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Abstract—This work extends the theory of a spherical magnetic dipole antenna with magnetic core by numerical results for practical antenna configurations that excite higher-order modes besides the main TE_{10} spherical mode. The multiarm spherical helix (MSH) and the spherical split ring (SSR) antennas are considered. It is shown that one should be particularly aware of the TM_{11} and TM_{20} modes, whose resonances spoil the Q in a range of permeability of the core predicted to be optimal by the single-mode theory. Practical ways of suppressing these higher-order modes are presented.

I. INTRODUCTION

A magnetic material core placed inside an electrically small magnetic dipole antenna — an impressed electric current radiating the TE_{10} spherical mode — decreases the stored magnetic energy inside the antenna, and thus significantly reduces its radiation Q. As shown in [1], for vanishing electrical size of the antenna the internal stored magnetic energy is inversely proportional to the relative permeability μ_{r} of the core, and, consequently, the Q can be written as

$$Q = \left(1 + \frac{2}{\mu_r}\right) Q_{\mathrm{Chu}}, \ ka \ll 1 \tag{1}$$

where k is the free-space wave number, a is the smallest sphere circumscribing the antenna, and Q_{Chu} is the Chu lower bound [2]

$$Q_{\mathrm{Chu}} = \frac{1}{(ka)^3} + \frac{1}{ka}, \ ka \ll 1. \tag{2}$$

Valid for vanishing electrical size of the antenna, the expression (1), however, is not directly applicable for finite values of ka due to internal resonances in the magnetic core. From an exact analytical expression derived in [3] for an arbitrary value of ka, assuming an impressed electric current on a spherical surface of radius a, it follows that the lowest Q is obtained not with the infinitely high permeability, but with an optimum permeability $\mu_{\mathrm{r}}^{\mathrm{opt}}$ satisfying the relation

$$\mu_{\mathrm{r}}^{\mathrm{opt}} = \frac{1}{\varepsilon_r} \frac{(2.816)^2}{(ka)^2} \tag{3}$$

where ε_{r} is the relative dielectric permittivity of the core. The optimum permeability corresponds to the core electrical radius $k_s a = \sqrt{\mu_r \varepsilon_r} ka = 2.816$, which is half-way below the first TE_{10}-mode resonance at $k_s a = 4.493$. In real antenna configurations, however, other modes whose resonances are within the range $k_s a < 4.493$ should also be taken into account. These are the $\mathrm{TM}_{1\mathrm{m}}$ modes with the resonance at $k_s a = 2.744$ and the $\mathrm{TM}_{2\mathrm{m}}$ modes with the resonance at $k_s a = 3.870$. As shown in [3], [4], if a TE_{10}-mode antenna excites the TM_{11} and TM_{20} parasitic modes, their resonances in the magnetic core destroy the nice $Q(\mu_{\mathrm{r}})$ beahviour predicted by the theory [3].

In this paper, the effect of each of the parasitic modes is isolated using two TE_{10}-mode antenna configurations free of the $\mathrm{TM}_{1\mathrm{m}}$ and $\mathrm{TM}_{2\mathrm{m}}$ modes, respectively. Practical ways to overcome the parasitic mode problems and approach the idealized case, and thus the Chu lower bound, with magentic dipole antennas are presented and discussed.

II. EXACT THEORY OF A TE_{10}-MODE ANTENNA WITH A MAGNETO-DIELECTRIC CORE

In this section, we briefly revisit main results of the exact theory of an ideal TE_{10}-mode antenna with a magneto-dielectric core presented in [3], [5]. The theory derives an exact analytical expression for the quality factor Q for an impressed electric current density on a spherical surface of radius a as

$$Q = Q_{\mathrm{Chu}} + Q_{\mathrm{TE}_{10}}^- = Q_{\mathrm{Chu}} + |C|^2 \hat{Q}_{\mathrm{TE}_{10}}^- \tag{4}$$

where $Q_{\mathrm{TE}_{10}}^-$ represents a contribution of the internal magnetic stored energy to the gross Q,

$$C = -\sqrt[4]{\frac{\varepsilon_{\mathrm{r}}}{\mu_{\mathrm{r}}}} \frac{1 + i/ka}{(k_s a) j_1(k_s a)} \tag{5}$$

and

$$\hat{Q}_{\mathrm{TE}_{10}}^- = -(k_s a) j_1^2(k_s a) + \frac{(k_s a)^2}{2} j_0(k_s a) y_0(k_s a) + \frac{k_s a}{2}. \tag{6}$$

