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Fresnel zones for ground-based antennas

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Fresnel Zones for Ground-Based Antennas

J. BACH ANDERSEN

Summary—The ordinary Fresnel zone concept is modified to include the influence of finite ground conductivity. This is important for ground-based antennas because the influence on the radiation pattern of irregularities near the antenna is determined by the amplitude and phase of the groundwave. A new definition of Fresnel zones which takes into account the phase shift of groundwave propagation is given and it is shown that these new zones are zones of greatest influence, at least when discontinuities of the electrical parameters of the ground are considered. Numerical results for the first Fresnel zone are given, and it is shown that the zone is much smaller for a finite conductivity than for an infinite conductivity, especially for small angles of elevation.

INTRODUCTION

WHEN ESTIMATING the influence of the surroundings on the radiation pattern of a given antenna often only very approximate methods are used. The ground is mostly considered to be homogeneous with a constant conductivity, and if possible inhomogeneities are outside the first Fresnel zone, they are considered to be negligible.

It has been questioned[1,2] whether the Fresnel zone concept really is applicable for UHF and microwave antennas because the cancelling of the contributions from the various zones would not be effective when the amplitude of the field varies markedly over the zones. This would be the case especially when the angle of elevation is small because the resulting ellipse is then very long and very narrow.

The Fresnel zone concept is also important for vertical HF and VHF antennas, in which case the ellipse surrounds the antenna, the ground-based antenna being situated in one of the foci. Also in this case inhomogeneities are neglected when they are outside the first Fresnel zone, the Fresnel zones being computed under the assumption that the ground is perfectly conducting.

It is evident that if, in this latter case also, the Fresnel zones should be a measure of zones of greatest influence, the finite conductivity of the ground should be taken into account in some way. This is seen by the fact that the groundwave, which is generally attenuated to a great extent, excites the disturbance field from any

[1] D. E. Kerr, "Propagation of Short Radio Waves," McGraw-Hill Book Co., Inc., New York, N. Y., p. 417; 1951.

[2] P. Beckman, "On the problem of the reflection of radio waves from rough surfaces," *Slaboproudy Obzer* (in Czech.), vol. 14, pp. 302–309; 1953.

obstacle or discontinuity in the vicinity of the antenna. It is the purpose of this paper to modify the Fresnel zone concept in such a way that the dependence on the ground conductivity is taken into account.

Theoretical Development

The boundary of a usual Fresnel zone around a ground-based source A (Fig. 1) is defined as the locus of D on the plane such that

$$AD + DB - AB = \nu \frac{\lambda}{2}, \qquad \nu = 1, 2, 3, \cdots \quad (1)$$

or

$$\phi_0(AD) - \phi_0(AH) = \nu \cdot \pi \quad (2)$$

where λ is the wavelength, $\phi_0(l) = k_0 \cdot l$ is the phase shift in propagation over a length l, and k_0 is the free space wavenumber.

The modified Fresnel zones are now defined in the following way:

The boundary of a modified Fresnel zone is the locus of D on the plane such that

$$\phi_1(AD) - \phi_0(AH) = \nu \cdot \pi, \qquad \nu = 1, 2, 3, \cdots \quad (3)$$

where $\phi_1(AD)$ is the phase shift in groundwave propagation from A to D and $\phi_0(AH)$ is the phase shift in spacewave propagation from A to H.

When the ground is perfectly conducting, (3) reduces to (2), and the modified Fresnel zones are equal to the usual Fresnel zones.

It is not evident from a physical viewpoint that this definition fulfills the needs mentioned in the Introduction, namely that the zones should be a measure of greatest influence. The only way to prove this is to study the solution to a problem where a discontinuity or inhomogeneity is introduced close to the antenna; the model chosen here is a ground which is flat and consists of two sections of different electrical properties.

