Method for determining a deadzone angle of a backlash in a mechanical drive-train system, method for controlling a drive motor controller as well as drive-train system

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Figure 4

Abstract: The invention relates to a method for determining a deadzone angle \( \delta \) of a backlash in a mechanical drive-train system (1), wherein the drive-train system (1) comprises a drive motor (2), a load (3) and a shaft (4) for connecting the drive motor (2) with the load (3), the method comprising the steps: estimating an interconnection torque \( T_\delta \) in the deadzone, determining the deadzone angle \( \delta \) depending on the interconnecting torque \( T_\delta \) by means of an adaptive estimator, wherein in the adaptive estimator the backlash is modelled in terms of a variable stiffness \( K_{BL} \) of the shaft (4).

Title: METHOD FOR DETERMINING A DEADZONE ANGLE OF A BACKLASH IN A MECHANICAL DRIVE-TRAIN SYSTEM, METHOD FOR CONTROLLING A DRIVE MOTOR CONTROLLER AS WELL AS DRIVE-TRAIN SYSTEM

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Description

Method for determining a deadzone angle of a backlash in a mechanical drive-train system, method for controlling a drive motor controller as well as drive-train system

The present invention relates to a method for determining a deadzone angle of a backlash in a mechanical drive-train system, wherein the drive-train system comprises a drive motor, a load and a shaft for connecting the drive motor with the load. Furthermore, the present invention relates to a method for controlling a drive motor of a drive train system. Moreover, the present invention relates to a controller for a drive-train system. Finally, the present invention relates to a drive train system.

Backlash is a common positioning-degrading phenomenon that shows up frequently in industrial machines. High-performance machine tools use backlash compensation algorithms to ensure accurate positioning of the tool. As such, estimation of the developing deadzones in gearing and other power transmission components is essential.

Developing backlash in coupling equipment due to wear is one of the main reasons for performance degradation in machine tool systems. Since high precision tool positioning is fundamental for obtaining the required quality of the machined end-products, backlash compensation solutions are used in nearly all modern Computer Numerical Control (CNC) algorithms. This necessitates the knowledge of the backlash offset angle in advance so that it can be used in the position controllers actuating the machine axes' motors. Under this prism, online estimation method may facilitate an automatic compensation solution for developing backlash. A substantial amount of research has been conducted on estimation of the backlash phenomenon for various systems over the past three decades. For example, the backlash in a gearing system can indirectly estimated by calculating the
bounce, i.e. the change of the speed of the driving part of the gear due to the backlash impact upon exiting the deadzone. Extended Kalman Filter (EKF) are used for estimating the backlash torque in a two-mass motor arm, based on torque and position measurements. The modelling of the backlash torque using a differentiable function is known, where a nonlinear observer is used for estimation of the backlash amplitude. This method was also validated experimentally, wherein a sliding mode observer is used for the estimation of the backlash torque. The backlash function parameters are identified offline based on this estimation. The effect of the deadband due to backlash in a closed loop motion system was studied using describing functions. Here, the functions parameters are calculated online using a static relation for the controller gains. Backlash in automotive powertrains can be described based on the position difference between drive motor and load. Kalman filter is used for backlash estimation with 10% error. A four-parameter model for describing backlash effect for generic linear cascade systems is also known. The backlash identification is treated as a quasi-linear problem using iterative algorithms. Minimization of a quadratic prediction error is known using position, velocity and torque measurements for offline identification of backlash torques in a vehicle driveline system.

In the previous approaches, the backlash phenomenon is described in terms of the resulting torque upon contact of the two moving parts of the coupling. In the cases where the deadzone angle is identified directly, this is done offline or around a linearization point of the system. The results show accuracy that prohibits the use of the estimates in any backlash compensation algorithms for systems where high precision is required.

It is an object of the present invention to provide a solution, how a deadzone angle in a drive-train system can be estimated in a more reliable manner.
According to the invention this object is solved by a method for determining a deadzone angle, by a method for controlling a drive motor, by a controller and by a drive-train system having the features according to the respective independent claims. Advantageous embodiments of the invention are the subject matter of the dependent claims.

A method according to the invention serves for determining a deadzone angle of a backlash in a mechanical drive-train system, wherein the drive train-system comprises a drive motor, a load and a shaft for connecting the drive motor with the load. The method comprising the steps: estimating an interconnection torque in the deadzone and determining the deadzone angle depending on the interconnecting torque by means of an adaptive estimator, wherein in the adaptive estimator the backlash is modelled in terms of a variable stiffness of the shaft.

