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case is the same as the isotropic case. The X wave case is under investigation.

ACKNOWLEDGMENT

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Creeping Wave Modes for a Dielectric Coated Cylinder

N. C. ALBERTSEN

Abstract—The spectrum of creeping waves that can exist on the surface of a dielectric coated cylinder is determined by the poles in the Green's function for the cylinder. Accounts of the position of these poles may be found in the literature, but a number of poles appear so far to have been overlooked. Although these poles may be of little interest for practical applications, they shed some light on peculiarities in some of the literature's results.

I. INTRODUCTION

The concept of creeping waves has been a long-established part of the geometrical theory of diffraction, and the waves' propagation properties on perfect conductors are well documented. The same can hardly be said about creeping waves on dielectric covered surfaces, although some early papers gave significant contributions to the understanding of their properties, e.g. Elliott [1]. In recent years the growing interest in advanced technologies has generated a number of papers investigating creeping wave propagation on dielectric covered surfaces, e.g. [2] and [3]. Surprisingly enough, it seems, however, that new discoveries remain to be made in this area.

The present study was prompted by an investigation of radiation properties for antennas on the European space shuttle HERMES for

the European Space Agency. The first step necessary for the calculation of a creeping wave is to find the poles for the Green's function, since these determine the propagation constant for the wave, whereas other properties of the wave are readily found once the poles are known. The analysis that follows is therefore devoted entirely to the problem of tracing the locations of the poles.

II. ANALYSIS AND NUMERICAL RESULTS

For a perfectly conducting cylinder, the poles of the Green's function are well known. They all lie along the Stokes' line for a Hankel function or its derivative. For a dielectric cylinder the problem is more complex, since it involves finding the roots of a transcendental expression. From [2] we cite this expression as

$$d(v) = H_v'^{(1)}(k_0 b) + iC H_v^{(1)}(k_0 b) \tag{1}$$

where C may be either

$$C_v^h = i \frac{k_1 \epsilon_0}{k_0 \epsilon_1} \frac{H_v'^{(1)}(k_1 b) H_v'^{(2)}(k_1 a) - H_v'^{(1)}(k_1 a) H_v'^{(2)}(k_1 b)}{H_v^{(1)}(k_1 b) H_v'^{(2)}(k_1 a) - H_v'^{(1)}(k_1 a) H_v^{(2)}(k_1 b)} \tag{2}$$

for transverse electric (TE) polarization or

$$C_v^s = i \frac{k_1 \mu_0}{k_0 \mu_1} \frac{H_v'^{(1)}(k_1 b) H_v^{(2)}(k_1 a) - H_v^{(1)}(k_1 a) H_v'^{(2)}(k_1 b)}{H_v^{(1)}(k_1 b) H_v^{(2)}(k_1 a) - H_v^{(1)}(k_1 a) H_v^{(2)}(k_1 b)} \tag{3}$$

for transverse magnetic (TM) polarization. Here k, ϵ, and μ with subscript 0 or 1 refer to free space or dielectric, respectively, a is the radius of the metallic cylinder inside the dielectric and b is the outside radius of the dielectric coating. For convenience the expressions in [2] have been rewritten using Hankel functions in (2) and (3). Let $d = b - a$ denote the thickness of the dielectric, then obviously $d = 0$ leads to $C_v^h = 0$, $C_v^s = \infty$, and the required roots reduce to those of the nondielectric case. It would therefore seem logical to assume that the root loci for increasing values of d would emanate from points on the Stokes' lines of $H_v^{(1)}(k_0 b)$ and $H_v'^{(1)}(k_0 b)$. This procedure leads to the root loci shown in [2, figs. 2 and 4]. An attempt to reproduce [2, fig. 6] revealed that the critical radius effect mentioned in [2] may have a simple explanation, viz. that a double root is involved. This then leads to the question of the form of the second root locus, which must have its origin at $v = \infty$ for $d = 0$. Since the roots lie far from the transition regions of the Hankel functions with arguments $k_1 a$ and $k_1 b$ we replace the Hankel functions in (2) and (3) by their Debye representations (see, e.g., [4]), and to get an estimate of the roots for $d \to 0$ we replace the arguments containing $k_1 a$ with a two-term Taylor expansion around $k_1 b$. As a result, (2) and (3) are simplified to

