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Efficient First-Principles Calculation of Phonon-Assisted Photocurrent in Large-Scale Solar-Cell Devices

Mattias Palsgaard,1,2,* Troels Markussen,1 Tue Gunst,2 Mads Brandbyge,2 and Kurt Stokbro1

1 Synopsys-QuantumWise A/S, Fruebjergvej 3, Postbox 4, DK-2100 Copenhagen, Denmark
2 Department of Micro- and Nanotechnology (DTU Nanotech), Center for Nanostructured Graphene (CNG), Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark

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We present a straightforward and computationally cheap method to obtain the phonon-assisted photocurrent in large-scale devices from first-principles transport calculations. The photocurrent is calculated using the nonequilibrium Green’s function with light-matter interaction from the first-order Born approximation, while electron-phonon coupling (EPC) is included through special thermal displacements. We apply the method to a silicon solar-cell device and demonstrate the impact of including EPC in order to properly describe the current due to the indirect band-to-band transitions. The first-principles results are successfully compared to experimental measurements of the temperature and light-intensity dependence of the open-circuit voltage of a silicon photovoltaic module. Our calculations illustrate the pivotal role played by EPC in photocurrent modeling to avoid underestimation of the open-circuit voltage, short-circuit current, and maximum power. This work represents a recipe for computational characterization of future photovoltaic devices including the combined effects of light-matter interaction, phonon-assisted tunneling, and the device potential at finite bias from the level of first-principles simulations.

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I. INTRODUCTION

Photovoltaics (PV) represents a promising technology as a replacement for the burning of fossil fuels. In the past couple of decades, many promising thin-film absorber materials have been discovered, all of them with unique strengths and weaknesses. CdTe and copper indium gallium selenide [CIGS, CuIn(1−x)Ga x (x)Se2] can produce high efficiencies, but include rare and toxic elements, while copper zinc tin sulfide (CZTS, CuZnSnS4) includes only nontoxic earth-abundant elements, but suffers from low efficiency and open-circuit voltage (VOC) [1–4]. Clearly, there is still room for the discovery of new materials to improve on the cost-efficiency relationship. The field of computational material science has seen massive progression, and as a result the difference between the system size and complexity attainable in simulations and experiments is becoming smaller every day. Recently, a review was published on the design of new materials using first-principles calculations [5]. Here, it is stressed how the abundance of candidate materials together with the lack of efficient devices highlight the need for efficient predictive device calculations. Continuum models are used extensively in the field of PV to extract benchmark parameters from measurements on devices and to predict the performance of new device geometries [6]. It is difficult to include important effects such as confinement of electrons and phonons, surface effects, and strain in the continuum models. These effects can, however, be captured using atomistic models based on density-functional theory (DFT). DFT combined with the nonequilibrium Green’s function (NEGF) formalism [7–9] has, for example, previously been used to improve a continuum-model study of transport through the interface between CZTS and the buffer material CdS, which is important for CZTS solar-cell efficiency [10,11]. In spite of the influx of new thin-film-based PV cells, silicon remains the market leader and about 90% of PV cells are still based on silicon, where large modules with high efficiency and stability can be produced [1].

Silicon has an indirect band gap and as such, in order to conserve momentum, absorption of a photon below the fundamental direct gap energy must be accompanied by the absorption or emission of a phonon. A number of recent studies also show that EPC plays a key role in the outstanding performance of PV cells based on direct-band-gap perovskites [12–15].
The study of phonon-assisted photon absorption from first principles is notoriously difficult as it involves a double sum over fine grids of \( \mathbf{k} \) points and complex two-excitation processes. Therefore state-of-the-art DFT calculations of phonon-assisted absorption have so far been limited to bulk crystals where the supercell contains only a few atoms [16–18], while empirical models have been used for studies of realistic devices [19]. Recently, Zacharias and Giustino [18] introduced a very efficient method for including phonon-induced absorption processes using a single supercell calculation in which the atoms are displaced away from their equilibrium positions. We recently adopted this special-thermal-displacement (STD) approach to study electron transport in silicon systems with over 1000 atoms, including electron-phonon coupling within the DFT NEGF formalism [20].

In this paper we apply STD to calculations of the first-order photocurrent in a 19.6-nm silicon \( p-n \) junction from DFT NEGF. In this way we are able to capture the phonon-assisted absorption over the indirect band gap of silicon and study directly the effect of temperature on the performance of the device. In the following section, we summarize the methodology used to calculate the photocurrent using the first-order Born approximation and the inclusion of EPC through the STD approach. Exhaustive derivations of the important equations can be found in previous publications [18,20–23].

