Three-Step Switching Frequency Selection Criteria for the Generalized CLLC-Type DC Transformer in Hybrid AC/DC Microgrid

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Three-Step Switching Frequency Selection Criteria for the Generalized CLLC-Type DC Transformer in Hybrid AC/DC Microgrid

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Abstract—For hybrid AC/DC microgrids, the widely applied high frequency DC transformer (DCT) is usually scheduled to operate at resonant frequency with 50% duty-cycle based semi-regulated control (DCSR) to ensure the power transmission (PT), improve its power-density and simplify the system-level control. However, for a real DCT, it may exist more than one resonant frequency that exhibit totally different features, which may seriously degrade the PT ability if the inappropriate switching frequency is selected. Meanwhile, the actual values of the DCT inductances and capacitances are also changing with the power, temperature, etc., which may lead to the practical resonant frequency’s variation. Thus, how to select the suitable switching frequency to guarantee the PT ability against the parameter variation becomes a challenge. In this paper, three-step switching frequency selection criteria (SFSC) are proposed. A generalized CLLC-type DCT (GCLLC-DCT) model is extracted to make the proposed approach available for the LLC-, CL-, symmetric CLLC- and asymmetric CLLC-type topologies. For convenience, the definition of the active power transmission ratio (APTR) is introduced to help evaluate the PT ability. First, the number, value and impact variables of the resonant frequency are derived and analyzed in Step I as the preliminary. Afterwards, the optimum resonant frequency is confirmed for the GCLLC-DCT as the criterion based on the APTR in Step II. At last, the criterion in Step III is given to finally determine the switching frequency against the parameter variations. The proposed SFSC method is also validated by the experiments.

Index Terms—CLLC, DC transformer, power transmission (PT), resonant frequency, switching frequency.

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NOMENCLATURE

ACT AC Transformer.
DCT DC Transformer.
BIC Bidirectional Interlinking Converter.
HFT High Frequency Transformer.
PT Power Transmission.
VCG Voltage Conversion Gain.
SFSC Switching Frequency Selection Criteria.
GCLLC-DCT Generalized CLLC-Type DCT.
APTR Active Power Transmission Ratio.
DCSR 50% Duty-Cycle Based Semi-Regulated Control.
HV High-voltage.
LV Low-voltage.
VH DC voltage at HV side of DCT.
VL DC voltage at LV side of DCT.
PT HFT voltage and current at LV side respectively.
V_r V_H and V_L
V_{CD} and i_{CD}
i_n and i_m
P_H Power at HV side of DCT.
P_L Power at LV side of DCT.
P_B Transmitted power.
P_{D_{ref}} Transmitted power reference.
f_r Resonant frequency.
f_s Switching frequency.
f_A Switching angular frequency.
f_B Switching angular frequency.
\sigma_T Turn ratio of the transformer.
\sigma_R Leakage inductances.
\sigma_L Magnetizing inductances.
\sigma_L\_{\text{act}} Series resonant capacitances.
R_{e_{0t}}, R_{e_{0f}} Rated value of \sigma_\text{.}
\sigma_{\text{A}} Actual value of \sigma_\text{.}
\sigma_{\text{D}} Designed value of \sigma_\text{.}
q, Q_1, Q_2 Intermediate variables.
Z_{eq} Equivalent impedance of HFT.
X_{eq} Equivalent reactance of HFT.
The BIC is responsible of regulating the transmitted power

\[ A_{PTR} \]

\[ A_{PTR}^* \]

\[ m \]

1. INTRODUCTION

Along with the increasing renewable DC sources and DC loads, the DC microgrid becomes popular in recent years [1]. However, its AC power from wind turbine and utility grid needs multiple AC/DC conversion steps to support the DC load, resulting in additional power loss and increased complexity of the DC microgrid [2]. Hence, the co-existing AC and DC sources form the hybrid AC/DC microgrid to improve system compatibility [3], [4].

A typical hybrid AC/DC microgrid [3]-[8]. The line frequency AC transformer (ACT) is deployed in [3] to realize the galvanic isolation and to match the voltage-grade. However, the bulky ACT hinders the efficiency and powerdensity of the hybrid AC/DC microgrid [5], [6]. Therefore, the high frequency DC transformer (DCT) has been proposed to replace the bulky ACT in the microgrid application to reduce weight and space occupation [7], [8].

Fig. 1. A typical hybrid AC/DC microgrid.

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![Fig. 1. A typical hybrid AC/DC microgrid.](image)

In hybrid AC/DC microgrid application, DCT and bidirectional interlinking converter (BIC) must work together to improve the energy utilization by transferring the power between the AC bus and the DC bus [8]. As depicted in Fig. 2, where \( V_H(V_L) \) & \( P_H(P_L) \) represent the DC voltage & power in the high-voltage (HV) side (low-voltage (LV) side) respectively, DCT and BIC are collaborated in the following way:

- The BIC is responsible of regulating the transmitted power \( (P_L) \) in Fig. 2) between the AC bus and the DC bus. Its power reference \( P_{ref} \) is achieved through communication bus of the energy management system (EMS) [9], [10]. This is a mature technique, and thus not further discussed here.
- Two main tasks must be realized in DCT. One is to guarantee the power transmission (PT) ability to cooperate with BIC to achieve the high efficiency. Another one is to maintain the voltage conversion gain (VCG), i.e., keep its HV voltage (i.e., \( V_H \) in Fig. 2) within the permitted range to ensure the well operation of BIC [8], [11].

![Fig. 2. Cooperation between the DCT and BIC.](image)

A typical DCT topology is depicted in Fig. 3(a), which involves two H-bridge converters and one resonant high frequency transformer (HFT) to reduce the operating power loss and to improve the power-density [8], [12], [13]. Based on the resonant HFT topologies shown in Fig. 3(b), the DCT can be classified as CLL-, LLC-, symmetric LLC- and asymmetric CLLC-DCT [14]-[16]. Actually, all these resonant DCT topologies can be explained with a generalized CLLC-DCT (GCLLC-DCT) model. However, there are rare reports to discuss the CLL-, LLC-, symmetric LLC-, and asymmetric LLC- DCT with a unified form.