$$C_v^h \approx -\frac{1}{\sqrt{\epsilon_r}} \frac{z}{k_1 d} \frac{e^{iz} - e^{-iz}}{e^{iz} + e^{-iz}} \tag{4}$$

and

$$C_v^s = \sqrt{\epsilon_r} \frac{z}{k_1 d} \frac{e^{iz} + e^{-iz}}{e^{iz} - e^{-iz}} \tag{5}$$

where $\epsilon_r = \epsilon_1 / \epsilon_0$ and

$$z = \frac{d}{b} \sqrt{(k_1 b)^2 - v^2}. \tag{6}$$

The branch of the square root is chosen such that z is positive for $v = 0$, and the branch cut is placed outside the region of v under consideration.
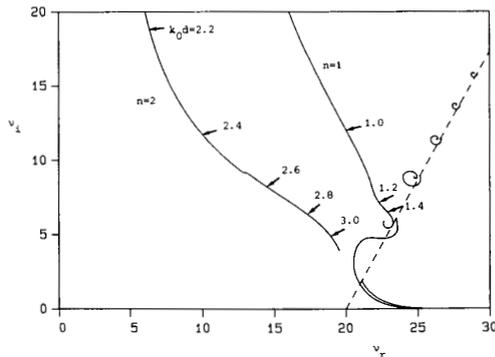
Fig. 1.   Root loci for TE case, $k_0 b = 20$, $\epsilon_r = 4$, $k_0 d = 0$ to π.

Fig. 2.   Root loci for TM case, $k_0 b = 20$, $\epsilon_r = 4$, $k_0 d = 0$ to π.

Since we shall be searching for roots near the Stokes' line for $H_v^{(1)}(k_0 b)$, we employ transition region representations for the Hankel functions in (1), even though these approximations are clearly not applicable in the limit $d \to 0$. In this limit, Debye approximations would be more appropriate, but to use these would be essentially to assume the dielectric to be locally flat and would provide a poor starting point for the root loci associated with creeping waves. Hence for the TE case

$$d(v) \approx \frac{2e^{-i\pi/3} \mathrm{Ai}(-\tau)}{m k_0 d \epsilon_r \cos(z)}$$

$$\times \left[z \sin(z) + k_0 d \epsilon_r \cos(z) \frac{e^{-i\pi/3} \mathrm{Ai}'(-\tau)}{m \mathrm{Ai}(-\tau)} \right] \quad (7)$$

and for the TM case

$$d(v) \approx \frac{2e^{-i\pi/3} z \mathrm{Ai}(-\tau)}{m k_0 d \sin(z)}$$

$$\times \left[\cos(z) + k_0 d z \sin(z) \frac{e^{-i\pi/3} \mathrm{Ai}'(-\tau)}{m \mathrm{Ai}(-\tau)} \right] \quad (8)$$

where $m = (k_0 b/2)^{1/3}$ and

$$\tau = 2m^2 e^{-i\pi/3} \left[i \sqrt{\left(\frac{z}{k_0 d} \right)^2 - \epsilon_r - 1} \right]. \quad (9)$$