Backlash shows up as the loss of engagement between the motor and the load due to the developing deadzone or gap in the coupling mechanism. A certain amount of backlash is required in mating gears in order to compensate for any thermal expansion, allow the incorporation of lubricants and compensate for any potential tolerances as regards installation inaccuracies. In particular, the clearance between the non-working flanks of a gear pair is referred to as the backlash. It is formed when the load flanks are touching. Backlash can describe the loss of engagement between the motor and the load. The interconnecting torque can be the torque of the load. Here, the deadzone angle is determined by means of an adaptive estimator, wherein in the adaptive estimator the backlash is modelled in terms of a variable stiffness of the shaft. In comparison to known deadzone models, in which the interconnecting torque is zero inside the deadzone and has a predetermined value outside the deadzone, the backlash can be estimated in a more reliable
manner. Furthermore, the method can be performed during operation of the drive-train system.

Preferably, the interconnecting torque is determined depending on the load velocity. The velocity of the load and in particular the angular velocity of the load can be measured. Therefore, the interconnecting torque can determined in a simple manner. Furthermore, the angular position of the load and the load or backlash friction can be measured. Moreover, the angular velocity and/or the angular position of the motor can be measured.

According to a further embodiment, depending on the load velocity, a load inertia and a load friction an error is determined and the interconnection torque is determined based on the error. Depending on the load velocity the dynamic of a state estimation error can be determined. Here, an injection signal can be determined, which reaches a sliding manifold and remains thereafter.

It is further advantageous, if the interconnecting torque is determined by means of a sliding mode observer. Preferably, a second-order sliding mode observer (SMO) is used. In particular, a super-twisting sliding mode observer (STSMO) is used for estimating the backlash torque. This value is used in an adaptive estimator design for finding the deadzone angle.

According to a further embodiment, the deadzone angle is determined by means of an adaptive estimator. For the estimation of the deadzone angle is based on a method for parameter estimation in nonlinearity parameterized systems. This method is presented in H. F. Grip, T. A. Johansen, L. Imsland, and G.-o. Kaasa: "Parameter estimation and compensation in systems with nonlinearly parameterized perturbations" Automatica, vol. 46, no. 1, pp. 19-28, 2010. The basic idea relates to estimating a perturbation of the system dynamics that depends on the unknown parameter and
then finding an adaptation law for estimating the parameter itself.

According to a further embodiment, the variable stiffness is modelled depending on a predetermined function. For example, a function can be used, which is based on an arctan-function. This function can include a parameter to adapt the slope for the change in the stiffness.

A further method according to the invention serves for controlling a drive motor of mechanical drive-train system, wherein the drive motor is controlled depending on the deadzone angle, which is determined by means of a method for determining the deadzone angle.

A controller according to the invention for a mechanical drive-train system is adapted for performing a method for determining the deadzone angle and/or a method for controlling the drive motor.

A drive-train system according to the invention comprises a controller according to the invention. Preferably, the drive-train system is a single axis mechanical drive-train system.

The preferred embodiments presented with respect to the method for determining the deadzone angle according to the invention and the advantages thereof correspondingly apply to the method for controlling the drive motor according to the invention, to the controller according to the invention as well as to the drive-train system according to the invention.

Further features of the invention are apparent from the claims, the figures and the description of figures. The features and feature combinations mentioned above in the description as well as the features and feature combinations mentioned below in the description of figures and/or shown in the figures alone are usable not only in the respectively
specified combination, but also in other combinations or alone without departing from the scope of the invention.

Now, the invention is explained in more detail based on preferred embodiments as well as with reference to the attached drawings.

These show in:

10 FIG 1 in a schematic drawing, a drive-train system comprising a drive motor, a shaft and a load;

FIG 2 a block diagram of an open-loop mechanical drive train system;

FIG 3 in a schematic drawing a deadzone angle of a backlash between a drive motor and the load;

FIG 4 a stiffness of the shaft varying between two values;

FIG 5 a backlash torque using a deadzone model and a varying stiffness model;

FIG 6 a real and an estimated backlash torque scaled by a load inertia;

FIG 7 a real and an estimated change of a deadzone angle; and

FIG 8 an estimation error of the deadzone angle.

In the figures, identical or functionally identical elements are provided with the same reference characters.

FIG 1 shows a drive-train system 1 that comprises a drive motor 2, a load 3 and a shaft 4. The drive-train system 1 is designed as a single axis drive train system. The angular
velocities of the motor $2$ and the load denoted by $\omega_m$, $\omega_l$, respectively, while $v_T$ is the tool linear velocity. The box 5 describes the gearing and shaft dynamics and the box 6 describes the linear axis dynamics.