Whereas the function $\hat{Q}_{\mathrm{TE}_{10}}^-(k_s a)$ is well-behaved and non-singular, the amplitude C goes to infinity at zeroes of the spherical Bessel function $j_1(k_s a)$. These values of $k_s a$ correspond to the internal TE_{10}-mode resonances in the magneto-dielectric spherical core, at which no radiation occur, and thus the quality factor Q is infinite. The typical behavior of Q versus the core permeability μ_{r} is illustrated in Fig. 1 for $ka = 0.254$ and three values of the core permittivity $\varepsilon_{\mathrm{r}} = 1$, 2, 8. The lowest Q in each case is achieved in the minimum below the first resonance. Thus, for a given antenna size ka and a core permittivity ε_{r} there is a minimum possible value of $Q = Q_{\mathrm{min}}(ka, \varepsilon_{\mathrm{r}})$ and a corresponding optimum value of $\mu_{\mathrm{r}} = \mu_{\mathrm{r}}^{\mathrm{opt}}$, which satisfies the relation in (3). The function
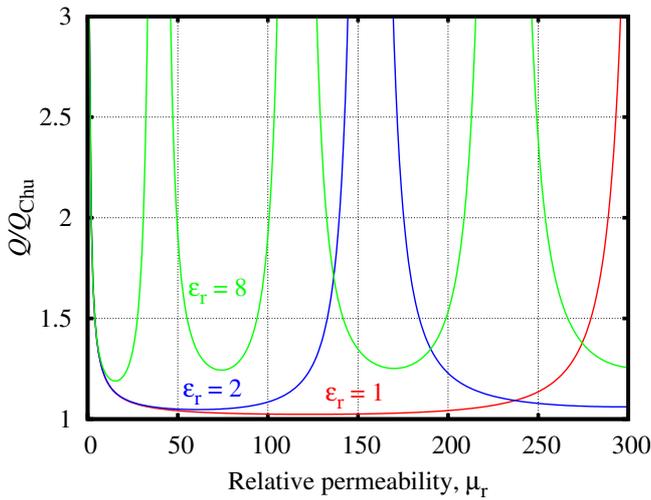
Fig. 1. Theoretical ratio Q/Q_{Chu} versus the core permeability μ_r for $ka = 0.254$ and three values of the core permittivity $\varepsilon_r = 1, 2, 8$.

$Q_{min}(ka, \varepsilon_r = 1)$ is a new lower bound for electrically small magnetic dipole antennas with magneto-dielectric core [3].

III. HIGHER-ORDER MODES IN REALISTIC ANTENNAS

In this section, results of numerical investigation of higher-order modes in realistic magnetic dipole antennas are presented; effects of the higher-order modes on the antenna quality factor Q are identified and discussed. The antennas consist of perfectly conducting (PEC) 1 mm wires wounded on a spherical surface of radius $r_0 = 40$ mm. The core of radius $r_c = 39$ mm is composed of a homogeneous lossless dispersion-less magneto-dielectric material. At the operating frequency $f_0 = 300$ MHz the electrical size of the antennas is $ka = 2\pi(r_0 + 0.5\,\text{mm}) = 0.254$. For the numerical simulations, the surface integral equation technique and the higher-order method of moments have been employed [6]. A delta-gap voltage generator is used to model the antenna excitation.