For $d \to 0$ we can now identify root loci emanating from $z = n\pi$ in the TE case and from $z = n\pi - \pi/2$ in the TM case. In Fig. 1 we show root loci for a TE case. It shows six root loci emanating from the Stokes' line of $H_v'^{(1)}(k_0 b)$ shown dashed plus two root loci of the new type corresponding to $n = 1$ and $n = 2$ in the complex $v = v_r + iv_i$ plane. We see that the first two "old" root loci move towards the real v axis, whereas the following root loci curl up, and that the first "new" root locus curls up at the position vacated by the second "old" root locus whereas all the following "new" loci move toward the real v axis. In Fig. 2 is shown root loci for a TM case. It shows 15 root loci emanating from the Stokes' line of $H_v^{(1)}(k_0 b)$ plus three root loci of the new type corresponding to $n = 1$, 2 and 3 in the complex v plane. Here only the first "old" root locus moves toward the real v axis, whereas all the following curl up. There is, however, a difference in the sense of direction between numbers 2 through 12 and the following. This leaves a gap which is filled by the "new" root locus with $n = 1$. The "new" root loci with $n > 1$ all move toward the real v axis.
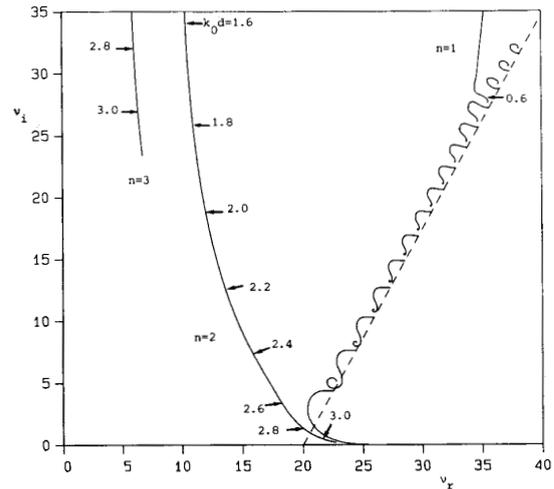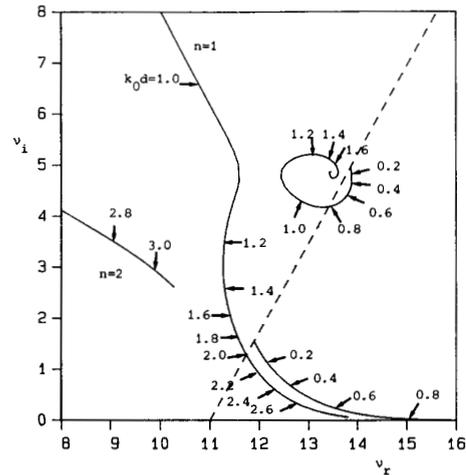

Fig. 3.   Root loci for TE case, $k_0 b = 11$, $\epsilon_r = 4$, $k_0 d = 0$ to π.
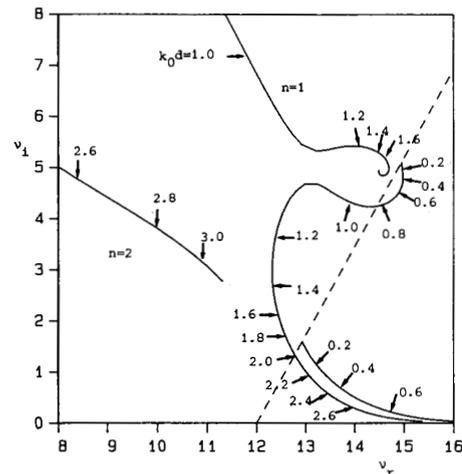

Fig. 4.   Root loci for TE case, $k_0 b = 12$, $\epsilon_r = 4$, $k_0 d = 0$ to π.

To analyze the critical radius effect mentioned in [2] also plots of root loci for $k_0 b = 11$ and 12 are shown in Figs. 3 and 4 for TE polarization. Besides the "new" root loci with $n > 1$, which all progress toward the real v axis, there are two root loci close to the real v axis. In Fig. 4 both these loci originate on the Stokes' line, whereas in Fig. 3 one originates on the Stokes' line and the other is a "new" locus with $n = 1$. It therefore seems that the number of Elliot type modes (root loci with small imaginary parts) is independent of the critical radius, for which the two root loci in question merge for a certain value of $k_0 d$ to form a double root.

III. CONCLUSION

A new system of poles for the Green's function for a dielectric coated cylinder has been found. In general, these poles correspond to creeping waves, which are strongly attenuated except for very thick coatings. For radii below a critical value, one of the new poles replaces one of those previously described in the literature and gives rise to a creeping wave of Elliott-type with low attenuation.

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