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Study of the geometrical resonances of superconducting tunnel junctions*

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The resonant cavity structure of superconducting Sn—Sn-oxide—Sn tunnel junctions has been investigated via photon-assisted quasiparticle tunneling. We find that the temperature-dependent losses at 35 GHz are determined by the surface resistance of the Sn films for reduced temperatures between 0.5 and 0.8. Our results are in very good agreement with the microscopic theory of Mattis and Bardeen for the surface resistance of Sn.

Electromagnetic waves can propagate in a superconducting tunnel junction with a phase velocity much smaller than the free-space velocity of light.[1] The resonant structure associated with such a junction manifests itself in experiments involving the ac Josephson effect[2-5] and photon-assisted tunneling.[6,7] These cavity modes play a particularly crucial role in high-precision measurements of $2e/h$ (via the microwave-induced steps) made with the use of Josephson tunnel junctions.[8] Although the existence of these modes has been well established, the nature of the losses determining the linewidth or Q has not been studied experimentally. Much interest in the various loss mechanisms has recently developed of the Josephson quasiparticle-pair interference current by Pedersen, Finnegan, and Langenberg.[9] To clarify the role of this interference term of the ac properties of a junction, it is essential to understand the other junction loss mechanisms. We report here the first detailed measurements of the geometrical resonance linewidth or Q. By comparing the experimental results with calculations based on an equivalent circuit representation of the junction similar to that of Scott,[10] we find that the dominant loss mechanism for reduced temperatures above 0.5 can be attributed to the surface resistance of the superconducting films which form the junction.

For a one-dimensional tunnel junction, the fundamental resonant frequency $\omega_R = \pi\bar{c}/L$ is determined by the junction length L and the phase velocity $\bar{c}$. The latter quantity is a function of the penetration depth λ of the electromagnetic fields, and $\bar{c}$ is therefore both temperature and frequency dependent.

For high microwave frequencies, a particularly convenient way to study the geometrical resonance is to vary ω_R by means of the temperature. The use of the photon-assisted tunneling phenomenon provides a simple and direct method for investigating the mode structure.

Hamilton and Shapiro[6] have shown that when a junction is coupled to external microwave fields, standing waves will be set up in the junction and therefore must be included in the theoretical description of the problem. Recently, Soerensen and Samuelsen[7] have shown that the relative step height, a_N, of the Nth photon-assisted step, when the applied frequency $\omega = n\omega_R$, is

$$a_N = (1/L)\int_0^L J_N^2(\alpha)\,dx, \tag{1}$$

with $\alpha = e v_{\mathrm{rf}}(x)/\hbar\omega$, where the rf voltage across the junction has the form

$$v_{\mathrm{rf}}(x) = v_0\cos(n\pi x/L). \tag{2}$$

The a_N are not very sensitive to the detailed spatial variation of $v_{\mathrm{rf}}(x)$ near $\omega = n\omega_R$, and for $N=0$ and $N=1$ [i.e., the steps at $2\Delta/e$ and at $(2\Delta - \hbar\omega)/e$] the dependence of the relative step amplitudes on $\langle\alpha^2\rangle$ is nearly independent of the spatial variation for small values of v_0.[11] (The brackets $\langle\,\rangle$ denote a spatial average.) Exact calculations of a_N have been carried out by assuming the voltage dependence described by Eq. (2).[7,11] Experimentally, a constant microwave field is coupled to the junction, and the observed height of the $N=0$ step, a_0, is used to determine $\langle\alpha^2\rangle$ and, hence, $\langle v_{\mathrm{rf}}^2\rangle$ in the junction at various temperatures.

A tunnel junction can be regarded as a section of transmission line with an equivalent circuit as indicated in Fig. 1. The line is terminated at each end by an effective impedance Z_0 which phenomenologically represents the radiation losses at the edges of the junction. For simplicity we take the impedance to be purely resistive, i.e., $Z_0 = R_0$.[12] The relevant junction parameters are obtained by solving the Maxwell and London equations in the two-fluid model. For a tunnel junction with thick films, the resistance r, the inductance l, and the capacitance c per unit length are, respectively,

$$r = 2R_s/w, \tag{3a}$$

$$l = \mu_0(2\lambda + d)/w, \tag{3b}$$

$$c = \epsilon_0\epsilon_r w/d. \tag{3c}$$

Here R_s is the real part of the surface impedance while λ is related to the imaginary part of the surface impedance X_s by the relation $X_s = \omega\mu_0\lambda$. The remaining quantities are ϵ_r, the relative dielectric constant of the oxide layer; w, the junction width; and d, the oxide thickness. The external microwave source appears as a constant current source coupled to the junction via the external waveguide impedance. The microwave currents induced at the edges of the junction will in general be a superposition of in-phase and out-of-phase components depending on whether the excited currents are in-phase or out-of-phase at the two junction edges. The symmetry of the external field is denoted even (in-phase) or odd (out-of-phase), corresponding to the parity of the junction mode numbers n [see Eq. (2)] which can be excited.

