



## Fatigue Evaluation Algorithms: Review

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*Publication date:*  
2010

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*

Passipoularidis, V., & Brøndsted, P. (2010). Fatigue Evaluation Algorithms: Review. Roskilde: Danmarks Tekniske Universitet, Risø Nationallaboratoriet for Bæredygtig Energi. Denmark. Forskningscenter Risoe. Risoe-R, No. 1740(EN)

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# FATIGUE EVALUATION ALGORITHMS: REVIEW

Risø-R-Report

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November 2009



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**Risø-R-1740(EN)**  
**November 2009**

Published on the internet July 2010

**Abstract (max. 2000 char.):**

A progressive damage fatigue simulator for variable amplitude loads named FADAS is discussed in this work. FADAS (Fatigue Damage Simulator) performs ply by ply stress analysis using classical lamination theory and implements adequate stiffness discount tactics based on the failure criterion of Puck, to model the degradation caused by failure events in ply level. Residual strength is incorporated as fatigue damage accumulation metric. Once the typical fatigue and static properties of the constitutive ply are determined, the performance of an arbitrary lay-up under uniaxial and/or multiaxial load time series can be simulated. The predictions are validated against fatigue life data both from repeated block tests at a single stress ratio as well as against spectral fatigue using the WISPER, WISPERX and NEW WISPER load sequences on a Glass/Epoxy multidirectional laminate typical of a wind turbine rotor blade construction. Two versions of the algorithm, the one using single-step and the other using incremental application of each load cycle (in case of ply failure) are implemented and compared. Simulation results confirm the ability of the algorithm to take into account load sequence effects. In general, FADAS performs well in predicting life under both spectral and block loading fatigue.

**ISSN 0106-2840**  
**ISBN 978-87-550-3835-6**

**Contract no.:**  
EFP 033001/33033-0077

**Group's own reg. no.:**  
1615114-00

**Sponsorship:**

**Cover :**

**Pages: 67**  
**Tables:**  
**References:**

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# 1 MOTIVATION

Fatigue design of Wind Turbine Rotor Blades, despite their steady increase both in size and weight, is performed using a number of simplifying assumptions. Such simplifications are accounted for by conservative safety factors to the detriment of product weight and cost. A number of those are:

- Uniaxial fatigue stress fields
- Fatigue characterization only of typical laminates
- Palmgren Miner rule for damage accumulation

There is substantial potential for optimizing the use of material while increasing the safety margins, if detailed fatigue design of Rotor Blades is performed. In order to accomplish this, an efficient life prediction methodology, taking into account the above mentioned parameters, should be developed. While no such methodology enjoyed general acceptance until now, a multitude of models applicable to limited or more general cases, are available. Nevertheless, this great variety can be misleading: The field of applicability, degree of complexity and efficiency of each theory should be considered in depth before its integration into a general life prediction algorithm. Naturally, the first step in this direction is a review on fatigue life prediction. The present report attempts such a review in order to provide different alternatives for building up such a detailed life prediction scheme.

## 2 INTRODUCTION

Fatigue life prediction in composite materials has been a subject of interest during more than three decades now. Numerous articles have been published, a substantial amount of experimental data on different materials is available and still there is no definite conclusion on specific predictive algorithms. This holds especially for the case of complex, variable amplitude stress fields. Amongst several works attempting a general discussion of the subject, one could find interesting books, e.g. [1], [2], [3] and recent review articles, e.g. [4], [5]. In the same time, a number of recent PhD dissertations on the subject are available, focusing on improving fatigue life prediction schemes for Wind Turbine Rotor Blades design, e.g. Wahl [6], Nijssen [7], Passipoularidis [8], Post [9].

All discussions agree that the problem of defining and modelling the damage mechanisms developing in anisotropic materials during fatigue has proved to be much more complicated than for their isotropic counterparts. Especially, the correlation of the micro-structural damage and the fatigue performance of general laminates under general loading conditions remains in large an unsolved problem. This impedes the development of applicable, engineering models that would require only a small amount of basic material characterization data to simulate the fatigue process. As a result, most fatigue models proposed until now are founded on an empirical or phenomenological basis. A general classification of fatigue modelling efforts adopted herein is the following:

- Empirical
- Phenomenological
- Mechanistic

As empirical are considered models that introduce a damage parameter with no physical interpretation as a means of accumulating fatigue damage until final failure of the material. Several examples of different levels of complexity are found in literature, with most typical example the widely used Palmgren-Miner rule [10]. Most empirical approaches have been introduced for metallic materials in an effort to predict fatigue damage and life under variable amplitude (block or spectrum) loading. Some amongst them account for non linear dependencies of fatigue life on parameters such as the load sequence, fatigue stress level etc.

In contrast to empirical models, phenomenological formulations try to correlate the fatigue damage state of the material with a physically measurable quantity like residual stiffness or strength. Again no special focus is put on the actual types of fatigue damage, since its effects are reflected in average value of said mechanical property. This category of models treats the fatigue life prediction problem from the macro down towards the meso- or even micro-scale. However, micro-damage is not treated as such but rather as a cause of change of material mechanical properties at the considered scale. Given the variety and various interactions between fatigue damage mechanisms, phenomenological modelling appears as an attractive and convenient solution to the problem. Nevertheless, as for the case of most empirical models, a number of parameters have to be experimentally derived for each laminate. This increases material characterization cost while limiting their applicability to specific materials and laminate architectures.

Mechanistic models is the third category of life prediction schemes for composites which will provide –hopefully– the missing link between actual fatigue damage mechanisms and macro-mechanical properties of general laminates under arbitrary stress conditions. These models attempt to solve the problem moving from the micro- to the meso- or macro-scale. This task is quite demanding considering the variety of mechanisms and is further complicated by the strong statistical character of the properties of the composite’s constituents. These are indeed the major reasons why such models still remain, after several years of development, in their infancy being applicable only in specific, simple laminate architectures and fatigue loading conditions. The theoretical advantage of such models, when fully developed, will be their ability to produce predictions of life requiring a comparatively small amount of experimental input from the basic elements of the composite (e.g. fibre, matrix and interface mechanical properties). However, such models are not explicitly considered in the present work since focus is put on life prediction under complex stress states on multi-axial composite laminates, which is still further away from their present applicability field.

An intermediate level of modelling the fatigue process between phenomenological and mechanistic approaches, or more correctly still phenomenological but also considering the direct consequences of fatigue damage mechanisms in the meso-scale of the laminate, are those often called laminate-to-lamina approaches. Such models can be obtained either from the micro level up by considering larger characteristic volumes, e.g. the micromechanical ‘Critical Element Model’ of Reifsnider [11], or from the macro level down combined with adequate failure criteria and degradation strategies, e.g. the ‘Generalized Material Property Degradation Model’ implemented by Lessard [12]. This kind of modelling is at present an optimal compromise between applicability and accurate modelling of the fatigue process requiring as input a limited amount of material characterization experimental data.

All of the models classified above and discussed further on, require the knowledge of some basic fatigue parameters which could be categorized into discreet modules. These constitute the building blocks of a possible generalized life prediction algorithm under arbitrary fatigue loads (i.e. complex stress states of variable amplitude). These modules are the following:

*S-N Curve Definition.* In order to obtain any life prediction under VA loads, the fatigue response of the material under constant amplitude fatigue is required. In composites there are doubts on the existence of a fatigue limit (e.g. see [13]). Furthermore, only a few fatigue tests can be performed at stress levels usually exceeding the actual fatigue loads to be encountered during operation (in order to have realistic testing times). Consequently, an adequate model for extrapolating or interpolating fatigue lives at any stress level must be assumed.

*Generalizing to Various R-ratios.* A wide variety of fatigue cycles of different maximum and minimum stress are to be encountered under realistic VA fatigue. In the same time, experimental cost limitations impose the definition of only a few S-N curves (usually 1-3) at characteristic stress ratios R (R=minimum stress/maximum stress). In order to obtain S-N curves at intermediate stress ratios, an adequate model must be used. Often, this is performed through the definition of a Constant Life Diagram (CLD), e.g. a linear Goodman type formulation.

*Damage Accumulation Metric (DAM).* Apart from the Palmgren-Miner rule mentioned earlier, a variety of damage accumulation metrics have been proposed up to date. Their main function is to assume a point at which fatigue failure under VA fatigue takes place. Naturally, input from the first two modules is required for DAM definition under VA fatigue.

*Fatigue Failure Criterion.* When considering the case of complex stress fatigue, an expression for adding up the contribution of each component of the stress tensor to failure of the composite must be assumed, using some kind of failure function. The variety of the failure mechanisms observed in composite materials, caused by their anisotropic nature, depending both on their lay-up and the loading characteristics, has led to the development of a variety of failure criteria for multiaxial static loading. Some of them could be generalized for the case of cyclic loading as well.

An additional module, defining the kind of input necessary for any life prediction scheme under VA fatigue, is the algorithm for analyzing irregular load-time series into a sequence of constant amplitude cycles. This requirement arises from the fact that all fatigue testing is performed (most usually) through application of load cycles of sinusoidal form. Consequently, in order to correlate the experimentally obtained fatigue characteristics of the material with the VA load applied, an adequate counting method must be applied on the latter.

Taking into account the highly stochastic behaviour observed in fatigue life of composites, several researchers have tried to develop statistical models for its prediction. In this direction several approaches have been attempted, including fatigue life statistical modelling (e.g. [14]), empirical (e.g. [15]) or phenomenological (e.g. [16]) damage metrics, or laminate to lamina methodologies (e.g. [17]). Nevertheless, the majority of life prediction methodologies under generalized cyclic loading remain deterministic, referring to expected values.

An engineering oriented review of the above topics will be presented further on. Special focus is put on life prediction of Wind Turbine Rotor Blade composites under plane stresses of variable amplitude. Following the discussion on life prediction schemes, it is evident that the simplest of all life prediction models is the S-N curve (Uniaxial stress field, Constant Amplitude, constant R ratio). This is the reason why the discussion begins with the different alternatives of S-N curve formulations. Proceeding to slightly more complex predictive methodologies (i.e. Uniaxial stress field, Variable Amplitude, constant R ratio) different damage accumulation metrics are discussed. Generalizing the above for the case of varying Stress Ratio, several published analytical methods or Constant Life Diagram implementations are presented. Further on, state of the art multiaxial fatigue failure criteria are discussed, as a prerequisite to upgrade the above methodologies to the level of making life predictions under complex stresses of variable amplitude. Finally, up-to-date lamina-to-laminate algorithms considering progressive damage of multidirectional laminates based on the properties of the constitutive ply are resumed.

All models developed until now for VA fatigue of composites have been formulated in the time domain. When stochastic loads are involved in design, e.g. the wind loads experienced by Wind Turbine Rotor Blades [18], fatigue calculations are based on load-time series obtained through simulations using the statistical characteristics of wind and the system's transfer function. These are subsequently used as input to simple or more sophisticated life prediction

methodologies. The output of those possibly feedbacks again design, affecting the structure's elastic properties, requiring in turn a number of iterations for concluding on the final structural design of the blade. While this procedure requires a lot of time, formulating the problem in the frequency domain, i.e. applying some spectral method, could drastically reduce computational effort. In that case, fatigue calculations would be performed directly on a Power Spectral Density function (PSD) obtained through the Fourier transform of the time based load series. A number of such methods, which have been introduced for fatigue calculations of metallic materials under stochastic loads, will be also reviewed.

## 2.1 S-N Curve Definition

Fatigue response of materials is usually defined in terms of simple functional relationships between cyclic stresses and corresponding number of cycles named S-N curves. In most cases they refer to a specific stress ratio R defined as:

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \quad (1)$$

$\sigma_{\min}$  and  $\sigma_{\max}$  are respectively the minimum and maximum cyclic stress. The stress ratio is a characteristic property defining whether the cycle is purely tensile ( $0 < R < 1$ ), alternating with dominant tension ( $-1 < R < 0$ ), alternating with dominant compression ( $-\infty < R < -1$ ), or purely compressive ( $1 < R < \infty$ ).

The equation chosen for the S-N curve is significantly affecting life prediction under variable amplitude fatigue as shown by the sensitivity analysis performed in [19]. The simplest way to derive an S-N curve is through linear regression on fatigue life data. The most commonly used relation is the following logarithmic (or log-log) equation. It is represented by a straight line in the  $\log(N)$ - $\log(\sigma_{\max})$  space:

$$\sigma_{\max} = KN^{\frac{1}{b}} \quad (2)$$

Another alternative is the semi-log or lin-log:

$$\sigma_{\max} = b \cdot \log(N) + K \quad (3)$$

Eq.(3) is drawn as a straight line in the  $\sigma_{\max}$ - $\log(N)$  space, and has been proved [20] to yield pessimistic life predictions in the very high cycles range. The slope b of both curves is a metric of the material's fatigue life sensitivity on stress level, i.e. high values of b introduce a highly sensitive dependence of life on stress level.

It is questionable whether it is appropriate to include static tests in the fitting of the S-N curve. If static data are excluded, the intercept of the regression curve with the stress axis is truncated, leading to incorrect predictions under low cycle fatigue, especially when Eq.(2) is implemented. If static tests are included

however, a wrong slope of the S-N curve could be imposed. A limited number of static tests performed at stress rates comparable to fatigue stress rate [21] indicate that despite the improvement, a difference still exists between the 1 cycle prediction by the S-N curve and the static strength data. Echtermeyer et al. [22] suggest that static and fatigue performances are not necessarily related to each other, so static tests should not be included in the S-N curve fitting.

Even though the log-log formulation is widely used in literature, alternative procedures for deriving the S-N curve have been proposed. In some cases phenomenological modelling along with simple statistical assumptions are used rather than direct fitting of fatigue life data.

In a review work on life prediction of composite materials [23] Sendekyj is referring to residual strength theories as a means for predicting life under constant amplitude fatigue, assuming the existence of a unique relationship between static strength and fatigue life. The following general S-N curve formulation is derived:

$$X_0 = \sigma_{\max} [1 + (N-1)f]^{-S} \quad (4)$$

The strength-life pairs  $(X_0, 1)$  and  $(\sigma_{\max}, N)$  are assumed to be the boundary conditions,  $X_0$  being the ultimate strength and  $\sigma_{\max}$  the maximum cyclic stress. Different alternatives are proposed for  $f$  and  $S$  parameters, according to the residual strength model assumed. Their values are fitted using the equivalent static strength procedure based on fatigue life and/or residual strength and/or static strength data. Alternative expressions are summarized in the following table.

Table 1 Expressions proposed by Sendekyj for  $f$ ,  $S$  is eq.(4)

	S	f
W1	$S_0$	1
W2	$S_0$	C
W3	$S_0$	$C(1-R)^G$
W3A	$S_0(1-R)^G$	$C(1-R)^G$
W4	$S_0 + D(1-R)^G$	$C(1-R)^G$
W4A	$S_0(1-R)^D$	$C(1-R)^G$

$S_0$ , C, G and D are constants. Expressions W3 to W4A take into account the dependency on stress ratio R.

Sutherland, Mandell et al [24] propose a three parameter equation based on the one proposed by Epaarachchi, [25] formulating an S-N curve flat at low cycles, steeper at medium and less steep at high cycles:

$$X_0 - \sigma_{\max} = \alpha \sigma_{\max} \left( \frac{\sigma_{\max}}{X_0} \right)^b (N^c - 1) \quad (5)$$

$a$ ,  $b$ ,  $c$  are fitting parameters while  $X_0$  is the ultimate strength of the material obtained at a strain rate similar to that of the fatigue tests.

Whitney [26] proposes a method for defining the S-N curve at specific reliability levels assuming Weibull probability distribution of fatigue life. Tests are performed at various stress levels and each data set is fitted to a Weibull distribution. Subsequently, the experimental fatigue lives at each stress level are normalized by their respective characteristic life and the resulting population is also assumed to follow a Weibull distribution. The latter has a shape parameter  $\alpha_f$  derived using Maximum Likelihood Estimators. The S-N curve equation is finally:

$$\sigma_{\max} = K \left[ \left( -\ln R(N)^{\frac{1}{\alpha_f b}} \right) \right] N^{-\frac{1}{b}} \quad (6)$$

Parameters  $K$  and  $b$  are derived through fitting of a log-log S-N formulation to the  $\sigma_{\max}$  – characteristic life data points at the corresponding stress level:

Rotem and Nelson [27] propose a generalized S-N formulation to account for vertical shift (through  $\alpha_i^s$ ) and slope change (through  $\alpha_i$ ) of the S-N curve due to temperature. Their model is based on data from a reference temperature  $T_0$ , to derive the S-N curve at an elevated temperature  $T$ :

$$\sigma_{i(T)}^u = \sigma_{i(T_0)}^s \left[ \alpha_{i(T)} \left( \frac{1}{\alpha_{i(T)}} - b_i \log N \right) - \alpha_i^s \right] N^{-\frac{1}{b}} \quad (7)$$

$\sigma_{i(T)}^u$  is the cyclic stress and  $\sigma_{i(T_0)}^s$  is the static strength at the reference temperature.

Xiao [28] proposes the power law S-N relation where the temperature dependence is accounted for by the factor  $b_T$ , being equal to the ratio of static strength at the considered temperature  $T$ , versus the static strength at a reference temperature. The equation is:

$$\frac{\sigma_{\max}}{X_0} = b_T \left( \frac{\sigma_0}{X_0} + \frac{1 - \frac{\sigma_0}{X_0}}{(1 + \tau N)^n} \right) \quad (8)$$

$\tau$ ,  $n$  are fitted parameters.  $X_0$  and  $\sigma_0$  are static strength and fatigue limit respectively.  $b_T$  is related to static strength as a function of temperature. The above equation refers to isothermal conditions. Under general non-isothermal conditions the temperature rise is calculated as a function of the area of the hysteresis loop, frequency and time. Fatigue strength is calculated using experimentally obtained iso-strength (in life vs temperature) diagrams using fatigue data at different temperatures.

Miyano presented in [29] a methodology for deriving S-N curves at arbitrary temperature and loading frequency:

$$\sigma_{bf}(T, f, t_{bf}) = \sigma_{bs}(T, t_{bs})_{t_{bs}=1/f} - \alpha \log N \quad (9)$$

The static term  $\sigma_{bs}(T, t_{bs})$  is obtained from static tests at various temperatures and stress ratios using time-temperature shift factor, in an analogy to creep strength. This way, a master curve can be constructed where static strength is given as a function of time to failure (or temperature through the shift factor) based on a reference temperature. This equation assumes the static strength term to be equal to the creep strength corresponding to one-time period of the cyclic test ( $=1/f$ ).

## 2.2 Generalizing to Various R-ratios

Several alternatives have been proposed to deal with the problem of generalizing fatigue life S-N curves to arbitrary cyclic loads. The parameter most frequently used to characterize cyclic loads is undoubtedly the stress ratio and for that reason many researchers focus in the development of models expressing fatigue life as a function of stress level and stress ratio. Furthermore, a number of studies have proved that considerable strain rate effects are evident during fatigue of composites, mostly related to their viscoelastic behaviour which is even more pronounced in GRPs. Different models, try to account for this effect, using mostly empirical or phenomenological relations for introducing, apart from the number of cycles, frequency (i.e. time) or temperature dependence in their models. Such methodologies at a single stress ratio have been previously discussed, while others generalizing their predictions for arbitrary stress ratio are presented in the following paragraphs.

Following the standard design practice in Wind Turbine Blades, interpolation or extrapolation of fatigue lives at stress ratios different from the ones experimentally derived is implemented through a Constant Life Diagram or CLD [30], [31]. CLD diagrams are linear or non-linear interpolation schemes between experimentally obtained S-N curves in the mean stress ( $\sigma_m$ ) – stress amplitude ( $\sigma_a$ ) coordinate system. They are visualized as lines connecting all points corresponding to a specific fatigue life. While CLD diagrams for metals are typically represented satisfactorily by the Goodman line [32] or Gerber parabola [33], the response of composites under varying stress ratio indicates a more irregular form of CLD. See for instance [34] or [35] for Glass Polyester and [7] and [36] for Glass Epoxy materials used in Wind Turbine Blades.

Even though the CLD is a convenient representation of the material's fatigue response for any combination of maximum cyclic stress and stress ratio, a number of questions remain. One of them regards the convergence of all CLD lines to the UTS and UCS in the area near  $R=1$ . In fact, it is questionable whether it is appropriate to use static properties in a fatigue life diagram instead of creep strength. Tests performed at various stress ratios by Mandell et al. [24] do not seem to support the former, so the creep strength of the material is used instead of static properties, being correlated with fatigue life by a constant frequency (see also [29], [37]). A second issue arises during high cyclic stresses (low cycle fatigue) which should be accounted for by the diagram: Some of the widely used S-N curves (e.g. lin-log or log-log) represent poorly the material's fatigue behaviour in this region. Consequently, when such cycles need to be accounted

for by the S-N curve, inclusion of static strength data to the S-N curve fitting should be considered. However it should be provided that static tests are performed at similar stress rates as cycling.

Investigations of the effect of various CLD implementations on life predictions under spectrum fatigue can be found in [36] for UD and in [37] for MD Glass Epoxy laminates.