In previous papers[3], [4] the radiation pattern over an inhomogeneous ground has been expressed as a surface integral in the following way:

$$A' = A + \text{const.} \times \iint_S H_{At} \cdot H_{Bt}' dS. \quad (4)$$

A' is the radiation pattern over the inhomogeneous ground, A is the radiation pattern over a homogeneous ground consisting of Medium 1 (see Fig. 2). H_{At} is the tangential, magnetic field from a dipole at A over homogeneous ground (Medium 1). H_{Bt}' is the tangential, magnetic field from a dipole at B over the actual, in-
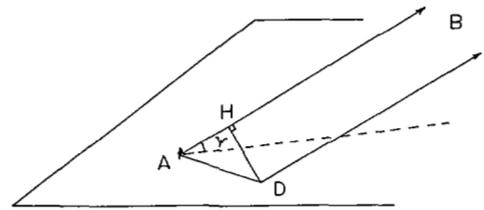
[3] J. B. Andersen, "The Skywave Field from a Vertical Dipole Over an Inhomogeneous, Flat Earth Consisting of Two Media," Tech. Note No. 2, AF 61(052)-503; October, 1961.

[4] J. B. Andersen, "An Extension to the Theory of a Dipole Over an Inhomogeneous Grounds," Tech. Note No. 4, AF 61(052)-503; November, 1962. Part of this and the first report is published in "Electromagnetic Theory and Antennas," E. C. Jordan, Ed., Pergamon Press, New York, N. Y.; 1963.

Fig. 1—Point-source A on plane surface, D locus of boundary of Fresnel zone, B point of reception, ψ angle of elevation.
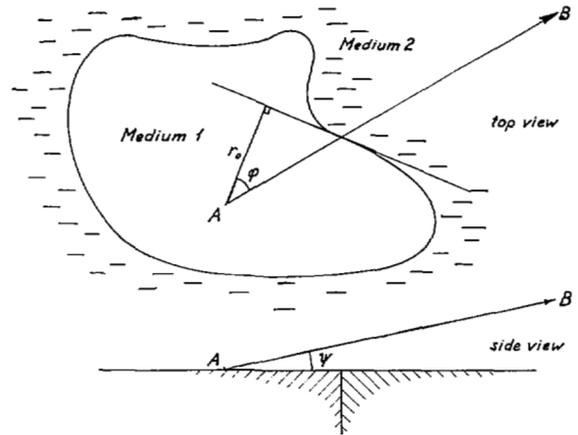

Fig. 2—Dipole on inhomogeneous ground.

homogeneous ground. The surface of integration S is over Medium 2. The constant in (4) is proportional to the difference in surface impedance between the various sections of ground.

Eq. (4) is useful only when the integrand can be approximated in some manner; it has been shown previously that a very good approximation is obtained when H_{Bt}' is replaced by H_{Bt}, the field from B over a homogeneous ground. In approximating in this manner, reflections from the boundary line are neglected, but these reflections are known to be very small when the angle of elevation ψ is small.

For an arbitrary boundary line, the integration could only be done numerically but here we approximate the boundary line by its tangent at the point where a line in the direction of propagation intersects the boundary. The surface integral is then converted to a line integral by means of the stationary phase method and the following results (for details see Andersen[3], [4]):

$$A' = A + \text{const.} \times \int_{t_0}^{\infty} G\left(\frac{t}{c}\right) \frac{e^{it}}{\sqrt{t}} dt; \quad (5)$$

$G(t/c) = G(p)$ is Sommerfeld's attenuation function.

$$t_0 = kr_0 \left[\sqrt{1 - \cos^2\psi \sin^2\phi} - \cos\psi \cos\phi \right] \quad (6)$$

$$p_0 = \frac{t_0}{c} = \frac{i}{2}(1 - \cos^2\psi \sin^2\phi)^{-1/2} kr_0 u^2(1 - u^2) \quad (7)$$

$$c = \frac{2(1 - \cos^2\psi \sin^2\phi - \cos\psi \cos\phi\sqrt{1 - \cos^2\psi \sin^2\phi})}{i \cdot u^2(1 - u^2)} \quad (8)$$

where k is the wave number $2\pi/\lambda$, and r_0, ϕ and ψ are defined in Fig. 2; also

$$u^2 = \frac{\epsilon_0}{\epsilon + i\dfrac{\sigma}{\omega}},$$

where ϵ and σ refer to Medium 1. The argument p in the attenuation function reduces to the well-known numerical distance when $\phi = 0$.