For traditional applications such as electric vehicle (EV) [17], battery charging/discharging [18], etc., the DCT is generally designed with wide voltage conversion gain (VCG) to operate as an independent converter system. Therefore, the closed-loop control scheme is essential to regulate DCT to its target voltage/power [19]-[21], which makes the DCT very robust to the parameter variations. However, since \( P_H \) is already regulated by the BIC, and the LV side voltage of DCT is maintained by the DC sub-grid in the hybrid AC/DC microgrid application, the relatively complex closed-loop control is not essential for DCT at high-frequency operation. Thus, the
control algorithm of the DCT is required to be as simple as possible to reduce the communication among the DCT, BIC and EMS and to simplify the systematic control. [8], [22].

Therefore, in the microgrid application, the 50% duty cycle based semi-regulated (DCSR) control is usually recommended for the DCT [8], [23]. However, the DCSR controlled GCLLC-DCT challenges the resonant parameters design regarding the PT and VCG. Therefore, a robust parameter design approach is presented in [24] to guide the design of the symmetric CLLC-DCT. This approach can guarantee the desired VCG in the full power range to against the parameter variations. However, when it is applied in the GCLLLC-DCT, the PT ability should be further improved to deal with the following problems:

- There may exist more than one resonant frequency \( f_r \) for the GCLLC-DCT, which is not discussed in depth [24]. Besides, the value of \( f_r \) derived in [23] and [24] is also not accurate enough for the GCLLC-DCT due to the magnetizing inductance is ignored in the symmetric CLLC-DCT. Thus, the number, accurate value and impact variables of \( f_r \) should be further derived and discussed.

- The PT ability of the resonant DCT may be seriously degraded at the mismatched \( f_r \), especially when \( f_r \) varies with the changing resonant parameters. Therefore, how to determine the optimum \( f_r \) from its available values should be further theoretically analyzed.

- It is known that the switching frequency \( f_s \) is generally selected as \( f_s = f_r \) for the DCT to cooperate with BIC to ensure the PT ability [8], [24]. However, the \( f_s \) is constant in the DCSR controlled DCT, while the \( f_r \) varies in practice. So, how to finally confirm \( f_s \) should be discussed to eliminate the negative impact induced by the mismatch between \( f_s \) and \( f_r \).

Therefore, three-step switching frequency selection criteria (SFSC) are proposed in this paper for the GCLLC-DCT. By establishing the general model, the active power transmission ratio (APTR) is introduced to evaluate the PT ability. Afterwards, the number, value and impact variables of the resonant frequency are derived as the preliminary. Also, the switching frequency is selected via considering the APTR characteristics under the different resonant frequency and the resonant parameter variations in practice. The rest of this paper is presented as: Section II establishes the general model of the resonant DCT and presents the problems induced by the parameter variations. The proposed SFSC is detailedly analyzed in Section III. The experimental results of Section IV further verify the theoretically analysis correctness. Conclusion is presented in Section V.

II. GENERAL MODEL AND PROBLEM DESCRIPTION OF THE RESONANT DCT

A. General model of the resonant DCT

a) General topology of the resonant DCT

The DCT analysis is actually its HFT analysis. When the power transmits between the LV side and the HV side, the equivalent topology of the resonant HFT circuit is depicted in Fig. 4 (a) and (b). Its circuit parameters are arranged as:

- Turn ratio of the transformer: \( n \).
- Resonant capacitances: \( C_1 \) and \( C_2 \).
- Magnetizing inductances: \( L_{m1} \) and \( L_{m2} \).
- Leakage inductances: \( L_{r1} \) and \( L_{r2} \).
- Equivalent resistances: \( R_{eqH} \) and \( R_{eqL} \), whose expressions are as follows:

\[
R_{eqH} = \frac{v_{CD}}{i_{CD}}\bigg|_{|p_x| = 0} = \frac{8V_H^2}{\pi^2 P_H} \quad (1a)
\]

\[
R_{eqL} = \frac{v_{AB}}{i_{AB}}\bigg|_{|p_x| = 0} = \frac{8V_L^2}{\pi^2 P_L} \quad (1b)
\]

Fig. 4. Equivalent topology of the resonant HFT when power is transmitted: (a) with LV→HV; and (b) with HV→LV. General topology of the resonant HFT when power is transmitted (c) with LV→HV; (d) with HV→LV.

In order to make the topology in Fig. 4(a) and (b) available for the CLL-, LLC-, symmetric CLLC-, and asymmetric CLLC-DCT, the “intermediate variables” \( g \), \( Q_1 \), \( Q_2 \), \( k \) and \( h \) are introduced, with the expressions listed below:

\[
g = n^2 C_{r2} / C_{r1} \quad (2a)
\]

\[
Q_1 = n^2 \sqrt{L_{r1} / C_{r1}} / R_{eqH} \quad (2b)
\]

\[
Q_2 = \sqrt{L_{r2} / C_{r2}} / (n^2 R_{eqL}) \quad (2c)
\]

\[
k = L_{m1} / L_{r1} = L_{m2} / L_{r2} \quad (2d)
\]

\[
h = L_{m1} / (n^2 L_{r1}) \quad (2e)
\]

For the commonly used transformer, \( L_{r2}/n^2 = L_{r1} \), and thus \( h = 1 \) here. The value of \( g \) can be utilized to denote the different resonant DCT topologies:

- When \( g \rightarrow 0 \), LLC-DCT will be introduced.
- When \( g = 1 \), symmetric CLLC-DCT will be achieved.
- When \( g = C_{st} \) and \( C_{st} \neq 1 \), where \( C_{st} \) is a positive constant, asymmetric CLLC-DCT will be obtained.
- When \( g \rightarrow \infty \), CLL-DCT will be employed.

Therefore, Fig. 4(c) and (d) can be regarded as the general model of the resonant DCT. In the following, CLL-, LLC-, symmetric CLLC- and asymmetric CLLC-DCT can be unified as GCLLC-DCT.

b) Mathematical model of the GCLLC-DCT

When the power transfers from AB(CD) to CD(AB) port, the topology in Fig. 4 can be simplified as Fig. 5. The equivalent impedance \( Z_{in} \) can be expressed as

\[
Z_{in} = v_{in} / i_{in} = R_{eq} + jX_{eq} \quad (3)
\]
where $v_{in}$ and $i_{in}$ indicate the equivalent input voltage and current of the GCLLC-HFT; $X_{eq}$ and $R_{eq}$ denote the equivalent reactance and resistance, respectively.

