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One-third (period three) harmonic generation in microwave-driven Josephson tunnel junctions

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One-third harmonic signals have been generated in the zero voltage state of a Josephson tunnel junction driven with a microwave current in the frequency range 8–20 GHz. The signal was as much as 50 dB above the noise level of the detector with a linewidth of less than 100 Hz. The junction parameters and microwave current were measured *in situ* in separate experiments. The subharmonic generation occurred for ranges of microwave current and frequency that were in reasonable agreement with the results of digital computer simulations.

Subharmonic generation in nonlinear, dissipative systems subjected to a periodic driving force was discovered more than 50 years ago in a moving coil loudspeaker.[1] Renewed interest in the phenomenon grew out of the recognition that such driven, anharmonic systems may exhibit chaos and that the transition from periodic to chaotic behavior often proceeds via subharmonic generation. One example is the current-biased Josephson tunnel junction in the presence of microwave radiation, which has been the subject of extensive digital and analog computer studies.[2] This system, which is analogous to the periodically driven damped pendulum, exhibits a rich variety of chaotic and subharmonic phenomena. One of the predictions[3] for a junction in the zero voltage state is the existence of narrow bands of even- and odd-subharmonic solutions in the microwave frequency-amplitude plane, often separated by regions of chaos. In this letter we report measurements of an odd-subharmonic, namely, one-third harmonic (period tripling) generation. The results are compared with computer simulations using parameters for the junction measured *in situ* in separate experiments.

The simulations are based on the resistively shunted junction model (insert Fig. 1) in which a junction with critical current I_0 is shunted with a capacitance C and a resistance R. The equation of motion for the phase difference ϕ across the junction is[2]

$$\frac{d^2\phi}{d\tau^2} + \frac{\alpha_0 d\phi}{d\tau} + \sin\phi = \rho + \rho_m \sin\Omega + \rho_n(\tau) . \quad (1)$$

Here, $\tau = \omega_{p0} t$ is the time normalized to the inverse of the maximum plasma frequency $\omega_{p0} = (2eI_0/\hbar C)^{1/2}$, $\alpha_0 = 1/Q_0 = 1/\omega_{p0} RC$ is the damping parameter, $\Omega = \omega_m/\omega_{p0}$ is the normalized microwave frequency, and ρ, ρ_m, and ρ_n are the dc bias, microwave current, and noise current normalized to I_0. The Nyquist noise current arises from the shunt resistor, and is assumed to be white over a frequency range that extends from zero to frequencies much greater than ω_{p0}. The magnitude of this noise is characterized by $\Gamma = 2ek_B T_{\text{eff}}/\hbar I_0$, where T_{eff} is the effective temperature of R. We have omitted the $\cos\phi$ term in Eq. (1), although its effect is not necessarily unimportant.[4]

We made measurements on underdamped Nb-NbO$_x$-Pb overlap tunnel junctions approximately $30 \times 30\ \mu\text{m}^2$ in area. The linear dimension was small compared with the Josephson penetration depth. The junction was incorporated into an inverted microstrip structure mounted in a vacuum can immersed in liquid ^{4}He. The junction was irradiated with a monochromatic microwave signal from a synthesizer in the frequency range 8–20 GHz, coupled in via an X-band waveguide. The subharmonic signals were coupled out via a semirigid coaxial waveguide to a low-noise field-effect transistor (FET) amplifier followed by a digital storage spectrum analyzer with a minimum resolution bandwidth of 100 Hz. To avoid saturating the amplifier with the pump signal we inserted low-pass filters between the junction and the amplifier. By measuring the suppression of the critical current as a function of microwave power and frequency we deduced that the microwave coupling at both the pump and subharmonic frequencies was strongly frequency dependent

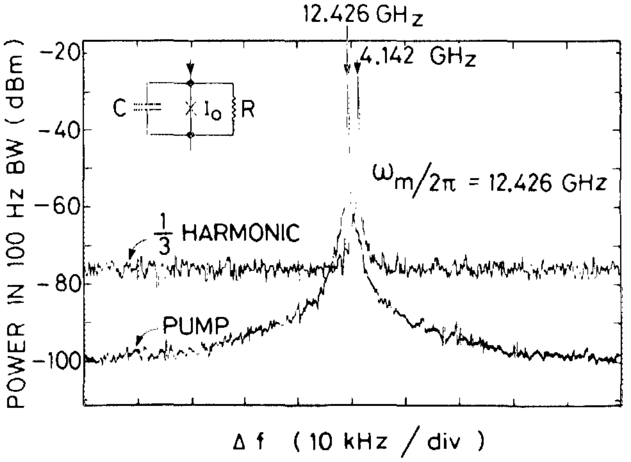

FIG. 1. Spectrum analyzer output showing one-third harmonic generation in a Josephson junction (S-6/7-4). For comparison of the linewidths the microwave pump signal at 12.426 GHz and the one-third harmonic signal emitted by the junction are displayed with the same frequency span. The resolution bandwidth is 100 Hz and the experimental parameters are $I_0 = 163\ \mu\text{A}$, $\omega_{p0}/2\pi = 14.45$ GHz ($\Omega = 0.86$), and $\rho_m = 1.28$. The insert shows the resistively shunted junction model.

