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Induced Kerr effects and self-guided beams in quasi-phase-matched quadratic media

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We show that quasi-phase-matching of quadratic media induces Kerr effects, such as self- and cross-phase modulation, and leads to the existence of a *novel class of solitary waves, QPM-solitons.*

The quasi-phase-matching (QPM) technique has been studied intensively in the context of second-harmonic generation [1]. It relies on a periodic modulation of the nonlinear susceptibility and/or refractive index, by which an additional (grating) wavevector is introduced, that can compensate for the mismatch between the wavevectors of the fundamental and second-harmonic waves. With the QPM technique, phase-matching becomes possible at ambient temperatures, and does not introduce spatial walk-off; the polarization with the largest nonlinearity can be used, and materials with strong nonlinearities can be explored, that are not phase matchable by angle or temperature tuning. The physics of QPM is well-known [2], but only recently have the experimental difficulties been overcome and stable techniques been developed [3]. The important question whether QPM can be used to support solitary waves in $\chi^{(2)}$ materials, has therefore not been addressed so far? To do this we derive effective equations that include both quadratic ($\chi^{(2)}$) and *induced cubic* nonlinearities. We use these to prove the existence of a *novel class of solitary waves, QPM solitons.*

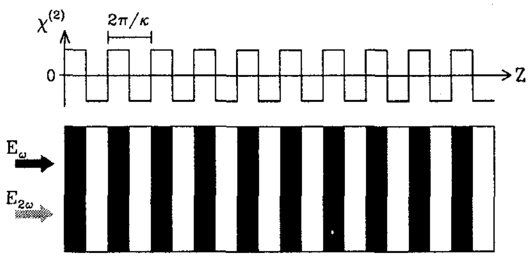

FIG. 1. Schematic presentation of a crystal with the typical square QPM modulation of the $\chi^{(2)}$ nonlinearity.

We consider the interaction of a CW beam (carrier frequency ω and wavenumber k_ω), and its second harmonic, as they propagate in a QPM $\chi^{(2)}$ slab waveguide, where only the $\chi^{(2)}$ susceptibility is modulated. The propagation along z of the slowly varying envelope of the fundamental, $W(x,z)$, and the second harmonic, $V(x,z)$, is governed by the normalized equations (see [1,2])

$$i\partial_z W + 2\partial_x^2 W + d(z) W^* V e^{-i\beta z} = 0, \qquad (1)$$
$$i\partial_z V + \partial_x^2 V + d(z) W^2 e^{i\beta z} = 0,$$

where $\beta = \Delta k |k_\omega| x_0^2$ is proportional to the mismatch $\Delta k = 2k_\omega - k_{2\omega}$. Walk-off is neglected; it is usually not present in QPM media, since perpendicular or parallel polarization states can be used. The transverse coordinate x is measured in units of the normalization parameter x_0, and z in units of the diffraction length $l_d = x_0^2 |k_\omega|$. The periodic modulation of the $\chi^{(2)}$ susceptibility is described by the grating function $d(z)$ with amplitude 1 and domain length π/κ (see Fig. 1). Expanding $d(z)$ in a Fourier series, $d(z) = \sum_n d_n e^{in\kappa z}$, and making the transformation $(W, V) = (w, v e^{i\tilde{\beta} z})$, we obtain

$$i\partial_z w + 2\partial_x^2 w + d_1 w^* v + w^* v \sum_{n\neq 1} d_n e^{i(n-1)\kappa z} = 0, \qquad (2)$$

$$i\partial_z v + \partial_x^2 v - \tilde{\beta} v + d_{-1} w^2 + w^2 \sum_{n\neq -1} d_n e^{i(n+1)\kappa z} = 0,$$

where $\tilde{\beta} = \beta - \kappa$. Assuming the most efficient QPM of the first order, $\kappa \approx \beta$, the effective mismatch $\tilde{\beta}$ is of order one or less (ideally 0), even though β might be large itself.