**Problem variations**

- **$X_{eq}$ and $R_{eq}$ expressions with LV→HV power flow**
  According to Fig. 4(c), $X_{eq}$ and $R_{eq}$ can be derived as
  
  \[ X_{eq} = X_{eq1} + \frac{1}{X_{eq2}} \]
  
  \[ R_{eq} = R_{eq1} + \frac{1}{R_{eq2}} \]

  \[ (4a) \]

  \[ (4b) \]

  where $\omega_r = 2\pi f_r$ and $f_r$ denotes the switching frequency.

  \[ \omega_r = \frac{1}{\sqrt{L_1 C_1}} \]

  \[ (5a) \]

  \[ \tau = \frac{1}{L_1 C_1} \]

  \[ (5b) \]

  \[ \sigma_1 = \frac{g^2(1 + k)(1 + 2k)}{Q_1} \]

  \[ (5c) \]

  \[ \sigma_2 = \frac{g^2(1 + k)(1 - k)}{Q_2} \]

  \[ (5d) \]

  \[ \sigma_3 = \frac{(1 + 2g)(1 + k) - g^2}{Q_2} \]

  \[ (5e) \]

- **$X_{eq}$ and $R_{eq}$ expressions with HV→LV power flow**
  The mathematical model when the power is transmitted with HV→LV is the same as that the power transmitted with LV→HV except for replacing $C_1$, $L_1$, $Q_1$ and $g$ by $C_2$, $L_2$, $Q_2$ and $1/g$ correspondingly. Hence, it is not further discussed here.

**B. Problem description of the GCLLC-DCT**

**a) Evaluation index of the PT ability for the GCLLC-DCT**

According to (3), by letting the imaginary part equal to zero, i.e., $X_{eq}=0$, the $f_r$ can be derived as:

\[ f_r = f_r = \frac{\omega_r}{2\pi} = \frac{\omega_r}{2\pi} \]

\[ (6) \]

where $\omega_r$ is the resonant angular frequency. This also means the reactive power $Q$ is zero, i.e.,

\[ P = \sqrt{P^2 + Q^2} \]

\[ (7) \]

where $P$ indicates the transmitted active power in GCLLC-DCT.

If (7) is hold, the following merits will be obtained:

- The unnecessary reactive power can be eliminated to reduce the conduction loss and to decrease the withstand voltage on the resonant capacitors [8].
- It can ensure the high efficiency and good PT ability of the DCT to cooperate with BIC in microgrid applications.

Therefore, the PT ability can be evaluated by the active power transmission ratio (APTR), which is defined as:

\[ A_{PT} = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{R_{eq}}{\sqrt{X_{eq}^2 + R_{eq}^2}} \]

\[ (8) \]

where, the desired range of $A_{PT}$ is defined as:

\[ A_{PT} \in [A_{PT1}, 1] \]

\[ (9) \]

where $A_{PT1}$ relies on the real system requirement.

**b) PT Problem induced by parameter variations in practice**

- **Parameter variations in practice**
  
  For convenience, a unified form is introduced to define the variation ranges of the actual inductances and capacitances in this paper:

  \[ L_{IA} \in \left[ (1-\zeta\%), (1+\zeta\%) \right] L_{IR} \]

  \[ (10a) \]

  \[ C_{IA} \in \left[ (1-\zeta\%), (1+\zeta\%) \right] C_{IR} \]

  \[ (10b) \]

  where $L_i$ represents $L_{IA}$, $L_{IR}$, $L_{IA}$ and $L_{IR}$; $C_i$ represents $C_{IA}$ and $C_{IR}$; $\zeta\%$ and $\zeta\%$ indicate variation ranges of the $L_i$ and $C_i$; $L_{IR}$ and $C_{IR}$ are the designed value of the $L_i$ and $C_i$ at the rated temperature & power status; $L_{IA}$ and $C_{IA}$ denote the actual value of $L_i$ and $C_i$, respectively.

  An example GCLLC-DCT in [24] is given in Table I to estimate the problems induced by the parameter variations.

<table>
<thead>
<tr>
<th>Parameters of an example GCLLC-DCT</th>
<th>$L_{IA}$</th>
<th>$L_{IR}$</th>
<th>$L_{IA}$</th>
<th>$L_{IR}$</th>
<th>Rated Power</th>
<th>$n$</th>
<th>$V_{IN}$</th>
<th>$C_{IA}$</th>
<th>$C_{IR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>50µH</td>
<td>225µH</td>
<td>1.4mH</td>
<td>5.6mH</td>
<td>6kW</td>
<td>2</td>
<td>380(760)V</td>
<td>45mF</td>
<td>11nF</td>
</tr>
</tbody>
</table>

Fig. 6 depicts the measured parameters deviation percentage of the example GCLLC-DCT. It can be observed that, the values of inductances and capacitances are changing in practice. This is consistent with the definition in [24].

**Fig. 6. Scheme of parameter variations vs temperature.**

**Fig. 7. Schematic diagram of $A_{PT}$ vs $f_r$.**

It is known that more than one resonant frequency $f_r$ are available for the GCLLC-DCT, and the changing inductances and capacitances make $f_r$ vary online. However, for the resonant DCT, its switching frequency $f_s$ is constant in the 50% duty cycle based semi-regulated (DCSR) control. Therefore, a typical schematic diagram of $A_{PT}$ vs $f_r$ with two resonant frequencies $f_{s1}$ and $f_{s2}$ is depicted in Fig. 7, where $f_{s1(2)}$ varies.
within $[1-\sqrt{3}, 1+\sqrt{3}]f_{i_{(2)}}$ due to the actual parameters change in practice, and $\nu$ depends on parameter variation ranges in (10).

It can be observed from Fig. 7 that the slight change of $f_{i_1}$ seriously decreases the $A_{PTR}$. This also means if the inappropriate $f_i$ is employed to cope with the parameter variations, the GCLLC-DCT may lose its PT ability, even resulting the failure cooperation with BIC.

PT problem induced by the parameter variations under the different $f_i$ selection

In the DCSR controlled GCLLC-DCT, the $f_i$ is usually selected as $f_i=f_{R}$, where $f_{R}$ is the resonant frequency achieved at the rated power [23], [24]. However, when operating under the light-load condition, the conventional $f_i$ selection approach may lower the PT ability. To verify this, the waveforms of $v_{in}$ & $i_{in}$ (i.e., $v_{A1}$ & $i_{A1}$) of the example GCLLC-DCT are given in Fig. 8(a) and (b) with $f_i<f_{R}$ and $f_i=f_{R}$ respectively at the light-load condition to compare the PT ability against the parameter variations. It can be observed from Fig. 8(a) that $v_{in}$ and $i_{in}$ are basically kept in phase, while in Fig. 8(b), the $v_{in}$ is ahead of $i_{in}$.