A. Multiarm Spherical Helix Antenna Excited by a Curved Dipole

The concept of a multiarm spherical helix (MSH) antenna radiating the TE_{10} spherical mode was first presented in [7]. Two symmetric hemispheres composed of multiarm wire helices are excited by a curved dipole with the generator in its mid-point (Fig. 2a). The lengths of the helical arms are adjusted so that the resonance frequency is kept constant for all values of μ_r. Besides the main TE_{10} mode, the antenna also radiates the TM_{11} and TM_{20} modes; and although these modes are relatively weak (< -20 dB for $\mu_r = 1$), their excitation dramatically degrades the quality factor Q, when a magnetic core is placed inside the MSH antenna. In Fig. 3, the Q/Q_{Chu} ratio for the ideal case described in Section II is compared to the numerical results for the MSH antenna with magnetic core [3]. The theory predicts that as μ_r increases the ratio Q/Q_{Chu} sharply drops below 1.05, then it stays at this low level in a broad range of permeability between $\mu_r \simeq 50$
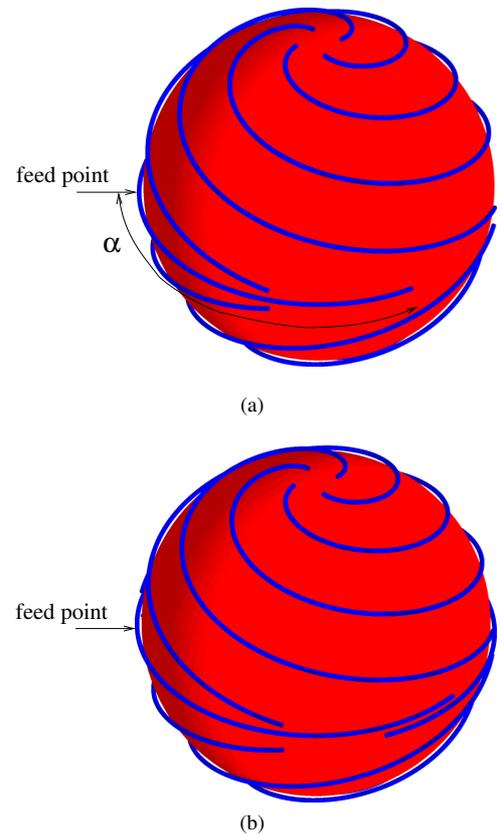

Fig. 2. Multiarm spherical helix (MSH) antenna with magnetic core excited by (a) a curved dipole [3] and (b) a split-ring resonator (SRR).
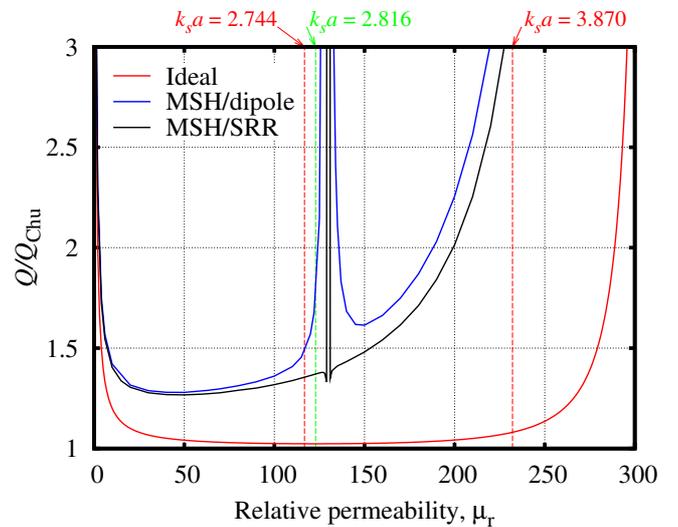

Fig. 3. Ratio Q/Q_{Chu} for an ideal TE_{10}-mode antenna and numerical results for the MSH antenna with dipole and SRR excitation.

