



## Analysis and Comparison of Typical Models within Distribution Network Design

Jørgensen, Hans Jacob; Larsen, Allan; Madsen, Oli B.G.

*Publication date:*  
2006

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*  
Jørgensen, H. J., Larsen, A., & Madsen, O. B. G. (2006). Analysis and Comparison of Typical Models within Distribution Network Design. Paper presented at Third International Workshop on Freight Transportation and Logistics, Altea, Spain, .

---

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

# Analysis and Comparison of Typical Models within Supply Chain Management

Hans Jacob Jørgensen, Allan Larsen and Oli B.G. Madsen

Centre for Traffic and Transport (CTT)  
Technical University of Denmark  
DK-2800 Kgs. Lyngby, Denmark

E-mail: ala@ctt.dtu.dk

14 March 2006

## 1 Introduction

Efficient and cost effective transportation and logistics plays a vital role in the supply chains of the modern world's manufacturers. Global distribution of goods is a very complicated matter as it involves many different distinct planning problems. The focus of this presentation is to demonstrate a number of important issues which have been identified when addressing the Distribution Network Design problem from a modelling angle. More specifically, we present an analysis of the research which has been performed in utilizing operational research in developing and optimising distribution systems.

## 2 Basics of Distribution Systems

Systems that are defined as *distribution systems* can be of extremely varying structure. Nonetheless, such systems need to contain a number of common properties. A distribution system needs to include one or more *customers* who are defined by having a demand on a given product. In addition, the system needs to include one or more *sources*, which are defined by producing or containing the product demanded by the customer(s). Finally, the system needs a connection between the source(s) and customer(s), which can accommodate a flow of the product from the source(s) to the customer(s) in order to obtain fulfilment of the demand. In terms of mathematical network theory, the distribution system can therefore be seen as a standard directed network containing nodes and directed arcs, where the set of sources and the set of customers can be seen as two distinct sets of nodes in the network which are connected by paths.

One of the very basic problems, which fulfil the properties of a distribution system, is the *Minimum Cost Network Flow Problem* (MCNFP). In this problem, the sources and customers are nodes in a general directed network. In addition to the sources and customers the total set of nodes might likewise contain a further subset of *intermediate* nodes. The directed links in the network are characterized by a (linear) cost in the amount of goods flowing through, and the resulting objective is therefore to guide the flow through the network which fulfils the customer demands while minimizing the flow costs. Although the MCNFP contains the basic properties to make it an example of a distribution system, the model is generally too simple to reasonably model actual real-life distribution systems and supply chains. To model any

real-life problem, a considerable number of modifications need to be performed, but exactly which depend on the specific instance and the level of detail needed, as the aim will always be to find the correct (best) trade-off between giving the model a proper level of detail to be realistic, but still keeping it simple enough to be solvable. In the following ten factors which need to be considered when constructing a mathematical model of a distribution system are presented.

### **1. Activity costs**

In the MCNFP the only cost to be minimized is the linear flow cost on the links, which is usually interpreted as the specific transportation cost. However, it is seldom reasonable to assume that this is the only cost of importance, and even a simple problem as the MCNFP includes a number of activities that are associated with considerable costs in a real-life situation. It will usually be reasonable to assume that the sources induce some kind of cost by supplying the goods, a cost which can be interpreted as production cost. Likewise, warehouse storage costs associated with the flow through the intermediate nodes are seen in the models.

### **2. The nature of the intermediate nodes (Distribution Centres (DC's))**

As described, a path needs to exist between the sources and the customers in the distribution system network. The question is then: Are there any intermediate nodes between the sources and the customers, and if so, what is the real-life interpretation of these nodes that are neither sources nor customers? Usually these will be considered as some kind of warehouse, hub or distribution centre. An intermediate node might in certain cases likewise be seen as an aggregation of several processes in the system, such as a combined storage and merging facility. In dynamic models, intermediate nodes can take on even further roles.

### **3. Solution and network structure**

An important decision is the structure of the flow of goods in the network. In the MCNFP the solution structure is general, as the flow can take any path through the network. In certain problems though, the flow in the network will always have a specific structure, for instance pass one DC on its way from source to customer. In this case, it can be advantageous to use a specifically fixed-echelon model of the network. The most frequently encountered fixed-echelon network is the 2-echelon network, which is a specifically structured network where 2 refers to the number of link "types" (source to DC and DC to customer). If the flows in the network are expected to be more complex, it can be necessary to define more complex general networks, and, in the most extreme case, a fully connected network. Fixed-echelon flow structures can be modelled on general networks, but the opposite is not possible.