Since the efficiency directly reflects the PT ability of the GCLLC-DCT, it has been measured in Fig. 8(c), which demonstrates Fig. 8(a) can ensure the higher efficiency.

Therefore, the conventional $f_i$ selection at the rated power may lower the efficiency during the light-load condition, as verified in Fig. 8. It should be further improved to guarantee the PT ability (i.e., APTR) to again parameter variations.

![Waveforms of $v_{in}$ & $i_{in}$ and corresponding efficiency at the light-load condition: (a) $f_i<f_{R}$; (b) $f_i=f_{R}$; (c) Efficiency.](image)

**C. Motivation**

The mismatch between $f_i$ and $f_s$ is inevitable in the DCSR controlled GCLLC-DCT of the hybrid AC/DC microgrid application. Therefore, the PT ability (i.e., APTR) may be seriously degraded if the inappropriate $f_i$ is selected to cope with the problems induced by the multiple $f_i$ and the changing resonant parameters. As a result, a three-step SFSC is proposed for the GCLLC-DCT to make (9) hold with the highest possibility to eliminate the negative impact induced by the mismatch between $f_i$ and $f_s$, with details summarized in Fig. 9.

![Flowchart of the proposed three-step SFSC.](image)

**III. PROPOSED THREE-STEP SFSC OF GCLLC-DCT**

In this section, the proposed three-step SFSC of the GCLLC-DCT will be presented detailedly. For step I, the number, value and impact variables of the resonant frequency will be analyzed in Section III-A. Step II presents the criterion to achieve the optimum resonant frequency based on the APTR in Section III-B. Afterwards, in Section III-C, the criterion to determine the switching frequency is confirmed based on the parameter variations as step III. At last, a design example is given in Section III-D to facilitate the procedure of the proposed three-step SFSC.

**A. Preliminary: the number, value and impact variables of $f_i$ in the GCLLC-DCT**

*a) Number of $f_i$ in the GCLLC-DCT*

According to Section II-B, the highest $A_{PTR}$ in (8) can be achieved by letting the reactive part to be zero, i.e.,

$$X_{eq} \left| L_{1}, L_{n}, C_{1}, \omega_{s} \right|_{AB-CD} \approx F(\omega_{s}) = y_{1} - y_{2} = 0$$  \hspace{1cm} (11a)

where

$$y_{1} = \sigma_{1} \omega_{s} + \sigma_{2} \omega_{s}^{2} \omega_{s}^{2} + \sigma_{3} \omega_{s}^{4}$$ \hspace{1cm} (11b)

$$y_{2} = \omega_{s} / \omega_{s}^{2}$$ \hspace{1cm} (11c)

Eqn. (11a) can be guaranteed by (6), i.e., $\omega_{s} = \omega_{s}$. It can be achieved at $y_{1}=y_{2}$. Since $\sigma_{2}>0$ (see 5(c)), there are four cases based on the plus-minus of $\sigma_{2}$ and $\sigma_{3}$, as indicated in Table II.

**Table II: Case Studies To Achieve $y_{1}=y_{2}$**

<table>
<thead>
<tr>
<th>CASE</th>
<th>$\sigma_{2}=0$</th>
<th>$\sigma_{2}&gt;0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{3}=0$</td>
<td>$\sigma_{3}\leq0$</td>
<td>$\sigma_{3}&gt;0$</td>
</tr>
<tr>
<td>$\sigma_{3}=0$</td>
<td>$\sigma_{3}\leq0$</td>
<td>$\sigma_{3}&gt;0$</td>
</tr>
<tr>
<td>$\sigma_{3}=0$</td>
<td>$\sigma_{3}\leq0$</td>
<td>$\sigma_{3}&gt;0$</td>
</tr>
</tbody>
</table>

NOTE: $m$ denotes the number of $f_{i}$ and $\omega_{s}$. 

![Case Studies](image)
For case I, it can be observed from Table II that, the number of \( f_j \) (or \( \omega_{rT} \)), defined as \( m \), involves three conditions to guarantee \( y_i = y_2 \):

- \( m = 3 \) at \( y_i = y_3 \), i.e., three resonant angular frequencies \( \omega_{rT1} \) , \( \omega_{rT2} \) and \( \omega_{rT3} \) (\( \omega_{rT1} < \omega_{rT2} < \omega_{rT3} \)) can be obtained.
- \( m = 2 \) at \( y_i = y_2 \), i.e., two resonant angular frequencies \( \omega_{rT1} \) and \( \omega_{rT2} \) (\( \omega_{rT1} < \omega_{rT2} \)) can be acquired.
- \( m = 1 \) at \( y_i = y_1 \), i.e., only one resonant angular frequency \( \omega_{rT1} \) can be achieved.

For case II, III and IV, \( m = 1 \), i.e., there is only one resonant angular frequency \( \omega_{rT1} \) to make \( y_i = y_1 \) hold.

The values of \( \omega_{rT1} \), \( \omega_{rT2} \) and \( \omega_{rT3} \) for the case I-IV are further derived in the following.

b) Value of \( f_j \) in the GCLLC-DCT

Eqa. (11a) can be rewritten as

\[
F(x) = \sigma_1 x^3 + \sigma_2 x^2 + \sigma_3 x - \sigma_4 = 0 \quad (x = \omega_r) \tag{12}
\]

It can be observed that (12) is a typical cubic equation, which can be solved by the corresponding mathematical approach [25]. To facilitate the procedure for deriving the value of \( \omega_{rT} \) in case I-IV, the following definitions are given:

\[
\begin{align*}
A &= \sigma_2 - 3 \sigma_1 \sigma_3 \\
B &= \sigma_2 \sigma_3 + 9 \sigma_1 \\
C &= \sigma_3^2 + 3 \sigma_2 \\
\Delta &= B^2 - 4AC
\end{align*}
\]

- If \( m = 3 \), i.e., (13d) satisfies

\[
\Delta = B^2 - 4AC < 0
\]