Using the junction model shown in Fig. 1, we have

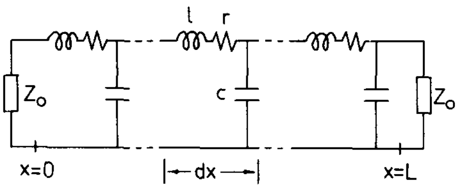

FIG. 1. Equivalent circuit of the tunnel junction strip line.

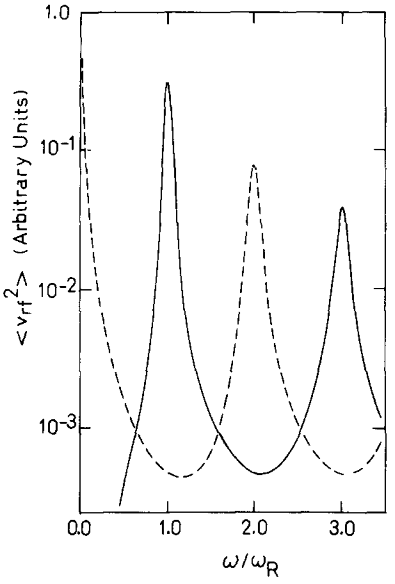

FIG. 2. Mean square of the microwave-induced voltage as a function of frequency for external fields of even symmetry (broken line) and odd symmetry (solid line) with $R_s/X_s = 0.05$ corresponding to a reduced temperature $t = 0.675$.

evaluated the voltage v_{rf} as a function of the coordinate x, the temperature, and the applied frequency. The microwave losses enter via the ratio R_s/X_s. This ratio was obtained from a calculation of the surface impedance $R_s + iX_s$ for Sn as a function of temperature and frequency with the use of the theory of Mattis and Bardeen.[13] The calculations were made for the case of a finite mean free path l and diffuse surface scattering.[14] For $l = 1000$ Å, R_s/X_s varies almost exponentially between 0.016 at a reduced temperature $t = 0.50$ and 0.19 at $t = 0.90$. (The reduced temperature t equals the ratio T/T_c, where T_c is the superconducting transition temperature.) Over this entire temperature range, R_s/X_s for $\omega = \omega_R$ and $\omega = 2\omega_R$ are the same within 10%.

In Fig. 2, the quantity $\langle v_{rf}^2 \rangle$ is shown plotted as a function of the applied frequency for a constant $t = 0.675$. From this figure, it is clear that near frequencies $\omega' = n\omega_R$ either an even or odd mode is sharply peaked, and therefore the resonant response does not depend critically on nonideal coupling to the external fields. The quality factor Q of the junction resonance [defined by $Q = 2\pi$(energy stored)/(energy dissipated per cycle)] can be expressed in terms of the junction parameters. If the film losses are dominant,

$$Q = \omega l / r \approx X_s / R_s. \tag{4}$$

The results of a series of measurements in the same junction are shown in Fig. 3. The observed $\langle v_{rf}^2 \rangle$ are plotted as a function of temperature for three different applied frequencies. The corresponding theoretical results obtained via the surface impedance calculations are also shown. The ratio between the barrier thickness and the dielectric constant, d/ϵ_r, has been used as an adjustable parameter. The solid lines in Fig. 3 are the calculated results for $d/\epsilon_r = 5.33$ Å. This choice of d/ϵ_r gives the best over-all agreement with the experimental results and is consistent with $d \approx 20$ Å and $\epsilon_r \approx 4$. For each of the three cases shown in Fig. 3, the temperature at which $\omega = \omega_R$ is indicated by an arrow.

As the applied frequency is increased, the peak position shifts toward lower temperatures. If the film losses are dominant, the amplitude of the induced rf voltage is proportional to Q, and, since Q increases as T decreases, the peaks in Fig. 3 become much sharper at lower temperatures. The agreement between the theoretical and experimental results is very good. We have also considered the effects of dielectric losses in the barrier in our calculations by varying the loss tangent and comparing the computed results with our experimental data. We have concluded that $\mathrm{Im}\,\epsilon_r/\mathrm{Re}\,\epsilon_r < 5 \times 10^{-3}$. The Q's due to the losses in the films as determined from Eq. (4) were between 15 and 30.