### **4. Infrastructure location**

A further important decision when formulating distribution models is whether the infrastructure elements of the network (links and nodes) are given beforehand. If not, the formulation needs to include the optimal location of the relevant elements. This will influence the cost function and the model formulation, which can become somewhat more complex.

### **5. Capacity**

In the basic MCNFP, the total surplus (capacity) at the sources equals the total demand of the customers, and the problem is basically to create equilibrium of goods in the system. This is of course an oversimplification of most real-life problems, as the sources will usually have associated a specific capacity, which is, as such, independent of the demands. If the capacities are not "much" bigger than the demands, they therefore need to be included as capacity constraints associated with

the sources in the model.

**6. The number of commodities**

In many problems a multitude of products or commodities are involved in the flow and if these different commodities cannot reasonably be aggregated to single flows it becomes necessary to include these in the model. Specific demands, capacities and flows therefore have to be modelled independently for the different commodities. The commodities might also have different characteristics regarding size, production and flow costs etc., which can therefore lead to further differences in the constraints needed.

**7. Direct shipments/demand fulfilment**

A special class of problems include direct shipments. In these models the customers need to have their demand fulfilled by specific sources. Whether this problem is at all a distribution problem is probably a matter of discussion, but from a strict modelling point of view, the direct shipment problem has many similarities to the multi-commodity distribution problem.

**8. Multiple transportation modes**

Various transportation modes can be involved in a distribution system or supply chain. If these modes differ considerably in for instance transportation time or transportation costs it can be necessary to include a modal choice for the flows in the network to properly optimise the system.

**9. Dynamics**

In a dynamic model the production can, in order to fulfil a given demand, occur in earlier time periods than the demand. The effect of including dynamic information into the model is the inclusion of storage possibilities where the goods can be stored between production and delivery. This will usually be carried out at the DC's.

**10. Nonlinear costs**

Naturally, costs which are linear in amount associated with transportation, production and storage are the simplest costs to model. However, the assumption that these costs are linear does not always describe the problem at hand satisfactorily. In this case nonlinear costs may have to be introduced.

### **3 Analysis and Comparison of Models**

The presentation will demonstrate the characteristics of a number of very different versions of the distribution network design problem. The work presented is based on the master thesis by Jørgensen [4]. Ten formulations for the supply chain model, all known from the literature, have been analysed and compared, and during the presentation, an overview of the similarities and the differences between the various models will be provided. The focus will be on demonstrating how the above stated issues have been addressed by different researchers, and to consider the effects of addressing these issues in different ways. The presentation will not focus on the actual mathematical formulations and potential solution methods, but rather the underlying distribution problems which have been the inspiration behind the development of the formulation or the solution method reported on. The overall idea is to provide a structured survey of the work which has been done within this field. This is done in order to provide an easy accessible overview of the most important issues involved in this problem as well as a providing a comprehensive reference list. These references include work by Barbarosoğlu and Özgür [1], Campbell [2], Crotxton et al. [3] and Prikul and Jayaraman [5].

## References

- [1] Barbarosoğlu G., Özgür D., "Hierarchical Design of an Integrated Production and 2-echelon Distribution System", *European Journal of Operational Research*, Vol. 118, 464-484, 1999
- [2] Campbell J.F., "Integer Programming formulations of discrete Hub Location Problems", *European Journal of Operational Research*, Vol. 72, 387-405, 1994
- [3] Croxton K.L., Gendron B., Magnanti T.L., "Models and methods for Merge-in-Transit Operations", *Transportation Science*, Vol. 37, No.1, 1-22, 2003
- [4] Jørgensen, H.J., "Supply Chain Models and their Applicability", *Master thesis, Centre for Traffic and Transport, The Technical University of Denmark, Kgs. Lyngby, Denmark, 2004*
- [5] Pirkul H., Jayaraman V., "Production, Transportation and Distribution planning in a Multi-Commodity Tri-echelon System", *Transportation Science*, Vol. 30, No.4, November 1996