and $\mu_r \simeq 200$, and finally rises again as μ_r approaches the value corresponding to the TE_{10}-mode resonance. In a close agreement with the theory the ratio Q/Q_{Chu} for the MSH antenna initially drops as μ_r increases. Then, however, the first spike occurs close to $\mu_r = 130$, or $k_s r_c = 2.8$, that is the
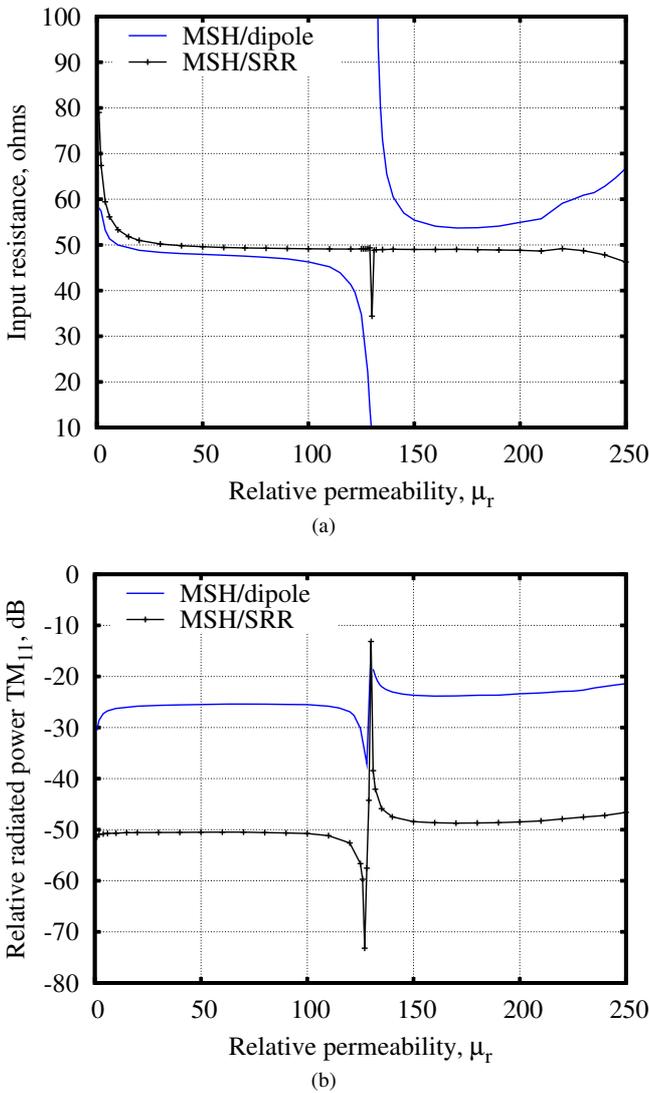
Fig. 4. Input resistance at resonance (a) and the relative radiated power of the TM_{11} mode (b) of the MSH antenna with magnetic core excited by a curved dipole and an SRR.

TM_{11} resonance in the magnetic core. Besides this resonance, the global behavior of the ratio Q/Q_{Chu} for the MSH antenna also differs from the theory — after reaching the minimum at $\mu_r = 45$, the Q/Q_{Chu} monotonically grows towards the TM_{20} resonance.

In the following, we consider two antenna configurations, in which either the TM_{11} mode or the TM_{20} mode is efficiently suppressed, and, thus, elucidate the effect of each of these modes on the antenna Q.

B. Multiarm Spherical Helix Antenna Excited by an SRR

To identify the source of the parasitic TM_{11} mode, an another excitation scheme for the MSH antenna in free space has been numerically modelled. It was found that the TM_{11} mode could be significantly suppressed (< -60 dB) by replacing the curved dipole with a closed loop. On the other hand, parasitic TE_{n0} modes with odd n become more pronounced,

which involves an increase in the stored magnetic energy, and, consequently, a higher Q. Moreover, the input resistance at resonance drops to a value that makes this configuration impractical.

An excellent alternative comprising advantages of the curved dipole and the loop is a split ring resonator (SRR). It allows us to tune the antenna to a desired input resistance at resonance, as the curved dipole does [7]–[9], and at the same time it provides a continuous loop current at the antenna equatorial plane, which ensures the TM_{11} mode suppression. The geometry of the MSH antenna with the SRR excitation and material core is sketched in Fig. 2b. Split rings constituting the SRR are identical and their length, quantified in angle units, is chosen so that the input resistance at resonance is close to 50 ohms in a large part of the core permeability range (Fig. 4a). As it is seen in Fig. 4b, the TM_{11} is significantly suppressed as compared to the MSH antenna with the dipole excitation, and although the TM_{11}-mode resonance is not entirely eliminated, its influence on the ratio Q/Q_{Chu} is highly localized (Fig. 3). In general, the main tendencies in the behavior of Q/Q_{Chu} remain the same, which leads us to a conclusion that the effect of the TM_{11} mode is limited to a range of core permeability in the vicinity of the TM_{11}-mode resonance.
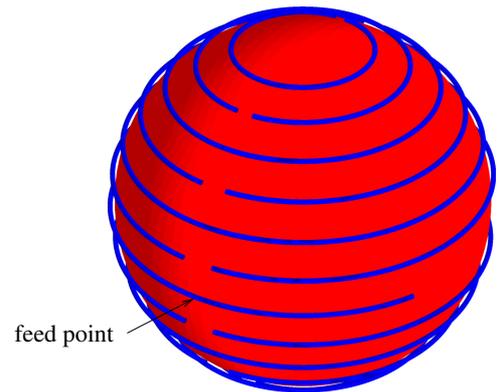

Fig. 5. Spherical split ring (SSR) antenna with magnetic core.