A typical single-axis machine tool consists of a linear axis, which positions the tool on a line. The axis is actuated by the drive motor $2$, which is typically connected to an angular-to-linear motion conversion system (e.g. a ball-screw axis). The combined elasticity, friction, damping and total mass of all the mechanical components that connect to the drive shaft $4$ motivate a collective description of the single-axis machine tool as a mechanical drive train. The latter consists of the drive motor $2$, one flexible shaft $4$ with damping and backlash and a generalised load $3$ with friction. The correspondence between the single-axis machine tool and the abstraction of the mechanical drive-train can be seen in FIG 1.

The drive motor $2$ can be a permanent magnet synchronous motor (PMSM), typically used for actuating linear axes in machine tools, especially for highly dynamic tasks. The motor is position-controlled with a controller $7$. The controller $7$ can comprise a cascade of a proportional (P) and a proportional-integral (PI) controller, used for the position and the velocity loops, respectively.

In general, the electrical closed-loop dynamics is much faster than this the mechanical system. Moreover, since the focus of this work is on the identification of backlash, i.e. on the level of the accelerations, and since the torque produced by the motor $2$ is measured, the closed-loop electrical dynamics of the motor will not be considered here. This does not affect the design or the estimation algorithms as it will become clearer later in the analysis.

The dynamics of the mechanical drive-train system $1$ reads:
\[ \omega_m = \frac{1}{J_m} u - \frac{1}{J_m} T_{F,m} - \frac{1}{N J_m} T_l \]  \hspace{1cm} (1)

\[ \dot{\theta}_m = \omega_m \]  \hspace{1cm} (2)

\[ \dot{\omega}_l = \frac{1}{J_l} T_{F,l} + \frac{1}{J_l} T_l \]  \hspace{1cm} (3)

\[ \dot{\theta}_l = \omega_l \]  \hspace{1cm} (4)

\( \omega_m \) is the motor angular velocity, \( \theta_m \) is the motor angular position, \( \omega_l \) is the load angular velocity and \( \theta_l \) is the load angular position, \( u \) is the torque command, \( T_{F,m} \) is the motor friction, \( T_{F,l} \) is the load friction or the backlash friction and \( T_l \) is the load torque. \( J_m \) is the motor inertia, \( J_l \) is the load inertia and \( N \) is the gearing ratio.

In the backlash-free case an interconnection torque or load torque \( T_l \) is given from:

\[ T_l = K_s \left( \frac{1}{N} \theta_m - \theta_l \right) + D_s \left( \frac{1}{N} \omega_m - \omega_l \right) \]  \hspace{1cm} (5)

\( K_s \) is the shaft stiffness, \( D_s \) is the shaft damping coefficient. \( K_s, D_s \) and \( N \) and are assumed to be known. The friction torques acting on the drive motor and the load express different frictional phenomena. \( T_{F,m} \) comes mostly from the contacting surfaces of the motor bearings while \( T_{F,l} \) describes the total Coulomb and viscous friction in the load. The two friction torques are modelled as described in the following equations:

\[ T_{F,m} = T_{C,m} sgn(\omega_m) + \beta_m \omega_m \]  \hspace{1cm} (6)

\[ T_{F,l} = T_{C,l} sgn(\omega_l) + \beta_l \omega_l \]  \hspace{1cm} (7)

\( T_{C,m} \) is the Coulomb friction on the motor and \( T_{C,l} \) is the Coulomb friction on the load. \( \beta_m \) is the motor viscous friction coefficient and \( \beta_l \) is the load viscous friction coefficient. \( T_{C,m}, T_{C,l}, \beta_m \) and \( \beta_l \) are considered known and constant. \( sgn(.) \) is the signum function defined as:

\[ sgn(\xi) = \begin{cases} 
1 & \text{if } \xi > 0 \\
0 & \text{if } \xi = 0 \\
-1 & \text{if } \xi < 0 
\end{cases} \]
FIG 2 illustrates a block diagram of the drive-train system in open loop. The block labelled "DZ" represents the backlash zone.

Backlash shows up as a loss of engagement between two moving parts (e.g. motor 2 and load 3) due to a developing gap (deadzone) in the coupling mechanisms as shown in FIG 3.

