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An Analysis Of Pole/zero Cancellation In LTR-based Feedback Design

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ABSTRACT.
The pole/zero cancellation in LTR-based feedback design will be analyzed for both full-order as well as minimal-order observers. The asymptotic behaviour of the sensitivity function from the LTR-procedure are given in explicit expressions in the case when a zero is not cancelled by an equivalent pole. It will be shown that the non-minimum phase case is included as a special case.
The results are not based on any specific LTR-method.

1. INTRODUCTION.

The main inspiration to this paper came from the two papers written by Z. Zhang and J.S. Freudenberg [1,2] which deal with an analysis of Loop Transfer Recovery for non-minimum phase plants when the LQG/LTR-method is used.
The LQG/LTR method used on non-minimum phase plants has been treated in an earlier paper by Stein and Athans [5], special the SISO case.

The target design has a more central role in the LTR-design, when it is used on a non-minimum phase plant. In the minimum-phase case we have asymptotic recovery [3,4,5], which make it possible to recovery a target loop to any prescribed level. The final controller will, of cause, depend on the target design [9]. For non-minimum phase plants, where asymptotic recovery normally doesn't exist, the resulting difference between the target and the full-loop transfer function, i.e. the recovery error, will depend of the target design [1,2,5,9]. It is therefore important to select the target design careful so the recovery error is minimized. Zhang and Freudenberg [2] have given a loop shaping design method for non-minimum phase plants. They have noted that it is possible to recovery the state-feedback properties exactly in the directions that are orthogonal to the RHP-zero directions, which have been used in the loop-shaping method.

In this paper it will be assumed that we have a target design with good properties in regard to the LTR-design. We will instead look at the LTR-step and give an analysis of the recovery error when a zero isn't cancelled by a pole in the controller. The non-minimum phase case is included in the analysis as a special case. The analysis is only treaded in the case when <u>one</u> zero isn't cancelled by a pole, which will make the equations more simple.

A RHP-zero z_1 is, of cause, impossible to cancel by a pole from a stable LTR-controller. When the LQG/LTR-solution is used [1,2] the controller will include a pole at $p_1 = -z_1$. However, if e.g. the eigenstructure assignment-LTR method [6] is used, the pole p_1 will be free to place in the LHP. This freedom in the selection of z_1 will be analyzed and explicit equations for the recovery error as function of p_1 will be derived for both full-order observers in section 3 as well as minimal-order observers in section 4..

2. THE LTR-METHODOLOGY.

Let the open loop plant G(s) have a minimal state-space realization S(A,B,C). Hence

$$G(s)=C\Phi(s)B, \quad \Phi(s)=(Is-A)^{-1} \qquad (2-1)$$

dim G(s) = m x r.

Let the state-feedback gain be denoted K, then the target loop shape (i.e. the loop to be recovered) is $G_{TFL}(s) = K\Phi(s)B$. If G(s) is minimum phase then [3,4]:

$$G_l(s) = C(s)G(s) \rightarrow K\Phi(s)B \qquad (2-2)$$

if

$$F(q) \rightarrow qBW \quad as \quad q \rightarrow \infty$$

$$\det(W) \neq 0, q \in R^+$$

where F is the full-order observer gain, C(s) is the model-based compensator and q is denoted the recovery parameter.
Following (2-2) $G_{TFL}(s)$ can be recovered

to any prescribed degree of accuracy. The LTR objective can be satisfied by a number of methods, e.g. LQG/LTR [3,4,5], eigenstructure method [6] or the method by Saberi and Sannuti [7].
Notice that eq. (2-2) is in practice never exactly achieveble and the result is a finite recovery error. Let this mismatch be denoted $E_I(s)$

$$E_I(s) = K\Phi(s)B - C(s)G(s) \qquad (2\text{-}3)$$

It is then easy to derive that [8]:

$$E_I = M_I(I+M_I)^{-1}(I+K\Phi(s)B) \qquad (2\text{-}4)$$

where the recovery matrix $M_I(s)$ is given by:

$$M_I(s) = K(Is-A+FC)^{-1}B$$

The prime interest of the designs lies in the input sensitivity function $S_I(s)$ for the full-loop. By using eq. (2-3) and (2-4) it is possible to express $S_I(s)$ as function of the sensitivity function for the target design $S_{TFL}(s)$. The following theorem can now be obtained:

Theorem 2.1.
Let the sensitivity functions for the target and the full-loop transfer function be given by:
$S_{TFL}(s) = (I+G_{TFL}(s))^{-1}$ and $S_I = (I+G_I(s))^{-1}$.
Then

$$S_I(s) = S_{TFL}(s)(I+M_I) \qquad (2\text{-}5)$$

Proof. Follows by simple manipulations of eq. (2-3) and (2-4), see [9,17].
Theorem 2.1 is very general, because it include also the case when a Luenberger observer is used, see [17], as well as the discrete-time case can also be described by an equivalent expression, [10,11]. In sec. 3 and 4 theorem 2.1 will be used in the analysis of pole/zero cancellation in LTR-based feedback design.

3. POLE/ZERO CANCELLATION.

For analysing the pole/zero cancellation in LTR-design, or to be more precise, to analyze the performance of LTR-based feedback designs when a zero is not cancelled by a pole in the controller.
For this analysis we will let the plant $G(s) = C(Is-A)^{-1}B$ be factorized into the following form:

$$G(s) = C(Is-A)^{-1}B_mB_z(s) \qquad (3\text{-}1)$$

$$G(s) = G_m(s)B_z(s)$$

where $G_m(s)$ is a minimum phase plant and $B_z(s)$ is a stable factor.
Eq. (3-1) include the case when a real RHP-zero z is placed at -z in the stable half plane. The equations for B_m and $B_z(s)$ are then given by [2]:

$$B_m = B - 2z\xi\eta^T \qquad (3\text{-}2)$$

$$B_z(s) = I - \frac{2z}{s+z}\eta\eta^T$$

The vectores ξ and η are the solutions of:

$$\begin{pmatrix} zI-A & -B \\ -C & 0 \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix} = 0, \quad \eta^T\eta = I$$

This factorization is an all-pass factorization, since $G(s)G(-s)^T = G_m(s)G_m(-s)^T$ and $B_z(s)B_z(-s)^T = I$. More general equations for B_m and $B_z(s)$ are derived for plants with more than one RHP-zero in [2].

Now let the sufficient LTR-condition in eq. (2-2) be used on $G_m(s)$:

$$F(q) \to qB_mW \text{ as } q \to \infty \qquad (3\text{-}3)$$

Out from this equation, it is possible to prove that there exist the following connection between the sensitivity functions for the full-loop S_I and the target-loop S_{TFL}, [1,2]:

Lemma 3.1.
Lets apply the recovery condition in eq. (3-3) on the plant G(s). The connection between S_I and S_{TFL} is given by:

$$S_I(s) = S_{TFL}(s)(I+E_S(s)) \qquad (3\text{-}4)$$

where $E_S(s)$ is given by the simple equation:

$$E_S(s) = K(Is-A)^{-1}(B-B_mB_z(s)) \qquad (3\text{-}5)$$

This result is similar in spirit to eq. (2-5), but eq. (2-5) from [9] is more general. This is shown in the next theorem.

Theorem 3.1.
Let the plant be factorized as in eq. (3-1) and let the observer gain F be selected as:

$$F(q) \to qB_MW \text{ as } q \to \infty$$

then

$$M_I(s) \to E_S(s) \text{ as } q \to \infty \qquad (3\text{-}6)$$

Proof. See appendix A.

E_S will therefore also be named the recovery matrix in the rest of this paper.

The general description by using the recovery matrix M_l in theorem 2.1 describe both the asymptotic properties of LTR-design as studied in [9,17] and the effect from plant zeros, as f.ex. RHP-zeros, which haven't been cancelled by controller poles. However, it is only the effect from one missing pole/zero cancellation we want to analyze in this paper.