C. Spherical Split Ring Antenna

The spherical split ring antenna (SSR), first presented in [7], is composed of multiple split rings on a spherical surface (Fig. 5). The central split ring serves as an excitation dipole, whose length defines the antenna input resistance at resonance. The resonance frequency is governed by both the number of split rings and a gap width in them. Here, we set the number of split rings to 17 and tune the resonance frequency to $f_0 = 300$ MHz for a given μ_r by adjusting the gap width. As in the previous case, the length of the excitation dipole is chosen to ensure close to 50 ohms input resistance at resonance.

The SSR antenna is free of the TM_{20} mode, and this drastically improves the behavior of the ratio Q/Q_{Chu} versus core permeability μ_r (Fig. 6). Besides the narrow spike corresponding to the TM_{11}-mode resonance, the Q/Q_{Chu} characteristic

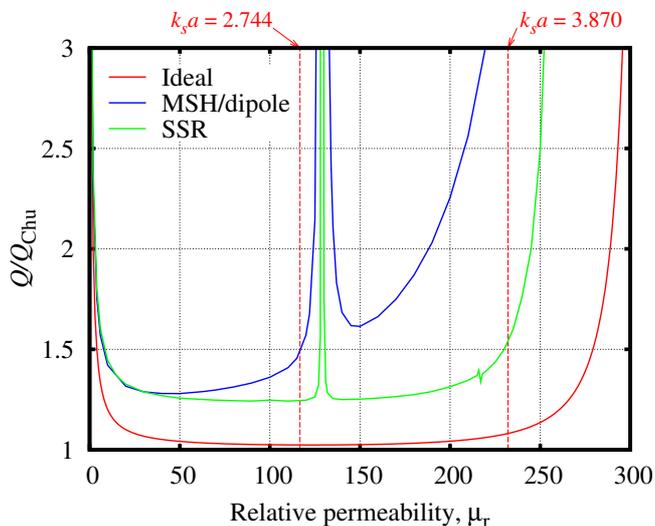| Antenna/Excitation	Free Space			Magnetic Core	
	TM_{11}	TM_{20}	Q/Q_{Chu}	μ_r	Q_{min}/Q_{Chu}
MSH/dipole	-30.6 dB	-23.0 dB	3.40	≈ 45	1.28
MSH/SRR	-51.4 dB	-23.1 dB	3.35	≈ 50	1.27
SSR/dipole	-31.6 dB	—	3.54	≈ 90	1.24

Fig. 6. Ratio Q/Q_{Chu} for the spherical split ring (SSR) antenna with magnetic core.

for the SSR antenna resembles the theoretical one with a flat low level between $\mu_r \simeq 50$ and $\mu_r \simeq 200$. An offset between the theoretical curve and the curve for the SSR antenna is attributed to a 1-mm gap between the wires (wounded at radius $r_0 = 40$ mm) and the core surface ($r_c = 39$ mm) as well as to the interspaces between wires, where the internal stored energy is not expelled by the magnetic core.

IV. CONCLUSIONS

Practical configurations of electrically small spherical magnetic dipole antennas with magnetic core are numerically investigated. In particular, we focus on the dependence of the antenna Q on the permeability μ_r of the magnetic core. It is shown that higher-order modes can significantly degrade the Q characteristic, as compared to the single TE_{10}-mode theory, not only in the vicinity of the corresponding resonances, but also away from them. This is due extra stored energy, besides that necessary to make an antenna resonance, brought to the system by higher-order modes.

Numerical results presented in this paper reveal that the TM_{11} mode has a very localized effect on the $Q(\mu_r)$ characteristic, whereas the influence of the TM_{20} mode can extend far beyond the resonance region. Consequently, there should be especially taken care of the latter one. Table I summarizes the results for the MSH and SSR antennas considered in this paper.

In our future work we will investigate the influence of an air gap between the impressed electric currents and the surface of the magnetic core on the antenna quality factor Q.

ACKNOWLEDGMENT

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