Typically, a Goodman type CLD is constructed based on the ultimate tensile and compressive strength of the material (UTS and UCS respectively) and the S-N curve at R=-1 stress ratio by assuming linear interpolation between them. The form of this CLD is shown in Fig.1 for the Glass Epoxy material used in [38]. The number of cycles for any combination of mean stress ( $\sigma_m$ ) and stress amplitude ( $\sigma_a$ ) is calculated separately for the region of tensile and compressive mean stress, by introducing  $\sigma_{eq}$  to the chosen R=-1 S-N equation:

$$\sigma_{eq} = \begin{cases} \frac{UTS \cdot \sigma_a}{UTS - \sigma_m} & \sigma_m > 0 \\ \frac{UCS \cdot \sigma_a}{UCS - \sigma_m} & \sigma_m < 0 \end{cases} \quad (10)$$

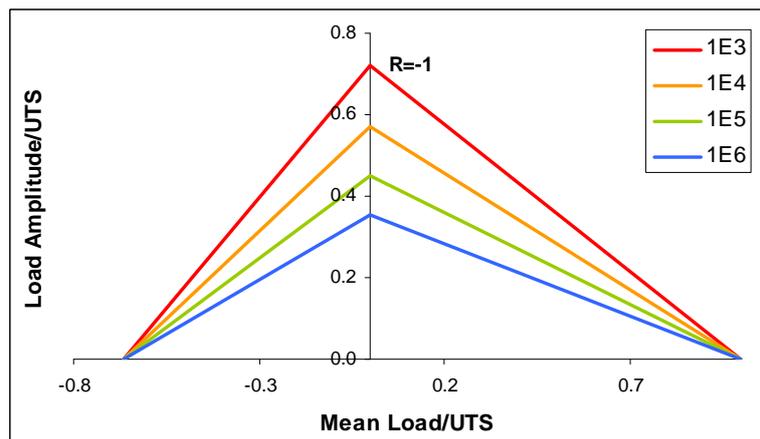


Fig. 1 Goodman type Constant Life Diagram

GL [30] proposes a slightly different CLD formulation with its peak located at equal distance between the UCS and UTS. The form of this symmetrical CLD is presented in Fig.2. The number of cycles for an arbitrary fatigue cycle, assuming  $b$  to be the slope of the S-N curve, is calculated through the equation:

$$N = \left( \frac{UTS + |UCS| - 2\sigma_m - UTS + |UCS|}{2\sigma_a} \right)^b \quad (11)$$

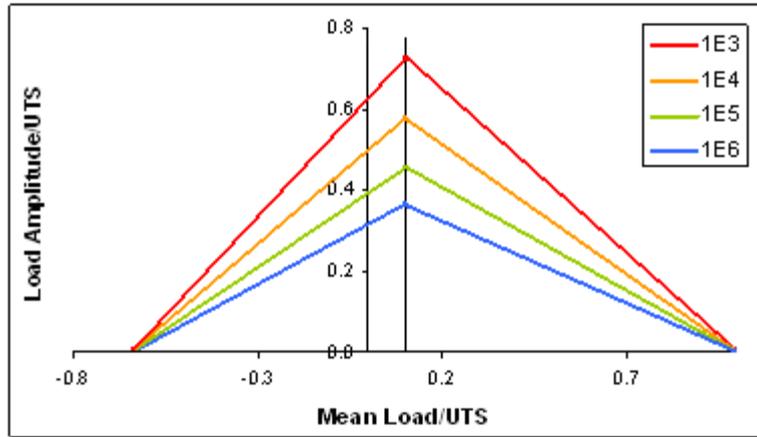


Fig. 2 CLD formulation proposed by GL [30]

More complex CLD diagrams can be assumed by including additional S-N curves. The CLD proposed by DNV [31] includes S-N curves in the tensile ( $R=0.1$ ) and compressive ( $R=10$ ) quadrant of the  $(\sigma_m-\sigma_a)$  plane as well. Its form is shown in Fig.3 while various procedures can be incorporated to derive the fatigue life under different cyclic stresses, see for instance [6], [39]. In general, a more detailed representation of fatigue behaviour in the CLD construction leads to improved predictions under fatigue life, as proposed by [35], [7], even though the cost associated with S-N curve determination at various stress levels can be prohibitive.

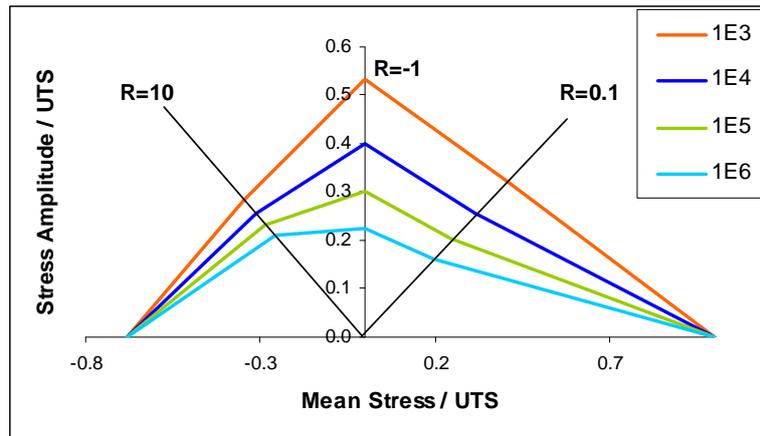


Fig. 3 CLD based on 3 S-N curves, as proposed by DNV [31]

An alternative CLD formulation is introduced by Adam et al. [40]. A three parameter function is proposed implying non linear interpolation between the available S-N curves and static strength values of the material. The result is the bell like CLD shown in Fig.4, which is not necessarily symmetrical around the  $\sigma_a$  axis. Its functional form is the following:

$$\frac{\sigma_a}{UTS} = f \left( 1 - \frac{\sigma_m}{UTS} \right)^u \left( \frac{UCS}{UTS} + \frac{\sigma_m}{UTS} \right)^v \quad (12)$$

$f$ ,  $u$  and  $v$  are parameters that depend on fatigue life.

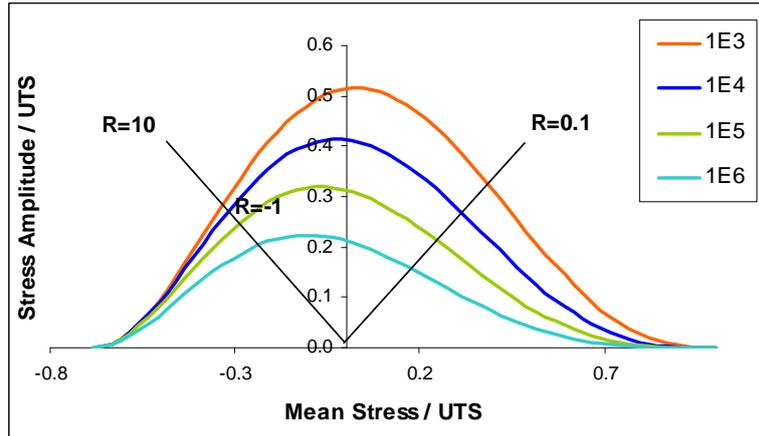


Fig. 4 The bell-shaped CLD proposed in [40]

Towo and Ansell [41] proposed a similar bell-shaped CLD formulation based on a third order polynomial equation fitted on S-N curve data at R=0.1 and R=-1.

Another non linear, asymmetrical CLD formulation has been proposed by Kawai and Koizumi [42] for Carbon Epoxy laminates. Its form is shown schematically in Fig.5.

$$-\frac{\sigma_a - \sigma_a^\chi}{\sigma_a^\chi} = \begin{cases} f \left( \frac{\sigma_m - \sigma_m^\chi}{UTS - \sigma_m^\chi} \right)^{2 - \frac{\sigma_{max}^\chi}{\sigma_B}} & UTS \geq \sigma_m \geq \sigma_m^\chi \\ f \left( \frac{\sigma_m - \sigma_m^\chi}{UCS - \sigma_m^\chi} \right)^{2 - \frac{\sigma_{max}^\chi}{\sigma_B}} & UCS \leq \sigma_m \leq \sigma_m^\chi \end{cases} \quad (13)$$

A critical stress ratio  $\chi$ , equal to the compressive versus the tensile static strength, is assumed.  $\sigma_m^\chi$ ,  $\sigma_a^\chi$ ,  $\sigma_{max}^\chi$  are mean, alternating and maximum cyclic stress at the critical stress ratio, expressed as functions of the unknown fatigue life N, while  $\sigma_B$  is the intercept of critical S-N line with the stress axis. The shape of the assumed CLD changes progressively from a straight line to a parabola with increased fatigue life.

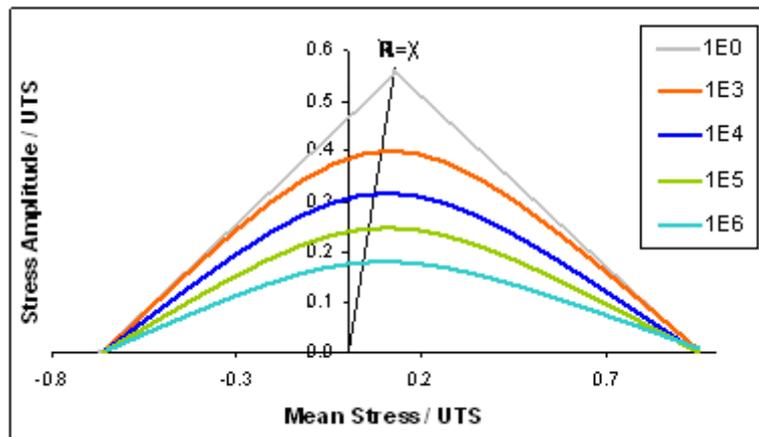


Fig. 5 The nonlinear CLD proposed by [42]

Boerstra [43] proposes different forms of CLD on either side of the  $\sigma_a$  axis, each described by a Gerber parabola. The slopes of the S-N curves are assumed to vary with mean stress  $\sigma_m$  according to an exponential relationship of the form:

$$b = b_0 \cdot \exp\left(-\frac{\sigma_m}{D}\right) \quad (14)$$

Another CLD form is implied when adopting the equivalent stress concept proposed by Brondsted et al in [44] where all fatigue cycles inside a spectrum are assumed to be described by a single S-N curve close to its average stress ratio. This simplistic CLD formulation can be interpreted by the following set of CLD lines, all having the same slope equal to -1. Its graphical presentation can be seen in Fig.6.

$$\sigma_a = \frac{\sigma_o}{N^k} - \sigma_m \quad (15)$$

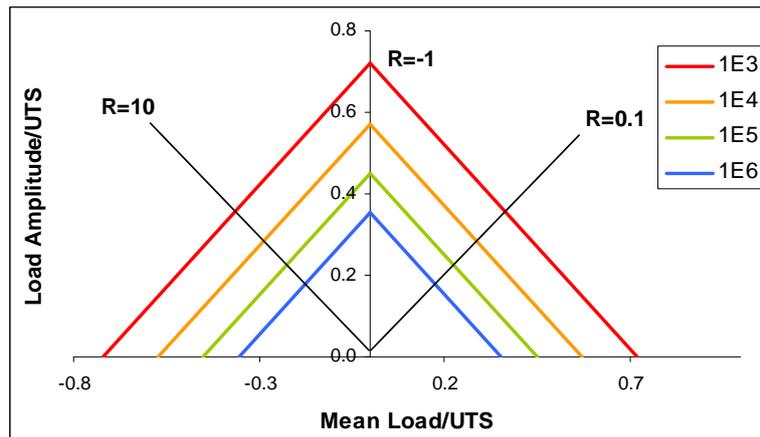


Fig. 6 CLD lines assuming a single S-N curve for any cycle

Alternative CLD formulations implying a similar behaviour as the latter in the purely tensile part of the graph have been discussed in [37].

Rotem in [45] suggests a methodology for deriving the S-N curve of a composite laminate at any stress ratio once its fatigue behaviour has been obtained at two stress ratios, one in the tensile failure mode region and the second in the compressive. The S-N relation for arbitrary stress ratio  $R$ , is obtained after calculating two of its points (referring to known fatigue lives, taken as  $10$  and  $10^6$  cycles) according to the following relations, assuming log-log S-N curve representation:

$$\sigma_{a1} = \frac{1-R_1}{\frac{R-R_1}{UTS} + \frac{1-R}{S_0 10^{6K}}} \quad \sigma_{a2} = \frac{1-R_1}{\frac{R-R_1}{UTS} + \frac{1-R}{S_0 10^{6K}}} \quad (16)$$

$S=S_0N^K$  is the known S-N curve at stress ratio  $R_1$  (tension dominated fatigue). The final form of the unknown S-N curve at  $R$  is:

$$\sigma_a = 10^{\frac{6 \log(\sigma_{a1}) - \log(\sigma_{a2})}{5}} N^{\log(\sigma_{a1}) - \frac{6 \log(\sigma_{a1}) - \log(\sigma_{a2})}{5}} \quad (17)$$

Similarly, S-N curves at the compression dominated region can be defined using the UCS and a known compressive S-N curve in the above relations.

Epaarachchi and Clausen [46] based as previously on a differential equation of residual strength degradation, suggested an S-N curve formulation which includes the stress ratio and fatigue frequency dependence:

$$\left( \frac{X_0}{\sigma_{\max}} - 1 \right) \left( \frac{X_0}{\sigma_{\max}} \right)^{0.6 - \psi |\sin \theta|} \frac{1}{(1 - \psi)^{1.6 - \psi |\sin \theta|}} f_B = \alpha (N^\beta - 1) \quad (18)$$

Parameters  $\alpha$  and  $\beta$  are material related, fitted on tensile fatigue tests while  $\theta$  is the smallest angle between fibre direction and loading,  $f_B$  is the cycling frequency  $\psi$  is the stress ratio for tensile and reversed loading fatigue and the inverse stress ratio for compression-compression.

A similar approach, aiming at the derivation of a master curve for all stress ratios  $R$ , has been proposed by Caprino and D'Amore in [47] implemented on flexural fatigue test data. The model is once more based on a residual strength degradation formulation, concluding in the following generalized S-N expression:

$$\left( \frac{X_0}{\sigma_{\max}} - 1 \right) \frac{1}{(1 - R)} = \alpha (N^\beta - 1) \quad (19)$$

When the left hand term of the above equation is plotted against  $(N^\beta - 1)$  a straight line is drawn having a slope of  $\beta$ . This way, using fatigue tests at various stress ratios, the parameters  $\alpha$  and  $\beta$  can be determined.

Finally, Miyano et al. generalized the model presented in the previous chapter to arbitrary stress ratio in [48] and [49]. The previously discussed master curve from static strength tests at different strain rates is constructed, along with another master curve for fatigue tests (at  $R=0$  stress ratio) for various testing frequencies. The S-N curve prediction is generalized to arbitrary stress ratio assuming linear dependence on  $R$ .

$$\sigma_f(t_f, f, R, T) = \sigma_{f:1}(t_f, f, T)R + \sigma_{f:0}(t_f, f, T)(1 - R) \quad (20)$$

$\sigma_f(t_f, f, R, T)$  is the fatigue strength depending on the creep strength  $\sigma_{f,1}(t_f, f, T)R$  (considered equal to the fatigue strength at  $R=1$  and arbitrary frequency  $f$ ) and the fatigue strength under  $R=0$   $\sigma_{f,0}(t_f, f, T)$ , derived for arbitrary frequency  $f$  through the fatigue master curve.

## 2.3 Damage Accumulation

### 2.3.1 Empirical Models

Various empirical relations for summing up fatigue damage at different stress levels and predicting failure have been proposed up to date. In general, non-linear formulations have been developed in a continuous effort to improve the – usually poor- performance of the linear theory discussed below. An interesting review of various cumulative damage models can be found in [50].

The best established amongst empirical damage accumulation metrics is the Palmgren-Miner rule [10], which assumes a linear dependence of fatigue damage  $D$  on the total life fraction spent during cycling.

$$D = \sum_{i=1}^k \frac{n_i}{N_i} \quad (21)$$

Eq.(21) refers to  $k$  blocks each consisting of  $n_i$  cycles with an expected fatigue life equal to  $N_i$ . Failure is usually assumed to occur when  $D=1$ .

While the Palmgren-Miner rule is a simple and straight forward approach in fatigue damage accumulation, its application on composites is rather problematic since it does not account for load sequence effects. Discussions on the efficiency of Palmgren-Miner rule are presented by Broutman & Sahu [51], Schaff and Davidson [52], Bond [53], Philippidis & Vassilopoulos [54], Gamstedt & Sjogren [55], Hosoi et al. [56] and others. The main conclusion from these investigations is the inefficiency of Palmgren-Miner rule to predict failure under block or spectrum fatigue, yielding in some cases conservative and in some others over-optimistic predictions. In view of this, different approaches of non-linear empirical fatigue damage have been proposed:

The non-linear Marco Starkey formula [57] is one of them:

$$D = \sum_{i=1}^k \left( \frac{n_i}{N_i} \right)^\alpha \quad (22)$$

Parameter  $\alpha$  is an experimentally fitted function of the stress amplitude. Using this theory sequence effects can be accounted for.

Another non-linear damage accumulation rule has been proposed by Owen & Howe [58] based on block loading fatigue observations:

$$D = \sum_{i=1}^k \left( A \left( \frac{n_i}{N_i} \right) + B \left( \frac{n_i}{N_i} \right)^2 \right) \quad (23)$$

Eq.3 has been later modified by Bond with one additional parameter  $c$  and validated using the WISPER [53] (on GRP laminates) and FALSTAF [59] (on CFRP) standardized spectra.

$$D = \sum_{i=1}^k \left( A \left( \frac{n_i}{N_i} \right) + B \left( \frac{n_i}{N_i} \right)^c \right) \quad (24)$$

Following the approach of Sabramanyan [60] based on the definition of equivalent damage lines converging to a knee point, Hashin & Rotem [61] have proposed a similar concept. This time the damage accumulation rule does not account only for load sequence but also for stress level dependency through the following equation:

$$D_i = D_{i-1}^{\frac{\log(\sigma_{\max,i})}{\log(\sigma_{\max,i-1})}} + \frac{n_i}{N_i} \quad (25)$$

## 2.3.2 Phenomenological Models

### 2.3.2.1 Residual Strength

Residual strength has been considered since early in the study of composites as a convenient property for expressing phenomenologically damage accumulation during fatigue. In contrast to empirical and other phenomenological formulations, residual strength theories inherently include a failure criterion: Failure is assumed to occur when residual strength degrades to the maximum applied stress. Additionally the 'Strength Life Equal Rank Assumption' or SLERA, introduced by Hahn & Kim [62] and named later on by Chou and Croman [63], provides a handy tool for deriving the probability distribution of either fatigue life or residual strength based on that of the static strength.

Up to date residual strength models have been extensively discussed by Sendekyj in [64], reviewed in [4], [5] and evaluated through implementation on common experimental data sets in [65]. An overview of the most characteristic models is presented in the present section.

Broutman and Sahu [51] have presented one of the first attempts for modelling static strength degradation of GFRP composites, in an effort to develop a modified Palmgren-Miner rule, which would account for load sequence effects. Their equation assumes linear degradation of strength up to failure:

$$X_r = X - (X - \sigma_{\max}) \left( \frac{n}{N} \right) \quad (26)$$

Their model has been proven [65] to overestimate strength degradation of WT blade composites under fatigue, especially in the range of higher fatigue lives. It has however performed satisfactorily as fatigue damage metric when incorporated into life prediction schemes under VA fatigue [see also 36].

Hahn and Kim [62] introduced the concept of rate of change of residual strength through the following rate type equation:

$$\frac{dX_r}{dt} = -AX_r^{-(c-1)} \quad (27)$$

The positive parameter  $A(\sigma)$  depends on the applied dynamic load  $\sigma(t)$  and the exponent  $c$  is a material constant. The behaviour of the above equation depends on parameter  $c$  being above or below unity, thus forcing the above equation to follow slow strength degradation becoming steeper as fatigue progresses or vice versa. Integration leads to:

$$X_r^c = X^c - cD(t - t_0) \quad (28)$$

Parameter  $D$ , which is the integral of  $A(\sigma)$  from  $t_0$  to  $t$ , in general depends on the characteristics of fatigue loading. In constant amplitude fatigue, appropriate parameters are the stress amplitude, stress ratio and frequency.

Yang et al have, since 1975, published several articles on residual strength [66]-[69] and in later works on stiffness degradation of composites due to fatigue [70], [71]. The most general form of their model, found in [72], is based on the following rate type equation:

$$\frac{d\left(\frac{X_r(n)}{\beta}\right)}{dn} = \frac{-\gamma n^\gamma f(\sigma_{\max}, X)}{c\left(\frac{X_r}{\beta}\right)^{c-1}} \quad (29)$$

The expression of  $f(\sigma_{\max}, X)$  in Eq.(29), is derived by applying the fracture condition after integration, which leads to:

$$X_r^c(n) = X^c - \frac{X^c - \sigma_{\max}^c}{(X^\omega - \sigma_{\max}^\omega)^\gamma} \beta^{\omega\gamma} (K\sigma_{\max}^b n)^\gamma \quad (30)$$

$c$ ,  $b$ ,  $K$ ,  $\omega$  and  $\gamma$  are model parameters derived through the equivalent static strength concept, i.e. attributing a fictitious static strength to each residual strength and/or fatigue life experimental datum through eq.(30) and trying to match the obtained probability distribution to that of static strength.

