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# HYPERCHAOTIC CIRCUIT with DAMPED HARMONIC OSCILLATORS

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## ABSTRACT

A simple fourth-order hyperchaotic circuit with damped harmonic oscillators is described. ANP3 and PSpice simulations including an eigenvalue study of the linearized Jacobian are presented together with a hardware implementation. The circuit contains two inductors with series resistance, two ideal capacitors and one non-linear active conductor. The Lyapunov exponents are presented to confirm the hyperchaotic nature of the oscillations of the circuit. The non-linear conductor is realized with a diode, a negative impedance converter and a linear resistor. The performance of the circuit is investigated by means of numerical integration of the appropriate differential equations.

## 1. INTRODUCTION

Hyperchaotic oscillations are defined as having more than one positive Lyapunov exponents (LE). Several 4<sup>th</sup> order hyperchaotic oscillators and circuits characterized by 2 positive Lyapunov exponents have been reported [1-12]. The increasing interest in hyperchaotic oscillators is stimulated by their possible application to secure communications [12-14]. In order to obtain hyperchaotic oscillations from an electronic circuit it should be at least either a 4<sup>th</sup> order one in the case of common passive nonlinearity, e.g. produced by diodes [4-6, 8-11], or a 3<sup>rd</sup> order one in the case of a hysteretic nonlinearity [3]. In 1997 a simple 4<sup>th</sup> order hyperchaotic oscillator based on non-linear coupling of two linear oscillators was proposed [4]. It consists of a linear stable resonance circuit with high Q and a linear unstable resonance circuit with low Q. These two resonance circuits are coupled by means of a diode. Further simple hyperchaotic circuits with unstable oscillators coupled by a non-linear device have been studied [9, 10]. However, almost all the hyperchaotic circuits proposed so far contain at least one unstable or active oscillator in the circuitry. Recently it was demonstrated that two unstable active linear negative resistance oscillators coupled by two diodes in anti parallel may control each other and show hyperchaotic performance with jumping between two chaotic attractors [6]. Unfortunately the circuit is difficult to realize in practice and the result becomes limit cycle behaviour controlled by the output swing of the op. amps.

Please note that active oscillators may be defined as amplifiers with unstable DC bias points.

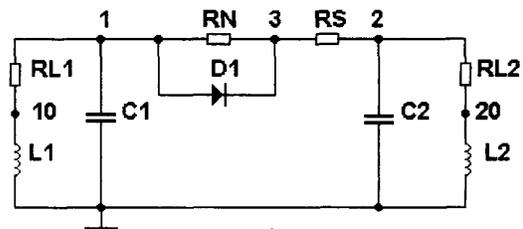
In order to overcome the problem with coupling of active oscillators a hyperchaotic circuit containing stable oscillators

coupled by an active non-linear conductor is suggested. The non-linear conductor composite is realized with a general-purpose diode, a negative impedance converter (NIC) and a linear resistor. The purpose of the NIC is to produce negative slope in the characteristic of the non-linear composite. This kind of configuration provides good temperature stability and better reproducibility of the hyperchaotic circuit. Also a linear series buffer resistor is included in the non-linear resistor circuit so that lower sensitivity to the manufacturing spread of the diode parameters is obtained.

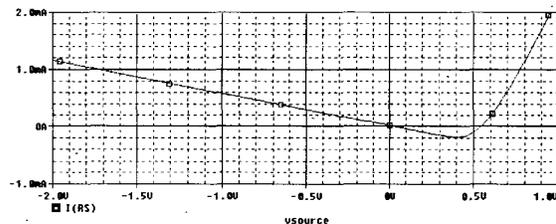
## 2. CIRCUIT DESCRIPTION

### 2.1 Circuit Diagram

The circuit diagram of the hyperchaotic circuit is shown in Fig.1.



**Figure 1.** Fourth-order hyperchaotic circuit with damped RLC oscillators.



**Figure 2.** Characteristic  $IG_N = f(V_{GN})$  of the non-linear conductor composite between node 1 and node 2 (PSpice calculation with diode 1N4148 and op amp uA741). The slopes in the two almost linear regions correspond to  $-1.75k\Omega$  and  $+162.5\Omega$ .

This circuit consists of two linear resonance circuits, (L1, RL1, C1) and (L2, RL2, C2) coupled by means of a non-linear conductor GN. The non-linear conductor is realized by a resistor RS in series with a parallel combination of a negative impedance converter (NIC, RN in Fig.1) and a general-purpose diode e.g. 1N4148. The current-voltage characteristic curve of the non-linear conductor is depicted in Fig.2.

