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Maximum Likelihood Estimation of the Attenuated Ultrasound Pulse

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***Abstract*—The attenuated ultrasound pulse is divided into two parts: a stationary basic pulse and a nonstationary attenuation pulse. A standard ARMA model is used for the basic pulse, and a nonstandard ARMA model is derived for the attenuation pulse. The maximum likelihood estimator of the attenuated ultrasound pulse, which includes a maximum likelihood attenuation estimator, is derived. The results of this correspondence are of great importance for deconvolution and attenuation imaging in medical ultrasound.**

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I. INTRODUCTION

In medical ultrasound, a short pressure pulse is emitted from a transducer. The ultrasound pulse then propagates in a narrow beam in the tissue. When the pulse arrives at inhomogeneities in the tissue, a part of the pulse is scattered back and received by the transducer. By mechanically or electronically changing the beam direction, an image of the acoustical properties of the tissue can be formed. Usually, only the envelope of the received signal is displayed. The attenuation of the tissue is not displayed directly. As the attenuation of the tissue is a clinically relevant feature, several attenuation estimation methods have been developed, e.g., the spectral-shift method and the spectral-difference method [3], [4]. However, none of these attenuation estimation methods are based on the maximum likelihood principle. Attenuation estimation is of interest in medical ultrasound for another reason. The resolution of the envelope-detected image is poor because of the extent of the ultrasound pulse. The resolution can be improved by deconvolution, e.g., [6], but an estimate of the attenuated ultrasound pulse is needed by the deconvolution algorithm. This applies to both the axial and to the lateral direction, but only the axial (1-D) case is treated in this correspondence. As the maximum likelihood estimate of the attenuated ultrasound pulse includes a maximum likelihood attenuation estimate, it is seen that there is a close connection between attenuation estimation and pulse estimation.

This correspondence is organized as follows. In Section II, a nonstandard ARMA model of the attenuated ultrasound pulse is derived. The maximum likelihood estimator of the attenuated ultrasound pulse in a constant attenuating medium is derived in Section III. Section IV presents an example, and the conclusion is given in Section V.

II. A PARAMETRIC MODEL OF THE ATTENUATED ULTRASOUND PULSE

The propagation of ultrasound waves takes place in three dimensions, but we consider 1-D effects only. The attenuated ultrasound pulse can be divided into two parts: a stationary basic pulse and a nonstationary attenuation pulse. The basic pulse consists mainly of the electromechanical response of the transducer and the scattering function; see [7, ch. 8].

The signal $y(n)$ received by the transducer is given by

$$y(n) = H_1(z)H_2(z, n)u(n) \tag{1}$$

where $n = 1, \dots, N$ denotes the discrete-time index, $z = e^{j\omega}$ the z-transform variable, $H_1(z)$ the stationary basic pulse, and $H_2(z, n)$ the nonstationary attenuation pulse. The radian frequency is denoted ω. The 1-D reflection sequence $u(n)$ is assumed Gaussian i.i.d. with zero mean and variance σ_0^2. A standard ARMA model is used for the basic pulse

$$H_1(z) = \frac{B(z)}{A(z)} \tag{2}$$

$$B(z) = 1 + \sum_{k=1}^{N_b} b_k z^{-k} \tag{3}$$

$$A(z) = 1 + \sum_{k=1}^{N_a} a_k z^{-k}. \tag{4}$$

Maximum likelihood estimation of the parameters of nonstationary models is possible in the time domain only. The following ARMA

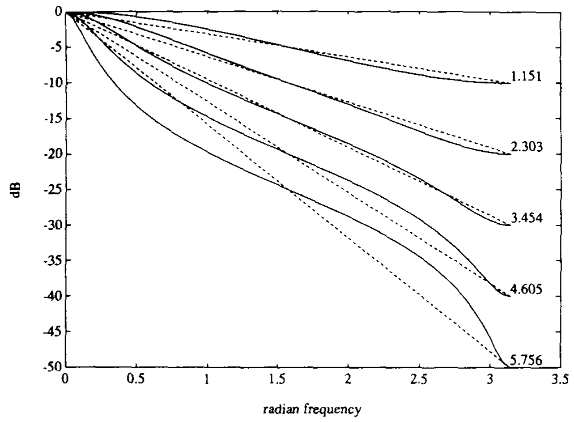

Fig. 1. Evaluation of the proposed ARMA model for the attenuation pulse. The solid curves represent the ARMA model. The dashed curves represent the linear-with-frequency characteristic. The values to the right of the curves are the $\alpha(n)$ values.