Alternatively, the backlash phenomenon can be interpreted as either a sudden change in the load inertia or even in the shaft stiffness. In general, the backlash torque is modelled after the restoring and damping torques in a mass-spring-damper system. While in the deadzone, these two torques are either vanishing or becoming very small, nearly zero. A number of studies have been carried out regarding the description of the torques due to backlash. The most intuitive and common one is the so-called deadzone model, in which the interconnection torque $T_i$ becomes zero inside the deadzone. Outside the deadzone the angle difference is offset by the width of the deadzone angle $\delta$. An approach is known, where the ratio of the angle differences over the deadzone width is considered. Dynamical models are known, which pertain to expressing the backlash torque as a sudden impact. The torque is again given by a mass-spring-damper system, where the elastic linear relative deformation of the two colliding coupling parts has its own stiff dynamics. A different dynamical model is known, in which a backlash angle $\delta$ is defined and its dynamics is used for calculating the impact torque. Most of the above-mentioned backlash models are effectively describing the phenomenon but they typically become difficult to use for control and estimation purposes due to their heavy discontinuities that often create numerical issues. This problem becomes even more apparent for non-constant deadzone, i.e. when the backlash angle $\delta$ increases over time due to wear.
A model for backlash, based on changing shaft stiffness, which is suitable for deadzone estimation and monitoring will be presented and will be compared to the deadzone model.

Defining the angular position and velocity difference between drive motor and load as:

\[
\Delta \theta = \frac{1}{N} \theta_m - \theta_l \\
\Delta \omega = \frac{1}{N} \omega_m - \omega_l
\]  

(9)  
(10) 

The interconnection torque \( T_i \) in the deadzone model is given by:

\[
T_i^{DZ} = \begin{cases} 
K_S (\Delta \theta - \delta \cdot sgn(\Delta \theta)) + D_S \Delta \omega & \text{if } |\Delta \theta| > \delta \\
0 & \text{if } |\Delta \theta| \leq \delta
\end{cases}
\]  

(11) 

The proposed model is based on a factorization of the backlash torque with respect to the shaft stiffness. The latter is very small, virtually zero, inside the deadzone and assumes its nominal value outside of it. The transition between the two extreme values of the stiffness is steep but smooth. The corresponding torque reads:

\[
T_i = \left[ \Delta \theta - \delta \cdot sgn(\Delta \theta) + \frac{D_S}{K_S} \Delta \omega \right] \cdot K_{BL}(\Delta \theta, \delta)
\]  

(12) 

\[
K_{BL} = \frac{K_S}{\pi} \left[ \pi + \arctan(\alpha(\Delta \theta - \delta)) - \arctan(\alpha(\Delta \theta + \delta)) \right]
\]  

(13) 

\( a \) is a large positive real number that parametrizes how stiff the change in the stiffness is. For \( a \to \infty \), it is clear that \( T_i \to T_i^{DZ} \). This can also be seen in FIG 4, where the stiffness \( K_{BL}(\Delta \theta, \delta) \) and the corresponding backlash torques are plotted for different values of the parameter \( a \). FIG 5 shows the backlash torque \( T_i^{DZ}, T_i \) for a sinusoidal \( \Delta \theta \) using a deadzone model and the varying stiffness models for different values of the parameter \( a \).

Here, the deadzone angle \( \delta \) should be estimation for dive-train system 1. This can be summarised in the following
problem formulation: Given the single-axis drive-train system described in equations (1) to (7) and the backlash model in equations (12) and (13), design an online dynamic estimator for the deadzone angle $\delta$ with the following requirements:

Maximum estimation error smaller than $10^{-3}$ rad and asymptotic convergence to the real parameter value.

The estimation of the deadzone angle $\delta$ belongs to the family of problems of online parameter estimation in systems with nonlinear parametrization. The approach followed here is based on a method for parameter estimation in nonlinearly parametrized systems presented in H. F. Grip, T. A. Johansen, L. Imsland, and G.-o. Kaasa: "Parameter estimation and compensation in systems with nonlinearly parameterized perturbations" Automatica, vol. 46, no. 1, pp. 19-28, 2010. The basic idea relates to estimating a perturbation of the system dynamics that depends on the unknown parameter and then finding an adaptation law for estimating the parameter itself.

The system dynamics is described by:

$$\dot{x} = f(t,x) + g(t,x)u(t,x) + d(t,x,\delta)$$  \hspace{1cm} (14)

where $x \in \mathbb{R}^n$ is measured, $f: \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^n$ and $g: \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^{n \times m}$ are the unforced system dynamics and time-varying input gain, respectively, which can be evaluated from the measurements. $\delta \in \mathcal{D} \subset \mathbb{R}$ is the unknown parameter, $u \in \mathbb{R}^m$ is the control input and $d: \mathbb{R}_{\geq 0} \times \mathbb{R}^n \times \mathcal{D} \to \mathbb{R}^m$ is a known matched disturbance vector, which can be evaluated if $\delta$ is known. The method pertains to finding an estimation $\hat{\phi}$ of the perturbation