## 2.2 Circuit Equations

The dynamics of circuit in Fig.1 are described by Eqns.(1):

$$\begin{aligned} C1 \times d/dt(VC1) &= -IL1 + IGN \\ L1 \times d/dt(IL1) &= +VC1 - RL1 \times IL1 \\ C2 \times d/dt(VC2) &= -IL2 - IGN \\ L2 \times d/dt(IL2) &= +VC2 - RL2 \times IL2 \end{aligned}$$

where VC1, VC2, IL1 and IL2 are the state variables (voltages of C1, C2 and currents of L1, L2). Please note that  $IGN = f(VC1 - VC2)$  is a non-linear function of the capacitor voltages. For simplicity, a two-segment piecewise linear model for the device GN, is represented as

$$IGN = f1(VGN) = Ga \times VGN \quad \text{for } VGN \leq Bp$$

$$IGN = f2(VGN) = Gb \times VGN + (Ga - Gb) \times Bp$$

$$\text{for } VGN \geq Bp$$

where Ga, Gb and Bp are the negative slope, the positive slope and the break-point voltage respectively.

Here  $Bp = 0.65V$  and  $Rd = 25\Omega$  at  $5mA$  are fixed from the diode parameters (e.g. 1N4148).

By introducing the following set of notations

$$x = VC1/Bp, \quad y = \rho \times IL1/Bp, \quad z = VC2/Bp,$$

$$\omega = \rho \times IL2/Bp, \quad \rho = \sqrt{L1/C1},$$

$$\tau = \sqrt{L1 \times C1}, \quad \varepsilon = C2/C1,$$

$$\mu = L2/L1, \quad R = RN \times Rd / (RN + Rd),$$

$$Ga = 1/(RN + RS), \quad Gb = 1/(R + RS),$$

$$a = \rho/(RN + RS), \quad b = \rho/(R + RS), \quad \theta = t/\tau,$$

$$du/dt = (1/\tau)(du/d\theta), \quad d = 1 - RS/RN,$$

we obtain the following set of differential equations describing the dynamics of the circuit Fig.1 as Eqns.(2):

$$dx/dt = -y - f(x-z)$$

$$dy/dt = x$$

$$\varepsilon \times dz/dt = -\omega + f(x-z)$$

$$\mu \times d\omega/dt = z$$

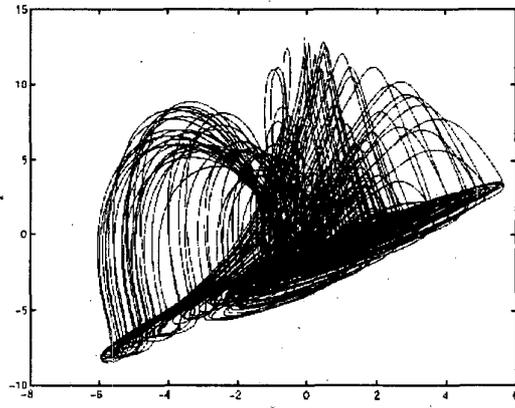
where

$$f(x-z) = a \times x + 0.5 \times b \times [\text{abs}(x-z-d) + (x-z-d)]$$

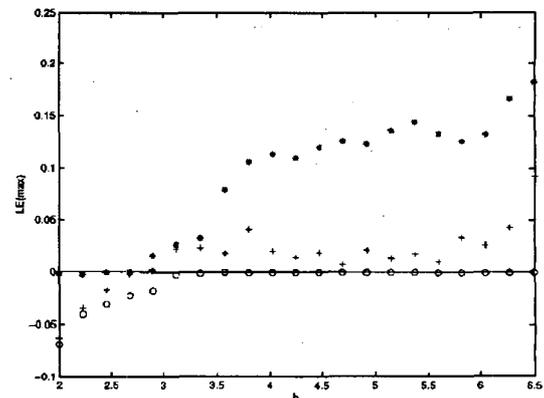
## 3. NUMERICAL SIMULATIONS

### 3.1 Equations and Lyapunov exponents

The dynamics of circuit as shown in Fig.1 are studied by numerical integration of the normalized differential equations. In order to check whether the circuit is hyperchaotic, the Lyapunov exponents of the system have been computed.