The three values of \( \omega_{rT} \) can be derived as

\[
\begin{align*}
\omega_{rT1} &= \sqrt{X_1} \quad X_1 = \min(x_1, x_2, x_3) \\
\omega_{rT2} &= \sqrt{X_2} \quad X_2 = \max(x_1, x_2, x_3) \\
\omega_{rT3} &= \sqrt{X_3} \quad X_3 = x_1 + x_2 + x_3 - x_1 - x_2
\end{align*}
\]

where

\[
\begin{align*}
x_1 &= \omega_r^2 [-\sigma_2 - 2 \sqrt{A \cos (\theta / 3)}] / (3 \sigma_1) \\
x_2 &= \omega_r^2 [-\sigma_2 + \sqrt{A \cos (\theta / 3) + \sqrt{3} \sin (\theta / 3)}] / (3 \sigma_1) \\
\theta &= \arccos T, \quad -1 < T < 1 \text{ and } T = 0.5 (2A \sigma_2 \sigma_3 - 3 \sigma_1 B) / \sqrt{A}
\end{align*}
\]

- If \( m = 2 \), i.e., (13d) satisfies that (A \( \neq 0 \)):

\[
\Delta = B^2 - 4AC = 0
\]

The two values of \( \omega_{rT} \) can be obtained as

\[
\begin{align*}
\omega_{rT1} &= \sqrt{X_1} \quad \text{where } X_1 = \min(x_1, x_2, x_3) \\
\omega_{rT2} &= \sqrt{X_2} \quad \text{where } X_2 = \max(x_1, x_2, x_3)
\end{align*}
\]

where

\[
\begin{align*}
x_1 &= -\sigma_2 / \sigma_1 + B / A \omega_r^2 \\
x_2 &= x_3 = -B \omega_r^2 / (2A)
\end{align*}
\]

- If \( m = 1 \), i.e., (13) satisfies one of the following conditions:

\[
\begin{align*}
A &= B = C = 0 \\
\Delta &= B^2 - 4AC > 0
\end{align*}
\]

If (20a) is satisfied, the value of \( \omega_{rT} \) can be derived as

\[
\omega_{rT1} = \sqrt{x_1} = \sqrt{x_2} = \sqrt{x_3}
\]

where

\[
\begin{align*}
x_1 &= x_2 = x_3 = -\sigma_2 \omega_r^2 / (3 \sigma_1) = -\sigma_4 \omega_r^2 / \sigma_4 = 3 \omega_r^2 / \sigma_4
\end{align*}
\]

Note that (22) is uniquely hold on the premise of \( \sigma_4 < 0 \) and \( \sigma_4 > 0 \), i.e., case I in Table II.

If (20b) is satisfied, the resonant frequency is achieved as

\[
\omega_{rT1} = \sqrt{x_1}
\]

where

\[
\begin{align*}
X_1 &= (\sigma_2 - Y_{13} \omega_r^3 - Y_{13} \omega_r^3) / (3 \sigma_1) \\
Y_{13} &= A \sigma_2 + 3 \sigma_1 - B \pm &\sqrt{B^2 - 4AC} / 2
\end{align*}
\]

(c) Impact variables of \( f_j \)

It can be observed from Section III-A-b that the value of \( \omega_{rT} \) depends on four unknown variables \( k, g, \omega_1 \) and \( Q_1 \). Therefore, the ranges of \( k, g, \omega_1 \) and \( Q_1 \) are discussed in the following.

For \( k \): According to (2d) & (10a), the actual value of \( k \) can be derived as:

\[
k = L_{nR} / L_{nI} = L_{mR} / L_{mI}
\]

It is noted that both \( L_{mR} \) and \( L_{mI} \) stem from one magnetic core. Therefore, when GCLLC-DCT is operating within its allowable temperature, the value of \( k \) in (25) will be stable.

For \( g \): According to (2a) and (10b), the actual value of \( g \) is rewritten as:

\[
g = n^2 \omega_{rT} \omega_{rT} / C_{rT2} / C_{rT1}
\]

Since the resonant capacitors are recommended to select the same material, \( C_{rT1} \) and \( C_{rT2} \) are varying with the same features. Therefore, the value of \( g \) is remained the same at any power & temperature conditions.

For \( \omega_1 \): According to (5a) & (10), the actual value of \( \omega_1 \) can be obtained to vary within following range:

\[
\omega_1 \in \left[ \frac{\omega_{1D}}{\sqrt{(1 + \zeta \%) (1 + \zeta \%)}} \right] \left[ \frac{\omega_{1D}}{\sqrt{(1 - \zeta \%) (1 - \zeta \%)}} \right]
\]

where \( \omega_{1D} \) is the designed value of \( \omega_1 \) at rated power and temperature.

For \( Q_1 \): By (2b), the value of \( Q_1 \) is inversely proportional to \( R_{eqR} \) and \( R_{eqI} \) varies from 0 to its rated value \( R_{eqR} \) based on the transmitted power. Therefore, the range of \( Q_1 \) is obtained as

\[
Q_1 \in [0, Q_{1max}]
\]

where \( Q_{1max} \) can be expressed as:

\[
Q_{1max} = n^2 \sqrt{(1 + \zeta \%) (1 - \zeta \%)} / [C_{rT2} (1 - \zeta \%)] / R_{eqR}
\]

Comparably, the variable range of \( Q_1 \) is much larger than \( k, g \) and \( \omega_1 \). Therefore, the number and value of \( \omega_{rT} \) largely depend on \( Q_1 \), as analyzed below:

- By substituting (5d) and (5e) into \( \sigma_1 \leq 0 \) and \( \sigma_5 \geq 0 \), it can be derived that Case I is hold on condition when \( Q_1 \) satisfies

\[
Q_1 \geq \max[\mathcal{R}_I(k, g), \mathcal{R}_O(k, g)]
\]

where

\[
\mathcal{R}_I(k, g) = \sqrt{g(1 + k) / [k^2 (1 + g) + 2k(g + 2) + 2 + g]}
\]
By substituting (5d) and (5e) into $\sigma_1 \leq 0$ and $\sigma_2 < 0$, Case II exists on the condition that
\[
R_1(k, g) \leq Q_1 < R_2(k, g)
\]
(31)
- By substituting (5d) and (5e) into $\sigma_2 > 0$ and $\sigma_3 < 0$, Case III exists on the premise that:
\[
R_2(k, g) \leq Q_1 < R_1(k, g)
\]
(32)
- By substituting (5d) and (5e) into $\sigma_2 > 0$ and $\sigma_3 < 0$, Case IV exists on the condition that
\[
Q_1 < \min[R_1(k, g), R_2(k, g)]
\]
(33)
The number and value of $\omega_{r_T}$ (i.e., $2\pi f_r$) as well as its main impact variable $Q_1$ are summarized in Table III.