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but always weak. All components of the experiment were carefully shielded to minimize room-temperature noise and interference from spurious signals.

We have observed one-third harmonic signal at zero dc bias in four junctions. For a given pump frequency, the signal appeared in a narrow range of pump power. An example at about 4 GHz is given in Fig. 1, with the pump signal at about 12 GHz superimposed for comparison. In this case the observed period three signal is 50 dB above the noise and has a linewidth of less than 100 Hz, limited by the resolution bandwidth of the spectrum analyzer. Thus any noise due to the one-third harmonic generation process introduced a linewidth broadening of less than 100 Hz. In other cases the one-third harmonic signal was weak and noisy or even intermittent, with a linewidth that sometimes exceeded 1 MHz. These noisy signals were generally observed near the boundaries of the regions in the ρ_m-Ω plane where we could observe one-third harmonic generation. The subharmonic signal generated in the zero voltage state was independent of bias current for $\rho < 1$, and disappeared abruptly at $\rho = 1$. Occasionally, we observed weak one-third harmonic signals on the one-third harmonic step at a voltage of $(\hbar\omega_m/2e)/3$.

To make a quantitative comparison of our results with simulations, we require independent estimates of the parameters I_0, ρ_n, ω_{p0} (i.e., C), $Q = \omega_p RC$ (i.e., R), and ρ_m. The measured escape rate from the zero voltage state due to thermal activation in the absence of microwaves[5,6] yielded the noise-free value of I_0 as well as the effective noise temperature $T_{\text{eff}} \approx 12$ K. We obtained ω_{p0} and Q from parametric half-harmonic excitation of the plasma resonance.[7] We detected the signal at the plasma frequency $\omega_p = \omega_{p0}(1-\rho^2)^{1/4}$ emitted by the dc-biased junction pumped weakly at a microwave frequency $2\omega_p$; from these results we deduced $\omega_p(I)$ and ω_{p0} with an accuracy of $\pm 2\%$. We estimated the Q of the plasma resonance at ω_p from the width of the resonance in terms of the bias current by comparing our data to digital computer simulations that included the effects of noise with $T_{\text{eff}} = 12$ K. The error in Q was $\pm 50\%$.

Finally, we estimated the normalized microwave current $\rho_m = I_m/I_0$ by measuring the suppressed critical current $I_0(I_m)$ as a function of microwave power at each of the relevant pump frequencies. Typical results are shown in Fig. 2. For this value of Ω, $I_0(I_m)/I_0$ varies slowly for $\rho_m < 0.1$, at which value there is a marked suppression. The results of simulations are also shown in Fig. 2; the values of the parameters are listed in the caption. We found that $I_0(I_m)/I_0$ jumped abruptly to a lower value for a small increase in ρ_m near 0.1. We have scaled the values of ρ_m by fitting this discontinuity to the abrupt drop in the measured critical current. Although the overall agreement between the experimental and simulated data is not exact we feel justified in using this procedure to obtain an estimate of ρ_m. The abrupt drop in our data is associated with the presence of the plasma resonance.

Having measured the relevant junction parameters *in situ* by independent experiments, we produced state diagrams for the one-third harmonic generation. An example is shown in Fig. 3, where we have indicated the regions in the