**Figure 3.** Hyperchaotic attractor observed from the circuit of Fig.1. Projection onto the (x-z)-plane. Attractor observed by numerical simulation of Eqns.(2) for  $\varepsilon = 0.31$ ,  $\mu = 0.33$ ,  $a = -0.62e-3$  and  $b = 6.0e-3$



**Figure 4.** Three largest Lyapunov exponents against parameter b

The circuit parameters of Fig.1 are fixed as  $C1 = 220nF$ ,  $L1 = 256mH$ ,  $RL1 = 15\Omega$ ,  $C2 = 68nF$ ,  $L2 = 84mH$ ,  $RL2 = 7.5\Omega$ ,  $RS = 1k\Omega$  (adjustable),  $RN = -1900\Omega$  and  $Rd = 25\Omega$ . For these parameters the normalized values are calculated as  $\varepsilon = 0.31$ ,  $\mu = 0.33$  and  $a = -0.62e-3$ . A typical phase-portrait is shown in Fig.3 for  $b = 6.0e-3$  and  $RS = 155\Omega$ .

The Lyapunov exponents characterizing the stability properties of the non-linear circuit have been computed from Eqns.(2). Fig. 4 depicts the three maximal Lyapunov spectrum ( $\lambda_{max}$  versus b) in which two maximal Lyapunov exponents are positive for certain values of b indicating the hyperchaotic nature of circuit oscillations of Fig.1.

### 3.2 ANP3 and PSpice simulations

In recent years, circuit simulators such as PSpice have been used for the simulation of the chaotic circuits. PSpice simulations of the hyperchaotic circuit (Fig.1) were carried-out using  $C1 = 220\text{nF}$ ,  $L1 = 256\text{mH}$ ,  $RL1 = 15\Omega$ ,  $C2 = 68\text{nF}$ ,  $L2 = 84\text{mH}$ ,  $RL2 = 7.5\Omega$ ,  $RS = 155\Omega$  and  $RN = -1900\Omega$ .

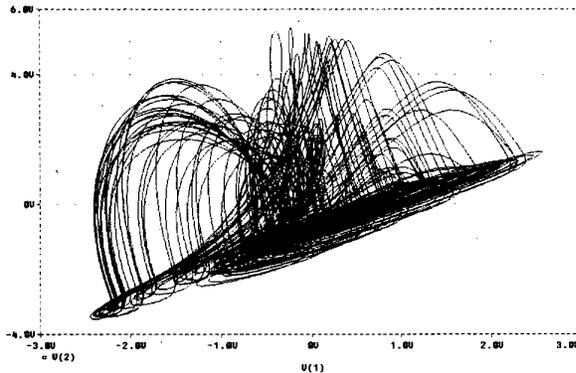


Figure 5. PSpice simulation result: Hyperchaotic attractor observed from the circuit of Fig.1. Projection onto the (VC1-VC2)-plane

For the implementation of the non-linear element GN, a negative impedance (-RN) is connected in parallel with a diode D1 (e.g. 1N4148,  $B_p = 0.65\text{V}$ ,  $R_d = 25\Omega$  at  $5\text{mA}$ ) and a linear resistor RS is connected in series with the (RN-D1)-parallel combination. The VC2 as function of VC1 trajectory is shown in Fig.5 for  $RS = 155\Omega$ . There is good agreement between the numerical simulation (Fig.3) and the PSpice simulation results.

#### ANP3 results with 1N4148 and $\mu\text{A}741$ small signal macro models

```
< 22> R20 -1.75e+3
POLES Sigma j*Omega Q f Hz
P 1.44666D+03 -4.32800D+03 -1.6 -6.89D+02
P 1.44666D+03 4.32800D+03 -1.6 6.89D+02
P 3.98097D+03 -1.14683D+04 -1.5 -1.83D+03
P 3.98097D+03 1.14683D+04 -1.5 1.83D+03
```

```
< 16> R20 2 0 +162.5
POLES Sigma j*Omega Q f Hz
P -5.49217D+02 0.00000D-01 0.50 0.00D-01
P -1.17296D+05 0.00000D-01 0.50 0.00D-01
P -3.85135D+02 -7.39772D+03 9.6 -1.18D+03
P -3.85135D+02 7.39772D+03 9.6 1.18D+03
```

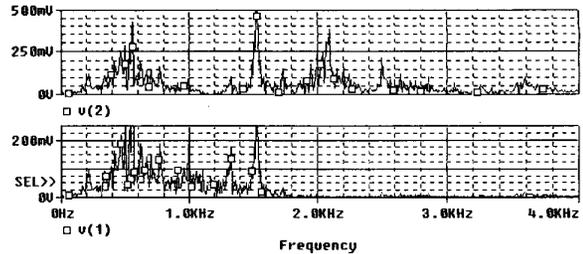
Introducing the linear values of the total resistance between node 1 and node 2 (Fig.2) ANP3 simulations gives the pole placements in the two regions [15]. It is seen that in the region with  $-1.75\text{k}\Omega$  we have two complex pole pairs in RHP corresponding to the two positive Lyapunov exponents and in the region with  $+162.5\Omega$  we have a complex pole pair and two real poles in LHP. As expected [5] we observe a very large negative real root ( $-1.226\text{E}+5$ ). The mechanism behind the chaotic behaviour seems to be the rapid switching between two complex poles in RHP and a large real pole in LHP.