Equations (2) include coefficients that are periodically varying with the period $2\pi/\kappa$. Thus the dynamics can be adequately described by averaged equations, if κ is sufficiently large. We consider therefore $\kappa \sim \beta \gg 1$, and expand the fields in Fourier series, $w = \sum_n w_n e^{in\kappa z}$, $v = \sum_n v_n e^{in\kappa z}$, where $w_n(x,z)$ and $v_n(x,z)$ are assumed to vary slowly compared with $e^{i\kappa z}$. Assuming that the higher harmonics are of order $\kappa^{-1} \ll 1$ or smaller, compared to the averages w_0 and v_0, we then take into account only the lowest order perturbation in the equations for the harmonics, and derive the relations,

$$w_n = (n\kappa)^{-1} d_{n+1} w_0^* v_0, \; v_n = (n\kappa)^{-1} d_{n-1} w_0^2. \qquad (3)$$

Inserting the harmonics (3) into the corresponding equations for w_0 and v_0, we obtain the average equations

$$i\partial_z w_0 + 2\partial_x^2 w_0 + d_1 w_0^* v_0 + (\gamma |w_0|^2 + \rho |v_0|^2) w_0 = 0,$$
$$i\partial_z v_0 + \partial_x^2 v_0 - \tilde{\beta} v_0 + d_{-1} w_0^2 + 2\eta |w_0|^2 v_0 = 0, \qquad (4)$$

where γ, ρ, and η are all of order κ^{-1}, and given by

$$\gamma = \kappa^{-1} \sum_{n\neq 0} \gamma_n / n, \; \rho = \kappa^{-1} \sum_{n\neq 0} \rho_n / n, \; \eta = \kappa^{-1} \sum_{n\neq 0} \eta_n / n,$$

with $\gamma_n = d_{n-1} d_{1-n}$, $\rho_n = d_{n+1}^* d_{n+1}$, and $\eta_n = d_{n+1} d_{-n-1}$. From Eqs. (4) follows the important result that the QPM grating induces *an effective cubic nonlinearity* in the form of self- and cross-phase modulation (SPM and CPM). However, the CPM term does not appear for the second

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harmonic, making the localized solutions and the system dynamics be different from earlier analyzed cases of competing nonlinearities [4].

In many physical applications the QPM grating is well approximated by the square function depicted in Fig. 1, for which $d(z)$ contains only odd harmonics, $d_{2n} = 0$ and $d_{2n+1} = 2/i\pi(2n+1)$. Then Eqs. (4) reduce to

$$i\partial_z w_0 + 2\partial_x^2 w_0 - i\chi w_0^* v_0 + \gamma(|w_0|^2 - |v_0|^2)w_0 = 0,$$
$$i\partial_z v_0 + \partial_x^2 v_0 - \tilde{\beta} v_0 + i\chi w_0^2 - 2\gamma |w_0|^2 v_0 = 0, \qquad (5)$$

where the nonlinearity coefficients are $\chi=2/\pi$, and $\gamma=\kappa^{-1}(1\text{-}8/\pi^2)$. Note the $\pi/2$ phase-shift in front of the quadratic terms and the opposite signs of the SPM and CPM terms ($\rho=\eta=-\gamma$). We will look for stationary solutions to Eqs. (5) in the form

$$w_0(x,z) = \bar{w}_0(x)e^{i\lambda z}, \; v_0(x,z) = i\bar{v}_0(x)e^{2i\lambda z}, \qquad (6)$$

where the real and exponentially localized profiles $\bar{w}_0(x)$ and $\bar{v}_0(x)$ decay monotonically to zero as $|x|$ increases. The real eigenvalue λ must satisfy $\lambda>\max\{0, -\tilde{\beta}/2\}$, in order for $\bar{w}_0$ and $\bar{v}_0$ to be exponentially localized. In the original variables, the stationary solutions correspond to self-guided beams with rapidly oscillating intensities given by $I_w=\bar{w}_0^2+\Delta I$ and $I_v=\bar{v}_0^2 - \Delta I$, where

$$\Delta I = \frac{8\bar{w}_0^2 \bar{v}_0}{\pi\kappa} \sum_{n=1}^{\infty} \frac{\cos(2n\kappa z)}{4n^2-1}. \qquad (7)$$

Since $d(z)$ is real, Eqs. (1) conserve the total power $P=\int_{-\infty}^{\infty}(I_w+I_v)dx=\int_{-\infty}^{\infty}(\bar{w}_0^2+\bar{v}_0^2)dx$.