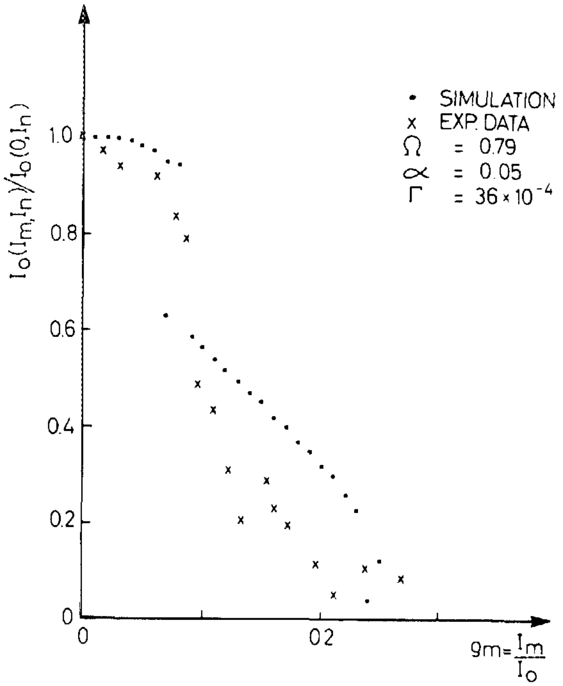

FIG. 2. Experimental and simulated suppression of the critical current vs the normalized microwave current in the junction (S-3008). The experimental points have been linearly scaled along the horizontal axis to fit the simulated points around $I_m/I_0 = 0.1$. Experimental parameters are $I_0 = 166$ μA, $\omega_{p0}/2\pi = 12.68$ GHz, $\omega_m/2\pi = 9.965$ GHz, $\alpha = 0.05$, and $T_{\text{eff}} = 12$ K. Parameters used in the simulation are $\Omega = 0.79$, $\alpha = 0.05$, and $\Gamma = 36\times 10^{-4}$.

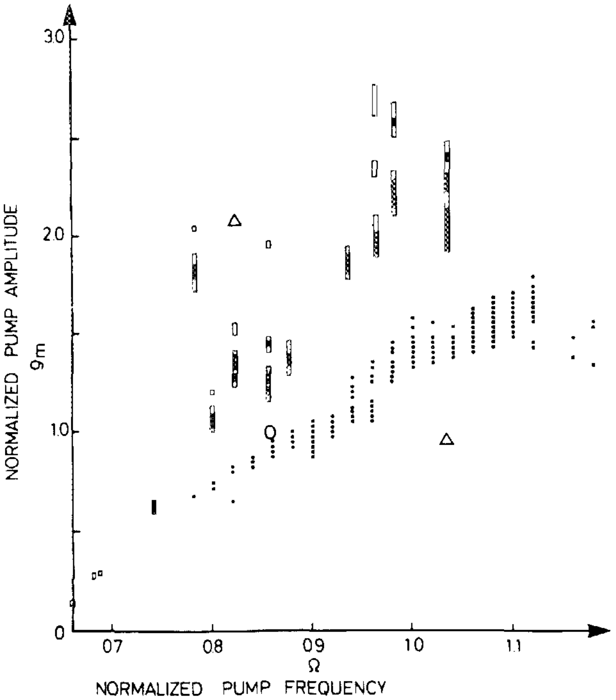

FIG. 3. Normalized microwave pump amplitude ρ_m vs normalized pump frequency Ω showing regions for which one-third (and one-half) harmonic signal occurs (junction S-6/7-4). Experimental: (■) narrow band one-third harmonic at $V = 0$, (□) broad band noisy or intermittent one-third harmonic at $V = 0$, (△) one-third and (○) one-half harmonic at $V = \hbar\omega_m/6e$ and $\hbar\omega_m/4e$, respectively. Experimental parameters are $I_0 = 163$ μA, $\omega_{p0}/2\pi = 14.45$ GHz, $\alpha = 0.05$, and $T_{\text{bath}} = 4.2$ K. Simulations: (●) narrow band one-third harmonic at $V = 0$ ($\alpha = 0.05$).

ρ_m -Ω plane where the one-third harmonic was observed. We emphasize that we can obtain data only when both ω_m and $\omega_m/3$ represent frequencies at which the junction is not too weakly coupled. Thus, the data in Fig. 3 do not imply that the one-third harmonic is nonexistent at frequencies between the bands shown. Rather, it may be that the data map out a single, broad band in the ρ_m -Ω plane in which the one-third harmonic exists. Points representing one-third and one-half harmonic signal on the subharmonic induced steps are also shown.

Figure 3 also shows the results of digital simulations of Eq. (1) for $I = 0$. We carried out the simulations using a second order Runge–Kutta technique with a time step $\Delta\tau = 1/32\ \Omega$ and initial conditions $\phi(0) = [d\phi/d\tau]_{\tau=0} = 0$. Each point in Fig. 3 indicates the existence of a one-third harmonic solution; the grid size was $\Delta\Omega = 0.02$ and $\Delta\rho_m = 0.025$. As with the experimental data, these points should most likely be regarded as mapping out a broad band in the ρ_m -Ω plane in which one-third harmonic generation occurs. The band in which the one-third harmonic exists in either the experiment or the simulations is probably the continuation of one of the bands predicted by Pedersen and Davidson[3] at lower values of ρ_m and Ω. Although the regions of the experimental and simulated data do not overlap, we note that they do occupy much of the same range of frequency, and that the ranges in microwave current differ by at most a factor of 2. We note that a change of 50% in the value of Q used in the simulations does not change the results materially.[8] Given the uncertainties in the experimental parameters which are measured independently rather than fitted, and the omission of noise and the cos ϕ term in the simulations,[4] the qualitative agreement between the two sets of data is quite satisfactory.

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