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Turbulence in a cusp *Q* device

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Spectral measurements are reported of plasma turbulence in the Cs plasma of a *Q* device, modified to a magnetic cusp geometry. The excitation mechanism for the fluctuations appears to be the centrifugal instability discussed by Chen. A transition from an f^{-5} to an f^{-3} power spectrum is observed as one moves from the hot plates to the midplane of the cusp.

I. INTRODUCTION

Spectra of low-β plasma turbulence have been measured, among others, by Bol[1] in the etude stellarator, by Chen and Bingham[2] in a Penning ionization gauge discharge, and by D'Angelo and Enriques[3] in a *Q* device. In all three cases the power spectra exhibited an f^{-5} frequency dependence at high frequencies. According to Chen,[2] a K^{-5} wavenumber dependence is to be expected on dimensional grounds, if the turbulence arises from excitation of drift waves.

We thought it worthwhile to measure the spectrum of low-β plasma turbulence in a *Q* device modified into a cusp geometry. One of the main motivations for the present study was our belief that measurements of turbulence spectra may be used as a diagnostic tool in determining what type of instability is to be held responsible for fluctuations observed in several types of plasmas, particularly in a few of geophysical interest. One example of this effect is provided by the OGO-5 fluctuation measurements in the dayside polar cusp of the magnetosphere.[4,5]

The paper is organized as follows. Section II contains a description of the experimental arrangement. Section III describes the experimental results. Section IV is devoted to a discussion of these results, and Sec. V contains the conclusions.

II. EXPERIMENTAL ARRANGEMENT

The measurements were carried out in a Cs plasma, produced by surface ionization of Cs neutral atoms on two tantalum plates (*Q* machine).[6] The plates, 3 cm in diameter, were spaced 50 cm apart and heated by electron bombardment to ~2200°K. The normal magnetic field configuration of a *Q* device was modified to obtain a cusplike geometry, with "point" cusps at the two cathodes and a "ring" cusp at the midplane of the device. The strength of the magnetic field at the cathodes could be varied between ~1000 and 2500 G. Figure 1 shows the measured (Hall probe) magnetic field strength distribution along the axis of the device. The plasma density in the region where most of the measurements were performed (i.e., less than 10 cm from the midplane of the cusp) was typically of the order of 10^{10} cm^{-3}, while near the generating plates it was almost one order of magnitude larger. Radial density profiles measured within ~10 cm from the midplane show very smooth plasma density distributions, with *e*-folding lengths (transverse to **B**) ~10 cm; a result hardly surprising, since typical values of the ion gyroradius at, say, 3–5 cm away from the null point, are of the order of several centimeters. Radial density profiles measured near the hot plates were, on the other hand, quite similar to those measured in normal *Q* devices operating with uniform magnetic fields of a few thousand Gauss. The neutral background pressure was typically 1–2 × 10^{-5} mm Hg.

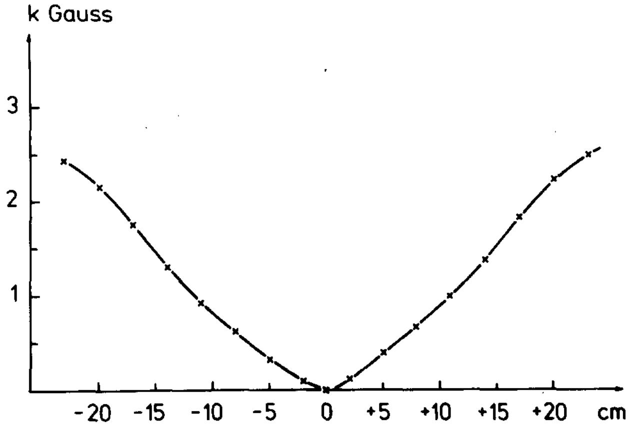

FIG. 1. The **B**-field strength along the cusp axis.

