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Integration of different CHP steam extraction modes in the stochastic unit commitment problem

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Appendix to *Integration of different CHP steam extraction modes in the stochastic unit commitment problem*

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I. Introduction to the Improved Hybrid Decomposition

In this document we explain in detail how the suggested improvements for the *scenario partition and decomposition method, variant 1 (SPDA1)* proposed in [1] are carried out. The improvements consist in applying heuristics to find a suitable number of scenario partitions or clusters for the specific problem and find a partly fixed first stage-decision to initialize the problem solution. These heuristics are based in the Progressive Hedging algorithm and rounding techniques. The Progressive Hedging algorithm was first introduced by [2] and has been applied to solve large-scale stochastic programming problems in different applications such as forest planning [3], resource allocation problems [4] and unit commitments problems [5]. The Progressive Hedging is an iterative process in which first the problem is solved for each scenario individually and the solutions obtained for the first-stage decisions are averaged for all scenarios. From these solutions a multiplier is created and afterwards, the problem is solved again for each scenario including this multiplier as a penalty in the objective function. Using a squared proximal term to calculate the distance between the first stage decision vector of each scenario and the average term of these, we can determine if the algorithm should stop. Later on, we use these values to initialize the solution of the two-stage stochastic programming problem. In our solution approach, in order to determine a suitable number of partitions, we take from Progressive Hedging the way of averaging the first-stage decision, making use of the squared proximal term to stop the algorithm. Furthermore, to initialize the solution of the problem, we use the rounding technique proposed in [6] where just those values close to 1 and 0 are fixed to initialize the solution. These values depend on two thresholds that we will name α and β.

II. Formulating the Hybrid Unit Commitment

In this section, the stochastic unit commitment (4a)-(4c) is reformulated to the *hybrid unit commitment* following the work done in [1]. The finite set of scenarios Ω is divided into $|P|$ different partitions. Therefore, the entire set of scenarios Ω is divided into different subsets named Ω_p which is comprised of all the scenarios $\omega \in \Omega$ that belong to partition $p \in P$. The *hybrid unit commitment* writes as follows.

$$\min_{\mathcal{X},\mathcal{Y}_\omega \gamma_p} \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}} \left(a_g x_{g,t} + C_g^{\mathrm{SU}} y_{g,t} + C_g^{\mathrm{SD}} z_{g,t}\right) + \sum_{p\in P} \rho_p \gamma_p \tag{6a}$$

$$\begin{aligned} \text{s.t. } \gamma_p \geq & \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^T} b_g p_{g,t,\omega} + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} C^{\mathrm{L}} L_{n,t,\omega}^{\mathrm{shed}} \qquad (6b)\\ & + \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{CHP}}}\sum_{m\in\mathcal{M}} a_{g,m}^{\mathrm{CHP}} u_{g,m,t,\omega} \\ & + \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{CHP}}}\sum_{m\in\mathcal{M}} b_{g,m}^{\mathrm{CHP}}\left(p_{g,m,t,\omega}^{\mathrm{CHP}} + \varphi_{g,m} q_{g,m,t,\omega}^{\mathrm{CHP}}\right) \\ & (\forall p \in P, \forall \omega \in \Omega_p) \\ & (1\mathrm{a}) - (1\mathrm{n}), (2\mathrm{a}) - (2\mathrm{v}) \qquad (6c) \end{aligned}$$

where ρ_p represents the probability attached to each partitions that is calculated as follows.

$$\rho_p = \sum_{\omega\in\Omega_p} \pi_\omega \qquad (\forall p \in P)$$

The auxiliary variable γ_p equals the worst-case system cost for partition p and therefore the second term in the objective function (6a) represents the expected value of the worst-case scenarios at each partition $p \in P$. To formulate the decomposition algorithm, we need to distinguish between the master problem and the subproblems. Both are formulated as in [1]. The master problem (MP) is formed by both first-stage and second-stage decisions. It solves one per partition $p \in P$ and for iteration i it writes as follows.

$$\min_{\mathcal{X}^i,\mathcal{Y}^i,\gamma_p} \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}} \left(a_g x_{g,t}^i + C_g^{\mathrm{SU}} y_{g,t}^i + C_g^{\mathrm{SD}} z_{g,t}^i\right) + \gamma_p \tag{7a}$$

$$\begin{aligned} \text{s.t. } \gamma_p \geq & \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^T} b_g p_{g,t,\omega}^i + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} C^{\mathrm{L}} L_{n,t,\omega}^{\mathrm{shed,i}} \qquad (7b)\\ & + \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{CHP}}}\sum_{m\in\mathcal{M}} a_{g,m}^{\mathrm{CHP}} u_{g,m,t,\omega}^i \\ & + \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{CHP}}}\sum_{m\in\mathcal{M}} b_{g,m}^{\mathrm{CHP}}\left(p_{g,m,t,\omega}^{\mathrm{CHP},i} + \varphi_{g,m} q_{g,m,t,\omega}^{\mathrm{CHP},i}\right) \\ & (\forall \omega \in \Omega_p^i) \\ & (1\mathrm{a}) - (1\mathrm{n}), (2\mathrm{a}) - (2\mathrm{v}) \quad (\forall \omega \in \Omega_p^i) \qquad (7c) \end{aligned}$$

