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Abstract — **We consider a method for designing 2-D constrained codes by cascading finite width arrays using predefined finite width periodic merging arrays. This provides a constructive lower bound on the capacity of the 2-D constrained code. Examples include symmetric RLL and density constrained codes. Numerical results for the capacities are presented.**

The 2-D codes considered are shift invariant constraints of finite extent (N, M). A constraint is defined by a set of *admissible* $N \times M$ configurations. The complement to this set is called the *forbidden* configurations. A configuration on an n by m rectangle is one having no forbidden configurations within the rectangle.

Let $F(n, m)$ be the number of admissible configurations on an n by m rectangle. Then the per symbol capacity, $C^{(2)}$ of the constraint is defined as follows.

$$C^{(2)} = \lim_{n,m \to \infty} \frac{F(n, m)}{nm}. \qquad (1)$$

Calkin and Wilf [3] presented a method giving tight bounds on the capacity for first order constraints, but they do not apply when $N \geq 2$ and $M \geq 2$ [4]. Etzion [1] studied 2-D SRLL(d, k) constraints and gave methods for constructing a merging array W given any two admissible arrays, X and Y.

Instead of designing the fixed width merging array, W after X and Y, we construct W first and then examine how X and Y may be constrained such that the (horizontal) cascading $WXWYW$ is admissible. These merging arrays are simpler and easier to identify (e.g. for synchronisation).

Consider a 2-D array, W of width $w \geq M - 1$ and infinite height. This merging array is repeated at intervals of $m + w$ columns horizontally. This leaves arrays of width m undefined in between two merging arrays. Let X and Y be 2-D arrays of width m such that they can be cascaded to form an admissible configuration $WXWYW$. Repeating this construction adding one array at a time the array may extend to define an admissible configuration in the entire plane.

Following Shannon [2] we describe the admissible arrays WXW by a finite state source with states of height $n \geq N - 1$.

The per symbol capacity for the resulting merged array is

$$H = \frac{H_W(m)}{m + w} \qquad (2)$$

where $H_W(m)$ is the capacity of the W boundary constrained array WXW in which X has a width of m.

Example A — For (Symmetric RLL) SRLL(d, k) we propose the following periodic merging array. Let x be a row of d 0s followed by d 1s. Let x_d be d identical rows of x on top of each other. Then W is given by alternating between x_d and its bitwise negation. We note that the finite state source for

WXW has $2d$ phases since W is periodic with period $2d$. We can generalise the construction by offsetting the phase of say the right merging array. Let W_ϕ denote W offset by the number of rows, ϕ, maximizing the capacity, H_ϕ, of WXW_ϕ.

In Table 1 results are given for the SRLL(2,3) constraint. The periodic merging arrays have $w = 4$ whereas Etzion's merging arrays have $w = 12$. Since the W boundary constrained arrays have 2 deterministic columns (beside W), we have compared bounds obtained using m and $m + 2$ columns, respectively.

m	11	12	13	14	15
$\frac{H(m)}{m+12}$	0.126	0.131	0.135	0.139	0.143
$\frac{H_W(m+2)}{m+6}$	0.0923	0.116	0.136	0.137	0.124
$\frac{H_\phi(m+2)}{m+6}$	0.131	0.123	0.136	0.137	0.138

Table 1: Lower bounds on capacity of SRLL(2,3)

Example B — Given N and M, the binary (d, k) density constraint is defined by $d \leq \sum_{i=1}^{N} \sum_{j=1}^{M} x_{ij} \leq k$ where $0 \leq d \leq k \leq NM$ and $x_{ij} \in \{0, 1\}$.

Table 2 gives the capacity using a checkerboard periodic merging array with $w = 2$ for the $(4, 5)$ density constraint with $N = M = 3$. The minimum width of a merging array for arbitrary arrays X and Y is not known. However by using a simple example we have shown that it can not be less than 8. Hence we also give the 'lower bounds' (in parentheses) using this value in Table 2. It is seen that the periodic merging array approach is superior in this example.

m	7	8	9	10	11
$\frac{H(m)}{m+8}$	(0.279)	(0.291)	(0.302)	(0.312)	(0.320)
$\frac{H_W(m)}{m+2}$	0.353	0.360	0.372	0.382	0.389

Table 2: Lower bounds on capacity for density(4,5)

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