III. EXPERIMENTAL RESULTS

Measurements of the fluctuation spectra were performed by means of a Langmuir probe, consisting of a tungsten wire 0.2 mm diam, with glass tube shielding. The exposed tip was 0.2 mm in length. The signal from the probe was fed, through a voltage follower with input impedance 1 MΩ, to a Hewlett–Packard spectrum analyzer. A typical power spectrum at ~3 cm from the midplane is shown in Fig. 2. Note the nearly $1/f^3$ frequency dependence. The same frequency dependence was observed in a number of spectra measured under the same conditions. On the other hand, spectra measured in the immediate vicinity of the hot plate exhibited the $1/f^5$ shape already reported in Refs. 1–3 and were characteristic of drift wave excitation. The same results were also obtained when plasma density fluctuations, rather than potential fluctuations, were measured. It was also found that $\tilde{n}/n \approx e\tilde{\phi}/\kappa T$, as expected for electrostatic fluctuations ($\tilde{n}$ and $\tilde{\phi}$ stand for density and potential fluctuations).

In order to understand the $1/f^3$ power spectra near the midplane, several other measurements were carried out. A double probe [Fig. 3(a)] was employed, which consisted of two "flat" tantalum disks separated by a Mylar disk. The double probe was movable across the plasma at 3 to

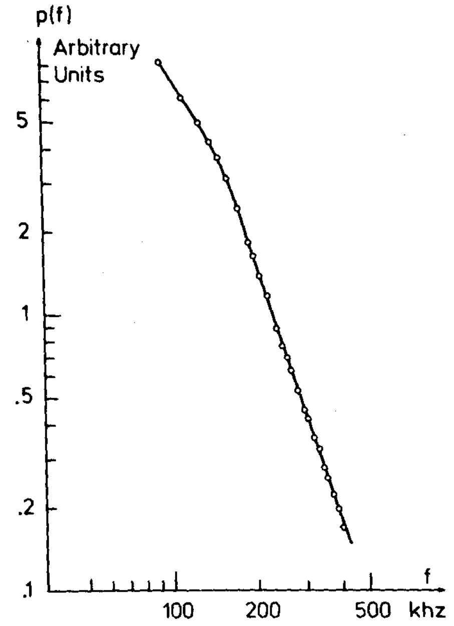

FIG. 2. A typical power spectrum taken at ~3 cm from the midplane of the cusp.

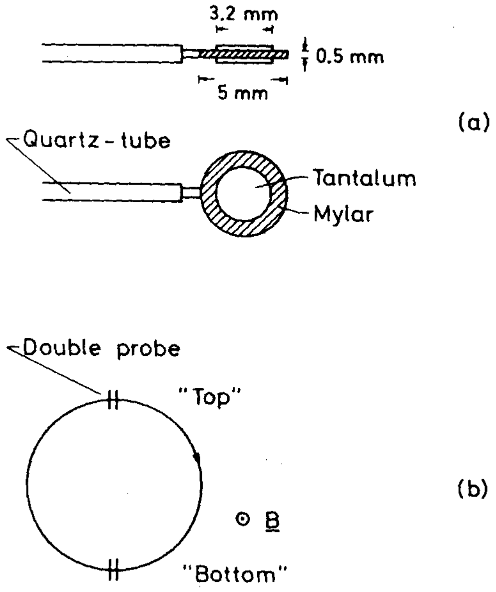

FIG. 3. (a) Double probe construction. (b) Schematic diagram showing two probe positions and the $\hat{r} \times \mathbf{B}$ direction (arrow).