II. METHOD

We calculate the photocurrent as a first-order perturbation to the electronic system caused by the interaction with a weak electromagnetic field. The electron-photon interaction is given by the Hamiltonian

\[
H' = \frac{e}{m_0} \mathbf{A} \cdot \mathbf{P},
\]

where \( \mathbf{A} \) is the vector potential and \( \mathbf{P} \) is the momentum operator. For a single-mode monochromatic light source, we have [21]:

\[
\mathbf{A} = e \left( \frac{\hbar \sqrt{\mu_r \varepsilon_r}}{2N\omega\varepsilon c} F \right)^{1/2} (b^\dagger e^{-i\omega t} + b e^{i\omega t}),
\]

where \( \mu_r \) is the relative permeability, \( \varepsilon_r \) is the relative permittivity, \( \varepsilon \) is the permittivity, \( \omega \) is the frequency of the light, \( F \) is the photon flux, \( N \) is the number of photons, \( b^\dagger \) and \( b \) are the bosonic creation and annihilation operators, and \( e \) is a unit vector giving the polarization of the light.

Using the standard Meir-Wingreen formula [24] and including only first-order terms in \( F \), we arrive at an expression similar to Fermi’s golden rule for the current into the left (\( L \)) and right (\( R \)) leads due to absorption of photons [23,25]:

\[
I_\alpha = \frac{e}{\hbar} \int_{-\infty}^{\infty} \sum_{\beta=L,R} [1 - f_\alpha(E)f_\beta(E - \hbar\omega)] T_{\alpha,\beta}^- (E) - f_\alpha(E) [1 - f_\beta(E + \hbar\omega)] T_{\alpha,\beta}^+ (E) dE,
\]

\[
T_{\alpha,\beta}^- (E) = N \text{Tr} \{ M^\dagger \tilde{A}_\alpha (E) M A_\beta (E - \hbar\omega) \},
\]

\[
T_{\alpha,\beta}^+ (E) = N \text{Tr} \{ M \tilde{A}_\alpha (E) M^\dagger A_\beta (E + \hbar\omega) \},
\]

where \( \alpha \in L, R \), \( f_\alpha \) is the Fermi-Dirac distribution function of lead \( \alpha \), \( A_\alpha = G T_\alpha G^\dagger \) is the spectral function of lead \( \alpha \), \( \tilde{A}_\alpha = G^\dagger T_\alpha G \) is the time-reversed spectral function of lead \( \alpha \), and the electron-phonon coupling matrix is

\[
M_{ml} = \frac{e}{m_0} \left( \frac{\hbar \sqrt{\mu_r \varepsilon_r}}{2N\omega\varepsilon c} F \right)^{1/2} e \cdot P_{ml}.
\]

The total photocurrent is then calculated as \( I_{ph} = I_L - I_R \). The retarded Green and advanced \( G^\dagger \) Green’s functions, the spectral broadening of the leads \( \Gamma_\alpha \), and the momentum operator \( \mathbf{P} \) are calculated self-consistently from DFT NEGF simulations of the silicon \( p-n \) junction device.

The temperature-dependent EPC is included through a single displacement of the atomic positions according to [18,20]

\[
u_{\text{STD}} (T) = \sum_{\lambda} (-1)^{\lambda-1} \sigma_\lambda (T) e_\lambda,
\]

where \( e_\lambda \) is the eigenvector of phonon mode \( \lambda \) and the Gaussian width \( \sigma \) is given by

\[
\sigma_\lambda (T) = \nu_0 \sqrt{2n_{\lambda} (T) + 1},
\]

where \( n_{\lambda} (T) \) and \( \nu_0 = \sqrt{\hbar/2M_\lambda \Omega_\lambda} \) are the Bose-Einstein occupation and vibrational characteristic length of mode \( \lambda \) with frequency \( \Omega_\lambda \). Phonon modes are obtained using a supercell method [26–28]. The configuration displaced according to Eq. (7) gives the correct thermal average of the Landauer conductance and the optical absorption for sufficiently large systems with many repetitions of the same unit cell [18,20].

In summary the total current at bias \( V \) and temperature \( T \) is calculated as follows:

\[
I (V, T) = I_0 [V, \mathbf{u}_{\text{STD}} (T)] + I_{ph} [V, \mathbf{u}_{\text{STD}} (T)],
\]

where \( I_0 \) is the dark current coming from the applied bias. By displacing the atomic coordinates once according
III. RESULTS AND DISCUSSION

Figure 1(a) shows the considered 19.6-nm silicon $p$-$n$ junction with transport along the [100] direction. The related local density of states is shown in Fig. 1(b) and we can see the typical $p$-$n$ profile along the device, with flat bands near the electrode indicating converged screening potentials [20,35]. Furthermore, we see that the calculated band gap is very close to the one observed in experiments on silicon [36].

