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Reflection of sound from finite-size plane and curved surfaces

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Outline

- General model
- Curved reflectors
- Finite size single reflectors
- Array of reflectors
- Conclusion
- Example of application

(Based on work done between 1982 and 1991, but not all has been published)

General model of sound reflection

$$\Delta L = L_{\text{refl}} - L_{\text{dir}} = \Delta L_a + \Delta L_m + \Delta L_k + \Delta L_s \quad (\text{dB})$$

attenuation from

a:	distance
m:	absorption (material)
k:	curvature
s:	reflector size

Characteristic distance, a^*

$$1/a^* = (1/a_1 + 1/a_2) / 2 \quad (\text{The harmonic average})$$

$$a^* = 2 a_1 a_2 / (a_1 + a_2)$$

Curved reflectors

References:

1982: Lydrefleksion fra konvekse og konkave cylinderflader. (In Danish). NAS-82, Stockholm. Proceedings pp. 71-74.

1985: Attenuation of Sound Reflections from Curved Surfaces. 24th Conference on Acoustics, Strbské Pleso. Proceedings pp. 194-197.

Geometrical analysis

$$R \cdot d\varphi = a \cdot d\beta / \cos \theta = a_1 \cdot d\beta_1 / \cos \theta$$

$$\Delta L_k = -10 \log \left(\frac{(a + a_2) d\beta}{(a_1 + a_2) d\beta_1} \right)$$

Attenuation due to curvature

$$\Delta L_k = -10 \log \left| 1 + \frac{a^*}{R \cos \theta} \right|$$

$$a^* = \frac{2a_1 a_2}{a_1 + a_2}$$

Measurement set-up using TDS technique

Fig. 6. Convex reflector

Fig. 7. Concave reflector

Comparing measured and calculated results

sketch

radius R

2.0 m

0.5 m

-2.0 m

-0.5 m

$a^*/R \cos \theta$

0.5

2.0

-0.5

-2.0

ΔL_{curv} (calc.)

-1.8 dB

-4.8 dB

3.0 dB

0.0 dB

ΔL_{curv} (meas.)

-2 ± 0.5 dB

-5 ± 1 dB

3 ± 1 dB

-1 ± 2 dB

Comments to results on curved reflectors

- Satisfactory agreement between measurements and theoretical model,
- but diffraction effects are seen as fluctuations in the case of $R = -0.5$ m
- so, the finite size should also be taken into account

Finite size single reflectors

References:

1986: Attenuation of Sound Reflections due to Diffraction. NAM-86, Aalborg. Proceedings pp. 257-260.

1992: Acoustic Design of Reflectors in Auditoria. Institute of Acoustics, Proceedings, Vol. 14: Part 2, pp.119-128.

Babinet's Principle

Reflection from a surface (a)
is equivalent to transmission through an aperture (b)
with same size and shape, surrounded by a rigid baffle

Kirchhoff-Fresnel approximation

Coordinate system has Origo in the point of geometrical reflection

$$\Phi = j \frac{Q}{8\pi\lambda} \int_A \frac{e^{-jk(r+s)}}{rs} (\cos(n, r) - \cos(n, s)) dA$$

$$\Phi \cong j \frac{Q}{4\pi\lambda} \frac{\cos \theta}{a_1 a_2} \int_A e^{-jk(r+s)} dA$$

Dimensions of the surface $\ll a_1$ and a_2

Transformation of variables

Rectangular aperture

$$u = \sqrt{\frac{2}{\lambda} \left(\frac{1}{a_1} + \frac{1}{a_2} \right)} \cdot \xi$$

$$v = \sqrt{\frac{2}{\lambda} \left(\frac{1}{a_1} + \frac{1}{a_2} \right)} \cdot \cos \theta \cdot \eta$$

$$\Phi \cong j \frac{Q}{8\pi} \frac{e^{-jk(a_1+a_2)}}{a_1 + a_2} (M - jN)$$

$$M - jN = \int_{u_1}^{u_2} e^{-j\frac{\pi}{2}u^2} du \int_{v_1}^{v_2} e^{-j\frac{\pi}{2}v^2} dv$$

$$M - jN = [C(u_2) - C(u_1) - j(S(u_2) - S(u_1))] \cdot [C(v_2) - C(v_1) - j(S(v_2) - S(v_1))]$$

Cornu's spiral

The Fresnel integrals: $C(v) = \int_0^v \cos\left(\frac{\pi}{2} z^2\right) dz$ $S(v) = \int_0^v \sin\left(\frac{\pi}{2} z^2\right) dz$

Result for an infinite large surface: —————

This is taken as the reference for the attenuation due to size

Rectangular reflector

Deviation from geometrical acoustics:

$$\Delta L_s = 10 \log(K_1 K_2)$$

$$\left. \begin{aligned} K_1 &= \frac{1}{2} \left((C(u_2) - C(u_1))^2 + (S(u_2) - S(u_1))^2 \right) \\ K_2 &= \frac{1}{2} \left((C(v_2) - C(v_1))^2 + (S(v_2) - S(v_1))^2 \right) \end{aligned} \right\} \text{(Two orthogonal sections)}$$

$$v_{1,i} = \frac{2}{\sqrt{\lambda a^*}} (e - 2b) \cos \theta \quad \text{(corresponds to left edge of plate)}$$

$$v_{2,i} = \frac{2}{\sqrt{\lambda a^*}} e \cos \theta \quad \text{(corresponds to right edge of plate)}$$