First, let the factorization in eq. (3-1) with B_m and $B_z(s)$ given by eq. (3-2) and (3-3) be generalized into a more useful form, for this analysis, as done in lemma 3.2.

Lemma 3.2.
Let the plant G(s) have the zeros: $z_1,\cdots,z_l$, and let $G_m(s)$ have the zeros: $z_{1m},z_2,\cdots,z_l$, where z_1 is real and z_{1m} is real and placed in the LHP. B_m and $B_z(s)$ are given by:

$$B_m = (B-(z_1-z_{1m})\xi\eta^T)\left|\frac{z_1}{z_{1m}}\right| \qquad (3\text{-}7)$$

$$B_z(s) = \left(I-\frac{z_1-z_{1m}}{s-z_{1m}}\eta\eta^T\right)\left|\frac{z_{1m}}{z_1}\right| \qquad (3\text{-}8)$$

where

$$\begin{pmatrix} z_1 I-A & -B \\ -c & 0 \end{pmatrix}\begin{pmatrix}\xi \\ \eta\end{pmatrix} = 0, \quad \eta^T\eta = I$$

Proof. Followed by a simple extension of eq. (3-2) and (3-3).

Note that the all-pass factorization is a special case of lemma 3.2. When $z_1 \neq -z_{1m}$, $G(s)$ and $G_m(s)$ wouldn't satified: $G(s)G(-s)^T = G_m(s)G_m(-s)^T$ as in the all-pass case.
Now lets use the factorization given by lemma 3.2 in the equation for the recovery error $E_S(s)$ in lemma 3.1 which will result in the following theorem:

Theorem 3.2.
Let the plant transfer function be given by $G(s) = C(Is-A)^{-1}B_mB_z(s)$ with B_m and $B_z(s)$ defined in lemma 3.2. The recovery matrix $E_S(s)$ defined in lemma 3.1 is then given by:

$$E_S(s) = \frac{z_1-z_{1m}}{s-z_{1m}}K(Iz_1-A)^{-1}B\eta\eta^T \qquad (3\text{-}9)$$

Proof. See [2] or appendix B.

It is clear that we must select $z_{1m}=z_1$ if z_1 is a LHP-zero, which will result in exact recovery, i.e. $E_S(s)=0$.
When z_1 is a RHP-zero, the selection of z_{1m} isn't quite clear. It is well known that the LQG/LTR-solution will place the pole at $-z_1$, [12]. It can simply be shown that the LQG/LTR method will have the following properties on $E_S(s)$:

Lemma 3.3.
Let the recovery error be given by theorem 3.2. The optimal choice for the pole z_{1m} in the LTR-design which will minimize E_S over the entire frequency range are:

$$z_{1m} = \pm z_1 \qquad (3\text{-}10)$$

The LQG/LTR solution will give $z_{1m}=z_1$ for minimum phase systems which result in exact recovery and $z_{1m}=-z_1$ for non-minimum phase systems which result in the following recovery matrix:

$$E_S(s) = \frac{2z_1}{s+z_1}K(Iz_1-A)^{-1}B\eta\eta^T \qquad (3\text{-}11)$$

The proof is simple and is therefore omitted here.

From lemma 3.3 we can see that E_S is nearly constant for $\omega < z_1$ and given by:

$$E_S(j\omega) \approx 2K(Iz_1-A)^{-1}B, \quad \omega<z_1 \qquad (3\text{-}12)$$

and it goes against zero for $\omega>z_1$. The recovery error will be placed at low frequencies op to $\omega=z_1$ when the LQG/LTR-solution is used on non-minimum phase plants. The recovery matrix is strong dependent on the RHP-zero z_1. A bad placed RHP-zero in connection with the target design K can result in large recovery error, which can spoile the entire design. In [2], loop-shape methods for the target design have been derived which try to minimize the effect from the RHP-zero on the final design.

Lets look again at the LTR-design. The LQG/LTR-method isn't the only one, a good alternative is the eigenstructure assignment LTR-method, ES-LTR, [6]. We have a freedom in the ES-LTR-method to place some of the poles in the observer which include z_{1m}. The zero z_{1m} is free to place in the LHP. Now let z_{1m} be placed at 0 and $z_{1m} \to -\infty$ which give the recovery matrix defined in lemma 3.4.