$$\phi = g(t,x)d(t,x,\delta)$$  \hspace{1cm} (15)

and then derive an adaptation law

$$\dot{\delta} = \rho(t,x,\hat{\phi},\delta)$$  \hspace{1cm} (16)
Regarding the drive-train system 1, since all the states are measured and the unknown parameter $\delta$ affects the dynamics of both the motor and the load in the same way, i.e. through the torque $T_{l,r}$ either of the subsystems in equations (1), (3) can be chosen to apply the method on. For simplicity, the load velocity dynamics is chosen. The system can be written in the form of equation (14) with $f(\omega_l) = \frac{1}{J_l} T_{r,l}$, $g = \frac{1}{J_l}$ and $d(x, \delta) = T_l$, where $T_l$ has been defined in equations (12), (13) and $x \triangleq [\omega_m \theta_m \omega_l \theta_l]^T$ is the state vector of the drive-train system 1. The method is divided in two parts: estimation of the perturbation $\phi$ and derivation of the adaptation law $\rho(x, \hat{\phi}, \delta)$.

A second-order sliding mode observer (SMO) is used for finding an estimate of $\phi$. In general SMOs can offer finite-time estimation of unmeasured states by using high-frequency injection signals in their design, which depend on the observer innovation term. By doing so, the estimation error dynamics reaches the sliding manifold, i.e. a manifold on which the error is zero, and remains thereafter. This provides at the same time an estimation of any unknown perturbations that affect the system dynamics. This can become clearer in the following:

The load velocity dynamics presented in equation (3) and a SMO given by

$$\hat{\omega}_l = -\frac{1}{J_l} T_{r,l} + \nu(\bar{\omega}_l)$$

are considered, wherein $\bar{\omega}_l = \omega_l - \hat{\omega}_l$ is the state estimation error and $\nu$ is an appropriate high frequency term depending on the innovation signal $\bar{\omega}_l$. Define the sliding manifold $S = \{\bar{\omega}_l \in \mathbb{R}: \bar{\omega}_l = \hat{\omega}_l = 0\}$. The estimation of the state estimation error reads:

$$\dot{\hat{\omega}}_l = \dot{\omega}_l - \dot{\hat{\omega}}_l = \frac{1}{J_l} T_l - \nu$$
If the error dynamics reach the sliding manifold, then \( \tilde{\omega}_i = 0 \) for all future times, which means that \( v = \frac{1}{j_i} T_l \). In other words, if the injection signal \( \mathbf{v} \) is designed such that the estimation error dynamics reaches the sliding manifold \( S \) and remains thereafter, then the unknown perturbation \( \phi = -\frac{1}{j_i} T_l \) is estimated by \( \mathbf{v} \). The design of \( \mathbf{v} \) is given in:

\[
v = k_1 |\tilde{\omega}_i|^2 \text{sgn}(\tilde{\omega}_i) + k_2 \int_0^t \text{sgn}(\tilde{\omega}_i(t)) dt
\]

where \( k_1, k_2 \) are positive gains. The resulting observer is called the STSMO and it was proven that for appropriate gains \( k_1, k_2 \) the term \( \mathbf{v} \) brings the observer error dynamics on the sliding manifold \( S \). Hence, the unknown perturbation can be estimated by equation (19), where

\[
\hat{\phi} = \mathbf{v}
\]
Assumption 2: There exists a positive real constant number $L > 0$ such that $\forall \delta_1, \delta_2 \in D,$

$$\frac{1}{J_l} |T_l(x, \delta) - T_l(x, \delta)| \leq L\sqrt{\sigma(x)}|\delta|$$

(22)

Assumption 3: There exist positive real numbers $T, \varepsilon$ such that $\forall t \in \mathbb{R}_{\geq 0},$

$$\int_t^{t+T} \sigma(x(t)) dt \geq \varepsilon$$

(23)

Theorem 1: Consider the dynamics of the load velocity expressed in the form of equation (14) with $g = \frac{1}{J_l}$ and $\dot{\alpha}(x, \delta) - T_l(x, \delta)$ defined in equations (11) and (13). The adaptive estimator

$$\dot{\delta} = \rho(x, \phi, \delta) = \text{Proj} \left[ \delta_m(x, \delta) \left( \phi - \frac{1}{J_l} T_l(x, \delta) \right) \right]$$

(24)

with $\gamma > 0$ being the adaptation gain, $\phi$ an asymptotic estimate of $\frac{1}{J_l} T_l$ and $\text{Proj}(.,.)$ the projection operator defined in V-A, ensures that the estimation error $\delta$ asymptotically converges to 0, uniformly in $x$ if assumptions 1 to 3 are satisfied.