<table>
<thead>
<tr>
<th>Case</th>
<th>$Q_1$</th>
<th>$\omega_{r_T}$</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$m=3$</td>
<td>$\omega_{r_{11}}, \omega_{r_{12}}, \omega_{r_{13}}$ based on (15)</td>
<td>$\Delta &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>$m=2$</td>
<td>$\omega_{r_{11}}, \omega_{r_{12}}$ based on (18)</td>
<td>$\Delta = 0$</td>
</tr>
<tr>
<td></td>
<td>$m=1$</td>
<td>$\omega_{r_{11}}$ based on (21)</td>
<td>$A = B = C = 0$</td>
</tr>
<tr>
<td></td>
<td>$m=1$</td>
<td>$\omega_{r_{11}}$ based on (23)</td>
<td>$\Delta &gt; 0$</td>
</tr>
<tr>
<td>II</td>
<td>$m=1$</td>
<td>$\omega_{r_{11}}$ based on (23)</td>
<td>$\Delta &gt; 0$</td>
</tr>
<tr>
<td>III</td>
<td>$m=1$</td>
<td>$\omega_{r_{11}}$ based on (23)</td>
<td>$\Delta &gt; 0$</td>
</tr>
<tr>
<td>IV</td>
<td>$m=1$</td>
<td>$\omega_{r_{11}}$ based on (23)</td>
<td>$\Delta &gt; 0$</td>
</tr>
</tbody>
</table>

Note: $A$, $B$, and $C$ refer to (13).

B. Step II. Criterion based on APTR of the GCLLC-DCT

Substituting (4) into (8), the value of $A_{PTR}$ can be deduced as
\[
A_{PTR} = \frac{R_q}{\sqrt{X_{eq}^2 + R_n^2}}
\]
(34a)

\[
= \frac{1}{\sqrt{Q^2(\sigma_1 \sigma_2 / \sigma_1 + \sigma_2 / \sigma_3 - 1 / \varepsilon^2) / (g^4 k^4) + 1}}
\]
(34b)

According to (34), the optimum $\omega_{r_T}$ determination is converted to select the optimum value of $\varepsilon$ (defined as $\varepsilon_m$) from the possible $\omega_{r_T}$ in Table III. $\varepsilon_m$ is expressed as
\[
\varepsilon_m = \omega_{r_T} / \omega_1
\]
(35)

Therefore, the criterion based on the APTR is transformed to find $\varepsilon_m$ to make the $A_{PTR}$ in (34a) locate within its desired range in (9) with the highest probability (i.e., the maximum shadow zone in Fig. 10 to offset the variation of $\varepsilon_m$).

By (34a), the value of $A_{PTR}$ is related with the selected resonant topologies (i.e., the value of $g$ in Section II-A). Therefore, in the following, the analysis will be carried out based on the topology.

a) $g = 0$, LLC resonant topology

Substituting $g = 0$ into (5c) ~ (5e), it can be derived that
\[
\sigma_1 = \sigma_2 = 0, \sigma_2 = 1 + k > 0
\]
(36)

Therefore, $\Delta = 0$ can be achieved based on (13d). According to Table III, two resonant frequencies exist at $g = 0$.

The characteristic of $A_{PTR}$ vs $\varepsilon$ is depicted in Fig. 11. Clearly, the range of the desired $A_{PTR}$ around $\varepsilon_2$ is more than ten times wider than that around $\varepsilon_1$. This also indicates that if $\varepsilon_2$ is selected, the permitted variation range of $\varepsilon$ will be widest to guarantee the desired $A_{PTR}$. Therefore, for the LLC-DCT, $\varepsilon_2$, corresponding to the maximum value of $\omega_{r_T}$, is recommended due to the highest probability to make $A_{PTR} \in [A_{PTR}^*, 1]$ hold.

b) $g = 1$, symmetric CLLC resonant topology

According to Table III, $\Delta < 0$, $\Delta = 0$, $A = B = C = 0$ and $\Delta > 0$ are possible in the symmetrical CLLC-DCT application. By (34), the curves of $A_{PTR}$ vs $\varepsilon$ are given in Fig. 12, where $\varepsilon_3$ in Fig. 12(a), $\varepsilon_3$ in Fig. 12(b), and $\varepsilon_3$ in Fig. 12(c), corresponding to the maximum value of $\omega_{r_T}$, can ensure the highest probability to achieve the desired $A_{PTR}$.

c) $g = C_{st}$, asymmetric CLLC resonant topology

The characteristics of $A_{PTR}$ vs $\varepsilon$ are similar to the results in Fig. 12 obtained by $g = 1$, and thus not further discussed here.

d) $g \rightarrow \infty$, CLL resonant topology

It can be derived from (5e) that $\sigma_3 < 0$. According to Table I, only case II and IV exist, i.e., $m = 1$ at $g \rightarrow \infty$. As given in Fig. 13, only one resonant frequency $\varepsilon_3$ is available.

Fig. 11. Characteristic of $A_{PTR}$ vs $\varepsilon$ @ $g = 0$.

Fig. 12. Characteristics of $A_{PTR}$ vs $\varepsilon$ when $g = 1$: (a) $m = 3$; (b) $m = 2$; (c) $m = 1$.

Fig. 13. Characteristics of $A_{PTR}$ vs $\varepsilon$ @ $g \rightarrow \infty$. 
The optimum $\varepsilon_m$ and corresponding $\omega_T$ under the different topologies are summarized in Table IV. It can be concluded from Table IV that, the maximum resonant frequency is recommended as the optimum one to ensure the widest APTR range within $[A_{\text{APTR}}, 1]$. This is regarded as the criterion based on APTR for the GCLLC-DCT.

**TABLE IV. RECOMMENDED OPTIMUM $\varepsilon_m$ AND CORRESPONDING $\omega_T$.**

<table>
<thead>
<tr>
<th>Topology</th>
<th>$m$</th>
<th>$\varepsilon_m$</th>
<th>$\omega_{\text{T}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g=0$</td>
<td>$m=2$</td>
<td>$\varepsilon_m=e_2$ in Fig. 11</td>
<td>$\omega_{\text{T}}=\omega_{T2}$</td>
</tr>
<tr>
<td>$g=1(C_{\text{a}})$</td>
<td>$m=2$</td>
<td>$\varepsilon_m=e_3$ in Fig. 12(a)</td>
<td>$\omega_{\text{T}}=\omega_{T3}$</td>
</tr>
<tr>
<td></td>
<td>$m=1$</td>
<td>$\varepsilon_m=e_1$ in Fig. 12(b)</td>
<td>$\omega_{\text{T}}=\omega_{T2}$</td>
</tr>
</tbody>
</table>

Note: $\omega_{T1}$, $\omega_{T2}$ and $\omega_{T3}$ refers to Table III.