Lemma 3.4.
The recovery matrix is given by:

$$E_S(s) = \frac{z_1}{s}K(Iz_1-A)^{-1}B\eta\eta^T \qquad (3\text{-}13)$$

when $z_{1m}=0$ and

$$E_S(s) = K(Iz_1-A)^{-1}B\eta\eta^T \qquad (3\text{-}14)$$

when $z_{1m}\to-\infty$.
Proof. Eq. (3-13) and (3-14) follows directly of theorem 3.2.

When z_{1m} is placed at origo, the recovery matrix will goes agains infinity at low frequencies, which is in the most cases unacceptable. On the other hand if z_{1m} go against minus infinity the recovery matrix will be constant and "equal" to the half of E_s for the LQG/LTR-solution in eq. (3-12) at low frequencies. By placing z_{1m} at $-\infty$ we can reduce the recovery matrix at low frequencies in return for an increasing recovery matrix at high frequencies in connection with the recovery matrix from the LQG/LTR-solution. So, the placing of z_{1m} is a trade-off between which frequency-ranges the recovery matrix should be minimized.
At last in this section it will be shown how it is possible to make asymptotic recovery for non-minimum phase plants.

Lemma 3.5.
Let the system G(s) be a non-minimum phase system with the RHP-zero z_1 and the eigenvector η. If the target loop $G_{TFL}(s) = K\Phi(s)B$ include a RHP-zero at z_1 with a eigenvector η, asymptotic recovery can be achieve, and z_{1m} will be free to select.

Proof. See [10,15,17].

4.0 LTR with minimal-order observers.

In this section the results from section 2 and 3 will be extended to include LTR with minimal-order observers.
In the following the notation of minimal-order observers is briefly introduced [13].

Let S(A,B,C) be partitioned as:

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \begin{matrix} m \\ n-m \end{matrix} \qquad (4-1)$$

$$C = (I_m \quad 0)$$

There is no loos of generallity of assuming that $C=(I_m \;\; 0)$, since any system can be transformed into this form. The minimal order observer for the system in eq. (4-1) is [14]:

$$\dot{z}(t) = Dz(t) + Gu(t) + Ey(t)$$

$$\hat{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} I & 0_m \\ V_2 & I_{n-m} \end{pmatrix} \begin{pmatrix} y(t) \\ z(t) \end{pmatrix} \qquad (4-2)$$

with

$$D = A_{22} - V_2 A_{12}$$

$$G = B_2 - V_2 B_1$$

$$E = A_{12} - V_2 A_{11} + A_{22} V_2 - V_2 A_{12} V_2$$

and V_2 is the observer gain matrix.

The feedback law is:

$$u(t) = -K\hat{x}(t) = -K_1 x_1(t) - K_2 \hat{x}_2(t) \qquad (4-3)$$

It is assumed that (C,A) is observable, which implies that (A_{12}, A_{22}) is observable [14].

Let the loop be broken at the plant input, then the open loop transfer functions are as follows [13]:

$$G(s) = \Phi_{11} B_1 + \Phi_{11} A_{12} (\Phi_{22}^{-1} - A_{21} \Phi_{11} A_{12})^{-1} (A_{21} \Phi_{11} B_1 + B_2)$$
$$= \Phi_{11} B_1 + \Phi_{11} A_{12} L \qquad (4-4)$$

$$G_{TFL}(s) = K_1 \Phi_{11} B_1 + K_1 \Phi_{11} A_{12} L + K_2 L \qquad (4-5)$$

$$G_I(s) = C(s)G(s)$$
$$= \left\{ I + K_2 (\Phi_{22}^{-1} + V_2 A_{12})^{-1} (B_1 - V_2 B_2) \right\}^{-1} (K_1 + K_2 (\Phi_{22}^{-1} + V_2 A_{12})^{-1} (V_2 \Phi_{11}^{-1} + A_{21}) (\Phi_{11} B_1 + \Phi_{11} A_{12} L) \qquad (4-6)$$

with
$\Phi_{11}(s) = (Is - A_{11})^{-1}$ and $\Phi_{22}(s) = (Is - A_{22})^{-1}$
Out from these three open-loop transfer functions the following lemma can be derived.