The second term in (5) represents the correction to the far-field pattern due to the presence of the second medium. When Medium 1 is perfectly conducting, $G(p) = 1$ and the integral in (5) reduces to a Fresnel integral with argument t_0. A fixed t_0 implies a constant correction, and $t_0 = \pi$ determines the first Fresnel zone, $t_0 = 2\pi$ the second Fresnel zone, etc. For fixed t_0 and ψ, (6) is the equation for a circle in polar coordinates (r_0, ϕ) (see Fig. 3). The ellipse corresponding to a Fresnel zone is inscribed in the circle in such a way that the boundary line is tangent to the ellipse. The position of the dipole is at one of the foci of the ellipse. When the boundary is outside the first Fresnel zone, $t_0 > \pi$ and the correction term in (5) is small.

One can think of the two terms in (5) as two vectors which when added give the resultant A'. When t_0 increases, the correction vector rotates around the endpoint of A with a decreasing amplitude, and when t_0 is infinite, A' is equal to A. This would give a figure corresponding to the well-known Cornu spiral for Fresnel integrals when the conductivity is infinite.

Here we define the complementary Fresnel integral as

$$CF(t_0) = \int_{t_0}^{\infty} \frac{e^{it}}{\sqrt{2\pi t}} dt \qquad (9)$$

and define the endpoints of the various Fresnel zones as the points where the tangent to the Cornu spiral is horizontal

$$\frac{d}{dt_0}\left\{ \mathrm{Im}\left[CF(t_0)\right] \right\} = 0, \qquad (10)$$

where $\mathrm{Im}[CF(t_0)]$ denotes the imaginary part of $CF(t_0)$. This leads to

$$t_0 = \pi, 2\pi, \cdots \qquad \text{(origin not included)},$$

which is in agreement with ordinary Fresnel zone theory.

It is now a natural extension of the Fresnel zone concept to define the various Fresnel zones for an arbitrary ground conductivity by

$$\frac{d}{dt_0}\left[\mathrm{Im}\int_{t}^{\infty} G(p)\, \frac{e^{it}}{\sqrt{2\pi t}} dt \right] = 0 \qquad (11)$$

or

$$\tan(t_0) = -\tan\left[\arg G(p_0)\right] \qquad (12)$$

instead of (10). The zeros of (1'), $t_0^{(1)}$, $t_0^{(2)}$, etc. determine curves in the (r_0, ϕ)-plane which correspond to the
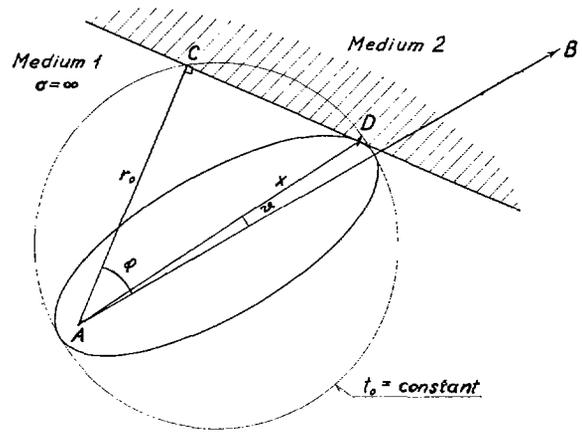

Fig. 3—Ordinary Fresnel zone is an ellipse when Medium 1 is infinitely conducting.

circle in the case, when the ground is infinitely conducting.

$t_0^{(1)}$ corresponds to the first Fresnel zone,

$t_0^{(2)}$ corresponds to the second Fresnel zone, etc.

The boundary of the first Fresnel zone according to (11) indicates approximately the place where the correction vector starts to rotate around the asymptotic value. The zone will not be an ellipse.