5 cm from the midplane of the device. Two positions of this probe are indicated as "top" and "bottom" in Fig. 3(b). The $\hat{r} \times \mathbf{B}$ direction is shown by the arrow, $\hat{r}$ being the radial unit vector. With an outwardly directed radial electric field, **E** (see Sec. IV), the $\hat{r} \times \mathbf{B}$ direction is, of course, the same as the $\mathbf{E} \times \mathbf{B}$ direction. With the probe 4 cm above the cusp axis (top) the signal on the side facing $\hat{r} \times \mathbf{B}$ is, at nearly all frequencies, ~30% higher than the signal on the opposite side. The same is true when the double probe is moved to 4 cm below the cusp axis (bottom). The difference in the two signals evidently cannot arise from different effective areas of the two probe sides. A further check to this effect was obtained by repeating the measurements, both in the top and bottom locations, with the double probe turned by 180°. What these measurements indicate is a clear anisotropy in the turbulence, the "preferred" direction being the same as the $\hat{r} \times \mathbf{B}$ direction.

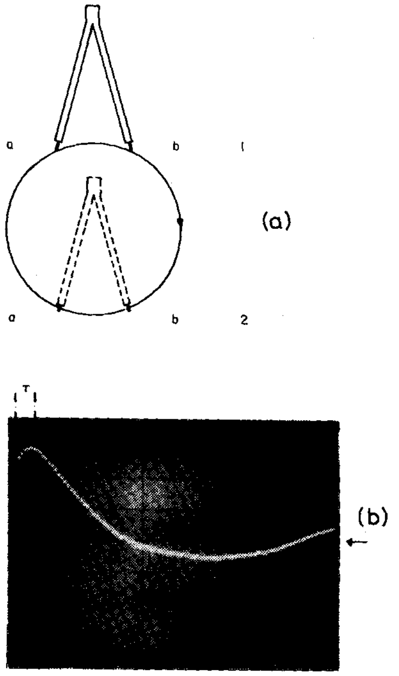

FIG. 4. (a) The "fork" probe shown at two positions in a plane normal to the axis of the device, ~3 cm from the midplane. (b) Cross correlogram obtained with the "fork" probe. The base line is indicated by an arrow. The time basis is 10 μsec/large div.

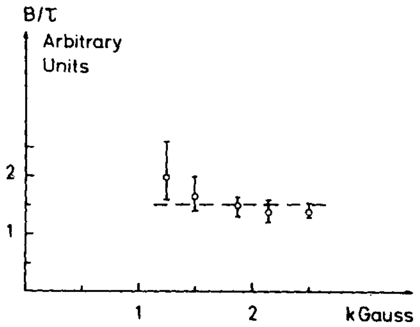

FIG. 5. B/τ as a function of the **B**-field strength. The delay time τ is obtained from figures of the type of Fig. 4(b). The **B** field is measured at the hot plate position.

To further test this point and in order to obtain more quantitative information, a "fork" probe was then used, of the type shown in Fig. 4(a). It consists of two separate Langmuir probes, mechanically connected and movable as indicated by the two positions, 1 and 2, shown in Fig. 4(a). Each probe has a (tungsten) tip 0.5 mm diam, with an exposed length of 0.8 mm. The two tips are 2 cm apart. The fork probe was nearly at the same location as the double probe of Fig. 3. The signals from the two tips of the fork, filtered through a bandpass filter (10 kHz–~100 kHz) were fed to a cross correlator. A typical cross correlogram is shown in Fig. 4(b) for "a delayed" in the crosscorrelation and the fork in position 1. The cross correlation shows an unmistakable shift, corresponding to a drift in the $\hat{r} \times \mathbf{B}$ direction. The same correlogram is obtained with the fork in position 2 and "*b* delayed." The time shifts in the correlograms indicate azimuthal phase velocities of ~10^5 cm/sec.

The time shift τ varies proportionally with the magnetic field strength. Figure 5 shows the measured ratio B/τ over the ("point" cusp) B field range ~1000 G to ~2500 G.

These data show a $v_{\rm ph} \propto 1/B$ dependence of the phase velocity on magnetic field strength.

In order to obtain information on the lifetime of the measured fluctuations, we performed, by means of the same correlator, autocorrelation measurements. The lifetime, τ_c, of the fluctuations turned out to be comparable to their periods.