**C. Step III. Criterion based on the parameter variations of the GCLLC-DCT**

The optimum $\varepsilon_m$ achieved by the criterion in step II depends on the resonant parameters $L_r$, $C_r$ and $R_{\text{eqH}(L)}$ (i.e., $Q_{1(2)}$ according to (1)), where $L_r$, $L_{\text{c1}}$, $L_{\text{c2}}$ and $L_{\text{m1}}$; and $C_r$ represents $C_{\text{c1}}$ and $C_{\text{c2}}$. Therefore, the impacts on $\varepsilon_m$ induced by the resonant parameter variations are further discussed below.

a) **The impact on $\varepsilon_m$ induced by variation of $L_r$ and $C_r$**

In practice, the actual values of $L_r$ & $C_r$ may change with the temperature and power, and thus they are not constant, as stated in Section II-B. According to (10) & (35), the actual $\varepsilon_m$ varies within $[\varepsilon_{\text{min}}, \varepsilon_{\text{max}}]$ in practice, and the values of $\varepsilon_{\text{min}}$ and $\varepsilon_{\text{max}}$ are

$$\varepsilon_{\text{min}} = \sqrt{(1-\xi\%)\omega_{sD}}$$

and

$$\varepsilon_{\text{max}} = \sqrt{(1+\xi\%)(1-\xi\%)\omega_{sD}}$$

where $\varepsilon_{\text{ref}}$ is the designed value of $\varepsilon_m$, and can be expressed as

$$\varepsilon_{\text{ref}} = \omega_{T} = \sqrt{L_{\text{m1}}C_{\text{c1}}}$$

$\omega_{T}$ refers to Table IV.

b) **The impact on $\varepsilon_m$ induced by the variation of $Q_{1(2)}$**

The characteristics of $\varepsilon_m$ vs $Q_{1(2)}$ are depicted in Fig. 14, where $\varepsilon_m$ is basically increasing with $Q_{1(2)}$ until it reaches a constant value. According to (1) and (2), it can be achieved that

$$Q_{1(2)} \propto \frac{1}{\omega_T^2}$$

Then, the $\varepsilon_m$ is up to the value of the optimal $\omega_{T}$ .

Therefore, Fig. 14 also indicates the characteristics of $\omega_{T}$ and the transmitted power.

It can be concluded from Fig. 14 that the optimal $\omega_{T}$ is increasing with the transmitted power, and thus the maximum value of $\omega_{T}$ is achieved at the rated power. Therefore, the curves $A_{\text{APTR}}$ vs $\varepsilon$ under various $Q_{1(2)}$ are depicted in Fig. 15, where the value of $\varepsilon_m$ will be decreased when $Q_{1(2)}$ is decreased, and the maximum value of $Q_{1(2)}$ also indicates that the transmitted power is at its rated value.

From Fig. 15, the shadow zone can satisfy $A_{\text{APTR}} \in [A_{\text{APTR}}, 1]$ for three curves; and only when $\varepsilon = \varepsilon_{\text{min}}$, the highest probability can be guaranteed to make $A_{\text{APTR}}$ locate within $[A_{\text{APTR}}, 1]$ under the decreasing $Q_{1(2)}$.

In summary, the criterion considering the resonant parameter variations can be summarized as

$$\omega_s = \varepsilon_{\text{min}} \omega_{1D} = \sqrt{(1-\xi\%)(1-\xi\%)\omega_T}$$

where $\varepsilon_{\text{min}}$ refers to (37a).

![Fig. 14. Characteristics of $\varepsilon_m$ vs $Q_{1(2)}$ with $g=0, g=1, g=C_{\text{d}}$ and $g \rightarrow \infty$.](image)

![Fig. 15. Characteristics of $A_{\text{APTR}}$ vs $\varepsilon$ with various $Q_{1(2)}$.](image)

**D. Design example**

Take the specification of the GCLLC-DCT in Table I as an example to facilitate the proposed SFSC procedure, $\xi\%$ and $\xi\%$ are 4% according to Fig. 6; $k=25$ and $Q_{1(2)}=0.1, 1.786$. The $f_1$ will be confirmed in the following steps.

**Step I:** Calculate the number and value of $f_1$ according to Table III & (6):

- $m=3$;
- $f_{11} = \omega_{T1} = 14.12 \text{ kHz}$;
- $f_{12} = \omega_{T2} / (2 \pi) = 19.68 \text{ kHz}$;
- $f_{13} = \omega_{T3} / (2 \pi) = 99.75 \text{ kHz}$.

**Step II:** Select the maximum value of $\omega_{T}$ based on Table IV:

- $\omega_{T} = \omega_{T3}$ are determined, i.e., $f_{1} = f_{13} = 99.75 \text{ kHz}$.

It is noteworthy that $f_{1} = 99.75 \text{ kHz}$ determined based on the impedance model is more accurate than the approximate value $f_{1} = 100 \text{ kHz}$ in [24] due to the ignored magnetizing inductance.

**Step III:** Confirm $f_1$ based on $f_{13}$ & (6) & (39):

- $f_{1} = 95.76 \text{ kHz}$.

These design steps of GCLLC-DCT are derived based on $g=1$. Adopting the same transformer specification in Table I, the other topologies of the GCLLC-DCT can be achieved by replacing the resonant capacitors as Table V, where the $f_s$ is derived in the same way with the above steps.

To facilitate the $f_s$ selection, a flowchart of the proposed three-step SFSC is given in Fig. 16. Based on the known specifications, the impedance model can be firstly established. By letting the imaginary part of the impedance model be zero, we can derive the accurate values and number of $f_s$. 

**Fig. 16. Flowchart of the proposed three-step SFSC.**

![Fig. 16. Flowchart of the proposed three-step SFSC.](image)
which is also regarded as Step I of the proposed SFSC. After confirming the best \( f_s \) based on APTR in Step II, the \( f_s \) is finally selected considering both the resonant parameter variations and power variations.