Inspired by Grid et al. and assuming constant deadzone angle $\delta$, the dynamics of the parameter estimation error can be written as

$$\dot{\delta} = \delta - \dot{\delta} = -\text{Proj} \left[ \delta_m(x, \delta) \left( \phi - \frac{1}{J_l} T_l(x, \delta) \right) \right]$$

(25)

Substituting the asymptotic estimate $\phi$ with the real perturbation $\phi$ in (25) yields

$$\dot{\delta} = -\text{Proj} \left[ \delta_m(x, \delta) \frac{1}{J_l} \left( T_l(x, \delta) - T_l(x, \delta) \right) \right]$$

(26)

In the following the shortened notation will be used
\[ V(t, \delta) = \frac{1}{2} \left( \frac{1}{\gamma} - \kappa \int_t^\infty e^{(t-\tau)} \sigma(x(\tau)) d\tau \right) \delta^2 \] (27)

where \( \kappa \) is a real positive number to be defined. The function is positive definite since

\[ \left( \frac{1}{\gamma} - \kappa \sup_{x \in \mathbb{R}} \sigma(x) \right) |\delta|^2 \leq V(t, \delta) \leq \frac{1}{\gamma} |\delta|^2 \quad \text{for} \quad \kappa < \frac{1}{\gamma \sup_{x \in \mathbb{R}} \sigma(x)}. \]

The time derivative of \( V \) along the trajectory of the estimation error reads:

\[
\dot{V}(t, \delta) = \frac{\partial V}{\partial t}(t, \delta) + \frac{\partial V}{\partial \delta}(t, \delta) = \\
= \frac{1}{2} \delta^2 \frac{\partial}{\partial t} \left[ -\kappa \int_t^\infty e^{(t-\tau)} \sigma(x(\tau)) d\tau \right] \\
- \delta \left( \frac{1}{\gamma} - \kappa \int_t^\infty e^{(t-\tau)} \sigma(x(\tau)) d\tau \right) \delta = \\
= \frac{\kappa}{2} \delta^2 \left[ \frac{\partial}{\partial t} \int_t^\infty e^{(t-\tau)} \sigma(x(\tau)) d\tau - \int_0^\infty \frac{\partial}{\partial t} e^{(t-\tau)} \sigma(x(\tau)) d\tau \right] \\
- \delta \frac{1}{\gamma} \text{Proj} \left[ \delta, \gamma \mu \cdot \frac{1}{f_l}(T_l - \tilde{T}_l) \right] \\
+ \delta \kappa \int_t^\infty e^{(t-\tau)} \sigma(x(\tau)) d\tau \cdot \text{Proj} \left[ \delta, \gamma \mu \cdot \frac{1}{f_l}(T_l - \tilde{T}_l) \right] \tag{28}
\]

Using the property

\[- \delta \frac{1}{\gamma} \text{Proj} \left[ \delta, \gamma \mu \cdot \frac{1}{f_l}(T_l - \tilde{T}_l) \right] \leq - \delta, \gamma \mu \cdot \frac{1}{f_l}(T_l - \tilde{T}_l) \]

as well as the property

\[ \int_0^\infty e^{(t-\tau)} \sigma(x(\tau)) d\tau \geq e^{-T} \int_t^{t+T} \sigma(x(\tau)) d\tau, \tau > 0 \]

equation (28) gives
\[ \dot{V}(t, \delta) = \frac{\kappa}{2} \delta^2 \left( \sigma(x) - e^{-T} \int_t^{t+T} \sigma(x(\tau))d\tau \right) \]
\[ - \delta \int_0^1 \mu \frac{1}{l_i} \frac{\partial T_i}{\partial \delta} (\delta + p \delta) dp \]

Using assumptions 1 to 3 in the inequality above and introducing \( M, \Sigma \) as upper bounds for \( |\mu(\chi, 5)|, |\sigma(\chi)| \), respectively yields

\[ \dot{V} \leq -\left( \mu - \frac{\kappa}{2} \right) \sigma(x) \delta^2 - \frac{\kappa}{2} \epsilon e^{-T} \delta^2 + \kappa \frac{\gamma}{l_i} \cdot M \cdot \Sigma \sqrt{\sigma(x)} |\delta|^2 = -\psi^T Q \psi \quad \text{(29)} \]

where

\[ \psi^T = [\delta, \sqrt{\sigma(x)}|\delta|] \]
\[ Q = \begin{bmatrix} \frac{\kappa}{2} \epsilon e^{-T} & -\frac{\kappa}{2} \gamma \cdot M \cdot \Sigma \\ -\frac{\kappa}{2l_i} \gamma \cdot M \cdot \Sigma & \mu - \frac{\kappa}{2} \end{bmatrix} \]