In concluding this section, we emphasize that these results were obtained (with plasma generated at both plates) when the measurements were performed several centimeters away from the midplane. With only one plate "on," the same results were obtained on that side of the device where the plasma was produced. Very close to the midplane (at less than $\sim$0.5 cm) the interpretation of the results was not as straightforward, particularly as far as the cross-correlation measurements are concerned. This may be expected, however, since the $\mathbf{E} \times \mathbf{B}$ drifts must be oppositely directed in the two halves of the device.

IV. DISCUSSION OF THE EXPERIMENTAL RESULTS

A satisfactory explanation of the results presented in Sec. III should account for:

(a) the magnitude of the measured azimuthal phase velocities ($v_{\rm ph} \approx 10^5$ cm/sec),

(b) the measured dependence of $v_{\rm ph}$ on the B field strength ($v_{\rm ph} \propto 1/B$),

(c) the observed azimuthal direction of propagation (same as $\hat{r} \times \mathbf{B}$),

(d) the frequency (power) spectra measured near the hot plate (f^{-5}) and in the vicinity of the null point of the cusp (f^{-3}).

It seems clear at the outset that electrostatic perturbations are involved, since the β of the plasma is very low ($\sim 10^{-6}$), and since we observe the relation to hold $\tilde{n}/n \simeq e\tilde{\phi}/\kappa T$.

The most satisfactory explanation of points (a) through (c) which we have been able to find, is in terms of a centrifugal instability (see the linear theory by Chen[7]). If a fully ionized plasma column with cylindrical symmetry is made to rotate under a radial electric field, the centrifugal force which results from the rotation can produce what essentially amounts to a Rayleigh–Taylor gravitational instability. With $K_{||} \simeq 0$ ($K_{||}$ being the component of the propagation vector parallel to the $\mathbf{B}$ field), the azimuthal phase velocity is synchronous with the mean ion momentum per unit mass ($\mathbf{v}_{\rm ph} = \mathbf{v}_{\rm E\times B} + \mathbf{v}_{i,d}$). At high densities, electron–ion collisions may allow $K_{||}$ to be large enough so that the drift mode is excited, synchronous with the mean electron momentum per unit mass ($\mathbf{v}_{\rm ph} = \mathbf{v}_{\rm E\times B} + \mathbf{v}_{e,d}$). $\mathbf{v}_{i,d}$ and $\mathbf{v}_{e,d}$ stand for ion and electron diamagnetic velocity, respectively.

Hartman and Munger[8] have experimentally studied the centrifugal instability in a Cs plasma device in which the radial temperature gradient across the generating hot plate was controllable. They found, with a rotational velocity nearly shear-free, the waves to be synchronous with the mean azimuthal ion momentum per unit mass.

With an outwardly directed, radial $\mathbf{E}$ field,[7] of magnitude $E \approx \kappa T/e\Lambda \approx 0.5$ V/cm (Λ being the e-folding length of the radial density profile) at the hot plate, we infer a $v_{\rm E\times B} \approx 10^5$ cm/sec at a few centimeters from the midplane of the cusp. We assume that the $\mathbf{B}$ field lines are (dc-wise) nearly equipotential, so that the $\mathbf{E}$ field at the hot plate is mapped out to the neighborhood of the cusp midplane. That this assumption must be essentially correct follows from a consideration of the maximum (parallel to $\mathbf{B}$) currents that a plasma with $n \approx 10^{10}$ cm^{-3} can sustain and from a value for the plasma resistivity, η, of $\sim$0.5 Ω cm.