Chou and Croman propose, on one hand a different wear-out model, including an additional free parameter [63], and on the other hand they introduce the sudden-death model [73], as a limiting case in the residual strength study. This latter is the case for which the residual static strength remains constant, i.e. independent of load cycles, until immediately prior to failure and then drops suddenly. The form of their wear-out degradation equation is the following:

$$X_r^\alpha = X_\gamma^\alpha - n_\gamma^{\alpha_\gamma} \left( \frac{n}{n_\gamma} \right)^{\alpha_\gamma} \quad (31)$$

$X_\gamma$  and  $n_\gamma$  are the static strength and fatigue life respectively, that give a value of  $(1-\gamma)$  in the cumulative distribution function (CDF). By assuming different values for parameter  $i$ , a family of degradation curves is obtained, ranging from gradual wear-out to sudden death behaviour

Adam et al. [74] introduced the *interaction model*, motivated by the apparent similarity of the residual strength curves at various stress levels, and the possible event of some appropriate normalized formulation. In this direction, they introduce the residual strength ratio as:

$$r = \frac{X_r - \sigma_{\max}}{X - \sigma_{\max}} \quad (32)$$

As well as the cycle (or log-time) ratio:

$$t = \frac{\log n - \log 0.5}{\log N - \log 0.5} \quad (33)$$

These two normalized quantities are combined under the appropriate boundary conditions, i.e. points (1,0) and (0,1), through the following expression:

$$t^x + r^y = 1 \quad (34)$$

$x$  and  $y$  are determined through fitting on residual strength data.

Reifsnider (e.g. [11]) proposed a *Critical Element Model* theory for modelling the fatigue process in composites based on the definition of critical (causing overall failure) and sub-critical (causing stress redistribution) elements. The general equation proposed for the critical elements can account for different failure modes through an adequately chosen failure criterion and for local stress concentrations. The equation proposed for the case when local effects are disregarded is:

$$X_{r(n)} = X - (X - \sigma_{\max}) \left( \frac{n}{N} \right)^y \quad (35)$$

Parameter  $\nu$ , expressing the non-linearity in the degradation of static strength, has been close to unity for the cases examined [11]. For WT blade materials however this parameter seemed to be a function of the fatigue damage state of the material [65].

Schaff and Davidson [52] have adopted in their study the above degradation equation. Parameter  $\nu$  is derived by fitting the occurring fatigue life probability distribution to a fatigue life sample. They also introduced the *effective number of cycles* quantity, necessary under variable amplitude fatigue, being the number of cycles that would have brought residual strength down to its current value if the material was subject to constant amplitude fatigue at the current block's stress level.

Epaarachchi and Clausen [75] proposed a frequency and stress ratio dependent residual strength formulation as damage accumulation metric under step VA fatigue, previously implemented for deriving the materials S-N behaviour under different fatigue conditions:

$$1 = \frac{\sum_{k=1}^n \left\{ \alpha \left( \frac{\sigma_k}{X_{r(n-1)}} \right)^{0.6-\nu|\sin\theta|} \left( \sigma_k (1-\psi_k)^{1.6-\nu_k|\sin\theta|} \right) \frac{1}{f_k^\beta} (n_k^\beta - 1) \left[ 1 - \left( 1 - \left( \frac{n_k}{N_{\sigma n}} \right)^\beta \right)^\beta \right] \right\}}{\alpha \left( \frac{\sigma_n}{X} \right)^{0.6-\nu|\sin\theta|} \left( \sigma_n (1-\psi_n)^{1.6-\nu|\sin\theta|} \right) \frac{1}{f_n^\beta} (N^\beta - 1)} \quad (36)$$

$\alpha$  and  $\beta$  are experimentally defined model parameters. As previously,  $\theta$  is the smallest angle between load and fibre in the laminate and  $\psi$  is the stress ratio  $R$  or its inverse value when  $R > 1$ .  $N_{\sigma n}$  is the residual life at level  $n$  after the loading at level  $(n-1)$  step.

Yao and Himmel [76], suggested a different residual strength degradation equation for the case of tensile loading, having the following form:

$$X_r = X - (X - \sigma_{\max}) \frac{\sin\left(\beta \frac{n}{N_f}\right) \cos(\beta - \alpha)}{\sin(\beta) \cos\left(\beta \frac{n}{N_f} - \alpha\right)} \quad (37)$$

Parameters  $\alpha$  and  $\beta$  are derived through experiments. Under compressive fatigue they propose a non-linear formulation of the form of Reisnider (Eq.(35))

It must be noted that any residual strength degradation equation available can be generalized to variable amplitude fatigue using the effective number of cycles during the application of subsequent single cycles or CA blocks. Nevertheless, attention must be paid when the residual strength is assumed constant through a part of the specimen's life. In that case, the zero degradation from one block to another makes numerically impossible the calculation of effective cycles within available accuracy limits. Consequently, when this kind of behaviour is assumed, different strategies for accumulating damage over sequences of variable amplitude must be implemented.

### 2.3.3 Stiffness Degradation

Several researchers proposed phenomenological formulations relating the observed stiffness degradation during fatigue to fatigue life. While the implementation of such models is similar to those considering residual strength, stiffness can be recorder non-destructively during cycling. Nevertheless, a failure criterion must be assumed, which is not as obvious as for residual strength. A number of researchers considered the ultimate strain during static strength test to be an adequate metric. The significant difference in the damage mechanisms and loading conditions however, make this assumption questionable, as suggested in [22], [20].

Hwang and Han [50], [77] proposed a model based on the fatigue modulus concept, i.e. the slope between the maximum stress vs strain at an arbitrary cycle and the origin of the stress-strain coordinate system. This metric expresses not only modulus degradation but also residual strains accumulating during cycling. The degradation equation is based on the following power equation:

$$\frac{dE}{dn} = -Acn^{c-1} \quad (38)$$

A and c are material constants. The failure criterion adopted upon integration of the above equation assumes failure when the fatigue modulus degrades up to the ultimate strain during static testing.

For the case of variable amplitude fatigue, amongst others, the following damage metric has been proposed [50]:

$$D = \frac{\frac{\sigma_{\max}}{X} \left( \frac{E_0}{E_n} - 1 \right)}{1 - \frac{\sigma_{\max}}{X}} \quad (39)$$

$E_0$  and  $E_n$  are the initial fatigue modulus and the fatigue modulus at cycle n respectively, while the damage parameter D is assumed equal to unity at failure.

Sidoroff and Subagio [78] proposed a different damage equation, considering no damage during compressive cycling and the following damage rate under tensile fatigue:

$$\frac{dD}{dn} = \frac{A \cdot (\Delta\varepsilon)^c}{(1-D)^b} \quad (40)$$

The variable D is equal to  $1-E/E_0$  and A, b, c are material constants defined experimentally.  $\Delta\varepsilon$  is the stain amplitude applied during fatigue. This model has also been implemented in FEM code by Van Paepegem and Degrieck [79]. Van

Paepegem implemented later on [80] a different stiffness degradation approach adopting the following equation, which distinguishes between the two stages of fatigue stiffness degradation, corresponding to damage initiation and damage propagation:

$$\frac{dD}{dn} = \begin{cases} c_1 \Sigma e^{-c_2 \frac{D}{\sqrt{\Sigma}}} + c_3 D \Sigma^2 \left(1 + e^{-c_5 (\Sigma - c_4)}\right) & \sigma \geq 0 \\ \left[ c_1 \Sigma e^{-c_2 \frac{D}{\sqrt{\Sigma}}} \right]^3 + c_3 D \Sigma^2 \left(1 + e^{-\frac{c_5}{3} (\Sigma - c_4)}\right) & \sigma < 0 \end{cases} \quad (41)$$

$c_i$  ( $i=1:5$ ) are model parameters and  $\Sigma$  is a quantity named failure index defined equal to  $E_0 \varepsilon / X$ ,  $E_0$  being the initial modulus,  $X$  the tensile or compressive strength and  $\varepsilon$  the current cycle's maximum strain.

Whitworth [81] proposed a different stiffness degradation equation for the case of CA fatigue, assuming the following change rate for the modulus:

$$\left( \frac{E\left(\frac{n}{N}\right)}{E_0} \right)^\alpha = 1 - H \left( 1 - \frac{\sigma_{\max}}{X} \right)^\alpha \left( \frac{n}{N} \right) \quad (42)$$

$H$  and  $\alpha$  are stress independent parameters. According to [82], fatigue damage for the case of VA cyclic loading can be accumulated according to the following formula which is used to calculate the damage contribution of each cycle of normalized range  $S$ :

$$D = \left( \frac{H(1-S)^\alpha}{1-S^\alpha} \right) \left( \frac{n}{N} \right) \quad (43)$$

In [83], another stiffness degradation model is proposed, further implemented to derive the statistical distribution of residual stiffness:

$$E(n) = E_N \left( -h \cdot \ln(n+1) + \left( \frac{E_0}{E_N} \right)^m \right)^{\frac{1}{m}} \quad (44)$$

$h$  and  $m$  are model parameters to be determined experimentally and  $E_N$  is the stiffness at failure assumed, as in previous cases, to take the value of ultimate strain at static test.

Yang et al. [70] assumed stiffness degradation rate of Graphite/Epoxy laminates under CA fatigue to be a power function of the number of cycles. When integrated, their model is expressed through the following equation, where  $a_1$ ,  $a_2$ ,  $a_3$  are parameters independent of the stress level, obtained through stiffness measurements during cycling.

$$E(n) = E_0 \left( 1 - (\alpha_1 + \alpha_2 \alpha_3 + \alpha_2 BS)(n)^{\alpha_3 + BS} \right) \quad (45)$$

B is a random variable used further on in the derivation of the stiffness statistical distribution and S is the applied stress level.

Lee, Fu and Yang proposed in a subsequent publication [84] the following stiffness degradation equation for spectral fatigue (at the  $n_k^{\text{th}}$  loading block):

$$E(n) = E_0 \left( 1 - \left( \frac{n}{n_k} \right)^{v(k)} \left( 1 - \frac{E(n_k)}{E_0} \right) \right) \quad (46)$$

The only parameter is  $v(k)$ , fitted on stiffness measurements during fatigue. The failure criterion assumed for defining failure stiffness and consequently fatigue life, is based on the following relation where S is the stress level and  $A_0$  is a fitted constant:

$$\varepsilon(N) = \frac{S}{E_0} + A_0 \frac{UTS}{E_0} \quad (47)$$

Brondsted et al [85] proposed a stiffness degradation equation implemented in the definition of stiffness controlled S-N curves of Glass/Polyester laminates based on the specific failure stiffness  $E_L$ , the latter being experimentally obtained. Their model, assuming CA fatigue, is:

$$\frac{E_{(n)}}{E_0} = 1 - \left( 1 - \frac{E_L}{E_0} \right) \frac{n}{N} \quad (48)$$

Post et al [86] suggested the implementation of a stiffness degradation equation on a statistical life predictive algorithm based on the critical element model philosophy of Reifsnider [11]. The strain controlled failure criterion assumes failure to occur as soon as fatigue strain equals the failure strain during the static test. The stiffness model has the form:

$$E_{(n)} = E_0 \left( \alpha e^{-bn} - cn + (1 - \alpha) \right) \quad (49)$$

$\alpha$ ,  $b$  and  $c$  are fitted parameters. The above equation claims to fit the modulus degradation during its first and second stage, i.e. not during its last stage of steep drop prior to failure.

## 2.4 Complex Stress Failure Criteria- Static

The application of failure criteria under complex stress states used in metals, such as the Mohr-Coulomb or Von-Mises theory, is not effective in composite materials due to their anisotropic nature. Trying to take this anisotropic behaviour into account, different researchers proposed a multitude of failure theories under biaxial static stress conditions, while different experimental setups for inducing such stress fields in composite laminates have been incorporated, to provide the necessary input for their validation. A review and categorization of failure criteria under multiaxial static loading can be found in [87].

Criteria suggesting the maximum stress or maximum strain at the two principal directions and in shear of a laminate in order to derive the failure surface in the  $\sigma_1, \sigma_2, \tau_{12}$  space have been proposed respectively by Stowell and Liu [88] and Waddoups [89].

Hill [90] proposed a failure criterion for metals with anisotropic strength properties induced by extreme deformation of their crystalline structure by a manufacturing process. The model was similar to the one proposed later by Azzi and Tsai [91] suggesting a different normal stress interaction parameter.

Norris [92] proposed a quadratic formulation combining the two stresses in the principal coordinate system and in-plane shear in a similar expression as that of Hill, leaving out the normal stress interaction term. Later [93] he introduced an additional, non-squared term. In an effort to account both effects of normal stress interaction and different strength in tension and compression, Hoffman [94] suggested a relation including both tensile and compressive strengths in the symmetry directions of the orthotropic material.

Several other quadratic or cubic failure functions have been proposed, with many amongst them being different adaptations of the general theory proposed by Tsai-Wu [95]. Its tensorial expression is:

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1 \quad (50)$$

The linear terms in the above relation take into account the difference between positive and negative stress induced failures, while the quadratic term defines an ellipsoid in the stress space.  $F_i$  and  $F_{ij}$  are strength tensors of the 2<sup>nd</sup> and 4<sup>th</sup> rank respectively and for the case when  $i=j$  are determined through simple static tests. The non-diagonal term  $F_{ij}$  ( $i \neq j$ ) (interaction term) on the contrary is defined through multi-axial tests and defines the orientation and position of the ellipsoid in the 3D stress space. Off-axis, tubular or cruciform specimens for instance could be used for its determination. The following stability equation has to be satisfied in order to have a closed failure surface.

$$F_i F_{jj} - F_{ij} \geq 0 \quad (51)$$

Different failure criteria are derived with different assumptions of the interaction term (see e.g. [96], [97]).

A recent attempt to access different biaxial static failure criteria proposed up-to-date, based on available experimental data is the 'World Wide Failure Exercise'

organized and coordinated by MJ Hinton (Defence Evaluation & Research Agency) and PD Soden, AS Kaddour (UMIST, Applied Mechanics Division) [98]. The final scope was to provide the composites community with accurate failure prediction methods for multidirectional laminates under complex static stresses. Complete methodologies to predict biaxial failure in the UD laminate and generalize it up to final failure (i.e. last ply failure) of a multi-directional lay-up are proposed. Specific degradation rules, in the form of knock down factors, are proposed for stiffness and/or strength components due to non-catastrophic failure of plies, depending on the failure mode observed. In total 14 such failure theories are considered and implemented. All of them can be found in [99], which was dedicated to the World Wide Failure Exercise. A comparative presentation of the methodologies is presented in [100], presenting the outcomes of the part A of the exercise.

The methodologies are implemented on 4 different material systems (2 Carbon/Epoxy and 2 Glass/Epoxy) and various predictions are obtained for a variety of lay-ups, discussed in [101]. The predictions from the various authors are presented in [102], which was dedicated to Part B of the World Wide Failure Exercise. A complete discussion of the part B of the exercise (i.e. the implementation and assessment of all models based on different experimental data sets of multidirectional laminates), grading the various models can be found in [103]. Five different criteria are used to in the grading procedure: Strength of a UD laminate under biaxial stress, Initial and Final failure, stress-strain behaviour simulation and ability to predict the general trend observed in the data.

The final part of the exercise (part C) has been presented in [104] where additional models presented by that time were included and recommendations for design application of the methodologies considered during the exercise were discussed [105].

The input provided by this discussion on the various aspects of each methodology along with the variety of materials and lay-ups considered provides a solid basis for understanding and modelling the consequences of damage during static loading in composite multidirectional laminates. Nevertheless, considering the numerous possible combinations of failure modes that can be integrated to progressive damage models, through simple knock-down factors applied on the elastic and strength properties, attention should be paid to keep a close correlation between assumed failure modes and the actual damage mechanisms observed during testing. Otherwise, even though accurate predictions may be fitted, the physical background of the model is discredited.

A review of existing methodologies modelling failure in composite materials has appeared quite recently by Orifici et al. [106]. An extensive review of failure predicting models is presented, categorizing them in terms of the failure mode predicted:

- Fibre failure
- Matrix failure
- Shear failure
- Ply level failure
- Delamination initiation

- Existing delamination growth

Fibre and matrix failure criteria are listed separately when different formulations are used in tension and compression. This work is valuable in assembling and categorizing all up-to-date failure theories for quasi-static loading.

## 2.5 Complex Stress Failure Criteria- Fatigue

Efforts for taking into account complex stress states in fatigue of composites are usually incorporated through modification of a static multiaxial criterion: The failure functions are expressed in terms of the fatigue strength of the composite rather than of its static strength. This concept has been used by Owen and Griffiths in their critical review of biaxial stress failure criteria [107].

In certain cases the failure criteria proposed for UD laminates can be generalized to MD lay-ups, the laminate however can only be treated macroscopically since otherwise additional assumptions for damage progression and degradation strategy for post failure behaviour of sub-critical elements (which are constrained between adjacent layers) should be assumed as well. Furthermore, the latter would cause plane stress redistributions between layers, constantly altering the stress field in each lamina, thus requiring the implementation of a damage accumulation assumption in order to obtain a realistic prediction of failure. This is in brief the lamina-to-laminate approach discussed in the following section.

### 2.5.1 Hashin & Rotem

Hashin and Rotem proposed in [108] a fatigue failure criterion for unidirectional laminates under constant amplitude fatigue, distinguishing between fibre and matrix failure modes:

$$\sigma_F = \sigma_F^u \left( \frac{\sigma_T}{\sigma_T^u} \right)^2 + \left( \frac{\tau}{\tau^u} \right)^2 = 1 \quad (52)$$

Transition from one mode to the other can be calculated so a critical off-axis angle is obtained. Terms  $\sigma_F$ ,  $\sigma_T$  and  $\tau$  refer to normal stresses in the fibre, transversely to the fibre and in-plane shear respectively. Superscript u denotes fatigue failure stresses which are defined as the product of the corresponding static strength times a dimensionless material fatigue function, the latter being in general a function of stress ratio, number of cycles and frequency. The three failure functions can be derived using three experimentally obtained S-N curves, one for the fibre direction and two off-axis angles  $\theta$ . Then, the fatigue functions for shear and transverse direction can be calculated by solving the 2x2 system of equations derived through the following relationship, relating the fatigue functions in the transverse direction  $f_T$  and shear  $f_\tau$  and the fatigue function of the two  $\theta^\circ$  off-axis laminates:

$$f'' = f_{\tau} \sqrt{\frac{1 + \left(\frac{\tau}{\sigma_T^u}\right)^2 \tan^2 \theta}{1 + \left(\frac{\tau}{\sigma_T^u} \frac{f_{\tau}}{f_T}\right)^2 \tan^2 \theta}} \quad (53)$$

## 2.5.2 Philip et al

The failure criterion for anisotropic materials proposed by Hill was implemented by Philip et al [109] to predict the fatigue behaviour of tubular specimens under combined axial-torsional fatigue stresses. The general formulation they propose as the extended Hill's criterion is:

$$\frac{\sigma_x^2}{X_{(N)}^2} - \sigma_x \sigma_y \left( \frac{1}{X_{(N)}^2} + \frac{1}{Y_{(N)}^2} - \frac{1}{Z_{(N)}^2} \right) + \frac{\sigma_y^2}{Y_{(N)}^2} + \frac{\tau_{xy}^2}{S_{(N)}^2} = 1 \quad (54)$$

Z is through-the-thickness strength of the laminate. No stress ratio dependence is assumed. Since only on-axis (tension) and shear stresses are applicable in their case, the equation used included only the on-axis and shear term.

## 2.5.3 Ellyin & El-Kadi

Ellyin and El-Kadi [110] suggested a different failure criterion based on strain energy density under cyclic loading. The latter, for the case of plane stresses, is expressed through the following relationship, where  $\bar{S}_{ij}$  are the transformed compliances of the unidirectional orthotropic lamina and  $\Delta$  in front of stress terms implies fatigue stress range:

$$\begin{aligned} \Delta W = & \bar{S}_{11} \left[ \frac{\Delta \sigma_x^2}{2(1-R_x)^2} \right] + \bar{S}_{22} \left[ \frac{\Delta \sigma_y^2}{2(1-R_y)^2} \right] + \bar{S}_{66} \left[ \frac{\Delta \tau_{xy}^2}{2(1-R_s)^2} \right] \\ & + \bar{S}_{12} \left[ \frac{\Delta \sigma_x \Delta \sigma_y}{(1-R_x)(1-R_y)} \right] + \bar{S}_{16} \left[ \frac{\Delta \sigma_x \Delta \tau_{xy}}{(1-R_x)(1-R_s)} \right] + \bar{S}_{26} \left[ \frac{\Delta \sigma_y \Delta \tau_{xy}}{(1-R_y)(1-R_s)} \right] \end{aligned} \quad (55)$$

According to Ellyin and El-Kadi, the strain energy density can be expressed as a function of the fatigue life under the plane cyclic stresses considered above, through the following power type relation:

$$\Delta W = kN^a \quad (56)$$

$k$  and  $a$  are fitted parameters depending in their case on the fibre orientation angle  $\theta$ . The criterion was evaluated using the experimental data of Hashin and Rotem [108]. The following expressions appeared to correlate satisfactorily with the experimental observations, where  $a$ ,  $\beta$  and  $b$  are constant parameters and  $k_0$  and  $a_0$  are the values obtained for  $0^\circ$  fibre orientation:

$$\begin{aligned}\alpha &= \alpha_0 + a\theta \\ \log(k) &= \log(k_0) + b\theta^\beta\end{aligned}\tag{57}$$

## 2.5.4 Plumtree & Cheng

Plumtree and Cheng [111] proposed another failure criterion based on energy density for predicting the fatigue behaviour of off-axis UD laminates, in a context similar to Ellyin & El-Kadi. They use the Smith Watson Topper parameter assuming that fatigue of off-axis UD is dominated by transverse matrix cracks. The proposed damage parameter is resembling to strain energy density:

$$\Delta W = \sigma_{22}^{\max} \Delta \varepsilon_{22} + \tau_{12}^{\max} \Delta \gamma_{12} / 2\tag{58}$$

Where  $\Delta$  indicates range of strains and the superscript max indicates maximum cyclic stress. A log-log curve is used to fit the damage parameter as a function of the number of cycles to failure, based on off-axis CA fatigue data. This master curve is subsequently used for predicting fatigue life of other UD coupons of different off-axis angle.

