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VECTOR SPACE REPRESENTATION OF ARRAY ANTENNA PATTERN SYNTHESIS PROBLEMS

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Abstract
Considering the difficulties of the non-linear optimization methods for the array and array fed reflector antenna power pattern synthesis problems, a novel approach has been developed to present the problem in a geographical way, which allows to derive efficient algorithms and to visualise the optimization process.

Introduction
Although many computational methods for array or array fed reflector pattern synthesis have been developed, some difficulties are still present with regard to main beam shaping, sidelobe suppression, cross-polar isolation, wide-band design, beam efficiency, non-super-gain solutions, local optimum solutions, etc. The main reason is probably that the synthesis problem is difficult to represent in a clear and general way which may cover all the problems mentioned above. This paper is devoted to the derivation of a vectorial representation of the problem, a separate paper [1] will deal with the optimization algorithm.

Vector Space Representation
In the case of fixed array or multifeed reflector configuration where only the element excitations (not their positions) can be adjusted, the synthesis procedure can be separated from the analysis [2][3]. The field in a far field sampling direction j is

$$E_j = <e_j, a^*> = \sum_{i=1}^{n} e_{ji} \ a_i \qquad (j=1, m) \qquad (1)$$

where m is the number of far field sampling directions and n is the number of elements. e_j (j=1,m) are the field vectors in n-dimensional Hilbert space. They characterise the radiation system and is obtained by the analysis procedure. Once the array structure is fixed, e_j's are fixed. a is the complex excitation vector. The synthesis problem is then to find the best excitation vector a according to a certain pattern specification.

In most applications, only the field magnitude (or power) pattern is of interest. The following discussion will concentrate on this kind of

application only. From equation (1), the field magnitude can be expressed as:

$$|E_j| = \; <F_j, a^*> \qquad\qquad (j=1, m) \qquad (2)$$

where $\qquad F_j = e_j <e_j, a^*>^* \; / \; | <e_j, a^*> |. \qquad\qquad (3)$

The right hand side of (2) is similar to that of (1) but the field vector e_j is replaced by a new vector F_j defined in (3) which has the same norm as e_j with a rotation related to the current direction of a. In the following discussion, F_j will be called field vector still.

Representation of the General Synthesis Problem
Equation (2) shows that the field magnitude is an inner productor between two vectors. If a is normalized for unit total input power, the projection length of F_j on a represents the field magnitude $|E_j|$. The synthesis problem is then to find the direction of the excitation vector a to get the right projection lengths from F_j's ($j=1,m$) weighted according to the specified pattern function. If the inner productor is real, the Euclidean angle Φ_j between the two vectors F_j and a^* is given by:

$$\cos \Phi_j = \; <F_j, a^*> \; / \; (||F_j|| \; ||a||). \qquad\qquad (4)$$

The problem can be represented graphically now. Fig.1 shows a simple case where three field vectors sampled from the main beam direction are F_1, F_2 and F_3. The projection lengths on a must be optimized to be equal (different main beam shape will be discussed later). The dashed vectors represent an initial situation where a is not in an optimum direction because the projection length from F_1 is very small. The solid vectors represent the best solution of this case where $|E_1|, |E_2|$ and $|E_3|$ are equal.

Necessary condition of the optimum solution
Based on the above representation the necessary condition of an optimum solution is given as the follows: An optimum excitation vector a must be normal to the super plane which is defined by the extremities of active field vectors. In the corresponding directions, the field magnitudes are then equal or weighted as required.

Sufficient Condition For Global Optimum Solution
The above condition will give a solution where the shape of the pattern agrees with the desired one but the gain of the antenna may not be maximized. Since the field vector F_j is rotating while the excitation vector a is changing direction, the super plane defined by a set of active field vectors is also changing. As mentioned above, at an optimum solution, the projections of all the active field vectors on a will be at the same point, that is the intersection point of a (or the extension of a, if a is normalized) and the super plane, see Fig.2. The

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distance between this point and the origin is also the distance between the super plane and the origin since **a** is in the normal direction to the plane. Therefore the sufficient and also the necessary condition of the global optimum solution is: The global optimum excitation vector **a** is in the direction of the normal to the super plane (as defined above) which has the biggest distance from the origin.

The sufficient condition of the global optimum solution reflects the non-linearity of the problem. One method may equalize the field magnitudes at all the sampling directions, which means the synthesized pattern has the exact value as the desired one at those directions, but it may not yield the best gain performance.
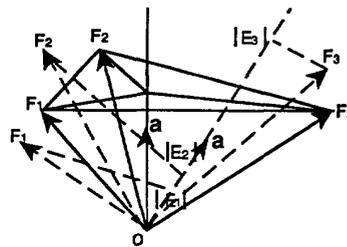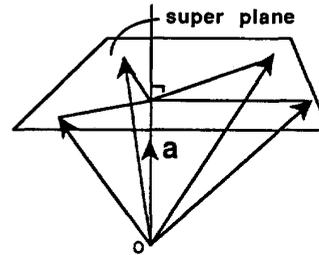

Fig. 1   Fig.2

Representation of Different Synthesis Parameters
1. Beam shaping: Different desired pattern values will be used to weigh the corresponding field vector's norms. The extremities of active field vectors which define the super plane will have the exact specified value after the optimization. The rest (if there is any) will be above the plane on the other side respect to the origin according to a maximin design concept [4][5], see Fig.3, or equally distributed on both sides of the plane according to the minimax design concept[3][6].
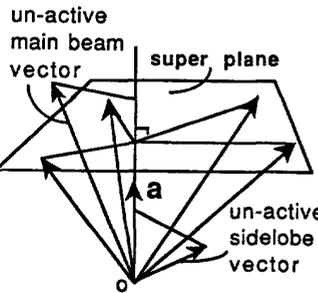

Fig.3

2. Sidelobe suppression: Wanted sidelobe levels will be used to weigh the corresponding sidelobe region field vector's norms. Their extremities will be either in the super plane or below, on the same side as the origin, see Fig.3.
3. Cross-polar suppression: Field vectors can be calculated at the far-field sampling directions where cross-polar level are specified. Those cross-polar field vectors will be treated as the sidelobe region field vectors.

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4. Wide-band design: Different field vectors have to be calculated at different frequencies in the band. The number of vectors will be multiplied by the number of frequencies.
5. Main beam efficiency and non-super-gain solution: Those two aspects will be fulfilled automatically by optimizing directly the field value of equation (2) instead of an error function or the directivity of the array.

Conclusions
The vector space approach is very powerful to represent the array pattern synthesis problems. It is not only general since many parameters are represented under one model, but also helps to visualize the problem. The proposed approach also provides a new support for optimization algorithms. Related developments will be found in a separate paper.

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