Lemma 4.1.
Let the plant be controlled by a minimal-order observer together with a state-feedback. The recovery error is then given by:

$$E_R(s) = G_{TFL}(s) - G_I(s)$$
$$= M_R(s)(I + M_R(s))^{-1}(I + G_{TFL}) \qquad (4-7)$$

where the recovery matrix M_R is given by:

$$M_R(s) = K_2 (Is - A_{22} + HA_{12})^{-1} (B_1 - HB_2).$$

Proof. See [17].

The recovery error defined in lemma 4.1 is an extension of eq. (2-4) for the full-order observer case. Based on lemma 4.1 and the proof of theorem 2.1 the following theorem can be derived.

Theorem 4.1.
Let the recovery error be defined by lemma 4.1 with $M_R(s)$ as the recovery matrix. The connection between the sensitivity functions $S_I(s)$ and $S_{TFL}(s)$ is then given by:

$$S_I(s) = S_{TFL}(s)(I + M_R(s)) \qquad (4-8)$$

Proof. It follows directly from theorem 2.1.

The results from the LTR-design of full-order observers in sec. 3 will now be extended to minimal-order observers. In the rest of this section, it will be assumed that $B_1 = 0$, if anything else is said. Results for the more general case with B_1 arbitrary haven't been derived at this time, but we are still working on it.

Theorem 4.2.
Let the system be rewritten as $G(s) = C(Is-A)^{-1}B_m B_z(s)$ and it is assumed that $C = (I_m \quad 0)$ and $B_1 = 0$. The recovery matrix defined by $E_S(s)$ in lemme 3.1 can now be rewritten as:

$$E_S(s) = K_2(Is-A_{22})^{-1}(B_2 - B_{2m}B_{2z}(s)) \quad (4-9)$$

The proof is omitted here.

Based on theorem 3.1, 4.1, 4.2, lemma 4.3 and the LTR-condition for minimal-order observers:

$$V_2 \rightarrow qB_{2m}W \quad \text{for } q \rightarrow \infty \quad (4-10)$$

it is simple to show that:

$$M_I(s) \rightarrow E_S(s) \quad \text{as } q \rightarrow \infty \quad (4-11)$$

Before the last theorem is given for the minimal-order observers case dealing with a factorization and the belonging recovery matrix $E_S(s)$, the system's zeros will be defined in lemma 4.2 for systems with $C = (I_m \quad 0)$.

Lemma 4.2.
The zeros of S(A,B,C) with $C = (I_m \quad 0)$ are defined by the z for which:

$$r[B_1 + A_{12}\Phi_{22}(z)B_2] < m \quad (4-12)$$

Proof. (See [16]).

Note in the case when $B_1 = 0$, the zeros of S(A,B,C) will be achieved in $S(A_{22}, B_2, A_{12})$. Based on this result it is now simple to derive the following theorem which is equvialent to lemma 3.2 and theorem 3.2 for the full-order observer case.

Theorem 4.3.
Let the plant G(s) have the zeros: $z_1, \ldots, z_j$ and let $G_m(s)$ have the zeros: $z_{1m}, z_2, \ldots, z_j$, where z_1 is real and z_{1m} is real and LHP. It is assumed that $B_1 = 0$. B_{2m} and $B_{2z}(s)$ are given by:

$$B_{2m} = (B_2 - (z_1 - z_{1m})\xi\eta^T)\left|\frac{z_1}{z_{1m}}\right| \quad (4-13)$$

$$B_{2z}(s) = \left(I - \frac{(z_1 - z_{1m})}{s - z_{1m}}\eta\eta^T\right)\left|\frac{z_{1m}}{z_1}\right| \quad (4-14)$$

