Downloaded from orbit.dtu.dk on: Nov 19, 2018

Technical University of Denmark

On single-row routing

Andersen, Lars

Published in:
IEEE Transactions on Circuits and Systems

Link to article, DOI:
10.1109/TCS.1980.1084751

Publication date:
1980

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Andersen, L. (1980). On single-row routing. IEEE Transactions on Circuits and Systems, 27(12), 1262-1263. DOI: 10.1109/TCS.1980.1084751

DTU Library
Technical Information Center of Denmark

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Letters to the Editor

On Single-Row Routing

LARS ANDERSEN

***Abstract*—A new lower limit on the street congestion in single-row routing is presented.**

I. Introduction

The single-row routing problem as presented in [1] and [2] includes several unsolved problems. This letter deals with only one of them.

Lower limits on the maximum street congestion are given both in [1] and [2]. In Section III it is shown that the limit given in [2] is often weaker than the one in [1], and a new lower limit is presented that may supersede these. This limit is based on the observation that in every interval covered by two or more nets one must be at the top and another at the bottom.

In this paper only nets connected to two and only two nodes will be considered. It is hoped that this limitation may be removed at a later time.

The use of the isomorphic mapping of a realization called the interval graphs was introduced in [2]. This and other terms and definitions from [2] are used in the following. However, for completeness most of the terms used later are defined in Section II.

II. Definitions

1) The *terminals* of a net are the nodes to which the net is connected.

2) A *basic interval* b_i is the open interval of the reference line between the two successive nodes i and $i+1$.

3) The *density* d_i of a basic interval is the number of nodes passing over and/or under b_i.

Note: Nets placed on the reference line (only nets between two successive nodes) are not incalculated in d_i because they do not influence the street congestion.

4) A *net* is said to *cover a node* if it passes over or under this node without being connected to it.

5) The *range* of a net is the set of nodes consisting of the nodes covered by the net and its terminals.

6) The *cutnumber of a node* c_i is the number of nets covering node i.

7) The *cutnumber of a net* q_N is the bigger of the cutnumbers of its terminals.

8) The *length* of a net l_N is the number of nodes in its range.

9) The *maximum street congestion* Q is the maximum number of nets that pass any basic interval on one side.

10) An *outer node* is a node that on one side is not covered by any net in a given realization.

11) An *outer net* in a basic interval b_i is a net that on one side is not passed by any other net within b_i.

12) Let the net N pass the basic interval b_i. Let the smallest cutnumber for the nodes in the range of N to the left of b_i be c'_{Ni} and the smallest to the right of b_i be c''_{Ni}. To every such b_i and N is connected a *condition number* y_{Ni} defined as $y_{Ni} \triangleq \max\{c'_{Ni}, c''_{Ni}\}$.

13) For every basic interval b_i with density $d_i > 1$ we define a Y_i as the smallest integer such that at least *two* nets have $y_{Ni} \leqslant Y_i$.

Manuscript received October 15, 1979; revised February 19, 1980.
The author is with the Institute of Circuit Theory and Telecommunication, Technical University of Denmark, Lyngby, Denmark.

III. Theorems

In [1] it was shown that

$$Q \geqslant [d_M/2] \tag{1}$$

where $d_M = \max_i d_i$ and $[x]$ is the smallest natural number not smaller than x.

In [2] it was shown that

$$Q \geqslant \max\{q_m, [q_M/2]\} \tag{2}$$

where $q_m = \min_N q_N$ and $q_M = \max_N q_N$.

Let the net N have $q_N = q_M$ and let node i be a terminal of N with $c_i = q_M$. The basic intervals b_{i-1} and b_i must have either

$$d_i = q_M \quad \text{and} \quad d_{i-1} = q_M$$

or

$$d_i = q_M \quad \text{and} \quad d_{i-1} = q_M + 1$$

or

$$d_i = q_M + 1 \quad \text{and} \quad d_{i-1} = q_M.$$

(The first case only if net N is on the reference line.) From this it is seen that

$$d_M \geqslant q_M$$

and

$$[d_M/2] \geqslant [q_M/2].$$

Therefore, (2) should be replaced by

$$Q \geqslant \max\{q_m, [d_M/2]\}. \tag{3}$$

Theorem 1.

For an outer node $c_i \leqslant Q$.

Proof: On one side this node is not covered by any nets. Therefore all the c_i covering nets must be in either the upper or the lower street.

Corollary 1: A net that is terminated in two outer nodes must have $q_N \leqslant Q$.

Theorem 2.

If the net N is an outer net in the entire basic interval b_i then it has a condition number not greater than Q in this interval:

$$y_{Ni} \leqslant Q.$$

Proof: It is sufficient to show that the range of N contains at least one node at each side of b_i with cutnumber $c_i \leqslant Q$.

Only the nodes to one side (left) of b_i are considered as the other side (right) can be treated in the same way.

Let N be an upward outer net in b_i. If the left terminal (T_L) of net N is an outer node then c_i of this node is $\leqslant Q$ according to Theorem 1.

If T_L is not an outer node then it is upward covered by at least one net (see Fig. 1). Such upward covering nets must have the right terminals (T') between T_L and b_i where N is the upward

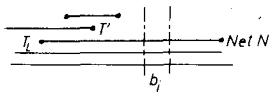

Fig. 1. Interval graph illustrating Theorem 2.