## 2.5.5 Reifneider & Gao

Reifneider and Gao [112] developed and implemented a micromechanical methodology for life prediction under complex stresses using the formulation of Hashin and Rotem discussed above. The stresses used however are calculated using the Mori-Tanaka method [113] which deals with the problem of the stress field around inhomogeneities (i.e. fibres in this case) located inside a matrix. The failure criterion is:

$$\begin{aligned}\langle \sigma_{11}^f \rangle &= X^f \\ \left( \frac{\langle \sigma_{22}^m \rangle}{X^m} \right)^2 + \left( \frac{\langle \sigma_{12}^m \rangle}{S^m} \right)^2 &= 1\end{aligned}\tag{59}$$

The denominators are fatigue failure functions of the fibre and unreinforced matrix material while the terms in brackets refer to the average stresses calculated with the Mori Tanaka method.

## 2.5.6 Fawaz & Ellyin

Fawaz and Ellyin propose in [114] a simplified approach for determining off axis S-N curves, based on a reference S-N curve and the off-axis static strength. As pointed out also by Awerbuch and Hahn [115] off axis S-N curves seem to follow a common trend when the data are normalized by the corresponding static strength. The final form of the model is:

$$\sigma_{(\alpha_1, \alpha_2, \theta, R, N)} = f_{(\alpha_1, \alpha_2, \theta)} \left[ g_{(R)} m_r \log(N) + b_r \right] \quad (60)$$

$\alpha_1$  and  $\alpha_2$  are the biaxiality stress ratio  $\sigma_y / \sigma_x$  and  $\tau_{xy} / \sigma_x$  respectively,  $\theta$  is the off-axis angle,  $R$  is the stress ratio and  $m_r$ ,  $b_r$  are the parameters of the reference S-N curve. The function  $g$  is introduced to account for different stress ratios, and function  $f$  is defined as the ratio of the static strength along the considered direction (under the specific biaxiality ratio) versus the static strength along the same direction under the reference loading parameters:

$$g_{(R)} = \sigma_{\max} \frac{1-R}{\sigma_{(\max)r} - \sigma_{(\min)r}} \quad f_{(\alpha_1, \alpha_2, \theta)} = \frac{\sigma_{x(\alpha_1, \alpha_2, \theta)}}{X_r} \quad (61)$$

$\sigma_{x(\alpha_1, \alpha_2, \theta)}$  is obtained by substituting the principal stresses (as a function of the applied cyclic stress and off-axis angle) to an adequate failure criterion.

The model is very simple and requires minimal experimental effort, nevertheless it is highly sensitive on the choice of the reference S-N curve.

## 2.5.7 Fujii & Lin

Fujii and Lin [116] implemented the failure criterion of Tsai and Wu to model the fatigue failure locus under plane stresses of different biaxiality ratios, induced through cyclic tests on tubular specimens (combined tension and torsion). The simplified form they used was:

$$\left( \frac{1}{X} - \frac{1}{X'} \right) \sigma_1 + \left( \frac{1}{XX'} \right) \sigma_1^2 + \left( \frac{1}{S^2} \right) \tau_{12}^2 = 1 \quad (62)$$

$X$  and  $X'$  are the tensile and compressive fatigue strength respectively.  $X$  is calculated from the experimental curve they derive by normalizing the maximum cyclic stresses with the corresponding static strength (under the same biaxiality ratio). However, a generic definition of compressive fatigue strength  $X'$  is not proposed. On the contrary  $X'$  is rather used as a free parameter in order fit the failure criterion to the experimental fatigue data.

## 2.5.8 Philippidis & Vassilopoulos

Philippidis and Vassilopoulos [117] developed their fatigue failure criterion named 'Failure Tensor Polynomial in Fatigue', based on the Tsai-Hahn equation [118]:

$$\left(\frac{1}{XX'}\right)\sigma_1^2 + \left(\frac{1}{YY'}\right)\sigma_2^2 + \left(\frac{1}{X} - \frac{1}{X'}\right)\sigma_1 + \left(\frac{1}{Y} - \frac{1}{Y'}\right)\sigma_2 + \left(\frac{1}{S^2}\right)\tau_{12}^2 - \sqrt{\frac{1}{XX'}\frac{1}{YY'}}\sigma_1\sigma_2 = 1 \quad (63)$$

The static strength terms initially considered by Tsai-Hahn are replaced directly by the material's fatigue strength, described by the S-N curves of the considered material at 0°, 90° and 45° the latter divided by 2 to simulate the fatigue behaviour under shear. Equal fatigue strengths are assumed under tension and compression. The final form of the proposed equation is:

$$\frac{\sigma_1^2}{X_{(N,R)}^2} + \frac{\sigma_2^2}{Y_{(N,R)}^2} + \frac{\tau_{12}^2}{S_{(N,R)}^2} - \frac{\sigma_1\sigma_2}{X_{(N,R)}Y_{(N,R)}} = 1 \quad (64)$$

The model was later implemented in predictions under spectrum fatigue [39] using a standardized spectrum as well as an irregular one obtained from aero-elastic simulation of a WT blade. A linear CLD based on three S-N curves was implemented to derive the S-N curves at different R ratios and the Palmgren-Miner rule was used as damage accumulation metric.

## 2.5.9 Aboudi

Aboudi proposed in [119] an extension of a previously presented micromechanical model for static strength of composites [120] under plane stresses. The latter is based on a representative unit cell representing a simplified model of the fibre and surrounding matrix area. The stress state in the fibre and matrix areas is defined based on the properties of the constituents and the necessary correlation between the strength quantities of the constituents and those macroscopically observed is attempted through a stress concentration matrix **B**.

The fatigue failure criterion proposed is based on the one proposed by Hashin and Rotem distinguishing between the fibre and matrix failure modes:

$$\begin{aligned} S_{11}^{(1)} &= X_f^{(F)} && \text{(fibre)} \\ \left( \left( \frac{S_{\beta\gamma}^{(\beta\gamma)}}{X_m^{(F)}} \right)^2 + \left( \frac{S_{\beta\gamma}^{(\beta\gamma)}}{S_m^{(F)}} \right)^2 \right) &= 1 && \text{(Matrix)} \end{aligned} \quad (65)$$

Where  $\beta\gamma=12, 21, 22$  are the three matrix areas of the representative volume. The  $X_f^{(F)}, X_m^{(F)}$  and  $S_m^{(F)}$  are fatigue failure functions of the fibre and matrix materials, depending on all fatigue parameters such as stress ratio, frequency and number of cycles. Since fatigue testing e.g. on fibre is difficult to perform, the respective fibre, transverse and shear properties of the UD ply are used, transformed through the (undefined) stress concentration matrix. The theory is implemented to UD and angle ply laminates of different types of composite

systems with relative success under simple loading conditions. Nevertheless, the generalization to varying loads is not considered.

## 2.6 Lamina to Laminate Approaches

### 2.6.1 Hashin and Rotem

The fatigue failure model initially proposed by Hashin and Rotem for off axis laminates [108] and further developed to account for fatigue failure of angle ply laminates [121] was later extended to the general condition of multidirectional laminates [122]. Classical lamination theory is implemented to derive all in plane stresses in each lamina, rotated to its principal coordinate system. Three failure modes are distinguished: One for fibre failure, one for matrix failure and one for delaminations. The criteria are formulated as follows:

$$\begin{aligned}
 \sigma_A^c &= \sigma_A^s f_A && \text{(fibre)} \\
 \left( \left( \frac{\sigma_T^c}{\sigma_T^s f_T} \right)^2 + \left( \frac{\tau^c}{\tau^s f_\tau} \right)^2 \right) &= 1 && \text{(Matrix)} \\
 \left( \left( \frac{\sigma_d^c}{\sigma_d^s f_d} \right)^2 + \left( \frac{\tau_d^c}{\tau_d^s f_d} \right)^2 \right) &= 1 && \text{(Delamination)}
 \end{aligned} \tag{66}$$

Subscripts A, T and  $\tau$  refer to fibre direction, transverse direction and to shear respectively. Superscript c denotes cyclic stresses, s static strength and subscript d refers to inter-laminar normal or shear stress and strength components.  $f_A$  is a fatigue function under fibre failure mode, depending on fatigue life N, stress ratio R and testing frequency, determined experimentally from cyclic tests in the fibre direction of the unidirectional material.  $f_T$  and  $f_\tau$  are the corresponding fatigue functions transversely to the fibre and in shear define through fatigue testing of the material in the transverse direction and under shear fatigue.

For the simple case of uniaxial load applied on a laminate, the methodology results in simplified expressions, relating the principal stresses in each ply to the stress developing in the ply expressed in the global coordinate system ( ${}_p\sigma_x^c$ ):

$$\begin{aligned}
 {}_p\sigma_A^c &= {}_p k_{xx} \cdot {}_p\sigma_x^c \\
 {}_p\sigma_T^c &= {}_p k_{yy} \cdot {}_p\sigma_x^c \\
 {}_p\tau^c &= {}_p k_{xy} \cdot {}_p\sigma_x^c
 \end{aligned} \tag{67}$$

The expressions  ${}_p k_{ij}$  are functions defined in [121] depending on the laminate's stiffness matrix, the layer's elastic properties and its orientation angle. Simplified expressions are derived for the fatigue failure criteria. The inter-laminar shear stress is calculated by the formula proposed by Puppo and Evensen in [123] while the corresponding strength quantities, including the corresponding fatigue function are defined through tests on  $\pm 15$  angle-ply laminates. The results show

a close correlation between inter-laminar shear to the shear fatigue properties of the lamina. Normal tension between layers can be neglected.

Failure events are treated as causes of stress redistributions. The consequent change in the complex stress field of each ply is re-evaluated through CLT, introducing another stage of fatigue after each failure. The model was implemented for the case CA tests on symmetrically balanced E-glass/Epoxy laminates with acceptable results.

### 2.6.2 Lawrence Wu

Lawrence Wu [124] proposed a modified version of the Tsai-Hill failure criterion for triaxial stress states in order to simulate fatigue of CFRP in finite elements. Its form is:

$$f^2 = \frac{3}{2(F+G+H)} \left( F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\sigma_{yz}^2 + 2M\sigma_{zx}^2 + 2N\sigma_{xy}^2 \right) \quad (68)$$

$\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \sigma_{zx}, \sigma_{yz}$  are cyclic stresses in the lamina and F, G, H, L, M, N are:

$$\begin{aligned} F &= \frac{1}{2} \left( \frac{1}{Y^2} + \frac{1}{Z^2} + \frac{1}{X^2} \right) & L &= \frac{1}{2Q^2} \\ G &= \frac{1}{2} \left( \frac{1}{Z^2} + \frac{1}{X^2} + \frac{1}{Y^2} \right) & M &= \frac{1}{2R^2} \\ H &= \frac{1}{2} \left( \frac{1}{X^2} + \frac{1}{Y^2} + \frac{1}{Z^2} \right) & N &= \frac{1}{2S^2} \end{aligned} \quad (69)$$

X, Y, Z, Q, R and S are normal and shear strength components of the material, assumed to be functions of the fatigue life.

Once the stresses are calculated and the above quantities defined as a function of cycles, the failure criterion can be arithmetically solved for life N. Final failure is assumed to occur when the failure criterion reaches unity. Even though the proposed methodology claims to predict life under CA and 3D fatigue stresses it neglects stress redistributions due to damage accumulation and assumes an FPF, which is usually extremely conservative.

### 2.6.3 Fawaz and Ellyin

The concept of Fawaz & Ellyin is generalized for multidirectional laminates under constant amplitude fatigue in [125]. The concept is an extension of their previously discussed approach for defining a generalized S-N expression for biaxial fatigue based on a reference S-N curve. The reference S-N curve this time (parameters m and b) refers to the constitutive lamina being constrained between off-axis plies while f is defined as previously as a function of the biaxiality ratios  $a_1, a_2$  through an appropriate static stress failure criterion:

$$\sigma(\alpha_1, \alpha_2, N) = f(\alpha_1, \alpha_2) [m \cdot \log(N) + b] \quad (70)$$

Using CLT the stresses in individual laminae are computed along with the corresponding biaxiality ratios. Using the above equation, the number of cycles to failure can be computed for each layer. The shorter life indicates the first ply failure after  $N_1^{0_1}$  cycles. Failure degradation is modelled at this point by neglecting its properties only in the direction normal to failure. The stress state is then altered accordingly for each ply (using again CLT) and new governing S-N equations are established. The number of cycles to failure of the laminate can be calculated from:

$$N = N_1^{0_1} + \sum_{i=2}^m n_i^{0_i} \quad (71)$$

$n_i^{0_i}$  is the remaining life of the  $i^{\text{th}}$  ply that fails and can be given by the following expression:

$$n_m^{0_m} = \left[ N_m - N_m \left\{ \sum_{i=1}^{m-3} \left( \left( \frac{n_i}{N_i} \right)^{\alpha_i} + \frac{n_{i+1}}{N_{i+1}} \right)^{\alpha_{i+1}} \right\} + \frac{n_{i+2}}{N_{i+2}} \right]^{\alpha_{m-1}} \quad (72)$$

The latter can be considered as a cumulative damage model taking into account also sequence effects through the non-linear exponents  $\alpha_i$ . Each  $N_i$  is the fatigue life that corresponds to the  $m^{\text{th}}$  lamina under the  $(i-1)^{\text{th}}$  cyclic stress applied on it and  $n_i$  is the number of cycles applied during the fatigue stage  $i$ .

It must be noted that no procedure for determining the exponents  $\alpha_i$  is proposed, while the extension of the model to VA cyclic loads would be challenging.

#### 2.6.4 Jen & Lee

The methodology developed by Jen and Lee [126], [127], is a CLT based approach using the Tsai-Hill failure criterion extended in order to account for failure under plane fatigue stresses. The proposed formulation is:

$$\left( \frac{\sigma_{11}}{\bar{\sigma}_{11}} \right)^2 + \left( \frac{\sigma_{22}}{\bar{\sigma}_{22}} \right)^2 - \left( \frac{\sigma_{11}\sigma_{22}}{\bar{\sigma}_{11}^2} \right) + \left( \frac{\sigma_{12}}{\bar{\sigma}_{12}} \right)^2 = 1 \quad (73)$$

The terms in the denominators are fatigue strength quantities in the principal directions and in plane shear assumed to be functions of the corresponding number of cycles and stress ratio, while frequency dependence is neglected. Different functions are implemented for each principal direction of the basic orthotropic ply depending on whether fatigue is in the tension or compression dominated regime. Their derivation is based on fatigue testing of the UD material at  $0^\circ$ ,  $90^\circ$  and  $45^\circ$  (for shear) at various stress ratios, even though no specific methodology (e.g. CLD assumption) is proposed. Consequently, the criterion is

divided into four parts according to the four possible combinations or the fatigue strength functions:

$$\begin{aligned}
\left(\frac{\sigma_{11}}{L_{(N,R_{11})}}\right)^2 + \left(\frac{\sigma_{22}}{T_{(N,R_{22})}}\right)^2 - \left(\frac{\sigma_{11}\sigma_{22}}{L_{(N,R_{11})}^2}\right) + \left(\frac{\sigma_{12}}{\tau_{(N,R_{12})}}\right)^2 &= 1 & |R_{11}| \leq 1 & \quad |R_{22}| \leq 1 \\
\left(\frac{\sigma_{11}}{I_{(N,R_{11})}}\right)^2 + \left(\frac{\sigma_{22}}{T_{(N,R_{22})}}\right)^2 - \left(\frac{\sigma_{11}\sigma_{22}}{I_{(N,R_{22})}^2}\right) + \left(\frac{\sigma_{12}}{\tau_{(N,R_{12})}}\right)^2 &= 1 & |R_{11}| \geq 1 & \quad |R_{22}| \leq 1 \\
\left(\frac{\sigma_{11}}{I_{(N,R_{11})}}\right)^2 + \left(\frac{\sigma_{22}}{t_{(N,R_{22})}}\right)^2 - \left(\frac{\sigma_{11}\sigma_{22}}{I_{(N,R_{11})}^2}\right) + \left(\frac{\sigma_{12}}{\tau_{(N,R_{12})}}\right)^2 &= 1 & |R_{11}| \geq 1 & \quad |R_{22}| \geq 1 \\
\left(\frac{\sigma_{11}}{L_{(N,R_{11})}}\right)^2 + \left(\frac{\sigma_{22}}{t_{(N,R_{22})}}\right)^2 - \left(\frac{\sigma_{11}\sigma_{22}}{L_{(N,R_{11})}^2}\right) + \left(\frac{\sigma_{12}}{\tau_{(N,R_{12})}}\right)^2 &= 1 & |R_{11}| \leq 1 & \quad |R_{22}| \geq 1
\end{aligned} \tag{74}$$

Classical lamination theory (CLT) is implemented to derive the stresses of each layer in its principal coordinate system. The failure functions having been defined, the failure locus for specific fatigue lives is drawn in the  $\sigma_{11}$ - $\sigma_{22}$  plane and the corresponding fatigue life under the combined stresses is calculated using linear interpolation between the different loci.

Fatigue damage is assumed to accumulate according to the linear Palmgren-Miner rule for each ply. In the case of failure of a ply, the stiffness reduction policy followed assumes zeroing of all stiffness components except from the fibre stiffness which remains unchanged.

The predictions of the model of Jen and Lee have been verified against CA fatigue test results on quasi-isotropic  $[0/45/90/-45]_{2S}$ , cross-ply  $[0/90]_{2S}$  and angle-ply  $[\pm 45]_{2S}$  Carbon/PEEK laminates, producing satisfactory predictions in the case of fibre dominated laminates and conservative predictions for the angle-ply laminate.

## 2.6.5 Shokrieh & Lessard

Shokrieh and Lessard proposed the 'generalized progressive fatigue damage model' for the simulation of the fatigue process in multidirectional laminates under general (3-D) loading conditions [12] using the properties of the constitutive UD ply. The model is based on a previously proposed methodology for UD laminates under multiaxial fatigue [128].

First, CLT is implemented to perform stress analysis of the multidirectional laminate under the current cyclic load. When the stresses and stress ratios at each principal direction of each lamina have been defined, the bell-shape CLD proposed by Gathercole et al. [40] is used to derive the number of cycles to failure corresponding to each principal stress component. Having calculated each cyclic stress and fatigue life they proceed by calculating the residual strength corresponding to the fibre, transverse and shear strength of each layer according to the interaction model of Adam et al. [74]. The stiffness characteristics of each

ply are assumed to change due to fatigue following a similar model as that of strength degradation. Subsequently, Hashin type failure criteria are used in order to distinguish between different modes of failure, where the strength is replaced by the residual strength previously calculated. For fibre fatigue failure in tension, the following equation is applicable:

$$\left\{ \left( \frac{\sigma_{xx}}{X_t} \right)^2 + \left( \frac{\frac{\sigma_{xy}}{2E_{xy}} + \frac{3}{4}\delta\sigma_{xy}^4}{\frac{S_{xy}^2}{2E_{xy}} + \frac{3}{4}\delta S_{xy}^4} \right)^2 + \left( \frac{\frac{\sigma_{xz}}{2E_{xz}} + \frac{3}{4}\delta\sigma_{xz}^4}{\frac{S_{xz}^2}{2E_{xz}} + \frac{3}{4}\delta S_{xz}^4} \right)^2 \right\}^{\frac{1}{2}} = 1 \quad (75)$$

$\sigma_{xx}$ ,  $\sigma_{xy}$ ,  $\sigma_{xz}$ , are the cyclic normal stress in the fiber direction, in-plane shear and out-of-plane shear respectively.  $X_t$  is the on-axis residual tensile strength of the UD ply,  $E_{xy}$  and  $E_{xz}$  are the residual shear moduli (in-plane and out-of-plane).  $S_{xy}$ ,  $S_{xz}$  are the corresponding residual shear strengths. Finally, parameter  $\delta$  is a constant, expressing the non-linear impact of the shear components on tensile fiber failure. Similar criteria are assumed for fatigue failure under fiber compression ( $\sigma_{xx} < 0$ ), fiber-matrix shearing ( $\sigma_{xx} < 0$ ), matrix tension, ( $\sigma_{yy} > 0$ ), matrix compression ( $\sigma_{yy} < 0$ ), normal tension ( $\sigma_{zz} > 0$ ) and normal compression ( $\sigma_{zz} < 0$ ).