In order to find the "ellipse" when the "circle" is known, we utilize the fact that CD (see Fig. 3) is a tangent to the ellipse, and from simple geometry, we derive that the distance to the point of tangency is equal to

$$x = \sqrt{\left(\frac{dr_0}{d\phi}\right)^2 + r_0^2}, \qquad (13)$$

and the angle θ is given by

$$\mathrm{tg}\,(\phi - \delta) = -\frac{1}{r_0}\frac{dr_0}{d\phi}. \qquad (14)$$

The procedure for finding the modified Fresnel zones can now be expressed in the following way:

1) The given data are $|u^2|$, $\arg(u^2)$ and ψ.
2) ϕ is made to run from $0°$ to $180°$.
3) c is found from (8) for each ϕ.
4) $r_0(\phi)$ is found from (11) for each ϕ.
5) A numerical differentiation of $r_0(\phi)$ yields x and θ by means of (13) and (14) and the Fresnel zone may be plotted.

These computations can easily be made by means of an electronic computer when a procedure for Sommerfeld's attenuation function exists. Such a procedure has been given by Christiansen[5] and is described in a report, where it is used for finding parametric curves for radiation fields from an antenna near a coastline.

[5] S. Christiansen and J. B. Andersen, "Parametric Curves for Radiation from a Vertical Dipole near a Coastline," Tech. Note No. 6, AF 61(052)-503; 1963.

Finally, it should be mentioned that the present extension of the Fresnel zone concept could just as well be based on the usual Kirchhoff method of integration over a homogeneous surface.[6]

If u is a z-component of a Hertzian vector, we could write

$$u(B) = u_0(B)$$
$$+ \frac{1}{4\pi} \int \int \left[\frac{e^{ikr}}{r} \frac{\partial u}{\partial n} - u \frac{\partial}{\partial n} \left(\frac{e^{ikr}}{r} \right) \right] dS.$$

This formulation is similar to the formulation in (4), which was based on the compensation theorem.

If u in the integral is approximated by

$$u_{z=0} = g(\rho) \frac{e^{ik\rho}}{\rho}$$

where ρ is the distance from A, and we neglect the slow variation of $g(x, y)$ compared to the exponential function, then the usual Fresnel zones are determined by the stationary phase of the integral.

When the dipoles are sufficiently elevated, $g(\rho)$ is a constant, but when the dipole at A is ground-based, $g(\rho)$ is given by Sommerfeld's attenuation function $G(p)$, and if this is included in the integral and the zones over which the phase is stationary are found, then (11) is derived again.

Thus it seems that it is an unnecessary complication to introduce a second medium and base the extension of the Fresnel zone concept on the influence of this second medium. However, this is done in order to verify the applicability of the Fresnel zone principle in a case where the actual ground is inhomogeneous.

It is not difficult to show that (12) is in agreement with (3), and the modified (as well as the ordinary) Fresnel zones are therefore zones of greatest influence.

NUMERICAL RESULTS

In order to determine the Fresnel zones it is necessary to solve the transcendental equation (12)

$$\tan (t_0) = U,$$

where

$$U = - \tan [\arg G(p_0)].$$

Curves of arg $G(p_0)$ as a function of the numerical distance can be found in the literature. The function U is shown in Fig. 4 together with $\tan(t_0)$ for various values of $|u^2|$ and arg (u^2). The intersection between the two curves determined the various Fresnel zones according to (12). The points marked (1) give the first Fresnel zone, (2) the second Fresnel zone, etc. The general behaviour of $U(t_0)$ is as follows: U is negative until the phase of G is $\pi/2$, where it goes from minus infinity to plus infinity, after which it decreases to a constant positive value for $t_0 \rightarrow \infty$.

