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Domain Discretization and Circle Packing

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1 Introduction

A *circle packing* is a configuration of circles which are tangent with one another in a prescribed pattern determined by some combinatorial triangulation where the vertices in the triangulation correspond to centers of circles and edges correspond to two circles (having centers corresponding to the enpoints of the edge) being tangent to each other. Three mutually tangent circles form a face of the triangulation. Since circle packings are dual to triangulations, circle packing method can be applied to domain discretization problems such as triangulation and unstructured mesh generation techniques.

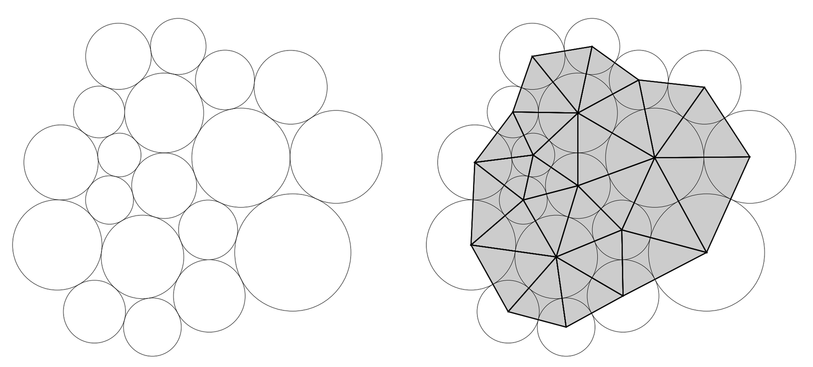

Figure 1: Circle packing with its corresponding triangulation. Figure provided courtesy of Ken Stephenson [2].

2 Problem Proposal

It is wished to analyze if circle packing techniques can be effectively applied to triangulation and mesh generation problems of bounded domains in $\mathbb{R}^2$ with Euclidean metric. Such analysis should consider the validity of such triangulations and the optimization and refinement of meshes. We consider 2 methods:

- A primitive triangulization and mesh generation algorithm to be described below
- A mesh generation method relying on discrete conformal mappings

3 Primitive Algorithm

One of the fundamental theorems for circle packing states that there exists a circle packing with respect to the Euclidean, hyperbolic, or spherical metric, preserving the combinatorics of some given topological simplex.

I wish to propose the reverse is true, that is, given a cloud of points in $\mathbb{R}^2$, I propose it is possible to construct a circle packing preserving the positions of the vertices. If so, then there is an underlying triangulation of the domain where the vertices of the triangulation correspond to the centers of the circles.

I am constructing a primitive algorithm that consists of the following steps:

- We assume that no two points are identical
- Give every vertex an initial radius that is 1/2 the minimum of the distances between all points
- Fix the radii of the circles that are touching
- Increase simultaneously the radii of the remaining circles until at least two more touch, repeat this step

This creates the initial structure. We fatten this skeleton by the following:

- For any two tangent circles, there are two "sides" of the tangent point to the edge that they join. Create a new circle on each side of some small radius that is tangent to both of the circles, omitting the sides that already have a 3rd circle tangent to those two.
- The radii of these circles are increased as was done for the initial structure.

3.1 Triangulation

This algorithm is still in its initial stages, so it is inconclusive as of yet whether or not this algorithm always gives a connected triangulation. In the future, it would be interesting to compare such triangulations with Delaunay triangulations in particular.

3.2 Mesh Generation

The primitive algorithm mentioned before can also be applied to triangular mesh generation, where the cloud of points is to be seen as the boundary set. Indeed the construction of the initial skeleton is reminiscent of construction of the empty mesh.

4 Conformal Mappings

We wish to create a method of mesh generation of Jordan domains in $\mathbb{R}^2$ using circle packings by the following:

- A planar (polygonal) domain is defined by choosing a set of vertices as the boundary
- Conformally map the domain to a standard shape, such as a disk
- Construct a uniform mesh (hexagonal, for instance) in the standard shape
- Invert the conformal map to produce mesh elements in the original domain

Kenneth Stephenson, University of Tennessee at Knoxville has created a program called *CirclePack* [2], which is a program that makes discrete analytic mappings using circle packings. It is hoped that this program can be utilized to implement this mesh generation technique, but as the program exists now, the uniform packing is created in the domain, and the program maps this uniform circle packing to a non-uniform circle packing in the disk. This is the opposite of what we want, so some extension of this algorithm should be created such that the inverse map can be calculated.

