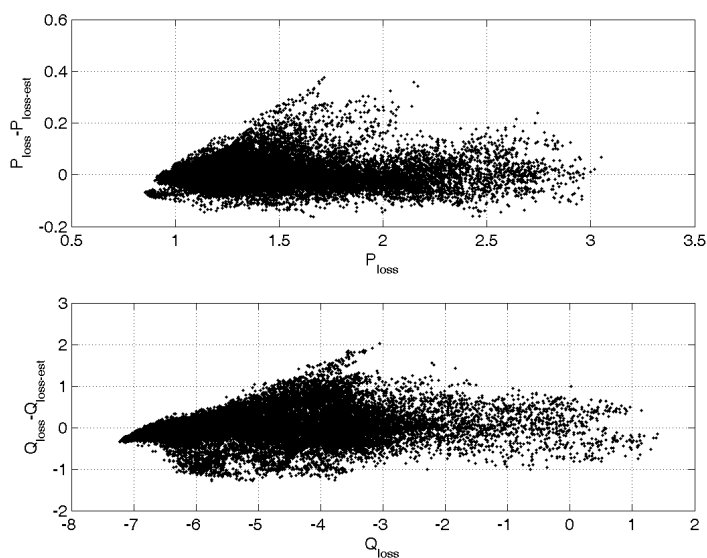
	$P_{load}$	$P_{CHP}$	$P_{Wind}$
$P_{load}$	1	-0.75	-0.16
$P_{CHP}$	-0.75	1	0.17
$P_{Wind}$	-0.16	0.17	1

**Table 3:** Cross correlation matrix

Figure 3 shows the active and reactive power losses from the load flow calculations and the losses which have been estimated with the Z-matrix from Table 1. The active power losses can be estimated very well whereas there are larger discrepancies in the estimate of the reactive power losses.



**Figure 3:** Actual and estimated losses from October 22<sup>nd</sup> to November 2<sup>nd</sup> 2006



**Figure 4:** Residuals plotted against the losses from April 6<sup>st</sup> 2006 to February 6<sup>st</sup> 2007

Figure 4 shows the residuals plotted against the actual power losses. The standard deviation of the residuals is 0.05 MW for the active power losses and 0.38 MVar for the reactive power losses. Given the simple structure of the model and the fact that the main intention of the model is to qualitatively describe the influence of the distributed generation on the losses, this is considered acceptable.

Table 4 shows the contribution of the three terms in (3.5) to the mean losses. The constant no-load term contributes with the highest active power losses which mainly come from iron losses in the transformers and

$S_{loss-no-load}$	$\bar{\mathbf{1}}^T \left( E[\bar{\mathbf{P}}] E[\bar{\mathbf{P}}]^T \right) \bullet \underline{\mathbf{Z}} \cdot \bar{\mathbf{1}}$	$\bar{\mathbf{1}}^T \text{cov}[\bar{\mathbf{P}}] \bullet \underline{\mathbf{Z}} \cdot \bar{\mathbf{1}}$	Sum
<b>0.825 - 7.14i</b>	<b>0.398 + 1.10i</b>	<b>0.159 + 0.822i</b>	<b>1.38 - 5.22i</b>

**Table 4:** The contributions to the mean losses of the system. Unit: [MVA]

a negative reactive power loss related to the charging currents of the underground cables. The next term corresponds to the losses in the system if all power flows were constantly at their mean values. This term is determined by the amount and location of consumption and distributed generation installed in the system. The last term is related to the variance in production and consumption. These are the extra losses that occur because the system is not operated with constant power flows. The additional active power losses make approximately 40 % of the losses related to the mean power flows. By optimal scheduling of the production and consumption, these losses could be reduced.

Table 5 shows the individual cross effects between the mean values. The highest contribution to this term comes from the load. The cross effects between the load and the distributed generation units contribute to reduction of the losses. It can, however, be seen that if the mean value of the wind production were set to zero, the active power losses would be expected to decrease with 28 kW.

	$P_{load}$	$P_{CHP}$	$P_{Wind}$
$P_{load}$	0.40 + 1.3i	-0.033 - 0.22i	-0.058 - 0.44i
$P_{CHP}$	-0.033 - 0.22i	0.034 + 0.43i	0.024 + 0.16i
$P_{Wind}$	-0.058 - 0.44i	0.024 + 0.16i	0.098 + 0.36i
Sum columns	<b>0.31 + 0.65i</b>	<b>0.025 + 0.37i</b>	<b>0.063 + 0.082i</b>
Sum row+col	<b>0.22 - 0.020i</b>	<b>0.015 + 0.31i</b>	<b>0.028 - 0.20i</b>
Sum all	<b>0.398 + 1.10</b>		

**Table 5:**  $\left( E[\bar{\mathbf{P}}] E[\bar{\mathbf{P}}]^T \right) \bullet \underline{\mathbf{Z}}$  Unit: [MVA]

The effect of the covariances is shown in Table 6. The highest contribution to this term comes from the wind, since the wind production has a relatively high variance and a low correlation with the load. If the variance of the wind production could be set to 0, the expected reduction in active power losses would be 120 kW. Although the variance of the CHP-units is highly correlated with the load, the total losses would also be slightly reduced, if the CHP units could be set to produce their mean output all the time.

	$P_{load}$	$P_{CHP}$	$P_{Wind}$
$P_{load}$	0.023 + 0.075i	-0.0056 - 0.038i	-0.0025 - 0.019i
$P_{CHP}$	-0.0056 - 0.038i	0.030 + 0.38i	0.0041 + 0.028i
$P_{Wind}$	-0.0025 - 0.019i	0.0041 + 0.028i	0.11 + 0.42i
Sum columns	<b>0.015 + 0.019i</b>	<b>0.029 + 0.37i</b>	<b>0.12 + 0.43i</b>
Sum row+col	<b>0.0068 - 0.038i</b>	<b>0.027 + 0.36i</b>	<b>0.12 + 0.44i</b>
Sum all	<b>0.159 + 0.82i</b>		

**Table 6:**  $\text{cov}[\bar{\mathbf{P}}] \bullet \underline{\mathbf{Z}}$  Unit: [MVA]

## 5 Conclusion / Discussion

A simple way of allocating distribution system losses to consumers, distributed generation units and the interaction between these has been proposed. Where other methods like marginal loss allocation and tracing focus on allocating the losses at a given point in time, this method also takes the covariance between production and consumption into account. The loss model used for the linear regression analysis was very simple, aggregating all consumers, all CHPs and all wind turbines in three lumped units and disregarding the reactive power flows. A more sophisticated approach could be made by making separate groups for each feeder and by using the complex currents as inputs rather than the active power. The statistical significance of the model could also be subject of further investigation.

For the system in the case study, it is concluded that both the CHPs and the wind turbines contribute to increasing the system losses. The contribution from the CHPs is, however, much smaller than the contribution from the wind turbines because of the high correlation between their production and the load demand.

## 6 Acknowledgments

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