In Fig. 2(a) we compare the photocurrent density calculated for a pristine silicon (neglecting zero-point motion) $p$-$n$ junction with that of a 300-K STD structure for $F = 1/\text{Å}^2 \text{s}$. The dashed lines indicate the energies of the indirect ($E^\text{indirect}_{g} = 1.135 \text{ eV}$) and direct ($E^\text{direct}_{g} = 2.853 \text{ eV}$) band gaps of the bulk silicon structure using the same calculational settings. First, we see that the obtained band gap agrees well with the experimental value (1.12 eV) [36], verifying our use of the GGA+$1/2$ exchange-correlation method. Second, we see that the inclusion of EPC through the STD results in an increase in photocurrent from the indirect transition of about two orders of magnitude. The photocurrent as a function of photon energy at 300 K compares well to previous calculations of the absorption coefficients in bulk silicon, where EPC was included in the same way [18]. Temperature effects on the electronic structure are included in the STD method, and indeed we see a finite photocurrent at photon energies below the bulk band gap corresponding to a reduction in the band gap with temperature. In addition to the finite-temperature renormalization of the band gap, we obtain the actual photocurrent of the transport setup assisted by band-to-band tunneling and including the device potential at finite bias. Unlike calculations on bulk silicon, we see here a small finite contribution to the photocurrent coming from the indirect transition even without EPC. This we trace back to symmetry breaking by the ultrathin $p$-$n$ junction and the resulting device potential along the [100] direction. This current contribution is analyzed in more detail in the Supplemental Material [37]. In simulations based on the bulk-silicon band structure, such transitions

![Diagram](https://via.placeholder.com/150)

**FIG. 1.** (a) Structure and cell used in the calculation of the 19.6-nm silicon $p$-$n$ junction, with $n = 2 \times 10^{19} \text{ cm}^{-3}$. (b) Local density of states along the transport direction of the silicon $p$-$n$ junction on a logarithmic scale.

![Diagram](https://via.placeholder.com/150)

**FIG. 2.** (a) Calculated photocurrent density for the pristine (blue dashed) and STD displaced (red solid) system, where $F = 1/\text{Å}^2 \text{s}$ and $V = 50 \text{ mV}$. (b) Calculated room-temperature current density as a function of voltage for the silicon $p$-$n$ junction in the dark ($F = 0$) and under illumination.
would be prohibited by the opposite symmetry of conductance and valence states if not assisted by phonons. This follows from the selection rule due to momentum conservation, which is a consequence of translational symmetry. However, in a p-n junction, the system is not translationally invariant in the transport direction, and hence the momentum in the transport direction is not a conserved quantity. The effects of phonon-assisted tunneling, temperature renormalization, and the device potential are all seen to play an important role in the quantitative photocurrent device characteristics. Regarding below-gap transitions, a similar result was seen previously when studying theoretically the phonon-assisted tunneling into graphene in a scanning-tunneling-spectroscopy setup [38]. Here a finite but strongly suppressed tunneling into the Dirac point of graphene is seen even below the threshold voltage of the phonon opening the inelastic channel. In order to generate an I-V curve, we need to calculate the total photocurrent under sunlight illumination. To that end we used the AM1.5 reference spectrum and integrated the spectral current densities for a certain applied bias.

The I-V curve of the silicon p-n junction is shown in Fig. 2(b), with and without the addition of photocurrent. A least-squares fit of the calculated data points to the usual expression for the current of a diode under illumination $I = I_{ph} + I_0[\exp(qV/nkBT) - 1]$ is added. The result looks very much like what is expected [39], with a photocurrent being a nearly constant contribution at all applied biases. The applied bias voltage where the illuminated I-V curve crosses the zero-current-density line and no current is generated is known as the open-circuit voltage ($V_{OC}$) and is a measurable parameter used to benchmark solar-cell performance. For crystalline-silicon PV cells, the open-circuit voltage is known to be in the range 0.55–0.60 V [40–42] at room temperature, in good agreement with the 0.54 V obtained from our calculation. The short-circuit current, obtained at zero applied bias, varies a lot depending on the quality and device geometry of the measured solar cell, making a direct comparison with our results difficult. The obtained 5 mA cm$^{-2}$ is, however, aligned with typical values published in the literature [41,42].