model is therefore proposed for the time-dependent attenuation pulse

$$H_2(z,n) = \frac{1-c(n)}{1+c(n)} \frac{1+c(n)z^{-1}}{1-c(n)z^{-1}}. \tag{5}$$

The deviations in the log-spectrum domain between the proposed ARMA model and the linear-with-frequency characteristic are illustrated in Fig. 1. The linear-with-frequency characteristic has been observed to be a good approximation for most soft biological tissues [4]. Considering that the linear-with-frequency characteristic already is an approximation of the physical case, the proposed ARMA model seems to be a reasonable compromise between modelling capacity and mathematical convenience. Requiring that the modelling error should be zero at

- $\omega = 0$ corresponding to $z = 1$
- $\omega = \pi/2$ corresponding to $z = j$
- $\omega = \pi$ corresponding to $z = -1$

results in

$$\left(\frac{1-c(n)}{1+c(n)}\right)^2 = \exp(-\alpha(n)) \tag{6}$$

$$\Updownarrow$$

$$\frac{1-c(n)}{1+c(n)} = \exp(-\alpha(n)/2) \tag{7}$$

$$\Updownarrow$$

$$c(n) = \tanh(\alpha(n)/4) \tag{8}$$

where $\alpha(n)$ is the cumulative attenuation coefficient at $\omega = \pi$. According to (8), we have that $|c(n)| < 1$ for all $\alpha(n)$ and $H_2(z,n)$ is therefore ensured to be minimum phase in agreement with the physics (see Sec. 5.5 of [1]). In practice, the linear-with-frequency characteristic and the assumption of large signal-to-noise ratio are often valid only in a certain frequency band. The received signal should, if this is the case, undergo a sampling rate conversion [2] so that only the usable frequency band is present in the discrete-time signal. The attenuation coefficient at $\omega = \pi$, which is denoted by $\alpha_1(n-1)$, is related to the cumulative attenuation coefficient:

$$\alpha(n) = \alpha_1(n-1) + \alpha(n-1). \tag{9}$$

If the received signal comes from a uniform area with constant attenuation α_1, we have that

$$\alpha(n) = (n-1)\alpha_1. \tag{10}$$

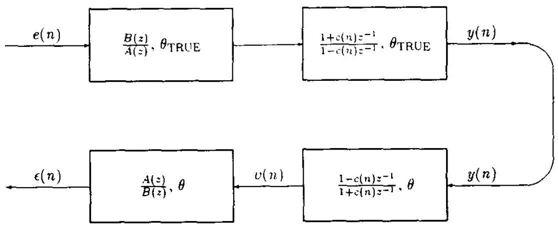

Fig. 2. Difference between $e(n)$ and $\epsilon(n)$.

III. Maximum Likelihood Estimation

If the basic pulse $H_1(z)$ is not known *a priori*, then we will assume it is minimum phase and estimate it along with the attenuation parameters. If the reflection sequence is nonGaussian distributed, then the minimum phase assumption can be relaxed [8], but this issue is not examined further in this correspondence. For the purpose of finding the likelihood function, a scaled version of the signal $u(n)$ is introduced:

$$e(n) = \frac{1-c(n)}{1+c(n)} u(n) \tag{11}$$

where $e(n)$ is independent Gaussian distributed with zero, mean and time-dependent standard deviation (see (7))

$$\sigma_e(n) = \exp(-\alpha(n)/2) \quad \sigma_0. \tag{12}$$

We then have that the received signal $y(n)$ is given by the convolution of a *monic minimum phase* filter, and the signal $e(n)$ (see (5))

$$y(n) = \frac{1+c(n)z^{-1}}{1-c(n)z^{-1}} \frac{B(z)}{A(z)} e(n). \tag{13}$$

The minus-log-likelihood function is (see Sec. 7.4 of [5])

$$V = \sum_{n=1}^{N} V(n) \tag{14}$$

where

$$V(n) = \frac{\epsilon(n)^2}{2\sigma_e(n)^2} + \log(\sigma_e(n)) + \log(2\pi)/2 \tag{15}$$