Whenever fatigue failure according one of these failure modes occurs, sudden degradation rules are imposed on the elastic and strength components of the corresponding ply. The rules for this degradation are summarized in the following table:

Table 2 Sudden degradation rules according to failure mode for the model of Shokrieh & Lessard

Failure Mode	Strength Matrix									Stiffness Matrix												
	$X_t$	$X_c$	$Y_t$	$Y_c$	$Z_t$	$Z_c$	$S_{xy}$	$S_{xz}$	$S_{yz}$	$E_{xx}$	$E_{yy}$	$E_{zz}$	$E_{xy}$	$E_{xz}$	$E_{yz}$	$\nu_{xy}$	$\nu_{xz}$	$\nu_{yz}$	$\nu_{yx}$	$\nu_{zx}$	$\nu_{zy}$	
Fiber Tension	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Fiber Compression	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Matrix Tension			0								0		0						0	0		
Matrix Compression				0							0								0	0		
Fiber-Matrix Shearing							0						0			0				0		
Normal Tension					0							0									0	0
Normal Compression						0															0	0

This modeling approach incorporates in a phenomenological way the effect of some important mechanisms as discussed by other authors, e.g. Reifsnider, on a more mechanistic base. These are mainly the strength and stiffness degradation of the plies due to fatigue, the failure degradation imposed and the distinction between different failure modes of fatigue failure. Stiffness changes are causing stress redistributions between plies, while residual strength provides both a damage accumulation metric and a fatigue failure parameter through its direct application on the failure criteria.

The required experiments for material characterization include static tests on the UD ply to define all its stiffness/static strength components as well as fatigue

tests (monitoring the stiffness degradation) to derive the CLDs in the fiber direction, transversely to the fiber and in shear, both in-plane and out-of-plane. Finally, residual strength tests are necessary in all principal directions of the basic ply, in order to fit the corresponding parameters. Simplified expressions and reduced experimental effort is required if plane stress fields are assumed.

The generalized progressive fatigue damage model is implemented in FEM code and verified against the experimental tests on pin/bolt loaded, Graphite/Epoxy multidirectional composite laminates of Herrington & Sabbaghian [129] with satisfactory predictions (see [130]).

## 2.6.6 Tserpes et al

Tserpes et al. [131] developed another 3-D progressive damage model in a context similar to that of Shokrieh and Lessard. The failure analysis adopted in this case assumes quadratic failure functions for matrix tensile and compressive cracking, fibre-matrix shear as well as for two modes of delamination (in tension and compression), the later related to the normal stress component and out-of plane shear stresses. The sudden degradation rules, due to the different failure modes, are imposed only on the stiffness properties of the corresponding ply as a means of stress redistribution after failure. In contrast to the model of Shokrieh considering zeroing of both strength and elastic properties, in this case they are assumed to degrade according to constant degradation factors, which are summarized in Table 3.

Table 3 Sudden degradation rules according to failure mode for the model of Tserpes et al.

Failure Mode	Stiffness Matrix											
	$E_{xx}$	$E_{yy}$	$E_{zz}$	$E_{xy}$	$E_{xz}$	$E_{yz}$	$\nu_{xy}$	$\nu_{xz}$	$\nu_{yz}$	$\nu_{yx}$	$\nu_{zx}$	$\nu_{zy}$
Fiber Tension	0,07											
Fiber Compression	0,14											
Matrix Tension		0,2		0,2		0,2						
Matrix Compression		0,4		0,4		0,4						
Fiber-Matrix Shearing				0								
Delamination Tens.-Compr.			0		0	0						

For modelling fatigue modulus degradation in each direction of the constitutive ply, a linear function of the life fraction is assumed.  $N_{fij}$  is the fatigue life in the  $ij$  direction,  $E_{ij}^S$  is the corresponding static modulus and  $A$  is a fitted parameter:

$$E_{ij}^F(n) = \left[ A \left( \frac{n}{N_{fij}} \right) \right] E_{ij}^S \quad (76)$$

Residual strength is assumed to degrade following a second order polynomial, in which  $T_{ij}^S$  and  $T_{ij}^F$  refers to static and residual strength respectively.  $B$  and  $C$  are fitted parameters.

$$T_{ij}^F(n) = \left[ B \left( \frac{n}{N_{fi}} \right)^2 + C \left( \frac{n}{N_{fi}} \right) \right] T_{ij}^S \quad (77)$$

The methodology has been implemented in FEM code and validated satisfactorily against experimental data from multidirectional laminates.

## 2.6.7 Himmel

Himmel introduced in [132] an engineering lamina to laminate methodology based on the 'critical element' philosophy to predict the fatigue behaviour of tubular  $\pm 45$  specimens under pure torsion cyclic loads.

The S-N formulation adopted is in the following form, where  $\alpha_0$ ,  $\beta$ ,  $\sigma_E$  and  $n_0$  are constant parameters fitted on the fatigue data:

$$\log(N + n_0) = \alpha_0 - \beta \log(\sigma_{\max} - \sigma_E) \quad (78)$$

In order to generalize the fatigue life estimation to arbitrary stress ratio, the Bell shape CLD of Harris initially proposed was finally rejected due to its numerical intensity. Instead a linear Goodman formulation based on the R=0.4 S-N curve was finally implemented.

The stiffness degradation model assumes linear dependence on the number of cycles n:

$$\frac{E_2(n)}{E_2} = 1 - \Delta E_2 \frac{n}{N} \quad \text{where} \quad \Delta E_2 = d(\sigma_{\max})^b \quad (79)$$

$\Delta E_2$  is the transverse stiffness degradation at failure, assumed to be a non-linear function of the applied external stress (torsional shear stress in their case). d and b are constants.

Subsequently, a plane stress fatigue failure criterion must be used. Himmel chooses the following quadratic Tsai/Wu type formulation, where the denominators denote residual strength components in the corresponding principal direction of the ply and  $a_{12}$  is a fatigue life function ideally defined through biaxial tests on UD laminates. Practically however,  $\sigma_{1,zf(N=1)}$  and  $\sigma_{2,zf(N=1)}$  are taken as the corresponding stress components on the UD layer calculated from static strength tests on the MD laminate.

$$\left( \frac{\sigma_1}{\sigma_{1,zf(N)}} \right)^2 + \left( \frac{\sigma_2}{\sigma_{2,zf(N)}} \right)^2 + \frac{\alpha_{12(N=1)}}{\sigma_{1,zf(N)} \sigma_{2,zf(N)}} \sigma_1 \sigma_2 \frac{\tau_{xy}^2}{S_{(N)}^2} + \left( \frac{\tau_{12}}{\tau_{12,zf(N)}} \right)^2 \leq 1 \quad (80)$$

Finally, the non-linear residual strength equation proposed by Reifsnider for the critical element is adopted, in order to model damage accumulation under varying cyclic stresses.

## 2.6.8 Passipoularidis & Philippidis

Another formulation following the same concept of progressive damage fatigue modelling has been proposed by Passipoularidis & Philippidis in [133].

Stiffness degradation due to fatigue is assumed to be described by the following expression, which applies with different fitting parameters  $k$ ,  $\lambda$  to all moduli of each ply in the laminate:

$$\frac{E_i(n)}{E_{i0}} = 1 - (1 - k) \left( \frac{n}{N} \right)^\lambda \quad (81)$$

Strength is degraded according to the linear strength degradation model of Broutman and Sahu, since the impact of choosing between different strength degradation formulations was previously shown to be limited [36] while significant cost is saved from residual strength characterization of the UD lamina.

The failure criterion adopted is based on the static failure criterion of Puck [134], [135], [136] and accounts for 5 different damage modes: The first two refer to fibre failure in tension and compression:

$$f_{E(\text{FF})}^T = \frac{1}{X_T} \left[ \sigma_1 + \left( \frac{E_1}{E_{f1}} v_{f12} m_{\text{of}} - v_{12} \right) \sigma_2 \right] \leq 1 \quad (82)$$

$$f_{E(\text{FF})}^C = \frac{1}{X_C} \left| \sigma_1 + \left( \frac{E_1}{E_{f1}} v_{f12} m_{\text{of}} - v_{12} \right) \sigma_2 \right| + (10\varepsilon_6)^2 \leq 1 \quad (83)$$

Regarding matrix -or inter fibre- failure (IFF), three distinct cases are taken into account by the failure criterion. The first one is mode A described by eq.(3) and is caused by tensile stresses in the transverse direction. The other two refer to compressive stresses in the transverse direction and the first one, called mode B (eq.(85)) initiates for relatively high values of in-plane shear and results in matrix cracking transversely to the normal stress direction that tends to close. When transverse compressive stress increases with respect to the shear stress the failure mode changes to a more damaging one, called mode C described by eq.(86), which causes cracks at a plane that is not perpendicular to the one defined by the in plane stresses. This explosive failure mode could lead to delaminations and/or local buckling.

$$f_{E(\text{IFF})}^A = \sqrt{\left( \frac{\sigma_6}{S} \right)^2 + \left( 1 - p_{\perp\text{II}}^{(+)} \frac{Y_T}{S} \right)^2 \left( \frac{\sigma_2}{Y_T} \right)^2} + p_{\perp\text{II}}^{(+)} \frac{\sigma_2}{S} + \left| \frac{\sigma_1}{\sigma_{1D}} \right| \leq 1 \quad (84)$$

$$f_{E(IFF)}^B = \frac{1}{S} \left( \sqrt{(\sigma_6^2) + (p_{\perp\perp}^{(-)} \sigma_2)^2} + p_{\perp\perp}^{(-)} \sigma_2 \right) + \left| \frac{\sigma_1}{\sigma_{1D}} \right|^6 \leq 1 \quad (85)$$

$$f_{E(IFF)}^C = \left[ \left( \frac{\sigma_6}{2(1 + p_{\perp\perp}^{(-)})S} \right)^2 + \left( \frac{\sigma_2}{Y_C} \right)^2 \right] \left[ \left( \frac{Y_C}{(-\sigma_2)} \right) + \left| \frac{\sigma_1}{\sigma_{1D}} \right|^6 \right] \leq 1 \quad (86)$$

$X_T$ ,  $X_C$ ,  $Y_T$ ,  $Y_C$  stand for the on-axis and transverse tensile and compressive strength respectively.  $E_1$  and  $\nu_{12}$  are the on-axis Young modulus and Poisson ratio of the UD ply while  $E_{f1}$  and  $\nu_{f12}$  are the respective quantities of the fibre. The term  $m_{of}$  accounts for a stress magnification effect caused by the difference between the moduli of fibre and matrix.  $S$  is the in-plane shear strength of the ply. The term  $\sigma_1/\sigma_{1D}$  accounts for matrix damage due to statistical fibre breakage before  $\sigma_1$  reaches its ultimate  $X_T$ , while factors  $p_{\perp\perp}^{(+)}$  and  $p_{\perp\perp}^{(-)}$  represent the slopes of the failure locus ( $\sigma_2$ ,  $\sigma_6$ ) at  $\sigma_2=0^+$  and  $\sigma_2=0^-$  respectively. Finally, parameter  $p_{\perp\perp}^{(-)}$  stands for the inclination of the above mentioned failure locus at zero transverse stress.

At inter-fibre failure of a ply, sudden degradation is imposed using a degradation coefficient, being a function of the inter-fibre failure effort (i.e. the value of the corresponding failure criterion):

$$\eta = \frac{1 - \eta_r}{1 + c(f_{E(IFF)} - 1)^{\xi}} + \eta_r \quad (87)$$

The discount policy according to each damage mode is resumed in the Table 4.

Table 4 Discount policy due to failure assumed by Passipoularidis & Philippidis

Failure Mode	Degradation Imposed
IFF(A)	$E_2 = \eta E_2$ $G_{12} = \eta G_{12}$
IFF(B)	$G_{12} = \eta G_{12}$
IFF(C)	$E_2 = 0.1 E_2$ $G_{12} = 0.1 G_{12}$

The model was implemented to CA fatigue data in on-axis and off-axis MD Glass/Epoxy laminates with satisfactory results.

## 2.6.9 Dzenis

Dzenis suggests in [137] a stochastic meso-mechanics model. Each ply of the multidirectional laminate is attributed stochastic (normally distributed) in-plane elastic characteristics. Based on these and on the randomly distributed thickness and orientation angle of each ply, the stochastic effective properties of the laminate can be calculated. Then, the externally applied plane stress field, assumed to be a quasi-stationary cyclic process, can be used to calculate the

mathematical expectations ( $\bar{\varepsilon}_i, \bar{\dot{\varepsilon}}_i$ ) and dispersions ( $D_{\varepsilon_i}, D_{\dot{\varepsilon}_i}$ ) of the laminate's strains and strain derivatives:

$$\begin{aligned}
 \bar{\varepsilon}_i(t) &= \int_0^t \bar{S}_{ij}(\tau) \bar{\sigma}_j(\tau) d\tau \\
 \bar{\dot{\varepsilon}}_i(t) &= \bar{S}_{ij}(t) \bar{\sigma}_j(t) \\
 D_{\varepsilon_i} &= \bar{S}_{ij}^2(t) K_{\sigma_j}(0) + \bar{\sigma}_j^2(t) D_{S_{ij}} \\
 D_{\dot{\varepsilon}_i} &= \bar{S}_{ij}^2(t) K_{\sigma_j}(0) + (\bar{\sigma}_j(t))^2 D_{S_{ij}}
 \end{aligned} \tag{88}$$

$K$  are the autocorrelation functions and  $\bar{S}_{ij}$  are current laminate compliances. Stresses and strains in the plies are subsequently calculated using CLT. Once the stochastic stress-strain field of each ply is calculated, the model proceeds with the calculation of the probability of failure, given by:

$$p(t) = 1 - \exp\left(-\int_0^t v(\tau) d\tau\right) \tag{89}$$

Where  $v(\tau)$  is the mathematical expectation of the mean number of excursions of the failure condition ( $\Xi$ ) by the value of the failure criterion used ( $\xi$ ) per unit time.

$$\begin{aligned}
 v(t) &= \frac{1}{2\pi} \sqrt{\frac{D_{\xi}}{D_{\xi} + D_{\Xi}}} \exp\left(-\frac{(\bar{\xi}(t) - \bar{\Xi}(t))^2}{2(D_{\xi} + D_{\Xi})}\right) \left(-\int_0^t v(\tau) d\tau\right) \\
 &\cdot \left\{ \exp\left(-\frac{\bar{\xi}(t)^2}{2D_{\xi}}\right) + \sqrt{\frac{2\pi \bar{\xi}(t)}{D_{\xi}}} \left(1/\Phi\left(-\frac{\bar{\xi}(t)}{\sqrt{D_{\xi}}}\right)\right) \right\}
 \end{aligned} \tag{90}$$

$\Phi$  is the Laplace function. Both  $\xi$  and  $\Xi$  are assumed to be stochastic processes described by their mathematical expectations  $\bar{\xi}, \bar{\Xi}$  and their dispersions  $D_{\xi}, D_{\Xi}$ . Three failure criteria are proposed, maximum stress, maximum strain and the Tsai-Hill failure function.

The above concept is applied on a mesovolume scale, i.e. to a structurally homogeneous part of the ply, small enough to satisfy the condition of stochastic homogeneity of the stress and strain fields. Stiffness degradation in each ply is then evaluated through the assumption that the relative fraction of broken mesovolumes is proportional to the probability of failure  $p(t)$  for each ply, introduced this way as a damage function.

The model uses explicit integration over each cycles thus claiming to account for effects such as the shape of the cycle, frequency etc. Theoretical predictions are produced for simple CA cyclic loads.

## 2.7 Counting Algorithms

Cycle counting is the first procedure any loading spectrum must undergo in order to be analyzed and processed in terms of constant amplitude fatigue cycles. Several methods have been proposed for counting the cycles of a spectrum. A review of such methods can be found in [138].

*Level Crossing Counting.* According to this method the load axis is divided into a number of preset stress levels and one counting is recorded each time the load exceeds one. When all level crossings have been recorded, cycles can be formed by constructing first the largest possible cycle, then the second largest etc until all level crossings have been used. This way the spectral loading is transformed into a series of decreasing amplitude cycles. Of course, once this -most damaging- counting scenario has been obtained, the cycles can be rearranged in any desired order inducing secondary load-sequence effects.

*Peak Counting.* As in the above case, the procedure focuses on the identification of peaks and troughs of the considered spectrum. Once all these have been obtained, the first cycle is constructed by combining the highest peak with the lower trough etc, until all peaks have been used.

Both peak and level crossing counting methods yield most damaging cycle counting results, since they focus on the construction of the largest cycles possibly obtained from a specific spectrum. Further on, they totally neglect the order of occurrence of each loading event and thus load sequence effects, which depending on the shape of the spectrum could have a considerable effect, cannot be taken into account.

*Simple Range Counting.* This most simple method considers a range, i.e. the difference between two successive load reversals, to be one half cycle. Even though the order of occurrence of loading events is retained during counting, large cycles having a major impact on fatigue analysis may not be recorded by this counting procedure if they include smaller load fluctuations which will divide them into several smaller ranges.

*Rainflow Counting.* This name represents a family of methodologies developed from the early 1960s in an effort to analyze a spectrum into loading cycles as accurately as possible, i.e. based on stress-strain events (hysteresis loops) occurring inside the material during fatigue. The method has been introduced almost simultaneously by Matsuishi and Endo [139] (accessible in English in [140]) and de Jonge [141] the latter calling the algorithm range-pair method.

A schematic interpretation of how a rainflow algorithm records cycles is shown in Fig. 62, in which a part of a loading spectrum (left) is interpreted in terms of stress-strain curves on the right hand side. The result counts three cycles (BC, EF and GH) while the segment AH that remains once closed cycles points are discarded is called residual.

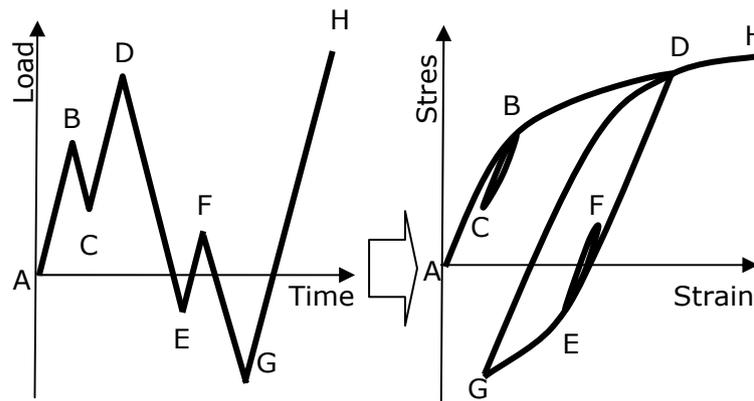


Fig. 7 Load time history and corresponding qualitative stress-strain graph, showing the hysteresis loops (BC, EF and DG) counted by Rainflow algorithm.

When all loops have been counted, it is possible that a number peaks and troughs (called residual) still remain. Different strategies can be followed in treating the residual.

Considering several alternatives for counting cycles of stress-strain histories Dowling proposed the Rainflow (or Range-pair method) to give the most accurate cycle content for fatigue analysis [142]. Actually the two algorithms give very similar results in most situations while the results are identical when the sequence is rearranged starting with the maximum peak or minimum valley.

Downing and Socie [143] proposed two slightly alternate algorithms. The first one, 'Algorithm I', requires rearrangement of the spectrum so that it starts with its maximum or minimum value. 'Algorithm II' counts a history of peaks and troughs, without rearranging them, in sequence as they occur. The closed loops are recorded immediately after completion while the remaining peaks and troughs are processed again.

Glinka & Kam [144] proposed an alternate Rainflow algorithm for counting long time-series by dividing them into smaller parts called blocks. No spectrum rearrangement is required according to their method.

Rychlik provided in [145] an alternative definition of the rainflow counting process based on comparing each peak in the spectrum with all the signal's previous and following down- and up-crossing respectively, resulting in counting full or half cycles depending on simple rules. This approach has been used in attempts to implement the rainflow algorithm in the frequency domain, as will be discussed later on.

Another alternative algorithm for counting the cycles using rainflow has been proposed by Hong [146]. The residual can either be treated as half cycles or rearranged so that it starts and ends with its maximum peak.

Amzallag [147], suggests two alternative ways of treating the residual: Either add the residual to itself and extract a number of full cycles (decomposition of residue into cycles) or reconstruct the residual as a loading sequence by inserting cycles into it simple rules.

The modification of Rainflow proposed by Anthes [148] is an attempt to keep the load sequence by considering rising half cycles that do not form a hysteresis loop as 'virtual hysteresis loops' that either close later by a decreasing segment or are combined with subsequent virtual loops. The reasoning is to appoint the damage events in an order of occurrence as accurate as possible.

A number of efforts have been recorded for the generalization of rainflow counting algorithm under multiaxial stress states. In the multiaxial case the identification of closed hysteresis loops is difficult, while out of phase cycling further complicates the problem. Consequently, most approaches use equivalent stress quantities to reduce the multiaxial problem to a uniaxial one, which restricts such solutions to isotropic materials.

Bannantine and Socie [149] considered the fatigue process in metals to be driven by shear stresses and used assumed reasonable to reduce the problem to uniaxial rainflow counting of the maximum shear component.

Wang and Brown [150] suggested a critical plane based approach to propose a multiaxial rainflow algorithm for non-proportional loadings.

Chu & Chu CC, [151] introduced the incremental damage concept, defining the cycles indirectly using two simple rules for tracking the damage rate in respect to the stress amplitude. This alternative definition of closed hysteresis loops can be generalized for multiaxial fatigue stresses provided an equivalent stress is assumed (to reduce the problem to a uniaxial one). The one they propose is the flow stress, representing the 'equivalent' hysteretic behaviour under such loadings.