[6] G. Boudouris, "Propagation Tropospherique," Centre de Documentation Universitaire, Paris, France; 1957.
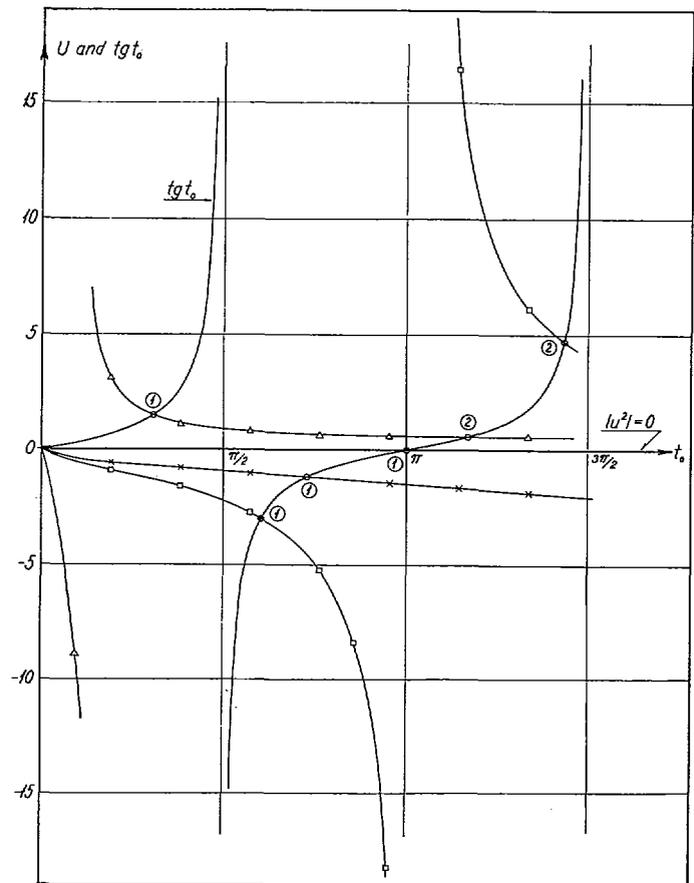

Fig. 4—$U = -\tan\{\arg G(p_0)\}$ and $\tan(t_0)$ as a function of t_0

	$\psi = 10°$,	$\phi = 0°$.
—×—	$|u^2| = 0.01$,	$\arg(u^2) = -10°$.
—□—	$|u^2| = 0.01$,	$\arg(u^2) = -70°$.
—△—	$|u^2| = 0.1$,	$\arg(u^2) = -70°$.

From Fig. 4 it is now easily seen how the Fresnel zones are modified by the finite conductivity of the ground. When $|u^2|$ is equal to zero $(\sigma = \infty)$, U equals zero and the points of intersection are $t_0 = \pi, 2\pi, \cdots$, etc. For finite $|u^2|$, the first point of intersection lies between zero and π, the second between π and 2π, etc. Thus the first Fresnel zone moves in around the antenna with no lower theoretical limit. There is a limitation though, because Sommerfeld's attenuation function is not valid unless $|u^2| < 0.2$.

An interesting, special case which is not shown on Fig. 4 is the case of a lossless, dielectric ground. Then the argument of $G(p_0)$ never reaches $\pi/2$, and U decreases to minus infinity for $t_0 \rightarrow \infty$. If $|u^2|$ is great enough, the points of intersection are $\pi/2, 3\pi/2, \cdots$, i.e., $\pi/2$ less than the value for a perfect conductor.

A computer program which gives the first Fresnel zone for arbitrary values of the parameters has been written in Algol. Because there is only one root of (12) in the interval $0 < t_0 \leqq \pi$, a regula falsi method could be used to determine the roots.