<table>
<thead>
<tr>
<th>Resonant Capacitors and ( f_s ) of Different GCLLC-DCT Topologies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topology</td>
</tr>
<tr>
<td>( g \to 0 )</td>
</tr>
<tr>
<td>( g = C_{sw} (C_{sw}=2) )</td>
</tr>
<tr>
<td>( g \to \infty )</td>
</tr>
</tbody>
</table>

A. Verification of PT ability in the GCLLC-DCT with the proposed SFSC

To verify the PT ability of the GCLLC-DCT achieved by the proposed SFSC, the experimental results have been measured in this sub-section when the power is transmitted from the HV side to the LV side.

- For the symmetric CLLC-DCT, the waveforms of the input voltage \( v_{in} \) and current \( i_{in} \) are given in Fig. 18. It can be observed the phase error between \( v_{in} \) and \( i_{in} \) is close to zero under both the light- and full-load conditions.
- For the asymmetric CLLC-DCT, the waveforms of \( v_{in} \) and \( i_{in} \) are similar to Fig. 18.
- For the CLL-DCT, the waveforms of \( v_{in} \) and \( i_{in} \) are depicted in Fig. 19. They are also kept in phase under both the light- and full-load conditions.
- For the LLC-DCT, the waveforms of \( v_{in} \) and \( i_{in} \) are similar to Fig. 19.

Since the efficiency directly reflects the APTR ability, the efficiency has been measured under both the light- and full-load conditions, as depicted Fig. 20. Apparently, the proposed SFSC exhibits satisfactory efficiency characteristics, especially during the light-load condition. If power is transferred from low voltage side to high voltage side, the similar results can be achieved.

In practice, the nonlinear relationship exists between \( g \) and efficiency, and this relationship may also change with power. Therefore, the efficiency of the symmetric CLLC-DCT at light-load condition outperforms that of the asymmetric CLLC-DCT, while at full-load condition it is the other way.

These results verify that the proposed SFSC overcomes the problems stated in Section II-B induced by the conventional \( f_s \) selection approach, and thus brings the good PT ability under both the light- and heavy-load conditions.

![Fig. 16 Flowchart of the proposed three-step SFSC.](image)

![Fig. 17 GCLLC-DCT based hybrid AC/DC microgrid prototype.](image)

![Fig. 18 Waveforms of \( v_{in} \) and \( i_{in} \) (a) @ light-load condition and (b) @ full-load condition for the symmetric CLLC-DCT.](image)

![Fig. 19 Waveforms of \( v_{in} \) and \( i_{in} \) (a) @ light-load condition and (b) @ full-load condition for the CLL-DCT.](image)
Cooperation verification of the BIC and GCLLC

When the proposed SFSC is applied in the GCLLC-DCT to cooperate with BIC, the cooperation performance is verified in this sub-section. Since all the GCLLC-DCT topologies in Section IV-A guarantee the good PT ability, only the topology with \( g=1 \) is introduced here.

Fig. 21 shows the waveforms of the BIC and GCLLC-DCT when the power is transmitted from high voltage side to the low voltage side. Apparently, the AC current of BIC is controlled in phase with the corresponding AC voltage. Therefore, there is no reactive power transmission, effectively ensuring the power factor. Besides, the \( v_{in} \) and \( i_{in} \) of the GCLLC-DCT are also kept in phase, guaranteeing the PT ability.

When the bidirectional power transient occurs, the results are given in Fig. 22. It can be observed that the power transients can be smoothly completed within 10ms. Besides, during the steady state shown in Fig. 22, the total harmonic distortion (THD) of the AC current is less than 3.2%.

These experimental results further demonstrate the effectiveness of the proposed SFSC for GCLLC-DCT in the hybrid AC/DC microgrid application.

APPENDIX

Note that (11a) is achieved by letting \( X_{eq}(L_{11}, L_{m1}, C_{r1}, \omega_{s}) |_{AB=CD} = 0 \) to ensure the highest \( A_{PFTR} \), i.e.,

\[
X_{eq}(L_{11}, L_{m1}, C_{r1}, \omega) |_{AB=CD} = \frac{L_{11}^{3} C_{r1}^{2} (\sigma_{1} \omega_{s}^{6} + \sigma_{2} \omega_{s}^{2} \omega_{1}^{2} \omega_{s}^{2} + \sigma_{3} \omega_{1}^{4} \omega_{s}^{2} - \omega_{s}^{2})}{\omega_{s}^{2} [R (\omega_{1}^{2} g^{2} C_{r1}^{2} + \omega_{1}^{2} g C_{r1} L_{11} + \omega_{s}^{2} g C_{r1} L_{m1} - 1)]} = 0
\]  
(A1)

According to (A1), it can be derived that

\[
\sigma_{1} \omega_{s}^{6} + \sigma_{2} \omega_{s}^{2} \omega_{1}^{2} \omega_{s}^{2} + \sigma_{3} \omega_{1}^{4} \omega_{s}^{2} - \omega_{s}^{2} = 0
\]  
(A2)

By dividing \( \omega_{1}^{2} \) at both sides of (A2), we can achieve that

\[
F(\omega_{1}) = \sigma_{1} \omega_{1}^{4} + \sigma_{2} \omega_{1}^{2} \omega_{s}^{2} + \sigma_{3} \omega_{1}^{2} \omega_{s}^{2} - \omega_{s}^{2} = 0
\]  
(A3)

Therefore, based on (A1) \(~ A3\), eqn. (11a) can be achieved.

According to (10), the maximum value and minimum value of \( \omega_{1} \) defined as \( \omega_{1\text{max}} \) and \( \omega_{1\text{min}} \) respectively can be obtained as:

\[
\omega_{1\text{max}} = \frac{1}{\sqrt{(1-\zeta)(1-\zeta)} L_{11} C_{r1}} = \frac{\omega_{1D}}{\sqrt{(1-\zeta)(1-\zeta)}}
\]  
(A4)

\[
\omega_{1\text{min}} = \frac{1}{\sqrt{(1+\zeta)(1+\zeta)} L_{11} C_{r1}} = \frac{\omega_{1D}}{\sqrt{(1+\zeta)(1+\zeta)}}
\]  
(A5)

where \( \omega_{1D} \) is expressed by

\[
\omega_{1D} = \frac{1}{\sqrt{L_{11} C_{r1}}} \quad \text{(A6)}
\]

Therefore, eqn. (27) is achieved.

REFERENCES