Langlais et al. [152] use again a critical plane based approach to reduce the order of the problem in the general case of non-proportional cycling. According to their modification, before discarding a counted cycle, information from all directions (channels) must be considered in order to choose the most damaging case.

A different concept has been used by Beste et al. In [153] according to which, cycles are counted for every possible linear combination of the plane stress components. Subsequently, the damage is assessed for each combination so that the most critical set of parameters is determined. This method is similar to the definition of an equivalent stress with the coefficients of the stress components optimized for maximum fatigue damage, once rainflow counting has been performed on each occurring series of the equivalent quantity. The procedure of course is computationally intensive.

## 3 FREQUENCY DOMAIN

### 3.1 Introduction

In general, fatigue calculations of composite structures undergoing random loads is performed in the time domain. The required input is load-time series which are produced by applying random loads of specific characteristics (e.g. stochastic wind loads) to the structure, component or structural detail using a simulation procedure. In this procedure a number of issues are inherently involved: The first has to do with the realistic representation of all the random load's characteristics to the time-load series generated. This may require the simulation of a long time period in order to include peaks that are more damaging but less frequent. A second issue is that fatigue calculations in the time domain are time consuming, especially regarding large structures with complicated geometry where the calculation must be performed at a large number of points. Furthermore, different fatigue load cases must each time be considered, which multiplies the effort and time, while the things become even more complicated when accounting for multiaxial fatigue in laminated structures and progressive damage approaches.

When the random input signal itself is available in the form of a power spectral density function or psd, i.e a scalar function that describes how the power of the time signal is distributed among frequencies, significant time could be saved if fatigue calculations could be performed directly in the frequency domain. Such a task requires the expression of cycles experienced by the material during fatigue as a function of the imposed load's statistical characteristics as well as the calculation of the fatigue damage induced by these cycles based on fatigue properties of the material and some sort of cumulative damage rule.

The former task can be performed by correlating the fatigue response of the material with the  $n^{\text{th}}$  order moments of the input signal's power spectral density (PSD), which are functions of the following form:

$$\lambda_n = \int_0^{\infty} f^n G(f) df \quad (91)$$

Where  $f$  is the frequency in Hz and  $G(f)$  is the one sided PSD of a stationary zero mean stress process  $x(t)$ . Various observable quantities of the signal  $x(t)$  can be related to spectral moments. For instance, the root mean square ( $S_{\text{RMS}}$ ), number of zero crossings with positive slope ( $\nu$ ) and rate of peak occurrences  $n_p$ , are expressed as functions of spectral moments, provided that the stochastic process is Gaussian:

$$\begin{aligned} S_{\text{RMS}} &= \sqrt{\lambda_0} \\ \nu &= \frac{1}{2\pi} \sqrt{\frac{\lambda_2}{\lambda_0}} \\ n_p &= \sqrt{\frac{\lambda_4}{\lambda_2}} \end{aligned} \quad (92)$$

It is also possible to define the irregularity factor  $\alpha$ , relating the rate of zero crossings to the rate of peak occurrences, as:

$$\alpha = \frac{\lambda_2}{\sqrt{\lambda_0 \lambda_4}} \quad (93)$$

In order to formulate a spectral fatigue damage method one must assume an S-N curve equation which must be fitted to experimental fatigue data for the material, e.g. the log-log S-N curve:

$$N = KS^m \quad (94)$$

In order to account for the different amount of fatigue damage induced by cycles of different characteristics a damage accumulation rule must be assumed. This is usually the previously mentioned Palmgren-Miner rule described by the following expression for the total fatigue damage after  $k$  cycles:

$$D = \sum_{i=1}^k \left( \frac{n_i}{N_i} \right) \quad (95)$$

In order to develop a spectral method for deriving the total fatigue damage directly through the signal's PSD, the probability distribution of the number of cycle ranges, corresponding to the considered spectrum, must be obtained. When the latter is known ( $P(S)$ ), it is possible to derive the number of cycles corresponding to a specific stress range  $S$ , at a given time period  $T$ , through the following expression:

$$n(S) = n_s TP(S) \quad (96)$$

$n_s$  is the expected number of range occurrences per unit time. Subsequently the expected fatigue damage over period  $T$  due to the stochastic load can be calculated through:

$$D = \int_0^{\infty} \frac{n(S)}{N(S)} dS = n_s T \int_0^{\infty} \frac{P(S)}{N(S)} dS = 1 \quad (97)$$

In the above expression,  $N(S)$  is known through the S-N curve of the material. The expected rate of stress ranges for instance can be reasonably assumed equal to the rate of peaks in the spectrum, which is calculated above as a function of the PSD moments. The problem often encountered, has to do with the calculation of the probability distribution of stress ranges corresponding to the signal's PSD. Even though analytical solutions have been approximated for certain types PSD types, the probability distribution of the stress ranges cannot be derived analytically directly from the signal's PSD, especially the ranges counted by Rainflow algorithm, providing the best possible count available. This is the major drawback of spectral methods. A indirect way to derive  $P(S)$  is by simulating long time series based on the stochastic signal and then perform counting of the

ranges obtained in the time domain, using a rainflow algorithm. The result is a simulated PDF for the counted ranges, according to which the fatigue damage can be calculated by the above expression Eq.(97).

In order to overcome or bypass this problem in an efficient way, thus improving the speed of fatigue calculations, several methods have been proposed for simple or more general cases of stochastic signals (i.e. different PSD functions). A review of those methods, referred to in general as spectral fatigue methods, is attempted in the following paragraphs.

A common approach in literature is assuming for different cases of stochastic processes. An overview of such methods is given below.

### 3.2 Narrow-band Processes

A narrowband process ( $\alpha \rightarrow 0$ ) is characterized by one dominant central frequency. A signal of this type is shown in the following figure. It can be seen that each peak is followed by a trough of nearly the same amplitude and opposite sign. This implies that the probability density function (PDF) of stress ranges can be approximated by that of peak occurrences, which can be calculated analytically.

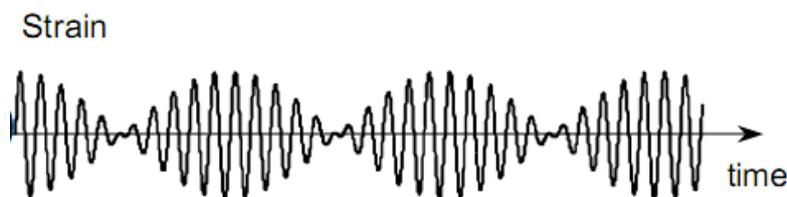


Fig. 8 : Narrow-band stochastic process

Provided that the input signal is Gaussian and narrow-band, the PDF of stress ranges is proved to follow a Rayleigh distribution depending only on the zero order moment [154], [155]:

$$p(S) = \frac{S}{\lambda_0} \exp\left(-\frac{S^2}{2\lambda_0}\right) \quad (98)$$

This formulation, known as Rayleigh approximation, and has been widely used in literature. Since a zero-mean stationary process of this kind corresponds to an alternating load having one peak frequency, the Rayleigh method formulates, in the frequency domain, the fatigue process of stochastic amplitude at  $R=-1$  under constant frequency. Different stress ratios can be accounted for through use of a Goodman type equation (CLD). The expected damage from a single cycle  $E[\Delta D_i]$  is:

$$E[\Delta D_i] = \frac{1}{K} 2^{\frac{m}{2}} S_{\text{RMS}}^m \Gamma\left(1 + \frac{m}{2}\right) \quad (99)$$

where  $S_{\text{RMS}}$  is the root-mean-square stress and  $\Gamma$  is the gamma function.

The Rayleigh approximation formulated above is derived under the assumption of Palmgren-Miner rule and log-log S-N curve. Nevertheless, in composites the former has been widely criticized for not taking into account sequence effects, while different expressions could be chosen for the latter.

In order to investigate the effect of S-N curve equation used, Sakrani considered in [156] narrow-band stochastic loads in damage accumulation using the Rayleigh approximation in FRPs assuming three S-N curve formulations (lin-log, log-log and bi-linear) along with the linear Palmgren-Miner rule. The conclusion was that for all S-N curves, predictions were (similarly) poor compared to the stochastic fatigue tests data, which were performed on different types of bolted joints.

Regarding the validity of the linear damage accumulation rule, Sakrani et al. [157], investigated the application of different non-linear damage accumulation methods (formulated in the time domain) validated against the spectral Rayleigh method. Residual strength and stiffness degradation based models were used for the comparison. Narrow band stochastic stress histories were simulated, concluding that similar (conservative) predictions are obtained for all damage rules.

Younesian et al [158] implemented the Rayleigh approximation method to steel train bogies and compared the results with a load time series derived through FEM simulation. Their results showed the Rayleigh method results to be less conservative than those from the simulation.

When the adjacent peaks are not perfectly correlated the Rayleigh approximation results deviate from the corresponding rainflow count. Yang [159] generalized the method for non-perfectly correlated signals, using a formulation that involves the hypergeometric function. Krenk for the same case [160] used a correction factor  $\lambda_k = \alpha^{m-1}$  ( $m$  is the S-N curve slope and  $\alpha$  the irregularity factor) for the damage predicted by the Rayleigh method, while a different correction factor is proposed by Winterstein and Cornell [161].

### 3.3 Broad-band Processes

Assuming equality of peak and cyclic range probability distributions for the general case of bi-modal or broad-band PSD functions usually large conservative errors occur since that fundamentally implies that one stress range per peak is assumed. This inefficiency becomes evident in the following figure, where a bimodal signal is displayed in the time domain. If the peak rate of the signal in the upper part of the figure is equated to its counted stress-range rate, the ranges shown in grey in the lower part would implicitly be assumed. This suggests a far worst scenario, leading to conservative life estimates.

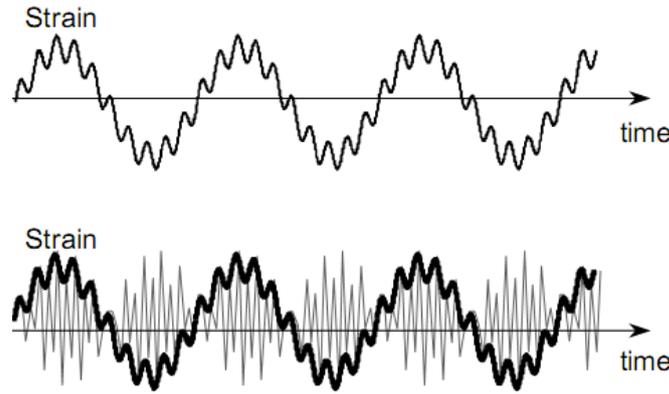


Fig. 9 Bi-modal narrow-band stochastic signal and signal count using the peak counting assumption (bottom).

Various efforts to develop spectral methods correcting this effect for this particular type of PSD have appeared up to-date. Braccesi [162] lately divided these approaches in three categories. The first engulfs theories that implement coefficients based on spectral moments to generalize the narrow-band approximation to other types of power spectral densities. The second includes theories trying to express the PDF as a function of basic probability distributions, whose parameters can be defined experimentally as functions of the spectral moments. Finally, efforts for deriving analytically the PDF of rainflow counted ranges are distinguished.

The presentation of methodologies regarding broad-band stochastic processes attempted herein distinguishes between (a) those that implement a correction factor to the narrow band approximation, (b) those that use analytical expressions for the PDF of peaks modified accordingly to express the PDF of the counted ranges, (c) combinations of standard distributions to describe the above PDF and (d) methods for analytically deriving the PDF of the counted cycles.

### 3.3.1 Correction Factors

Wirsching [163] introduced an indirect approach to deal with the problem of fatigue damage accumulation under broad-band stochastic loading. The idea was to approximate the stochastic load by a narrow-band to simplify the procedure and using modification factors to amend the initial result:

$$E[\Delta D_i] = \text{Cor} \frac{1}{K} 2^{\frac{m}{2}} S_{\text{RMS}}^m \Gamma\left(1 + \frac{m}{2}\right) \quad (100)$$

The correction factor  $\lambda_w$  is a function of the bandwidth parameter  $\varepsilon = \sqrt{1 - \alpha^2}$  and the S-N curve slope  $m$ :

$$\text{Cor} = (0.926 - 0.033m) + (0.074 - 0.033m)(1 - \varepsilon)^{1.587m - 2.323} \quad (101)$$

Madsen et al. used [] for the same purpose another correction factor:

$$\text{Cor} = (0.93 + 0.07\alpha^5)^m \quad (102)$$

Madsen et al. suggested that under boad-band processes it is not appropriate to pair every positive peak with an equal negative valley [164]. Alternatively, they proposed the counting of the number of up-crossings of a random level. Their narrow-band approximation results in a correction factor for the expected damage, equal to the irregularity factor  $\alpha$ . The expected damage is:

$$E[D] = \alpha n_p \frac{1}{K} 2^{\frac{m}{2}} S_{\text{RMS}}^m \Gamma\left(1 + \frac{m}{2}\right) \quad (103)$$

In the same work, the expected damage for the case of Range counting algorithm instead of Rainflow is derived and found to be related to both the irregularity factor and the S-N curve slope, through a correction factor  $\text{Cor} = \alpha^m$ .

It is interesting to note that according to Tovo [165] the damage induced by performing Rainflow counting has a lower bound, which is the damage calculated by the above expression (using range counting  $\text{Cor} = \alpha^m$ ), and an upper bound which is the damage assuming level up-crossing ( $\text{Cor} = \alpha$ ). In that sense Eq.(103) provides a non-conservative boundary solution. The problem of course is that the spread between these two bounds is usually quite large. This is especially true for composites where  $m$  is in the order of 10. The expected damage approximating rainflow counting that he proposes [183] is based on an intermediate value of the correction factor:

$$\text{Cor} = \alpha [b + (1 - b)\alpha^{k-1}] \quad (104)$$

where the parameter  $b$  is approximated by:

$$b = \min\left\{\frac{\alpha_1 - \alpha}{1 - \alpha_1}, 1\right\} \quad \text{where} \quad \alpha_1 = \frac{\lambda_1}{\sqrt{\lambda_0 \lambda_2}} \quad (105)$$

Another correction factor based on the Rayleigh approximated PSD is also proposed by Ortiz and Chen [166], where  $b = m/2$ :

$$\text{Cor} = \frac{1}{\alpha} \left( \frac{m_2 m_b}{m_0 m_{2+b}} \right)^{\frac{1}{2}} \quad (106)$$

Lutes et al. [167] uses the Rayleigh approximation to propose the Single Moment (SM) method (based on a single PSD moment) in order to improve the Rayleigh method results for bimodal psd functions. Their correction factor is:

$$\text{Cor} = \lambda_{2/m}^{m/2} \quad (107)$$

Jiao and Moan [168] extended the narrow-band approximation for the case of stochastic processes with two separate peaks in the PSD distribution. The correction factor is:

$$\text{Cor} = \frac{v_p}{v_y} \left[ \lambda_1^{2+m} \left( 1 - \sqrt{\frac{\lambda_2}{\lambda_1}} \right) + m \sqrt{\pi \lambda_1 \lambda_2} \frac{\Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{m}{2} + 1\right)} \right] + \frac{v_2}{v_y} \lambda_2^{m/2} \quad \text{where,}$$

$$v_p = \lambda_1 v_1 \sqrt{1 + \frac{\lambda_2}{\lambda_1} \left( \frac{v_2}{v_1} \sqrt{1 - \frac{m_{1,2}^2}{m_{0,2}/m_{2,2}}} \right)^2} \quad v_y = \sqrt{\lambda_1 v_1^2 + \lambda_2 v_2^2} \quad \lambda_1 = \frac{m_{0,1}}{m_{0,1} + m_{0,2}} \quad (108)$$

$$\lambda_2 = \frac{m_{0,2}}{m_{0,1} + m_{0,2}} \quad v_1 = \sqrt{\frac{m_{2,1}}{m_{0,1}}} \quad v_2 = \sqrt{\frac{m_{2,2}}{m_{0,2}}}$$

$m_{ij}$  stands for the  $i^{\text{th}}$  order moment of the narrow band process represented by the corresponding peak  $j$  of the bimodal PSD function and  $m$  is the slope of the log-log S-N curve equation.

A similar concept was adopted by Kihl at al. [169] based on narrow band, normally distributed loads to predict broad-band, non-Gaussian stochastic loads by multiplying the expected value for damage of the former by a factor  $\lambda$ , defined numerically through simulation using Rainflow analysis.

### 3.3.2 Analytical Expressions

While retaining the peak counting assumption, various researchers tried to express the peak (and counted ranges) distribution for the case of broadband PSD functions. An analytical expression to this end has been proposed and is known as the Rice distribution [170]:

$$p(S) = \frac{1}{\sqrt{2\pi}\sigma} (1 - \alpha^2)^{\frac{1}{2}} \exp\left[\frac{-S^2}{2\sigma^2(1 - \alpha^2)}\right] + \frac{S}{2\sigma^2} \alpha \left[ 1 + \text{erf}\left(\frac{S}{\sigma\sqrt{2\alpha^{-2} - 2}}\right) \right] \exp\left(-\frac{S^2}{2\sigma^2}\right) \quad (109)$$

where  $\text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-t^2} dt$

where  $\sigma$  is the RMS value of the stochastic signal (or else its zero<sup>th</sup> moment). This equation for a narrow-band ( $\alpha=1$ ) and a wide band signal ( $\alpha=0$ ) reduces to the Rayleigh and Gaussian PDF distribution respectively.

Wirsching and Light [171] based on the above distribution proposed an approximation of the fatigue damage caused by a broad-band stochastic fatigue process  $x(t)$ , assuming the peak approximation in the broad-band signal under consideration. The incremental damage is:

$$E[\Delta D_i] = \frac{1}{K} 2^{\frac{3m}{2}} \frac{S_{RMS}^m}{1+\alpha} \left[ 2S_{RMS} \Gamma\left(1 + \frac{m}{2}\right) + \frac{(1-\alpha^2)^{\frac{m+2}{2}}}{\sqrt{\pi}} \Gamma\left(\frac{1+m}{2}\right) - \alpha \int_0^{\infty} v^{\frac{m}{2}} \operatorname{erfc}\left(\frac{\alpha\sqrt{v}}{\sqrt{1-\alpha^2}}\right) e^{-v} dv \right] \quad (110)$$

where  $\operatorname{erfc}(u) = 1 - \frac{2}{\sqrt{\pi}} \int_0^u e^{-t^2} dt$

$\alpha$  is the irregularity factor discussed above. The integral in the above expression can only be evaluated numerically.

Also based on the Rice distribution, Chaudhury and Dover in [172], proposed the following expression for fatigue damage, through the assumption of a fixed value for the error function ( $\operatorname{erf}=1/2$ ) of the peaks PDF. The expression for its expected value:

$$E[D] = \eta_p \frac{1}{K} (2\sqrt{2}\sigma)^m \left\{ \frac{(\sqrt{1-\alpha^2})^{m+2}}{2\sqrt{\pi}} \Gamma\left(\frac{m+1}{2}\right) + 0.75\alpha \Gamma\left(\frac{m+2}{2}\right) \right\} \quad (111)$$

Using the above formulation they suggested an 'weighted average stress range', equivalent to a cyclic range that would induce the same fatigue damage when applied for the same number of cycles as during the stochastic process. According to their data from offshore structures under different sea states, the damage calculated by the above expression is close to the one obtained through Rainflow counting of a simulated time-series.

Kam and Dover [173] proposed in a subsequent publication a different expression for the error function, being a 7<sup>th</sup> order polynomial of the irregularity factor for  $\alpha < 0.96$  and  $\operatorname{erf}(x)=1$  for  $\alpha > 0.96$ . Kam also proposes in [174] other possible formulations for the error function and performs a comparison between different models in the crack growth rate under different broad-band signals.

Abdo and Rackwitz [175] analytically derived the expected damage given above by Kam and Dover, yielding:

$$E[D] = \eta_p \frac{1}{K} (2\sqrt{2}\sigma)^m \left\{ \frac{(\sqrt{1-\alpha^2})^{m+2}}{2\sqrt{\pi}} \Gamma\left(\frac{m+1}{2}\right) + \alpha \Gamma\left(\frac{m+2}{2}\right) T_v\left(\frac{\alpha}{\sqrt{1-\alpha^2}} \sqrt{v}\right) \right\} \quad (112)$$

$T_v$  is the Student's central t-distribution with  $v=m+2$  degrees of freedom. Abdo and Rackwitz also suggested an expression to derive the expected value only of positive peaks, since negative peaks do not assumingly contribute to damage.