To analyze the direct impact of EPC on the device, we plot in Fig. 3(a) the I-V curve under illumination for the perfectly symmetric (noninteracting) silicon system together with that of the system where atomic positions are displaced according to STD [Eq. (7)] for different temperatures. Without EPC the short-circuit current density is underestimated by about 25%. Importantly, the open-circuit voltage is significantly underestimated without EPC and one obtains $V_{OC} = 0.49$ V at 300 K. The open-circuit voltage of a PV device is an important performance indicator that defines its quality and is typically very well controlled, so a $\sim 10\%$ loss at room temperature

![FIG. 3. (a) Calculated I-V curves with (solid) and without (dashed) EPC for different temperatures. (b) Calculated open-circuit voltage as a function of temperature with linear fits. The green star markers are experimental measurements from [40]. The inset shows a close-up near the experimental measurements. In all calculations and experimental values shown in this figure, a light intensity of 1 sun was used.](image)

![FIG. 4. (a) Open-circuit voltage as a function of light intensity at 352 K. The calculated results including EPC were extrapolated from the fitted line in Fig. 3(b), since we did not perform calculations at the exact temperature measured in [40]. (b) Calculated power density as a function of applied voltage with (solid) and without (dashed) EPC for different temperatures.](image)
is substantial. This highlights the importance of including phonon effects in device calculations of indirect semiconductor PV devices.

We will now analyze the temperature dependence of the device characteristics, which can often be extracted from experiments. The short-circuit current is largely constant while the open-circuit voltage is degraded for higher temperatures, due to the increased probability of carriers tunneling through the p-n junction via electron-phonon scattering events at bias voltages below the band gap. In the case with no EPC, the increased current at higher temperatures comes only from the temperature dependence of the Fermi-Dirac distribution. Comparing with the results where EPC is ignored (dashed lines), we see that the open-circuit voltage is systematically underestimated and that the error is larger at high temperature. It is not surprising that the inclusion of EPC is more important at higher temperatures where the phonon population is higher. The short-circuit current is also constant for the case without EPC, but it is too low at all temperatures. Using the results shown in Fig. 3(a), we can extract the open-circuit voltage, which is often measured in experiments on solar-cell devices, as a function of temperature. The open-circuit voltage as a function of temperature with and without the inclusion of EPC is shown in Fig. 3(b). A linear fit was performed on both data sets using least-squares fitting, and the best fit was plotted alongside the data points. For both cases, we get the expected linear temperature dependence. The open-circuit voltage extrapolated to the \( T = 0 \) K point is often used in experiments to extract the activation energy of the dominant recombination path. Here we get 1.1 V ± 0.025 V with EPC and 1.08 V ± 0.029 V without EPC, both slightly below our calculated band gap. In Fig. 3(b) we also compare the calculated results with experiments carried out by Huang et al. [40], where the open-circuit voltage of a crystalline-silicon PV module was measured under simulated solar irradiation while controlling the cell temperature. The calculated results including EPC agree nicely with the experimental measurements. The calculated open-circuit voltages without the inclusion of EPC are much lower than the experimental values at all temperatures. Experimental measurements of \( V_{OC} \) performed at temperatures in the range 100–300 K under 1.1 sun illumination [41] also agree nicely with the values calculated including EPC (not shown).

Lastly, we will analyze the trends with light intensity. In Fig. 4(a) we show the calculated open-circuit voltage for different intensities of the light source at a temperature of 352 K and compare with experimental values from Huang et al. [40]. Again we see that the results where EPC is included through STD agree nicely with experimental measurements. The best agreement is seen at 0.2 sun, which is to be expected given the assumption of a weak field going into Eq. (1). On the other hand, the open-circuit voltages calculated while neglecting EPC do not agree with the experimental values. In fact, even the results calculated at 1 sun are below the experimental values, where only an intensity of 0.2 sun is used. This underlines the pivotal role played by EPC in PV devices. Figure 4(b) shows the generated power density for the p-n junction with and without the inclusion of EPC. We see that the maximum power and maximum power point are both underestimated in the case without EPC.

IV. CONCLUSION

We have presented a computationally cheap method to calculate the phonon-assisted photocurrent in large-scale devices from first principles. Previous studies of phonon-assisted optical absorption using state-of-the-art methods have been limited to bulk systems of high symmetry, where only a handful of atoms are considered due to the computational cost. Here we study a 19.6-nm-long silicon p-n junction under working conditions with an applied bias. The calculated current density as a function of photon energy is similar in shape to the temperature-renormalized bulk optical absorption of silicon calculated in previous studies using a similar description of EPC [18]. Our results agree nicely with experiments both for values of the open-circuit voltages and for trends in how they scale, with temperature and light intensity. The phonon interaction has a significant impact on the device characteristics, highlighting the need for photocurrent transport calculations including phonon coupling when considering devices using indirect semiconductors as the absorber material. The combination of device simulations with photon coupling as well as phonon coupling, through special thermal displacements, offers an appealing way forward in the difficult problems of combined light-matter interaction, phonon-assisted tunneling, temperature renormalization, and the nonequilibrium device potential in quantitative first-principles simulations.

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