From inequality (29) it can be seen that \( \dot{V} \) is negative definite if \( \kappa \) is chosen as

\[ \kappa < \frac{2M}{1 + 2\epsilon l_i \Sigma^2 \gamma^2} \quad \text{(30)} \]

and by Lasalle-Yoshizawa theorem the equilibrium point \( \dot{\delta} = 0 \) is uniformly asymptotically stable, which completes the proof.

The final steps of the design concern satisfying assumptions 1 to 3. Selecting \( \mu(x, \delta) \) as

\[ \mu(x, \delta) = \frac{1}{k_5^2} \frac{\partial T_1}{\partial \delta}(x, \delta) \quad \text{(31)} \]

Condition (21) is satisfied with

\[ \sigma(x) = \frac{1}{k_5^2} \left( \frac{\partial T_1}{\partial \delta} \right)^2 = \frac{1}{\pi^2} \left[ \chi_1(x, \delta) + \chi_2(x, \delta) \right]^2 \quad \text{(32)} \]
where $X_1, X_2$ are defined as

$$X_1(x, \delta) = \text{sgn}(\Delta \theta) \cdot \left[ \pi + \arctan\left( \alpha(\Delta \theta - \delta) \right) \right]$$

$$X_2(x, \delta) = \left[ \Delta \theta - \delta \cdot \text{sgn}(\Delta \theta) + \frac{P_s}{K_s} \Delta \omega \right] \cdot \left\{ \frac{\alpha}{1+|\alpha(\Delta \theta - \delta)|^2} + \frac{\alpha}{1+|\alpha(\Delta \theta + \delta)|^2} \right\}$$

Since $x$ and $\sigma(x)$ are bounded and $D$ is compact, it is easy to show that there exists $L > 0$, such that condition (22) holds.

The inequality $\int_t^{t+T} \sigma(x(\tau)) d\tau \geq \varepsilon$ expresses a type of persistence of excitation (PE) condition. From equations (32) to (34) it can be seen that this condition does not hold if during the time interval $[t, t + T]$ the system is always within the deadzone. This, however, is expected, since in that case there is no engagement between motor and load, hence no information about the stiffness of the shaft connecting them.

The adaptation law for the parameter estimate $\dot{\delta}$ is finally given by:

$$\dot{\delta} = \text{Proj}\left\{ \delta, \gamma \frac{1}{K_s} \frac{\partial T_l}{\partial \delta}(x, \delta) \left[ \phi - \frac{1}{J_l} T_l(x, \delta) \right] \right\}, \gamma > 0$$

where

$$\frac{\partial T_l}{\partial \delta}(x, \delta) = \frac{K_s}{\pi} \left[ X_1(x, \delta) + X_2(x, \delta) \right]$$

It is interesting to note that the selection of the specific $\mu(\hat{\delta})$ function results into a gradient-type adaptation law, which is very common in the literature of adaptive techniques. Although for nonlinearly parametrized systems it does not always guarantee parameter convergence as it does for linear-in-the-parameters systems, it is a natural first choice for the adaptation law.

In the algorithm for the backlash angle estimation, $\omega_m$, $\vartheta_m$, $\omega_t$, $\theta_t$ and $T_{F,t}$ are measured. The output is the deadzone angle $\delta$. In a first step, a STSMO for the load velocity is designed.
according to equations (17) and (19) is designed. In a second step, the backlash torque according to equation (20) is estimated. In a third step, the adaptive estimator for the deadzone angle \( \delta \) according to equations (33) to (36) is designed.

The drive-train system was simulated so that the performance of the estimation algorithm could be assessed. A PI controller is used to regulate the drive motor velocity into following a sinusoidal profile \( \omega_m^{\text{ref}} = \Omega \sin(\nu t) \). A 5\% change in the deadzone angle \( \delta \) is considered for the evaluation of the algorithm. The velocity measurements are afflicted with white Gaussian noise \( \omega \sim \mathcal{N}(0, \sigma^2_{\text{meas}}) \). High precision absolute position encoders are used and the error due to quantization is ignored. The compact set 9 is the real axis interval \([0, 1]\), the estimator is initialized at \( \delta(0) = \delta_0, \gamma \) is chosen to be 0.1 and the sampling time is 2 ms.