### 3.3.3 Using a Combination of Distributions

The second category includes semi-empirical methods that approximate the stress range distribution through a combination of basic distributions for the peaks. The most characteristic method in this category is Dirlik's formula [176], using for this purpose one exponential and two Rayleigh distributions. The formula is:

$$\begin{aligned}
 P(S) &= \frac{1}{2\sqrt{\lambda_0}} \left[ \frac{G_1}{Q} e^{-z/Q} + \frac{G_2 Z}{R^2} e^{-z^2/(2R^2)} + G_3 Z e^{-z^2/2} \right] \\
 Z &= \frac{S}{2\sqrt{\lambda_0}} \quad G_1 = \frac{2(x_m - \gamma^2)}{2\sqrt{\lambda_0}} \quad R = \frac{\gamma - x_m - G_1^2}{1 - \gamma - G_1 + G_1^2} \\
 G_2 &= \frac{1 - \gamma - G_1 + G_1^2}{1 - R} \quad G_3 = 1 - G_1 - G_2 \\
 Q &= \frac{1.25(\gamma - G_3 + G_2 R)}{G_1} \quad x_m = \frac{\lambda_1}{\lambda_0} \sqrt{\frac{\lambda_2}{\lambda_4}} \quad \gamma = \frac{m_2}{\sqrt{m_0 m_4}}
 \end{aligned} \tag{113}$$

The formula of Dirlik has been applied to fatigue life prediction of WT rotor blades by Bishop et al. [177], [178]

Another alternative for the same approach was proposed by Lutes (see Appendix of [179]) by approximating the distribution of the peaks of a normal process by a linear combination of a Gaussian and a Rayleigh distribution. The resulted incremental fatigue damage in this case is:

$$E[\Delta D_i] = \frac{1}{K} 2^{\frac{3m}{2}} S_{RMS}^m \left[ \frac{(1-\alpha)}{(1+\alpha)} \left( 1 - \frac{\alpha}{3} - \frac{\alpha^2}{4} \right) \frac{1}{\sqrt{\pi}} \Gamma\left(\frac{1+m}{2}\right) + \frac{2\alpha}{(1+\alpha)} \Gamma\left(\frac{1+m}{2}\right) \right] \tag{114}$$

Sakai and Okamura [180] considered the case of PSD functions with two dominant vibration modes and used two Rayleigh distributions to evaluate fatigue life by applying appropriate weighting factors. The two frequencies however must be well separated. The predicted fatigue life is:

$$T = \frac{2^{\left(\frac{1+m}{2}\right)} \pi K}{\Gamma\left(\frac{1+m}{2}\right)} \frac{1}{\omega_1 + \omega_2} \left[ \omega_1 \frac{\lambda_{0.1}^{1+\frac{m}{2}}}{\lambda_{2.1}} + \omega_2 \frac{\lambda_{0.2}^{1+\frac{m}{2}}}{\lambda_{2.2}} \right] \tag{115}$$

$\lambda_{ij}$  stands for the  $i^{\text{th}}$  moment of the  $j$  Rayleigh distribution

Fu and Cebon [181] also focused on bimodal psd functions. As Sakai et al. they assumed two Rayleigh distributions for the ranges corresponding to the two peaks of the psd.

$$T = \frac{2\pi K}{\omega_1 A + (\omega_2 - \omega_1) B}$$

$$A = \frac{1}{\lambda_{0,1} \lambda_{0,2}} \int_0^\infty \left\{ e^{-S^2/2\lambda_{0,2}^2} S^{-m} \left[ \int_0^\infty (Sy - y^2) e^{-\left(\frac{1}{2\lambda_{0,1}} + \frac{1}{2\lambda_{0,2}}\right)y^2 + \left(\frac{1}{\lambda_{0,1}}\right)Sy} dy \right] \right\} dS \quad (116)$$

$$B = 2^{-m/2} \lambda_{0,2}^{-m} \Gamma\left(1 - \frac{m}{2}\right)$$

Zhao and Baker in [182] assumed the stress range distribution to be a combination of a Weibull and a Rayleigh distributions. Specific values are attributed to the Weibull shape and scale parameters depending on the value of the irregularity factor. The expected damage at cycle N is:

$$E[D] = \frac{1}{K} N 2^m \left[ w \Gamma\left(\frac{b+m}{b}\right) \alpha^{-\frac{m}{b}} + (1-w) (\sqrt{2})^m \Gamma\left(\frac{2+m}{2}\right) \right] \quad (117)$$

Where the weighting factor w is:

$$w = \frac{1-\alpha}{1 - \frac{\sqrt{2}}{\sqrt{\pi}} \Gamma\left(1 - \frac{1}{b}\right) \alpha^{\frac{1}{b}}} \quad (118)$$

### 3.3.4 Counting

All previously discussed spectral methods try to solve the problem of deriving the PDF of the stress ranges from a broadband PSD, using different assumptions based on the correlation between the distribution of peaks and the distribution of stress ranges. A number of methods on the other hand, focus directly on the problem of counting the stress ranges from the PSD, proposing solutions for the theoretical derivation of its PDF. The most adequate solution to the problem should incorporate counting of the hysteresis loops applied on the material. This is, as already discussed, the most accurate way of transforming a random sequence of peaks and troughs to fatigue cycles and is usually performed through rainflow counting. This derivation however is very complex and no analytical solution has been proposed to date.

An alternative to the rainflow counting is range counting, assuming a half cycle event between each peak and trough of the stochastic signal. This half cycle is subsequently paired with a same half cycle of the opposite direction, thus forming one full cycle. In that case the problem is the derivation of the probability distribution between two neighbouring extremes of the spectrum which has been approximated analytically [183], with the following result, as presented by Tuna [184]:

$$p(\mu, S) = \frac{S}{4\alpha\sigma^3\sqrt{2\pi(1-\alpha)}} \exp\left(\frac{-\mu^2}{2\sigma^2(1-\alpha)}\right) \exp\left(\frac{-S^2}{8\sigma^2\alpha}\right) \quad (118)$$

$\mu$ ,  $S$  is the mean stress and stress range, while  $p(\mu, S)$  is joint probability function

Despite the analytical expression available for the range counting spectrum, as discussed in [185], their result in terms of fatigue damage is the lower bound of the rainflow counting result which expresses the inefficiency of Range counting and leads to non-conservative predictions. This is why special interest is shown in developing techniques to calculate the probability distribution of rainflow-counted stress ranges.

The most widely cited approach in this direction is the procedure proposed by Rychlik [145]. Based on this definition of the rainflow process Rychlik et al. attempted to approximate the probability of each spectrum's peak to satisfy the counting criterion through a Markov chain [186]. Using this assumption one can numerically calculate the conditional probability of the signal crossing a certain level having already crossed it before (see [187],[188]).

In an effort to simplify the above process, Bishop and Sherratt [189] proposed to derive the transition probability matrix using the Kowalewski probability density function for the dependence between adjacent peaks, though they note this is an approximate solution.

Olagnon [190] based on the same approach as above, performed computations leading to algorithmic expressions for the elements of the Markov chain transition matrices. This way the probability of each peak satisfying the criteria proposed by Rychlik can be calculated based on the construction of a transition (from-to) probability matrix, which is in turn based on the probability of a peak or trough being of a specific level.

### 3.3.5 Complex Stresses

Taking into account complex stress effects during stochastic fatigue of isotropic materials, in the frequency domain, has been investigated by a number of researchers. The first approach has been proposed by Preumont and Piefort [191] based on Von Mises equivalent stress. Since the stochastic process defined by its well known quadratic relation is not Gaussian and zero mean, in order to be reasonably approximated by one of the spectral methods discussed previously, they transform its mean-square value into the frequency domain. This way the multiaxial stress problem is reduced to a uniaxial one.

The use of Von-Mises equivalent stress is reasonable when the in-plane stresses are in-phase (constant principal directions) as discussed in [192], however when stress components are treated as independent random processes the principal directions change and different fatigue criteria must be implemented, usually based on a (randomly changing) critical plane where the shear stress is maximized, or criteria involving stress tensor invariants. Such an approach is considered by Pitoiset et al [193] who implement Matake's and Crossland's failure criteria in the frequency domain.

In [194] Pitoiset and Preumont compare time domain spectral fatigue predictions with the previously proposed spectral method based on Von-Mises equivalent stress and a frequency domain formulation of the multiaxial rainflow count procedure proposed Dressler et al [153], [195]. The latter is obtained through transformation of the (scalar) equivalent stress, defined by any possible linear combination of the stress components, to the frequency domain and implementation of the same procedure as for the Von-Mises stress case. The authors conclude that significant reduction of computational time is achieved this way.

## 4 REFERENCES

- [1] Fatigue of Composite Materials, Reifsnider, ISBN 0444705074
- [2] Fatigue in Composites-Harris ISBN 1-85573-608-x
- [3] Mayer Design of Composite Structures against fatigue, ISBN 0-85298-957-1
- [4] Degrieck J, Van Papaegem W, Fatigue damage modeling of fibre-reinforced composite materials: Review, *Appl. Mech. Rev.*, 54(2001), pp 279-300.
- [5] Post NL, Case SW, Lesko JJ, Modeling the variable amplitude fatigue of composite materials: A review and evaluation of the state of the art for spectrum loading, *Int. J. Fatigue*, 30(2008), pp 2064-2086.
- [6] Wahl N, Spectrum fatigue lifetime and residual strength for fiberglass laminates, Ph.D. Thesis, Montana State University, 2001
- [7] Nijssen R.P.L., 'Fatigue Life Prediction and Strength Degradation of Wind Turbine Rotor Blade Composites'. Ph.D. Thesis, Faculty of Aerospace Engineering, (2006), T.U. Delft
- [8] Passipoularidis VA, Residual strength and life prediction in composite materials after fatigue, Mechanical Engineering & Aeronautics dept, University of Patras, 2008.
- [9] Post NL, Reliability based design methodology incorporating residual strength prediction of structural fiber reinforced polymer composites under stochastic variable amplitude fatigue loading, Ph.D. Thesis, Virginia Polytechnic Institute and State University, 2008.
- [10] Miner MA, Cumulative damage in fatigue. *J. Appl. Mech.* 12(1945), A159-A164.
- [11] Reifsnider KL, The Critical Element Model: A modeling philosophy, *Eng. Fract. Mech.* 25(1986), pp 739-749
- [12] Shokrieh M.M., Lessard L.B., Progressive fatigue damage modelling of composite materials, Part I: Modeling, *J. Compos. Mater.*34(2000), pp.1056-1080
- [13] Michel AS, Rolf K, Martens HJ, Fatigue strength of carbon fibre composites up to the gigacycle regime (gygacycle-composites), *Int. J. Fatigue*, 28(2006), pp 261-270.
- [14] Whitney, J.M. (1981). Fatigue Characterization of Composite Materials, *Fatigue of Fibrous Composite Materials*, ASTM STP 723, pp. 245-260.
- [15] Kam TY, Tsai SY, Chu KH, Fatigue reliability analysis of composite laminates under spectrum stress, *Int. J. Solids Structures*, 34(1997), pp 1441-1461.
- [16] Passipoularidis VA, Philippidis TP, Strength degradation due to fatigue in fiber dominated Glass/Epoxy composites: A statistical approach, *J. Compos. Mater.*, 43(2009), pp 997-1013.
- [17] Diao X, Lessard LB, Shokrieh MM, Statistical model for multiaxial fatigue behavior of unidirectional plies, *Comp. Sci. Technol.*, 59(1999), pp 2025-2035.
- [18] IEC 61400-1 Ed3.0, Wind Turbines - Part 1: Design requirements (2005)
- [19] Nijssen R.P.L., 'Fatigue Life Prediction and Strength Degradation of Wind Turbine Rotor Blade Composites'. PhD Thesis, Faculty of Aerospace Engineering, (2006), T.U. Delft.
- [20] Echtermeyer AT, Engh B, Buene L, Influence of matrix and fabric, in *Design of composite structures against fatigue*, Mayer RM ed, 1996, Wiltshire, pp 33-50
- [21] Nijssen RPL, Kreause O, Philippidis TP, Benchmark on lifetime prediction methodologies, OPTIMAT BLADES project report, OB\_TG1\_R012, (<http://www.kc-wmc.nl/optimat/ blades/ Publications>)
- [22] Echtermeyer AT, Hayman E, Ronald KO, Comparison of fatigue curves for glass composite laminates, in *Design of composite structures against fatigue*, Mayer RM ed, 1996, Wiltshire, pp 209-226
- [23] Sendekyj G.P., 1991, 'Life Prediction for Resin-Matrix Composite Materials', in *Composite Material Series*, Vol. 4, 10. Amsterdam: Elsevier, pp. 431-483.
- [24] Mandell JF, Samborsky DD, Wahl NK, Sutherland HJ, Testing and analysis of low cost composite materials under spectrum loading and high cycle fatigue conditions. ICCM14, SME/ASC, 2003, paper 1811
- [25] Epaarachchi JA, Calausen PD, An empirical model for fatigue behaviour prediction of glass fibre-reinforced plastic composites for various stress ratios and test frequencies, *Composites: Part A*, 34(2003), pp 313-326.
- [26] Whitney JM. 'Fatigue characterization of composite materials', *Fatigue of fibrous composite materials*, ASTM STP 723, 1981, pp 133-151
- [27] Rotem A, Nelson HG, Fatigue behavior of Graphite-Epoxy laminates at elevated temperatures, *Fatigue of Fibrous Composite Materials*, ASTM STP 723 (1981), pp 152-173

- [28] Xiao XR, Modeling of load frequency effect on fatigue life of thermoplastic composites, *J. Compos. Mater.* 33(1999), pp 1141-1158
- [29] Miyano Y, McMurray MK, Enyama J, Nakada M, Loading rate and temperature dependence on flexural fatigue behaviour of a stain woven CFRP laminate, *J. Compos. Mater.*, 28(1994), pp 1250-1260
- [30] Germanischer Lloyd, Regulations for the Certification of Wind Energy Conversion Systems, Hamburg, 2003.
- [31] DNV Standard DNV-OS-J102, Design and manufacture of wind turbine rotor blades, offshore and onshore wind turbines, 2006
- [32] Goodman J, *Mechanics applied engineering*, London: Longman, Green & Co, 1899
- [33] Gerber WZ, *Bayer architect Ing* 1874;6, 101
- [34] Philippidis TP, Vassilopoulos AP, Complex stress state effect on fatigue life of GRP laminates, Part I experimental, *Int. J. Fatigue* 24(2002), pp 813-823
- [35] Sutherland HJ, Mandell JF, Optimized constant life diagram for the analysis of fibreglass composites used in wind turbine blades, *J. Sol. Energy Eng.*, 127(2005), pp 563-569
- [36] Passipoularis VA, Philippidis TP, A study of factors affecting life prediction of composites under spectrum loading, *Int. J. Fatigue*, 31(2009), pp 408-417
- [37] Nijssen RPL, van Delft DRV, van Wingerde AM, Alternative fatigue lifetime formulations for variable amplitude loading, *J. Sol. Energy Eng.*, 124(2002), pp 396-403
- [38] OPTIMAT BLADES project
- [39] Philippidis T.P., Vassilopoulos A.P., 'Life Prediction Methodology for GFRP Laminates under Spectrum Loading', *Composites: Part A*, 35(2004), pp. 657-666.
- [40] Gathercole N., Reiter H., Adam T., Harris B., 'Life prediction for fatigue of T800/5245 carbon-fibre composites: I. Constant amplitude loading', *Fatigue* (1994), 16, pp. 523-532.
- [41] Towo AN, Ansekk MP, Fatigue of sisal fibre reinforced composites: Constant life diagrams and hysteresis loop capture, *Comp. Sc. Techn.* 68(2008), pp915-924
- [42] Kawai M, Koizumi M, Nonlinear constant fatigue life diagrams for carbon/epoxy laminates at room temperature, *Composites part A*, 38(2007), pp 2342-2353
- [43] Boerstra GK, The multiscope model: A new description for the fatigue strength of glass fibre reinforced plastic, *Int. J. Fatigue*, 2007(29), 1571-1576
- [44] Brondsted P., Andersen S.I., Lilholt H., 'Fatigue Damage Accumulation And Lifetime Prediction Of GFRP Materials Under Block Loading And Stochastic Loading', *Proceedings of the 18th Risoe International Symposium on Material Science 1997*. Eds. Andersen S.I., Brondsted P. et al., pp. 269-278
- [45] Rotem A, The fatigue behavior of composite laminates under various mean stresses, *Comp. Struct.* 17(1991), pp113-126
- [46] Epaarachchi JA, Calausen PD, An empirical model for fatigue behaviour prediction of glass fibre-reinforced plastic composites for various stress ratios and test frequencies, *Composites: Part A*, 34(2003), pp 313-326.
- [47] Caprino G, D'Amore A, Flexural fatigue behaviour of random continuous-fibre-reinforced thermoplastic composites, *Comp. Sci. Technol.* 58(1998), pp 957-965
- [48] Miyano Y, Nakada M, McMurray MK, Muki R, Prediction of flexural fatigue strength of CRFP composites under arbitrary frequency, stress ratio and temperature, *J. Compos. Mater*, 31(1997), pp 619-638
- [49] Miyano Y, Nakada M, Kudoh H, Muki R, Prediction of tensile fatigue life for unidirectional CFRP, *J. Compos. Mater*, 34(2000), pp 538-550
- [50] Hwang W, Han KS, Cumulative damage models and multi-stress fatigue life prediction, *J. Compos. Mater.*, 20(1986), pp 125-153
- [51] Broutman L.J., Sahu S., 'A New Theory to Predict Cumulative Fatigue Damage in Fibreglass Reinforced Plastics', *Composite Materials: Testing and Design (2nd Conference)* 1972. Eds. Corten HT (American Society for Testing and Materials) STP 497, pp. 170-188.
- [52] Schaff J.R., Davidson B.D., 'Life Prediction Methodology for Composite Structures. Part I- Constant Amplitude and Two Stress Level Fatigue', *J Compos Mat*, 1997, 31(2), pp. 128-157.
- [53] Bond I.P., 'Fatigue Life Prediction for GRP subjected to variable amplitude Fatigue', *Composites: Part A*, 1999, 30, pp. 961-970
- [54] Philippidis T.P., Vassilopoulos A.P., 'Life Prediction Methodology for GFRP Laminates under Spectrum Loading', *Composites: Part A*, 35(2004), pp. 657-666
- [55] Gamstedt E.K., Sjögren B.A., 'An Experimental Investigation of the Sequence Effect in Block Amplitude Loading of Cross-Ply Composite Laminates', *Int. J. Fatigue* (2002), 24, pp. 437-446.

- [56] Hosoi A., Kawada H., Yoshino H., 'Fatigue Characteristics of Quasi-Isotropic CFRP Laminates Subjected to Variable Amplitude Cyclic Two Stage Loading', *Int J Fatigue* (2006), 28, pp. 1284-1289.
- [57] Marco SM, Starkey WL, A concept of fatigue damage, *Transactions of the ASME*, 76(1954), pp 627-632
- [58] Owen MJ, Howe RJ, The accumulation of damage in a Glass-Reinforced Plastic under tensile and fatigue loading, *J. Phys. D: Appl. Phys.* 5(1972), pp 1637-1649.
- [59] Bond IP, Farrow IR, Fatigue life prediction under complex loading for XAS/914 CFRP incorporating a mechanical fastener, *Int. J. Fatigue*, 22(2000), pp 633-644.
- [60] Sabramanyan S, A cumulative damage rule based on the knee point of the S-N curve, *J. Eng. Material & Tech.* 98(1976), pp 316-321.
- [61] Hashin Z, Rotem A, A cumulative damage theory of fatigue failure, *Mater. Sci. Eng.*, 34(1978), pp 147-160.
- [62] Hahn H.T., Kim R.Y., 'Proof Testing of Composite Materials', *J Compos Mat* (1975), 9, pp. 297-311
- [63] Chou P.C., Croman R., 'Residual strength in fatigue based on the strength-life equal rank assumption', *Journal of. Composite Materials* (1978), 12, pp. 177-194.
- [64] Sendekyj G.P., 1991, 'Life Prediction for Resin-Matrix Composite Materials', in *Composite Material Series*, Vol. 4, 10. Amsterdam: Elsevier, pp. 431-483.
- [65] Philippidis T.P., Passipoularidis V.A., 'Residual Strength After Fatigue in Composites: Theory vs Experiment', *Int J Fatigue*, 2007, 29(12), pp. 2104-2116.
- [66] Yang J.N., 'Fatigue and Residual Strength Degradation for Graphite/Epoxy Composites Under Tension-Compression Cyclic Loading', *Journal of. Composite Materials*, 1978, 12, pp. 19-39.
- [67] Yang J.N., Jones D.L., 'Statistical Fatigue of Unnotched Composite Laminates', in 'Advances in Composite Materials', ICCM-III, 1980, 1, pp. 472-483.
- [68] Yang J.N., Liu M.D., 'Residual strength degradation model and theory of periodic proof tests for Graphite/Epoxy laminates', *Journal of. Composite Materials*, 1977, 11, pp. 176-203.
- [69] Yang J.N., Sun C.T., 'Proof Test and Fatigue of Unnotched Composite Laminates', *Journal of. Composite Materials*, 1980, 14, pp. 168-176.
- [70] Yang J.N., Jones D.L., Yang S.H., Meskini A., 'A Stiffness Degradation Model for Graphite/Epoxy Laminates', *J. Compos Mater*, 1990; 24, pp. 753-769.
- [71] Lee L.J., Fu K.E., Yang J.N., 'Prediction of Fatigue Damage and Life for Composite Laminates Under Service Loading Spectra', *Compos Sci & Tech*, 1996, 56, pp. 635-648.
- [72] Yang J.N., Du S., 'An Exploratory Study Into the Fatigue of Composites Under Spectrum Loading', *Journal of Composite Materials*, 1983, 17, pp. 511-526.
- [73] Chou P.C., Croman R., 'Degradation and Sudden Death Models of Fatigue of Graphite/Epoxy Composites', *Composite Materials: Testing and Design* (5th conference), ASTM STP 674 (1979), pp. 431-454
- [74] Adam T., Dickson R.F., Jones C.J., Reiter H., Harris B., 'A Power Law Fatigue Damage Model for Fiber-Reinforced Plastic Laminates', *Proc Instn Mech Engrs* (1986); 200(C3), pp. 155-166
- [75] Epaarachchi JA, Clausen PD, A new cumulative fatigue damage model for glass fibre reinforced plastic composites under step/discreet loading, *Composites Part A*, 36(2005), pp 1236-1245
- [76] Yao WX, Himmel N, A new cumulative fatigue damage model for fibre-reinforced plastics, *Compos. Sci. Technol*, 60(2000), 59-64
- [77] Hwang W, Han KS, Fatigue of composites - Fatigue modulus concept and life prediction, *J. Compos. Mater.* 20(1986), pp 154-165
- [78] Sidoroff F, Subagio B, Fatigue damage modeling of composite materials from bending tests, 6<sup>th</sup> Conf on Composite Materials (ICCM-VI) & 2<sup>nd</sup> European Conf on Composite Materials (ECCM-II): Vol 4 proc. (1987), London, UK, Elsevier, pp 4.32-4.39.
- [79] Van Paepegem W, Degriek J, Experimental setup for numerical modeling of bending fatigue experiments on plain woven glass/epoxy composites, *Compos. Struct.* 51(2001), pp1-8
- [80] Van Paepegem W, Degriek J, A new coupled approach of residual stiffness and strength for fatigue of fibre reinforced composites, *Int. J. Fatigue* 24(2002),pp 747-762
- [81] Whitworth HA, modeling stiffness reduction of Graphite/Epoxy composite laminates, *J. Compos. Mater.*, 21(1987), pp 362-372
- [82] Whitworth HA, Cumulative damage in composites, *J. Eng. Mater. Technol.* 112(1990),pp 358-361
- [83] Whitworth HA, A stiffness degradation model for composite laminates under fatigue loading, *Compos. Struct.* 40(1998),pp 95-101
- [84] Lee LJ, Fu KE, Yang JN, Prediction of fatigue damage and life for composite laminates under service loading spectra