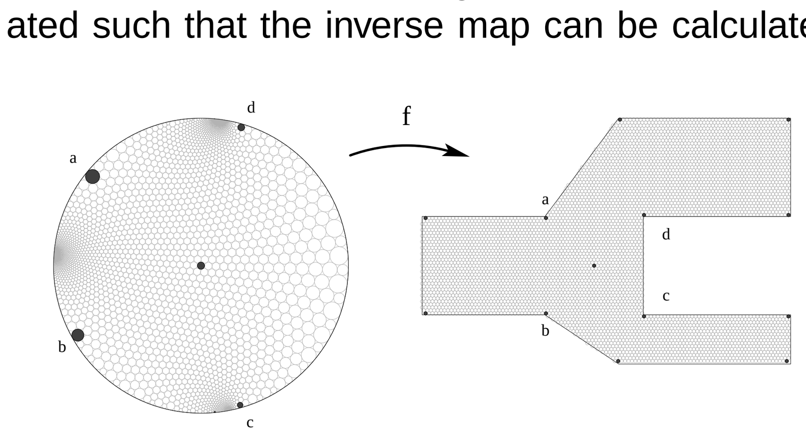

Figure 2: Discrete analytic mapping using circle packing from the disk $\mathbb{D}$ to a polygonal domain. Figure provided courtesy of Ken Stephenson [2].

4.1 Constrained Mesh

A constrained mesh can be achieved using such a circle packing by placing a demand on the radii of the boundary circles or by placing demands on the angle sums at each boundary vertex.

5 Optimization

Since the mesh generation first mentioned is quite primitive, optimization of such a mesh is vital to its effectiveness. Two options are:

- Put a bound on the radius of a circle to bound edge length and angles in faces
- Combine two tangent circles to be one, corresponding to collapsing an edge in the mesh.

With respect to the conformal mapping procedure, mesh refinement can be achieved in an obvious way by reducing the size of the circles in the regular mesh.

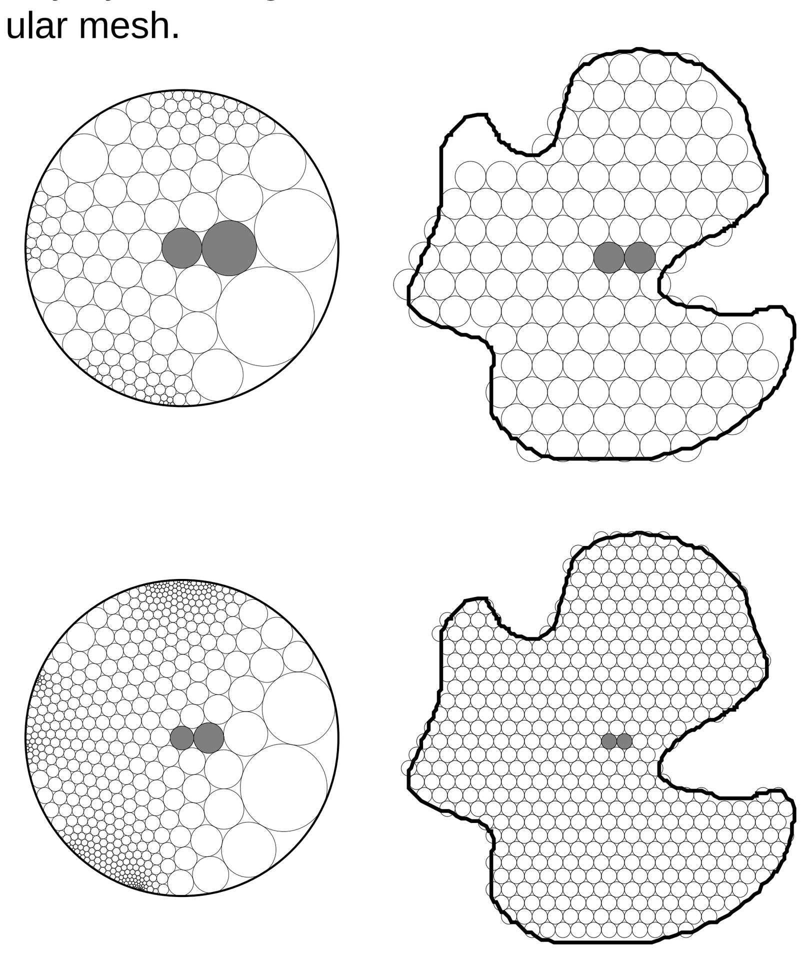

Figure 3: Refinement of a discrete analytic mapping defined by a circle packing. Figure provided courtesy of Ken Stephenson [2].

6 Conclusion

- The primitive triangulation and mesh generation algorithm needs to be formalized by way of proof.
- Augment the *CirclePack* program to perform the desired function
- The triangulation and mesh generation algorithms should be compared to existing techniques including Delaunay triangulation.
- In general, these ideas are very primitive and lack rigorous proof of their existence and estimations on their quality.
- Succcessful meshing of planar domains can, in the future, be extended to parametric surface meshing and remeshing techniques.

References

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