By using B_{2m} and $B_{2z}(s)$ in the recovery matrix $E_S(s)$ given by theorem 4.2, will result in the following equation:

$$E_S(s) = \frac{z_1 - z_{1m}}{s - z_{1m}}K_2(Iz_1 - A_{22})^{-1}B_2\eta\eta^T \quad (4-14)$$

In the case when $B_1 = 0$ the results for the minimal-order observer case are equvialent to the full-order observer case. Therefore, a further analysis of the minimal-order observer case is omitted here. Results for the analysis of the more general case with B_1 arbitrary isn't quite finished at this moment but we are still working on it. These results will be publish in a forthcoming paper.

5.0 Summarising remarks.
An analysis of pole/zero cancellation in LTR-based feedback design has been done in this paper for both full-order as well as for minimal-order observers. The connection between the two recovery matrices M_I and E_S has been derived, and equivalently recovery matrices has also been found for LTR-design of minimal-order observers. However, the results for minimal-order observers aren't quite finished.
As said in the introduction, the results are not based on any specific LTR-method. However, by using the factorization of the system G(s) given by lemma 3.2, it is possible to include the freedom from the selection of z_{1m} in the ES-LTR-method into the LQG/LTR-method. This can very easily be done by selecting the state-weight matrix as: $Q = qB_m B_m^T$, where B_m is computed by using lemma 3.2.
A more general LTR-design method for non-minimum phase plants has been derived in [17], based on H^∞ theory.

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Appendix A.
Proof of theorem 3.1.

Lets define $M_E(s)$ as:

$$M_E(s) = M_I(s) - E_S(s) \qquad (A1)$$

and prove that $M_E \rightarrow 0$ when $q \rightarrow \infty$ if F is selected as in eq. (3-3).

$$\begin{aligned} M_E(s) &= K(Is-A+FC)^{-1}B - K(Is-A)^{-1}(B-B_mB_z(s)) \\ &= K(Is-A)^{-1}\{(I+FC(Is-A)^{-1})^{-1}B - B + B_mB_z(s)\} \\ &= K(Is-A)^{-1}\{B_mB_z(s) - FC(Is-A)^{-1} \\ &\quad (I+FC(Is-A)^{-1})^{-1}B\} \\ &= K\Phi(s)\{B_mB_z(s) - F(I+C\Phi(s)F)^{-1}C\Phi(s)B\} \end{aligned} \qquad (A2)$$

$F \rightarrow qB_mW$ for $q \rightarrow \infty$ is now used in eq. (A2):

$$\begin{aligned} M_E(s) &= K\Phi\{B_mB_z(s) - qB_mW(I+C\Phi qB_mW)^{-1}C\Phi B_mB_z(s)\} \\ &= K\Phi B_m\{I - qW(I+G_m(s)qW)^{-1}G_m(s)\}B_z(s) \\ &= G_m(s)(I+qWG_m(s))^{-1}B_z(s) \\ &\rightarrow 0 \text{ as } q \rightarrow \infty \end{aligned}$$

Appendix B.
Proof of theorem 3.2.

$$\begin{aligned} B - B_mB_z(s) &= B - (B-(z_1-z_{1m})\xi\eta^T)\left(I - \frac{z_1-z_{1m}}{s-z_{1m}}\eta\eta^T\right) \\ &= \frac{z_1-z_{1m}}{s-z_{1m}}(Is-A)\xi\eta^T \end{aligned} \qquad (B1)$$

$$\begin{aligned} E_S(s) &= K(Is-A)^{-1}(B-B_mB_z(s)) \\ &= \frac{z_1-z_{1m}}{s-z_{1m}}K(Is-A)^{-1}(Is-A)\xi\eta^T \\ &= \frac{z_1-z_{1m}}{s-z_{1m}}K\xi\eta^T \\ &= \frac{z_1-z_{1m}}{s-z_{1m}}K(Iz_1-A)^{-1}B\eta\eta^T \end{aligned} \qquad (B2)$$

because $K\xi = K(Iz_1-A)^{-1}B\eta$