Rectangular reflector

Attenuation due to size – simplified model

Design frequency:

$$f_g = \frac{c a^*}{2 S \cos \theta}$$

$c = 344$ m/s is speed of sound
 a^* is characteristic distance
 S is area of reflector
 θ is angle of incidence

Measurements using pulse gating technique

22 mm hardboard, 0.6 m * 0.6 m, $a_1 = 6.0$ m, $a_2 = 4.0$ m, $\theta = 0^\circ$

Comment to first results on single reflectors

- A design frequency was derived from the theory and confirmed by the measurements
- However, this new design frequency is one octave lower than that previously suggested by L. Cremer (1953)
- Cremer agreed that the new design frequency is correct in his last conference paper at ICA 1989

Measurement results

Angle of incidence = 30° , $a_1 = 3.0$ m, $a_2 = 3.0$ m

Angle of incidence = 30° , $a_1 = 3.0$ m, $a_2 = 3.0$ m

Angle of incidence = 0° , $a_1 = 4.0$ m, $a_2 = 2.4$ m

Rotation of reflector

- to demonstrate that the theoretical model is orthogonal
-

Rotation 30°

Angle of incidence = 30° , $a_1 = 3.0$ m, $a_2 = 3.0$ m

Reflector array

References:

1988: Bølgeteoretiske metoder. (In Danish). Romakustisk prosjektering, NIF kursus, Geilo. (17 p).

1990: The Design of an Array of Reflectors for Improved Ensemble on a Concert Hall Platform. NAM-90, Luleå. Proceedings pp. 129-134.

1990: Attenuation of Sound Reflections from an Array of Reflectors. 29th Conference on Acoustics, Strbské Pleso. Proceedings pp. 231-234.

1991: Design of New Ceiling Reflectors for Improved Ensemble in a Concert Hall. Applied Acoustics 34, pp. 7-17.

Reflector array

$$K_1 = \frac{1}{2} \left[\left(\sum_{i=1}^n (C(v_{2,i}) - C(v_{1,i})) \right)^2 + \left(\sum_{i=1}^n (S(v_{2,i}) - S(v_{1,i})) \right)^2 \right]$$

$$v_{1,i} = \frac{2}{\sqrt{\lambda a^*}} (e_1 - 2b_i - (i-1)m_1) \cos \theta \quad \text{(left edges of plates)}$$

$$v_{2,i} = \frac{2}{\sqrt{\lambda a^*}} (e_1 - (i-1)m_1) \cos \theta \quad \text{(right edges of plates)}$$

High frequencies:
One plate dominates

Middle frequencies:
Several plates contribute,
but out of phase

Low frequencies:
All plates contribute, and
add almost in phase

Measurement results

Plate dimensions: 0.23 m * 0.20 m, $\theta = 30^\circ$, $a_1 = 3.0$ m, $a_2 = 2.0$ m

Plate dimensions: 0.23 m * 0.20 m, $\theta = 30^\circ$, $a_1 = 3.0$ m, $a_2 = 2.0$ m

Plate dimensions: 0.23 m * 0.20 m, $\theta = 30^\circ$, $a_1 = 3.0$ m, $a_2 = 2.0$ m

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Plate dimensions: 0.23 m * 0.20 m, $\theta = 30^\circ$, $a_1 = 3.0$ m, $a_2 = 2.0$ m

Reflector array - Parameter study

Best position

Worst position (near centre)

Panel size: 1.8 m * 1.8 m, angle of incidence = 45° , $a_1 = a_2 = 7.1$ m

Density of array: $\mu = 50\%$

Reflector array - Parameter study

Best position

Worst position (near centre)

Panel size: 1.8 m * 1.8 m, angle of incidence = 45° , $a_1 = a_2 = 7.1$ m

Density of array: $\mu = 25\%$

Reflector array - Parameter study

Best position

Worst position (near centre)

Panel size: 1.8 m * 1.8 m, angle of incidence = 45° , $a_1 = a_2 = 7.1$ m

Density of array: $\mu = 12.5\%$

Reflector array - Parameter study

Best position

Worst position (near centre)

Panel size: 1.8 m * 1.8 m, angle of incidence = 45° , $a_1 = a_2 = 7.1$ m

One single plate, only

Reflector array – A design guide

$$\Delta L_s = 20 \log \mu \quad (\text{dB})$$

$$f_g = \frac{c a^*}{2 S \cos \theta}$$

Design guide compared to *best* position

Design guide compared to *worst* position

Conclusion

- The *same design frequency* can be used for single reflectors and reflector arrays, but with opposite meaning:
- The useful range for single a reflector is *above* the design frequency
 - i.e. reflectors should be large
- The useful range for a reflector array is *below* the design frequency
 - i.e. reflectors in the array should be small

Example of application

- Danish Radio Concert Hall
 - Originally from 1945, but refurbished 1989 in order to improve the acoustic conditions for the musicians on stage
- Large reflectors introduced on the side walls of the stage
- New suspended reflector array with many small plates, slightly bent to avoid gaps between rows of reflectors

1991: Design of New Ceiling Reflectors for Improved Ensemble in a Concert Hall.
Applied Acoustics 34, pp. 7-17.