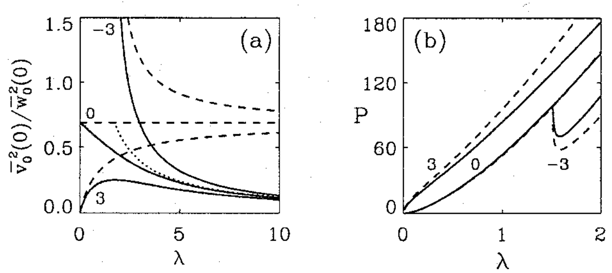

FIG. 2. Soliton families of the QPM-system (5) for κ=10 (solid curves) and the zero-order approximation (γ=0, dashed curves). (a) Ratio between the peak intensity of the second harmonic, $\bar{v}_0^2(0)$, and the fundamental, $\bar{w}_0^2(0)$, vs. λ. The dotted curve shows the asymptotic result $\chi^2/18\gamma\lambda$. (b) Power vs. λ. The value of $\tilde{\beta}$ is indicated at each pair of curves.

We have numerically found the solutions (6) for the coefficients that correspond to the square grating. In Fig. 2 we show the characteristic quantities for κ=10, compared with the results from the zero-order approximation (γ=0). From the ratio of the peak intensities in Fig. 2a, the cubic correction terms are seen to have a significant effect. In the zero-order approximation this ratio is a constant for $\tilde{\beta}$=0, which we find numerically to be 0.6865. However, in the QPM system with the induced Kerr effects ($\gamma\neq$0), this ratio tends to $\chi^2/18\gamma\lambda$ for $\lambda\gg$1. From Fig. 2b we see that there is a power threshold for the existence of solitons for negative $\tilde{\beta}$, which occurs close to the cut-off at $\lambda=\max\{0, -\tilde{\beta}/2\}$. The induced Kerr effects are seen to *increase* this threshold power. However, for positive $\tilde{\beta}$ the Kerr effects *decrease* the power required for generating a solitary wave with a certain propagation constant λ. Stability analysis of the zero-order equations (γ=0) has been developed in [5]. We expect that the results apply also for $\gamma\neq$0, so that the solitary waves are stable for $dP/d\lambda$>0, and unstable for $dP/d\lambda$<0.

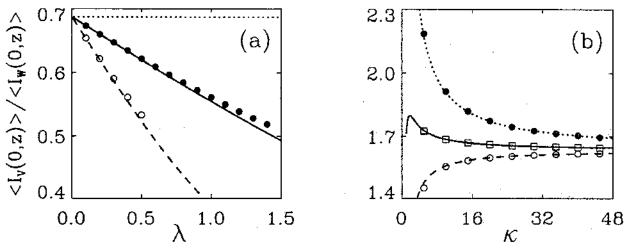

FIG. 3. Theoretical (curves) and measured (points) characteristics of the QPM solitons for $\tilde{\beta}$=0. (a) Ratio of the average peak intensities vs. λ for κ=4 (dashed curve, circles) and κ=10 (solid curve, dots). The dotted line corresponds to γ=0. (b) Mean (solid curve, squares), max. (dotted curve, dots) and min. (dashed curve, circles) values in a period of the fundamental peak intensity $I_w(0,z)$ vs. κ for λ=0.4.

Using QPM solitons as initial condition we solve Eqs. (1) numerically for the square grating, and $\tilde{\beta}$=0. After a steady state is reached we record the peak intensities in a whole number of periods π/κ, and calculate the average, and max. and min. values in a period. The measured ratio of the average peak intensities of the excited solitons, shown in Fig. 3a, deviates clearly from the prediction of the zero-order approximation (γ=0), but is in excellent agreement with the theory that takes into account the induced Kerr effect. Also the measured max. and min. values, shown in Fig. 3b, are in perfect agreement with our theory.

In conclusion, we have shown that self-guided beams in QPM $\chi^{(2)}$ slab waveguides are strongly influenced by induced Kerr effects, such as SPM and CPM. We have numerically found the new families of rapidly oscillating, spatial solitary waves in the physically relevant regime of the QPM domain length and the phase-mismatch.

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