For each value of ψ, $|u^2|$ and arg (u^2), kr_0 was found for $\phi = 0°, 10°, \cdots, 180°$. These nineteen values were
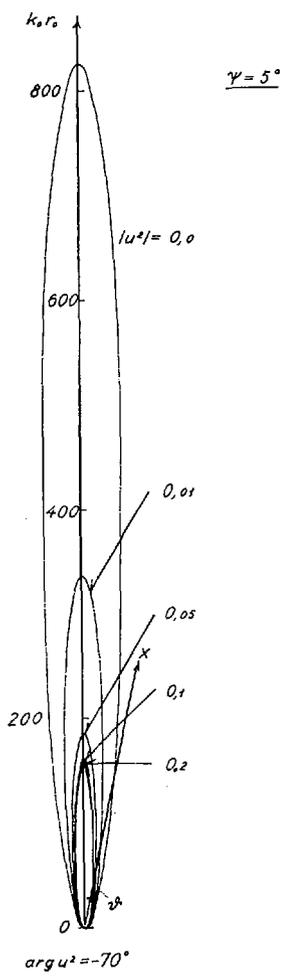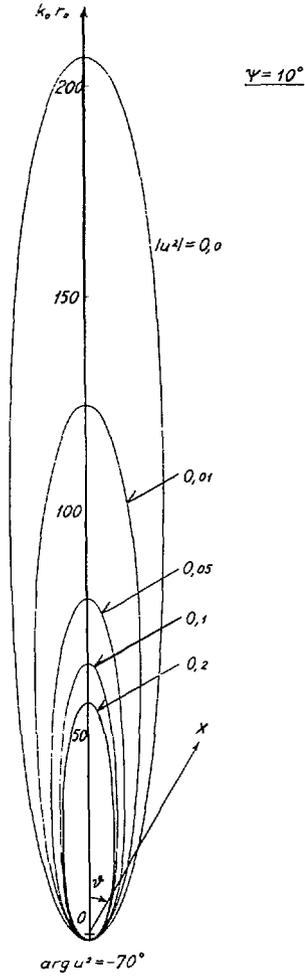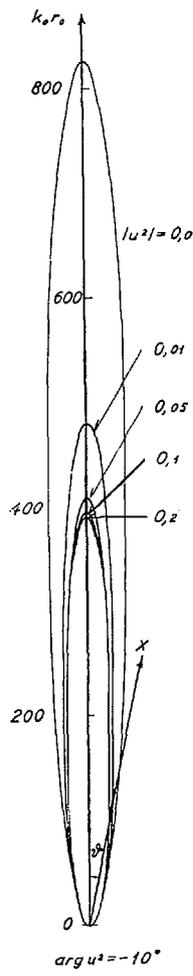
Fig. 5—First Fresnel zone for $\psi = 5°$.

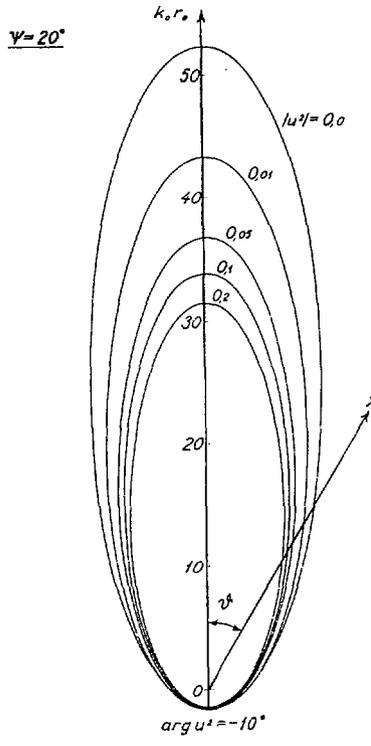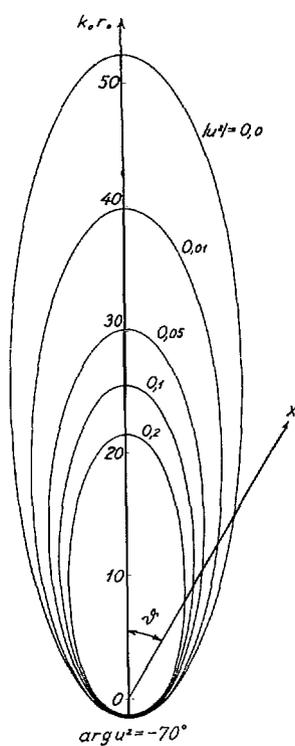Fig. 6—First Fresnel zone for $\psi = 10°$.

Fig. 7—First Fresnel zone for $\psi = 20°$.

then used in a numerical differentiation procedure to find x and δ according to (13) and (14). Some numerical results are displayed in Figs. 5–7 for $\psi = 5°$, $10°$ and $20°$, arg $(u^2) = -10°$ (a lossy dielectric), and arg $(u^2) = -70°$ (a good conductor), and $|u^2| = 0$, 0.01, 0.05, 0.1, 0.2.