#### ANP3 results with ideal op amp model and perfect diode model:

```
< 45> :D1 0 3 D1N4148 :diode OFF, GD=0
POLES Sigma j*Omega Q f Hz
P 1.45216D+03 -4.32894D+03 -1.6 -6.89D+02
P 1.45216D+03 4.32894D+03 -1.6 6.89D+02
P 3.99123D+03 -1.14570D+04 -1.5 -1.82D+03
P 3.99123D+03 1.14570D+04 -1.5 1.82D+03
```

```
< 75> RD1 0 3 0.5664 :diode ON
POLES Sigma j*Omega Q f Hz
P -5.28152D+02 0.00000D-01 0.50 0.00D-01
P -1.22627D+05 0.00000D-01 0.50 0.00D-01
P -3.70244D+02 -7.39858D+03 10. -1.18D+03
P -3.70244D+02 7.39858D+03 10. 1.18D+03
```

#### Spectrum for V(1) and V(2), Pspice



## 4. HARDWARE EXPERIMENTS

The circuit has been built in hardware (Fig. 6).

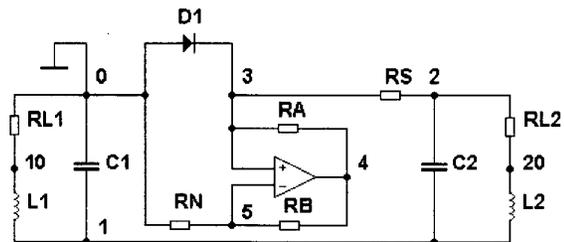


Figure 6. Hardware implementation. Please note that node 1 and the reference node are switched because the negative resistor is a three terminal element.

The structure proposed by Matsumoto et.al. [2] contains 2 op amps, 2 diodes, 2 ideal coils, 2 ideal capacitors and 8 resistors i.e. 1 grounded active resonator, 1 floating passive resonator and 1 grounded active nonlinear coupling.

The structure proposed here by Lindberg et.al. contains 1 op amp, 1 diode, 2 coils with loss resistors, 2 ideal capacitors and 4 resistors i.e. 1 grounded passive resonator, 1 floating passive resonator and 1 grounded active nonlinear coupling.

Fig. 7 shows the result with LM301AJ as op amp (compensation capacitor 40pF) and 1N4148 as diode. Laboratory decade boxes used for the coils and capacitors. RL1 measured to 14.8Ω and RL2 measured to 7.2Ω. RA = 85.0Ω, RB = 80.7Ω, RN = 1901Ω and RS = 155.8Ω. The negative resistance become Rneg = -RN×RA/RB = -2kΩ if ideal op amp is assumed. Close agreement with PSpice simulations has been found.

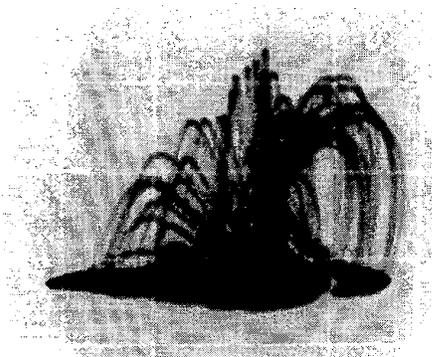


Figure 7. Oscilloscope:  $y = V(2)$ ,  $x = V(1)$  with RA = 85.0Ω and RB = 80.7Ω. Scale: x unit = 1V, y unit = 2V.

## 5. CONCLUSIONS

We have designed and investigated a damped linear oscillators based fourth-order hyperchaotic circuit. Two positive Lyapunov exponents characterize the circuit's oscillations. Due to the usage of stable oscillators coupled through a non-linear resistor, this circuit has better reproducibility, and higher stability. Further due to the inclusion of a large buffer resistor RS, the circuit has lower sensitivity to the manufacturing spread of the diode parameters. Also this circuit can be experimentally realized easily and further can be used for synchronization and ensuing secure communication applications.

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