- [85] Brondsted P, Andersen SI, Lilholt H, Fatigue performance of Glas/Polyester laminates and the monitoring of material degradation, 9th International Conference on Mechanics of Composite Materials, 17-20 October 1995, Riga, Latvia
- [86] Post NL, Bausano J, Case SW, Lesko JJ, Modeling the remaining strength of structural composite materials subjected to fatigue, *Int. J. Fatigue*, 28(2006), pp 1100-1108
- [87] Echaabi JE, Trochu F, Gauvin R, Review of failure criteria of fibrous composite materials, *Pol. Compos.* 17(1996), pp 786-798
- [88] Stowell EZ, Liu TS, On the mechanical behaviour of fibre reinforced crystalline materials, *J. Mech. Phys. Solids*, 9(1961) pp242-260
- [89] Waddoups ME, Advanced composite material mechanics for the design and stress analyst, Report F2M, 4763, General Dynamics, Fort Worth Division, 1967
- [90] Hill R, A theory for the yielding and plastic flow of anisotropic metals, *Proc. Roy. Soc. A*193, pp 281-297
- [91] Azzi VD, Tsai SW, Anisotropic strength of composites, *Exptl. Mech.* 5(1965), pp 283-288
- [92] Norris CB, McKinnon PF, compression, tension and shear tests on yellow-poplar plywood panels of sizes that do not buckle with tests made at various angles to the face grain, Rep. No 1328(1956), Forest Products Laboratory, Madison, Wisc
- [93] Norris CB, Strength of orthotropic materials subjected to combined stresses, Rep. No 1816(1962), Forest Products Laboratory, Madison, Wisc
- [94] Hoffman O, Brittle strength of orthotropic materials, *J. Compos. Mater.*, 1(1967), pp 200
- [95] Tsai W, Wu EM, A general theory for anisotropic materials, *J. Compos. Mater.*, 5(1971), pp 58-80
- [96] Theocaris PS, Philippidis TP, Failure criteria for composite materials based on the tensor polynomial theory. A critical assessment, in: *Advanced Composites in Emerging Technologies*, ed. Paipetis SA, Philippidis TP, Amatec Publishing Co, 1991
- [97] Tsai SW, Hahn HT, *Introduction to composite materials*, Technomic Publishing Co, 1980
- [98] Hinton MJ, Soden PD, Predicting failure in composite laminates: The background of the exercise, *Comp. Sci. Technol.*, 58(1998), pp 1001-1010
- [99] *Composite Science and Technology*, Vol 58(1998), Issue 7
- [100] Soden PD, Hinton MJ, Kadour AS, A comparison of the predictive capabilities of current failure theories for composite laminates, *Comp. Sci. Technol.*, 58(1998), pp 1225-1254
- [101] Soden PD, Hinton MJ, Kadour AS, Lamina properties, lay-up configurations and loading conditions for a range of Fibre-Reinforced composite laminates, *Comp. Sci. Technol.*, 58(1998), pp 1011-1022
- [102] *Composite Science and Technology*, Vol 62(2002), Issue 12-13
- [103] Hinton MJ, Kadour AS, Soden PD, A comparison of the predictive capabilities of current failure theories for composite laminates, judged against experimental evidence, *Comp. Sci. Technol.*, 62(2002), pp 1725-1797
- [104] *Composite Science and Technology*, Vol 64(2004), Issue 12-13
- [105] Soden PD, Kadour AS, Hinton MJ, Recommendations for designers and researchers resulting from the world-wide failure exercise, *Comp. Sci. Technol.*, 64(2004), pp 589-604
- [106] Orifici AC, Herszberg I, Thomson RS, Review of methodologies for composite material modelling incorporating failure, *Comp. Struct.* 86(2008), pp 194-210
- [107] Owen MJ, Griffiths JR, Evaluation of biaxial stress failure surfaces for a glass fabric reinforced polyester resin under static and fatigue loading, *J. Mater. Sci.*, 13(1978), pp 1521-1537
- [108] Hashin Z, Rotem A, A fatigue failure criterion for fibre reinforced composite materials, *J Compos Mater* 7, 448-464
- [109] Francis PH, Walrath DE, Sims DF, Weed DN, Biaxial fatigue loading on notched composites, *J. Compos. Mater.*, 11(1977), pp 488-501
- [110] Ellyin F, El-Kadi H, A fatigue failure criterion for fibre composite materials, *Compos. Struct.* 15(1), 61-74
- [111] Plumtree A, Cheng GX, A fatigue damage parameter for off-axis unidirectional fibre reinforced composites, *Int. J. Fatigue* 21, 849-856
- [112] Reifsnieder KL, Gao Z, A micromechanics model for composites under fatigue loading, *Int J Fatigue*, 13, 149-156
- [113] Mori T, Tanaka K, Average stress in matrix and average elastic energy of materials with misfitting inclusions, *Acta Metall.* 23(1973), pp 571-574
- [114] Fawaz Z, Ellyin F, Fatigue failure model for fibre reinforced materials under general loading conditions, *J. Compos. Mater.*, 28, 1432-1451

- [115] Awerbuch J, Hahn HT, Off-axis fatigue of Graphite/Epoxy composite, in: Fatigue of fibrous composite materials, ASTM STP 723, 1981, pp243-273
- [116] Fujii T, Lin F, Fatigue behaviour of a plain-woven glass fabric laminate under tension/torsion biaxial loading, J. Compos. Mater., 29(1995), pp 573-590
- [117] Philippidis TP, Vassilopoulos AP, Fatigue strength prediction under multiaxial stress, J Compos. Mater. 33, 1578-1599
- [118] Tsai SW, Hahn HT, Introduction to composite materials, Technomic Publishing Co, 1980
- [119] Aboudi J, Micromechanics prediction of fatigue failure of composites, Reinf. Plast. Compos., 8(1989), pp150-166
- [120] Aboudi J, Micromechanical analysis of the strength of unidirectional fiber composites, Comp. Sci. Technol. 33(1988), pp 79-96
- [121] Rotem A, Hashin Z, fatigue failure of angle ply laminates, AIAA Journal, 14(1976), pp 868-872
- [122] Rotem A, Hashin Z, fatigue failure of multidirectional laminate, AIAA Journal, 17(1979), pp 271-277
- [123] Puppo AH, Evensen HA, Interlaminar shear in laminated composites under generalized plane stress, J. Compos. Mater. 4(1970), pp 204-220
- [124] Lawrence Wu CM, Thermal and mechanical fatigue analysis of CFRP laminates, Compos. Struct. 25, 339-344
- [125] Fawaz Z, Ellyin F, A new methodology for the prediction of fatigue failure in multidirectional fiber-reinforced laminates, Composite Sc. & Tech. 53(1995), pp47-55
- [126] Jen MHR, Lee CH, Strength and life in thermoplastic composite laminates under static and fatigue loads. Part I: Experimental, Int. J. Fatigue, 20(9), 605-615
- [127] Jen MHR, Lee CH, Strength and life in thermoplastic composite laminates under static and fatigue loads. Part II: Formulation, Int. J. Fatigue, 20(9), 617-629
- [128] Shokrieh M.M., Lessard L.B., 'Multiaxial Fatigue Behaviour of Unidirectional Plies Based on Uniaxial Fatigue Experiments- part I. Modeling', Int J Fatigue 1997, 19(3), pp. 201-207
- [129] Herrington PD, Sabbaghian M, Fatigue failure of composite bolted joints, J. Compos. Mater. 27(1993), pp 491-512
- [130] Shokrieh M.M., Lessard L.B., Progressive fatigue damage modelling of composite materials, Part II: Material characterization and model verification, J. Compos. Mater. 34(2000), pp.1081-1116
- [131] Tserpes KI, Papanikos P, Labeas G, Pantelakis Sp, Fatigue damage accumulation and residual strength assessment of CFRP laminates, Compos. Struct. 63(2004), pp 219-230
- [132] Himmel N, Fatigue life prediction of laminated polymer matrix composites, Int. J. Fatigue, 24(2002), pp 349-360
- [133] Passipoularidis V.A., Philippidis T.P., 'A Life Prediction Methodology for Composites Based on Progressive Damage Mechanics', Eccomas Thematic Conference on Mechanical Response of Composites, Composites 2007, Porto-Portugal, 12-14 September 2007.
- [134] Puck A. Schürmann H., 'Failure Analysis of FRP Laminates by Means of Physically Based Phenomenological Models', Composite Science and Technology, 2002, 62, pp. 1633-1662.
- [135] Puck A., Kopp J., Knops M., 'Failure Analysis of FRP Laminates by Means of Physically Based Phenomenological Models', Composite Science and Technology, 2002, 62, pp. 371-378.
- [136] Puck A., Schürmann H., 'Failure Analysis of FRP Laminates by Means of Physically Based Phenomenological Models', Composite Science and Technology, 1998, 58, pp. 1045-1067
- [137] Dzenis YA, Cycle-based analysis of damage and failure in advanced composites under fatigue 2. Stochastic mesomechanics modelling, Int. J. Fatigue 25(2003), pp 511-520
- [138] ASTM E 1049-85 (reapproved 1997). Standard Practices for Cycle Counting in Fatigue Analysis, ASTM International, West Conshohocken, PA, [www.astm.org](http://www.astm.org).
- [139] Matsuishi M., Endo T., 'Fatigue of Metals subjected to varying stress', in Proceedings of the Kyushu Branch of Japan Society of Mechanics Engineering, Fukuoka, Japan (1968), pp37-40 (in Japanese).
- [140] Endo T., Matsuishi M., 'The Rainflow Method in Fatigue - the Tatsuo Endo Memorial Volume', Murakami Y. (Ed)., ISBN 0-7506-0504-9, July, 1991
- [141] De Jonge, J.B., 'The analysis of load-time histories by means of counting methods', in 'Helicopter Fatigue design guide', F. Liard (ed.), AGARD-AG-292, November, 1983
- [142] Dowling, N.E., 'Fatigue Failure Predictions for Complicated Stress-Strain Histories', Report No AD736583, 1972, Defence Technical Information Center
- [143] Downing S.D., Socie D.F., 'Simple Rainflow Counting Algorithms', Int J Fatigue (1982), 4, pp. 31-40

- [144] Glinka G, Kam JC, Rainflow counting algorithm for very long stress histories, *Int. J. Fatigue*, 9(1987), pp 223-228
- [145] Rychlik I, A new definition of the rainflow cycle counting method, *INT. J. Fatigue*, 9(1987), pp 119-121
- [146] Hong N, A modified rainflow counting method, *Int. J. Fatigue*, 13(1991), pp 465-469
- [147] Amzallag C, Gerey JP, Robert JL, Bahuaud J, Standardization of the rainflow counting method for fatigue analysis
- [148] Anthes, R.J., 'Modified rainflow counting keeping the load sequence', *Int. J Fatigue*, 19(1997), pp. 529-535
- [149] Bannantine JA, Socie DF, Multiaxial fatigue life estimation technics, in: *Advances in Fatigue Lifetime Predictive Technics*, ASTM STP 1122, 1991, pp249-275
- [150] Wang CH, Brown MW, Life prediction techniques for variable amplitude multiaxial fatigue part1: Theories, *J. Eng. Mater. Technol.* 118(1996), pp367-370
- [151] Chu CC, A new incremental fatigue method, in: *Fatigue and Fracture Mechanics*, ASTM STP 1389, 2000, pp 67-78
- [152] Langlais TE, Vogel JH, Vhase TR, Multiaxial cycle counting for critical plane methods, *Int. J. Fatigue* 25(2003), pp641-647
- [153] Beste A, Dressler K, Kotzle H, Kruger W, Multiaxial rainflow-A consequent continuation of Professor Tatsuo Endo's work, *Butterworth-Heinemann*, 1992
- [154] Miles JW, On structural fatigue under random loading, *J Aeronautical Sci*, 21(1954), pp753-762
- [155] Bendat J, Probability functions for random responses, *NASA report CR-33* (1964)
- [156] Sakrani S, Michaelov G, Kihl DP, Meach JE, Stochastic fatigue damage accumulation of FRP laminates and joints, *J. Struct. Eng.* 125(1999), pp 1423-1431
- [157] Sakrani S, Michaelov G, Kihl DP, Bonanni DL, Comparative study of nonlinear damage accumulation models in stochastic fatigue of FRP laminates, *J. Struct. Eng.* 127(2001), pp 314-322
- [158] Younesian D, Solhmirzaei A, Gachloo A, Fatigue life estimation of MD36 and MD523 bogies based on damage accumulation and random fatigue theory, *Journal of Mechanical Science and Technology* 23(2009), pp2149-21569
- [159] Yang JN, Statistics of random loading relevant to fatigue, *J. Engineering Mechanics*, 100(1974), pp 469-475
- [160] Krenk S, A double envelop for stochastic processes, *Trans. ASME, J. Basic Engineering*, D-87-2 (1965), pp 398-404
- [161] Winterstein SR, Cornell CA, Fatigue and fracture under stochastic loading, *Proc. 4<sup>th</sup> Int. Conf. On Structural Safety and Reliability*, 3(1985), pp 745-749
- [162] Braccesi C, Cianetti F, Lori G, Pioli D, Fatigue behaviour analysis of mechanical components subject to random bimodal stress process: Frequency domain approach, *Int. J. Fatigue*, 27(2005), pp 335-345
- [163] Wirsching PH, Light MC, Fatigue under wide band random loading, *J. Struct. Div. ASCE*, 106(1980), pp 1593-1607
- [164] Madsen PH, Frandsen S, Hollay WE, Hancen JC, Dynamic analysis of wind turbine rotors for lifetime prediction, *RISO Cotract report 102-43-51*, 1983
- [165] Cycle distribution and fatigue damage under broad-band random loading, *Int. J. Fatigue*, 24(2002), pp 1137-1147
- [166] Ortiz K, Chen NK, Fatigue damage prediction for stationary wideband processes, *Proc. 5<sup>th</sup> Int. Conf. On Application of Statistics and Probability in Soil and Struct. Eng.* 1987
- [167] Lutes LD, Larsen CE, Improved spectral method for variable amplitude fatigue prediction, *J. Struct. Eng.* 116(1990), pp 1149-1164
- [168] Jiao g, Moan t, Probabilistic analysis of fatigue due to Gaussian load processes, *Probabilistic Eng. Mechanics*, 5(1990), pp76-83
- [169] Kihl DP, Sakrani S, Beach JE, Stochastic fatigue damage accumulation under broadband loadings, *Int. J. Fatigue*, 17(1995), pp 321-329
- [170] Rice SO, Mathematical analysis of random noise, *Bell Syt Tech J*, 23(1944), pp282-332, 24(1945), pp 46-156. Reprinted in: *Selected Papers on Noise and Stochastic Processes*, Wax N, ed. (Dover, New York, 1952)
- [171] Wirsching PH, Light MC, Probability based fatigue design criteria for off-shore structures, 1979, *American Petroleum Institute*
- [172] Chaudhury GK, Dover WD, Fatigue analysis of offshore platforms subject to sea wave loadings, *Int. J. Fatigue*, 7(1985), pp 13-19
- [173] Kam LCP, Dover WD, Fast fatigue assessment procedure for offshore structures under random stress history, *Proc. Inst. Civ. Eng. Part II, Research and Theory*, 85(1988), pp 689-700

- [174] Kam JCP, Recent development in the fast corrosion fatigue analysis of offshore structures subject to random wave loading, *Int. J. Fatigue*, 6(1990), pp 458-468
- [175] Abdo ST, Rackwitz R, Discussion to JCP Kam, WD Dover, Fast fatigue assessment procedure for offshore structures under random stress history, *Proc. Of the Institution of Civil Engineers, Part 2*, 87(1989), pp 645-649
- [176] Dirlik T, Application of computer in fatigue analysis, PhD thesis, University of Warwick, 1985
- [177] Bishop NWM, Wang R, Lack L, A frequency domain fatigue predictor for wind turbine blades including deterministic components, 17<sup>th</sup> British Wind Energy Association Conference, Warwick, 1995
- [178] Bishop NWM, Hu Z, Wang R, Quarton D, Methods for the rapid evaluation of fatigue damage on the Howden HWP330 wind turbine, 15<sup>th</sup> British Wind Energy Association Conference, York, 1993
- [179] Sakrani S, Influence of high frequency components on fatigue of welded joints, *Int. J. Fatigue*, 12(1990), pp115-120
- [180] Sakai S, Okamura H, On the distribution of rainflow range for Gaussian random processes with bimodal PSD, *JSME International Journal, Series A*, 38(1995), pp 440-445
- [181] Fu TT, Cebon D, Predicting fatigue lives for bi-modal stress spectral densities, *Int. J. Fatigue*, 22(2000), pp 11-21
- [182] Zhao W, Baker MJ, A new stress range distribution model for fatigue analysis under wave loading, *Environmental Forces of Offshore Structures and their Prediction*, Kluwer Academic Publishers, 1990, pp 271-291
- [183] Bernasciutti D, Tovo R, Spectral methods for lifetime prediction under wide-band stationary random processes, *Int. J. Fatigue* 25(2005), pp 867-877
- [184] Butler D, The distribution of amplitudes and mean values between two neighbouring extremes in a Gaussian broad band random vibration, *English Electric Company*(1965), Technical Memo. W/M (64) u 9075
- [185] Tuna JM, Fatigue life prediction for Gaussian random loads at the design stage, *Fatigue Frct. Engng Mater. Struct.*, 9(1986), pp 169-184
- [186] Tovo R, Cycle distribution and fatigue damage under broad-band random loading, *Int. J. Fatigue*, 24(2002), pp 1137-1147
- [187] Rychlik I, Rain flow cycle distributions for ergodic load processes, *SIAM J. Appl. Math.*, 48(1988), 662-679
- [188] Lindgren G, Rychlik I, Rain flow cycle distributions for fatigue life prediction under Gaussian load process, *Fatigue Fract. Engng Meter. Struct.*, 10(1987), 251-260
- [189] Frendahl M, Rychlik I, Rainflow analysis: Markov method, *Int. J. Fatigue* 15(1993), pp 265-272
- [190] Bishop NWM, Sherratt FS, A theoretical solution for the estimation of rainflow ranges from power spectral density data, *Fatigue Fract. Engng Mater. Struct.*, 13(1990), pp 311-326
- [191] Olagnon M, Practical computation of statistical properties of rainflow counts, *Int. J. Fatigue*, 16(1993), pp 306-314
- [192] Preumont A, Piefort V, Predicting random high-cycle fatigue life with finite elements, *ASME J. Vib. Acoustics*, 116(1994), pp 245-248
- [193] Sines G, Ohgi G, Fatigue criteria under combined stresses or strains, *ASME J. Eng. Mat. Technol.*, 103(1981), pp 82-90
- [194] Pitoiset X, Rychlik I, Preumont A, Spectral methods to estimate local multiaxial fatigue failure for structures undergoing random vibrations, *Fatigue Fract. Engng Mater Struct*, 24(2001), pp 715-727
- [195] Pitoiset X, Preumont A, Spectral methods for multiaxial random fatigue analysis of metallic structures, *Int. J. Fatigue*, 22(2000), pp 541-550
- [196] Dressler Km Kottgen VB, Kozle H, Tools for fatigue evaluation of non-proportional loading, *Proc. Fatigue Design 95, Helsinki Finland 1995*

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