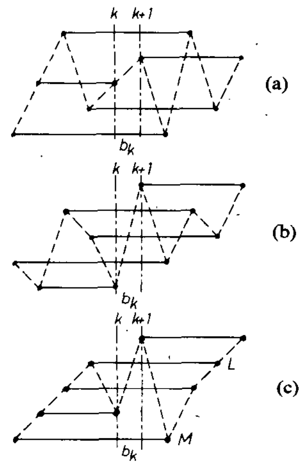

Fig. 2. Interval graph illustrating Theorem 3. (a) Two outer nets. (b) Two outer nodes. (c) One outer net and one outer node.

outer net. If T' is an outer node it has a $c_i \leqslant Q$. If T' is upward covered then the reasoning can be repeated. As there is a finite number of nodes between T_L and b_i it follows that this repetition will end with some node in the range of N that is an outer node and, therefore, with $c_i \leqslant Q$.

Theorem 3.

$$Y_M \leqslant Q$$

where $Y_M = \max Y_i$ for all $i = 1,2,\cdots,n-1$ where $d_i > 1$

Proof: For each basic interval b_k the set of surrounding nodes $(k, k+1)$ must fall in one of the following categories (see Fig. 2):

a) No outer nodes,
b) Two outer nodes, or
c) One outer node.

In each case it will be shown that there are two nets with $y_{Nk} \leqslant Q$ and, therefore, $Y_k \leqslant Q$.

Case a: As none of the nodes k and $k+1$ are outer nodes, they must be covered at both sides by two outer nets. Theorem 2 tells that $y_{Nk} \leqslant Q$ for both nets.

Case b: As both the nodes k and $k+1$ are outer nodes Theorem 1 tells that $c_k \leqslant Q$ and $c_{k+1} \leqslant Q$. All nets passing b_k will, therefore, have $y_{Nk} \leqslant Q$.

Case c: As only one node is an outer node (say node $k+1$), the other node (k) must be covered on both sides (both upward and downward). One of these nets is an outer net in b_k while some other is crossing the reference line inside b_k.

The outer net (M) has $y_{Mk} \leqslant Q$ (Theorem 2). One of the nets crossing the reference line within b_k is an outer net (say net L) in the neighboring basic interval b_{k-1}. Therefore, $y_{Lk-1} \leqslant Q$ which means that a node in its range to the left of b_{k-1} (and thereby to the left of b_k) has a cutnumber $c_i \leqslant Q$. The node $k+1$ is in the range of net L to the right of b_k and its $c_{k+1} \leqslant Q$ because it is an outer node. In other words, also $y_{Lk} \leqslant Q$.

The requirement $q_m \leqslant Q$ in (2) and (3) is the result of Corollary 1.

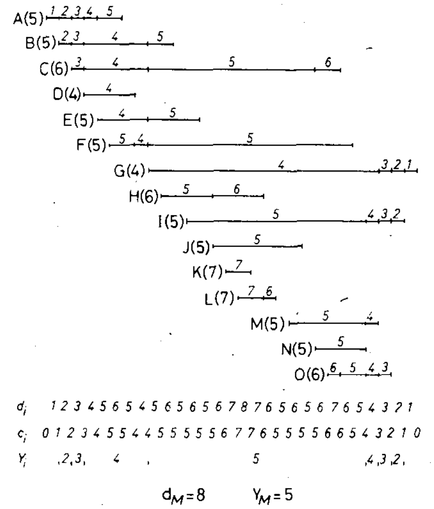

Fig. 3. d_i, c_i, q_N, y_{Ni}, Y_i, d_M, and Y_M for Example 2 of [2].

Theorem 3 is the result of observations on all basic intervals and their outer nets.

Although it is not proven that $q_m \leqslant Y_M$ it is suggested that (2) and (3) should be replaced by

$$Q \geqslant \max\{Y_M, [d_M/2]\}. \tag{4}$$

IV. Discussion

Fig. 3 shows the original interval graph of Example 2 from [2] with the y_{Ni}'s written on each net and with the d_i, c_i, and Y_i indicated below.

From this it is seen that $Y_M = 5$ and that it will be futile to search for realizations with $Q = 4$.

References

[1] B. S. Ting, E. S. Kuh, and I. Shirakawa, "The multilayor routing problem: Algorithms and necessary and sufficient conditions for the single-row, single-layer case," *IEEE Trans. Circuits Syst.*, vol. CAS-23, pp. 768–778, Dec. 1976.

[2] E. S. Kuh, T. Kashiwabara, and T. Fujisawa, "On optium single-row routing," *IEEE Trans. Circuits Syst.*, vol. CAS-26, pp. 361–368, June 1979.

On the Periodic Steady-State Problem by the Newton Method

DAVID A. ZEIN, SENIOR MEMBER, IEEE

***Abstract*—The methods by which linear sensitivity circuits have been used to evaluate the sensitivity matrix, $\partial x(T)/\partial x(0)$, can be improved. The sensitivity matrix is used in the computation of the Jacobian of $f(x(0)) = x(0) - x(T) = 0$, ($T$ being the period) in a periodic steady-state problem. It is shown here that the sensitivity matrix can be obtained from the solution of a linear homogeneous matrix equation which is simply derived by differentiating the state equations or a mixture of algebraic-differential equations arising from any formulation. This simplification makes the method easier and more practical to implement in a general purpose CAD program. Examples are given.**

Manuscript received May 22, 1979; revised January 28, 1980.

The author is with the IBM Data System Division, Hopewell Junction, NY 12533.

0098-4094/80/1200-1263$00.75 © 1980 IEEE