$$\epsilon(n) = \frac{A(z)}{B(z)} v(n) \tag{16}$$

$$v(n) = \frac{1-c(n)z^{-1}}{1+c(n)z^{-1}} y(n). \tag{17}$$

The prediction error $\epsilon(n)$ is, unlike the signal $e(n)$, a function of the assumed parameter vector θ. The statistics of the signal $e(n)$ are a function of the true parameter vector θ_{TRUE}. The difference between $\epsilon(n)$ and $e(n)$ is illustrated in Fig. 2. In Fig. 2, it is also illustrated that the signal $v(n)$ is a function of the assumed parameter vector θ and that the statistics of the signal $y(n)$ are a function of the true parameter vector θ_{TRUE}. The maximum likelihood estimate θ_{MLE} is the value of θ that minimizes the minus-log-likelihood function V. The maximization of the likelihood function is done by a Gauss–Newton-type algorithm; see [9] for details. The algorithm in this section is extendable to multiple independent received 1-D signals and to the case of varying attenuation [9] as well.

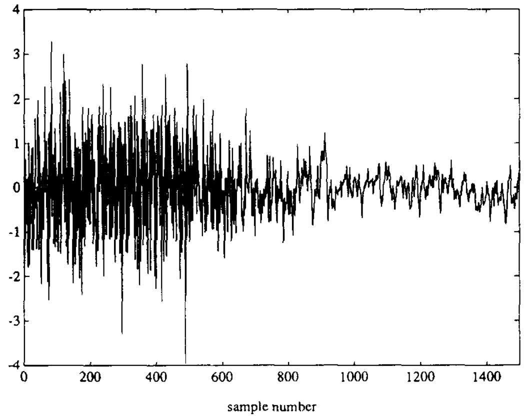

Fig. 3. Generated signal $y(n)$.

IV. EXAMPLE

The synthetic generated signal $y(n)$ shown in Fig. 3 has the following data:

$$N = 1500$$
$$H_1(z) = 1$$
$$\sigma_0 = 1$$
$$\alpha(1) = 0$$
$$\alpha_1(n) = \begin{cases} 0.01 & \text{for } 501 \le n \le 1000 \\ 0 & \text{otherwise} \end{cases}$$
$$\alpha(1500) = 5$$

Using the fixed values

$$H_1(z) = 1$$
$$\alpha(1) = 0$$

and the initial guess

$$\alpha_1 = 0$$
$$\sigma_0 = 2$$

it took six iterations to find

$$\alpha_1 = 0.0100 \quad \pm \quad 0.0004$$
$$\sigma_0 = 0.9918 \quad \pm \quad 0.0514$$

for $501 \le n \le 1000$. The values after $\pm$ are the estimated standard deviations. The spectral-difference approach [3], [4] resulted in

$$\alpha_1 = 0.0089 \pm 0.0006$$

using the two nonoverlapping segments $501 \le n \le 750$ and $751 \le n \le 1000$.

V. CONCLUSION

The information utilized by the maximum likelihood attenuation estimator is essentially the same information utilized by a short-time Fourier-transform-based method like the spectral-difference approach. However, because the received signal is nonstationary, even in the case of constant attenuation, the spectral-difference approach has to use overlapping and small segments if the spectral-difference approach is to perform as well as the maximum likelihood approach.

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Conditions for Third-Order Stationarity and Ergodicity of a Harmonic Random Process

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***Abstract*—The finite data estimates of the complex third-order moments of a signal consisting of random harmonics are analysed. Conditions for the third-order stationarity and ergodicity are obtained. Explicit formulas for the estimation error and its variance, as well as their limiting large sample values are derived. A special case relevant to quadratic phase coupling is considered, and these results are stated for this case. The variance is shown to comprise an ergodic and a nonergodic part.**

I. INTRODUCTION

The bispectrum has been shown to be a very useful tool in signal processing in the recent past, especially in nongaussian signal processing, gaussian noise cancellation, detecting phase relations among the harmonic components of a signal and in the study and identification of nonlinear and nonminimum phase systems [1]–[4]. One of the issues that arises in the practical implementation of any algorithm that uses the bispectrum or alternately the third-order moments is whether their estimates based on a finite observation interval of the signal are consistent or not. Brillinger or Rosenblatt [5], [6] have obtained fairly general results on the asymptotic statistics of the kth-order spectral estimates. However, a formal treatment of the exact statistics of the bispectral estimates of a harmonic random

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