The junction resonant response near $\omega = 2\omega_R$ was also observed by using external frequencies of 69.28 and 69.42 GHz. The peaks occurred at temperatures in good agreement with those expected from theory by using the value of d/ϵ_r obtained at $\omega = \omega_R$ and by taking into account the dispersion due to the frequency dependence of λ. Evidence of a reproducible fine structure and a sharpening of the main peak was observed; however, because of experimental difficulties at these fre-

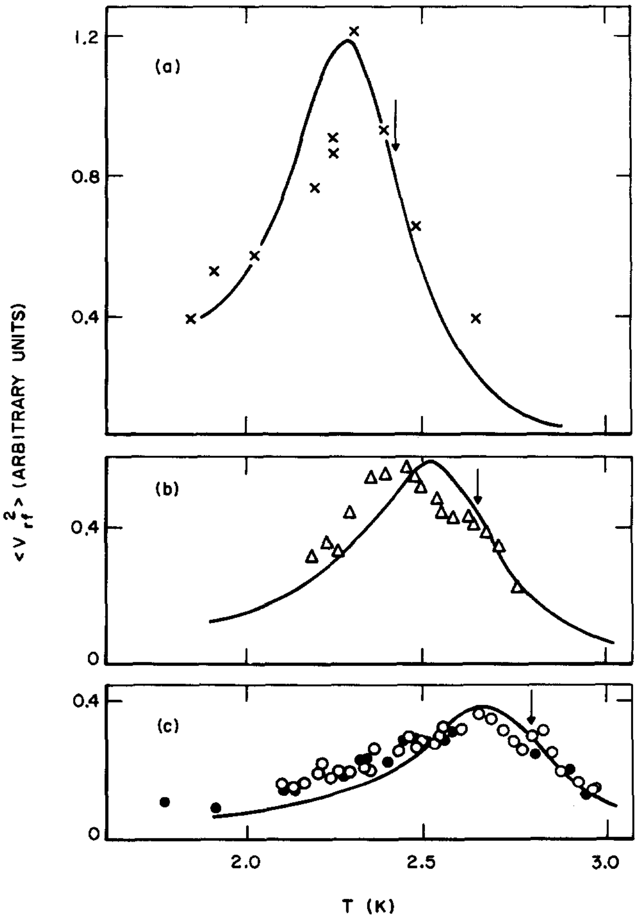

FIG. 3. Experimental and theoretical junction response as a function of temperature. The solid lines are theoretical results with junction parameters $L = 0.26$ mm, $w = 0.08$ mm, and $d/\epsilon_r = 5.33$ Å. (a) $f = 34.70$ GHz, external cavity at resonance; (b) $f = 33.93$ GHz, external cavity detuned; (c) $f = 33.33$ GHz, external cavity at resonance (full circles), external cavity detuned (open circles). The arrow in each case indicates the point $\omega_R(T) = \omega$.

quencies, it is not meaningful to make a detailed comparison between theory and experiments.[15]

Some preliminary experiments on resonant Pb—Pb-oxide—Pb junctions at 4.2 K have been carried out by Finnegan and Toots[16] using the microwave-induced Josephson steps to study the fundamental geometrical resonance ($\omega = \omega_R$) at 9 GHz. In these experiments, the Q was obtained by fitting a Lorentzian curve to the observed frequency-dependent response of the junctions to external microwave radiation. The observed Q's were about 110. Applying our theoretical model to a Pb tunnel junction (assuming a mean free path limited by the film thickness on the order of 1500 Å), we find $Q = 150$, which is in rather good agreement with the observed results for Pb—Pb-oxide—Pb junctions.

In conclusion, we have shown that the high frequency losses in resonant superconducting tunnel junctions are dominated by losses in the superconducting films and not by losses in the dielectric oxide barrier. These losses in the films are adequately described by the surface impedance which is strongly temperature dependent. We believe that the dominant microwave losses in a Josephson tunnel junction are also due to the surface resistance of the superconducting films and therefore should have important consequences in furthering our understanding of the electrodynamics of Josephson junctions. Our results indicate it should be possible to control the Q of a tunnel junction by using suitable superconducting alloys for the films.

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[15] At 69 GHz, the waveguide system was oversized. Also, at these frequencies the sample heating was a serious problem.

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