It is seen that in general the size and form of the zones are different from the usual ones ($|u^2| = 0$). The change is greatest in the direction of propagation ($\phi = 0$) and is very small in the backward direction ($\phi = 180°$). For small angles of elevation ($\psi = 5°$), the length of the zone is only about $\frac{1}{4}$ of the length for a perfect conductor, when arg $(u^2) = -70°$. Normally, the change is greater for a conductor than for a dielectric for the same $|u^2|$.

An example which shows the applicability of the Fresnel zone concept is shown in Fig. 8. The ordinate is the radiation pattern amplitude (in db relative to a perfectly conducting ground) from a vertical dipole which is placed on a lossy ground at a distance r_0 from a perfectly conducting medium. The projection of the direction of propagation is normal to the boundary between the two media. The frequency is 10 Mc, $\sigma = 10^{-2}$ mho/m and $\epsilon_r = 7$ for the medium around the dipole, and $\psi = 5°$ and $10°$. It is evident from the curves that the Fresnel zone defined in the text (a) is a much better measure of the limit of greatest influence than the usual Fresnel zone (b). When kr_0 is greater than $kr_{Fr(a)}$, $|A'|$ only deviates about 1 db from its asymptotic value.
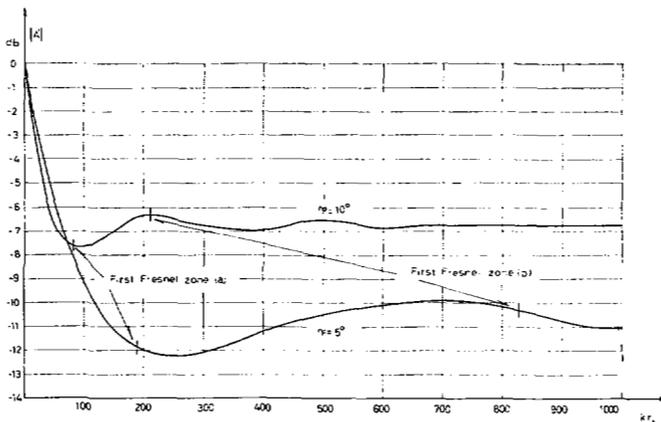

Fig. 8—Radiation pattern amplitude for a vertical dipole placed on a lossy ground ($\epsilon_r = 7$, $\sigma = 10^{-2}$ mho/m, $f = 10$ Mc) in a distance r_0 from a perfectly conducting medium. (a) corresponds to end of first Fresnel zone as given in Figs. 5 and 6. (b) corresponds to the usual Fresnel zones.

Conclusion

An evaluation of the influence of finite ground conductivity on Fresnel zones has been given. It is shown that all the Fresnel zones move in around the antenna when the ground is lossy, and the first Fresnel zone, which is of primary interest, is in general much smaller than the one computed for a perfectly conducting ground. The higher-order Fresnel zones are modified to a smaller extent.

The numerical results for the first Fresnel zone show that the most radical change takes place when the angle of elevation is small and when the numerical value of the complex dielectric constant is small. For equal numerical values of the complex dielectric constant, the change of the first Fresnel zone compared to a perfect conductor is greatest when the phase of the dielectric constant corresponds to a good conductor.

The results given can be applied when it is of interest to know what part of the antenna surroundings influence the radiation pattern. The antenna is supposed to be a vertical ground-based antenna, and the present Fresnel zone concept is useful for antennas operating in the HF and VHF range.

It must be emphasized that the present extension of the Fresnel zone concept is based on a flat ground with electrical discontinuities, it is not directly applicable to irregular terrain unless the irregularities are such that an effective surface impedance can be ascribed to the surface. Furthermore, only the phase of the groundwave enters in this calculation because the phase determines the distance from the antenna to the point, outside of which the contributions from the various zones tend to cancel due to phase relationships. When the boundary line between the two media is outside the first Fresnel zone, the radiation pattern deviates only little from the radiation pattern over homogeneous ground. The magnitude of the groundwave is also important, because it is responsible for the amplitude of the deviations. Thus we can conclude that a finite conductivity diminishes the influence of the surroundings in two ways: the important area of reflection around the antenna gets smaller, and the magnitude of any disturbance fields gets smaller than for a perfectly conducting ground. Only the former has been considered here in a quantitative manner.