FIG 6 shows the real torque \( J_1 \dot{\phi}, J_1 \dot{\dot{\phi}} \) applied in the system according to the deadzone model and its estimation by the STSMO. A small lag can be observed in the estimation of \( \dot{\phi} \), which, however, does not affect the performance of the algorithm. Increasing the gains \( k_1, k_2 \) of the observer reduces the delay in estimation but makes the method more sensitive to measurement noise.

The real deadzone angle \( \delta \) and estimated deadzone angle \( \hat{\delta} \) as well as the estimation error \( \delta \) are shown in FIG 7 and 8, respectively. It can be seen from the plots that the deadzone angle \( \delta \) is estimated in less than 1 s (200-400 ms) and in sufficient accuracy. Specifically, the average steady state estimation error is less than \( 2 \cdot 10^{-6} \) rad, which is smaller than the order of magnitude for positioning precision in machine tool applications \( (10^{-3} \) rad). Larger sensor noise has a direct impact on the speed of convergence and the steady state deviation.
A method for deadzone angle estimation in a single-axis drive-train with backlash has been presented. After the interconnecting torque acting on both motor and load is estimated using a STSMO, an adaptive algorithm is designed for estimating the deadzone angle $\delta$. The method was tested in a simulation framework, where the adaptive estimator was able to track the real value of the deadzone angle $\delta$, as well as a 5% increase in the backlash. The parameter convergence was achieved in less than 1 s and with precision in the order of $2 \cdot 10^{-6}$ rad. Such precision allows for use of the estimated parameter in backlash compensation algorithms, typically present in machine-tool applications. Moreover, detection of changes in the deadzone angle $\delta$ can infer a measure of wear assessment in the mechanical components (i.e. gearing, coupling mechanisms etc.) of the system.
Claims

1. Method for determining a deadzone angle (δ) of a backlash in a mechanical drive-train system (1), wherein the drive train-system (1) comprises a drive motor (2), a load (3) and a shaft (4) for connecting the drive motor (2) with the load (3), the method comprising the steps:
   - estimating an interconnection torque (T₁) in the deadzone,
   - determining the deadzone angle (δ) depending on the interconnecting torque (T₁) by means of an adaptive estimator,
   - wherein in the adaptive estimator the backlash is modelled in terms of a variable stiffness (Kₛₗ) of the shaft (4).

2. Method according to claim 1, characterized in that, the interconnecting torque (T₁) is determined depending on the angular load velocity (ω₁).

3. Method according to claim 2, characterized in that, depending on the angular load velocity (ω₁), a load inertia (Jᵢ) and a load friction (Tᵢ) an error is determined and the interconnection torque (T₁) is determined based on the error.

4. Method according to 2 or 3, characterized in that, the interconnecting torque (T₁) is determined by means of a sliding mode observer.

5. Method according to any one of the preceding claims, characterized in that, the deadzone angle (δ) is determined by means of an adaptive estimator.

6. Method according to any one of the preceding claims, characterized in that, the variable stiffness (Kₛₗ) is modelled depending on a predetermined function.

7. Method for controlling a drive motor (2) of drive-train system (1), wherein the drive motor (2) is controlled
depending on the deadzone angle \((\delta)\), which is determined by means of a method according to claims 1 to 6.

8. Controller (7) for a mechanical drive-train system (1), which is adapted for performing a method according to claims 1 to 6 and/or a method according to claim 7.

9. Drive-train system (1) comprising a controller (7) according to claim 8.

10. Drive-train system (1) according to claim 9, characterized in that, the drive-train system (1) is a single axis mechanical drive-train system.
INTERNATIONAL SEARCH REPORT

A. CLASSIFICATION OF SUBJECT MATTER

INV. G05B19/404 B25J9/16
ADD.

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

G05B B25J

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

EPO-Internal, WPI Data

C. DOCUMENTS CONSIDERED TO BE RELEVANT

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<th>Category</th>
<th>Citation of document, with indication, where appropriate, of the relevant passages</th>
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Further documents are listed in the continuation of Box C. See patent family annex.

Date of the actual completion of the international search 19 January 2018

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<td>X,P</td>
<td>DIMITRIOS PAPAGEORGIOU, MØGENS BLANKE, HANS HENRI KNIEMANN, JAN H. RICHTER: &quot;Backash estimation for industrial drive-train systems&quot;&lt;br&gt;I FAC-PAPERONLINE,&lt;br&gt;vol. 50, no. 1, 14 July 2017 (2017-07-14), pages 3281-3286, XP002777194,&lt;br&gt;ISSN: 2405-8963&lt;br&gt;page 3281 - page 3286</td>
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