It thus easily appears that points (a) through (c) can be understood in terms of the centrifugal instability. Whether the mode excited is traveling at $\mathbf{v}_{\rm ph} = \mathbf{v}_{\rm E\times B} + \mathbf{v}_{i,d}$ or at $\mathbf{v}_{\rm ph} = \mathbf{v}_{\rm E\times B} + \mathbf{v}_{e,d}$ cannot be determined with certainty from the cross-correlation measurements of Sec. III. The reason is that the measured e-folding lengths of the density profiles ($\Lambda \approx 10$ cm) give a $|\mathbf{v}_{i,ed}| \approx 2 \times 10^4$ cm/sec (i.e., considerably smaller than $\mathbf{v}_{\rm E\times B}$) at a few centimeters from the cusp midplane. It can be said, however, that close numerical agreement with the measured $\mathbf{v}_{\rm ph}$ obtains if the waves are taken to travel azimuthally at the mean *ion* momentum per unit mass.

The question finally remains (point d) as to why, in contrast to the f^{-5} frequency power spectra of normal drift wave turbulence, do we measure an f^{-3} spectrum near the cusp midplane. An answer to this question may be found by noticing that the dimensional arguments used by Chen[2] in deriving the K^{-5} wave spectra do apply for wavelengths of the perturbations in the range $L > \lambda > l_i$, where L is a length of the order of the linear size of the plasma and l_i is the ion gyroradius. Now, with $v_{\rm ph} \approx 10^5$ cm/sec, a frequency $f \approx 150$ kHz (see Fig. 2) corresponds to a wavelength $\lambda \approx 0.7$ cm, and thus all frequencies $f \gtrsim 150$ kHz in our Fig. 2 correspond to wavelengths *smaller* than the ion gyroradius ($l_i \approx 5$ cm at $B \approx 150$ G). For wavelengths smaller than the ion gyroradius the ion dynamics should no longer be much affected by the magnetic field (although the electron dynamics, of course, still is) and thus the wave motion should begin, as it were, resembling an ion-acoustic motion. It may be worth noticing in this connection that the kinetic energy density in an ion-acoustic motion is $\frac{1}{2}m_i n_0 \tilde{v}^2 \propto (\omega/K)^2 \tilde{\phi}^2 \propto \tilde{\phi}^2$ (since $\omega/K \approx C_s$); whereas the kinetic energy density in a drift-wave-type motion is $\frac{1}{2}m_i n_0 \tilde{v}^2 \propto E^2 \propto K^2 \tilde{\phi}^2$. Thus, a flattening from the f^{-5} spectrum, when λ becomes smaller than l_i, seems plausible enough. Against this interpretation one may object that the spectrum shown in Fig. 2 is *flatter* (rather than steeper) at frequencies $f \lesssim 150$ kHz and thus no f^{-5} spectrum is seen at the lower frequencies. One should realize, however, that with $l_i \approx 5$ cm and a plasma transverse linear size $L \approx 10$–15 cm, the wavelength interval $L > \lambda > l_i$ of Chen's turbulence theory hardly exists at all at the location in the cusp where the spectrum was taken ($B \approx 150$ G). It is comforting that spectra taken farther away from the cusp midplane, but not quite close to the hot plate, do show the transition from f^{-5} to f^{-3} occurring at $\lambda \approx l_i$. In addition, it should be noted, that the same phenomenon seems to have been present in a spectrum of plasma turbulence, taken in a rotating Cs plasma device (Fig. 5 of Ref. 9), although its significance was not pointed out by the authors.

V. CONCLUSIONS

In this paper we have presented spectral measurements of plasma turbulence in a Q machine device, modified into a cusp geometry. Near the midplane of the cusp the frequency power spectra are of the type f^{-3}, whereas in the vicinity of the generating plates we observe the f^{-5} frequency dependence characteristic of drift wave turbulence. The f^{-3} type spectrum seems to be associated with wavelengths smaller than the ion gyroradius, for which Chen's theory of drift wave turbulence should no longer hold. For $\lambda < l_i$, the ion dynamics in the turbulent motion must become relatively unaffected by the **B** field.

A gravitational instability[7] driven by an $\mathbf{E} \times \mathbf{B}$ generated centrifugal force seems to provide the most plausible mechanism for generating the turbulence.

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