Where $\mathcal{X}^i$=$\{x_{g,t}^i,\ y_{g,t}^i,\ z_{g,t}^i\}$ and $\mathcal{Y}^i$=$\{u_{g,m,t,\omega}^i, v_{g,m,m',t,\omega}^i,\ p_{g,t,\omega}^i,\ L_{l,t,\omega}^{\mathrm{shed},i},\ W_{f,t,\omega}^{\mathrm{spill},i},\ p_{g,m,t,\omega}^{\mathrm{CHP},i},\ q_{g,m,t,\omega}^{\mathrm{CHP},i},\ s_{g,t,\omega}^i,\ \phi_{n,t,\omega}^i : \forall\omega \in \Omega_p^i\}$. One subproblem (SP) per scenario $\omega \in \Omega_p$ is

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solved determining the second-stage decision variables.

$$\min_{\mathcal{Y}^i_\omega} \sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^T} b_g p^i_{g,t,\omega} + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} C^{\mathrm{L}} L^{\mathrm{shed,i}}_{n,t,\omega} \tag{8a}$$
$$+\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{CHP}}}\sum_{m\in\mathcal{M}} a^{\mathrm{CHP}}_{g,m} u^i_{g,m,t,\omega}$$
$$+\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}^{\mathrm{CHP}}}\sum_{m\in\mathcal{M}} b^{\mathrm{CHP}}_{g,m}\big(p^{\mathrm{CHP},i}_{g,m,t,\omega} + \varphi_{g,m} q^{\mathrm{CHP},i}_{g,m,t,\omega}\big)$$
$$\text{s.t.} \ \ (1f)-(1n),(2a)-(2v) \tag{8b}$$

Where $\mathcal{Y}^i_\omega$=$\{u^i_{g,m,t,\omega}$,$v^i_{g,m,m',t,\omega}$, $p^i_{g,t,\omega}$, $L^{\mathrm{shed},i}_{l,t,\omega}$, $W^{\mathrm{spill},i}_{f,t,\omega}$, $p^{\mathrm{CHP},i}_{g,m,t,\omega}$, $q^{\mathrm{CHP},i}_{g,m,t,\omega}$, $s^i_{g,t,\omega}$, $\phi^i_{n,t,\omega}\}$.

III. Solution Approach

The solution algorithm is described in the following. Note that the master problems (7a)-(7c) and subproblems (8a)-(8b) for each partition $p \in P$ are solved in parallel and that they are called instances of the SPDA1 algorithm.

1) Initialize iteration $j = 0$. Select the initial number of partitions k^j applying hierarchical clustering to the set of scenarios Ω.
2) Create k^j parallel instances of the SPDA1 algorithm.
3) Initialize iteration $i := 0$ and set $\Omega^i_p = \emptyset$.
4) Solve the master problem and return the optimal solution found for the vector of first stage decisions $\mathcal{X}^i_p$. Obtain the Lower Bound (LB) as $\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}}\big(a_g x^i_{g,t} + C^{\mathrm{SU}}_g y^i_{g,t} + C^{\mathrm{SD}}_g z^i_{g,t}\big) + \gamma_p$.
5) Solve the subproblems (SP) with the first-stage decision variables fixed at $\mathcal{X}^i_p$. Once all the subproblems are solved, obtain the scenario ω' that yields the highest system cost. Include this scenario in the reduce set of worst-case scenarios ($\Omega^{'}_p$) such that $\Omega^{'i+1}_p = \Omega^{'i}_p \cup \{\omega'\}$ and obtain the Upper Bound (UB) as $\sum_{t\in\mathcal{T}}\Bigg(\sum_{g\in\mathcal{G}}\big(a_g x^i_{g,t} + C^{\mathrm{SU}}_g y^i_{g,t} + C^{\mathrm{SD}}_g z^i_{g,t}\big) + \sum_{g\in\mathcal{G}^T} b_g p^i_{g,t,\omega} + \sum_{n\in\mathcal{N}} C^{\mathrm{L}} L^{\mathrm{shed,i}}_{n,t,\omega} + \sum_{g\in\mathcal{G}^{\mathrm{CHP}}}\sum_{m\in\mathcal{M}}\Big[a^{\mathrm{CHP}}_{g,m} u^i_{g,m,t,\omega} + b^{\mathrm{CHP}}_{g,m}\big(p^{\mathrm{CHP},i}_{g,m,t,\omega} + \varphi_{g,m} q^{\mathrm{CHP},i}_{g,m,t,\omega}\big)\Big]\Bigg)$.
6) Check convergance. If $|\mathrm{UB}-\mathrm{LB}| \le \xi$, where ξ is the tolerance value, the iterative process i stops. If $|\mathrm{UB}-\mathrm{LB}| > \xi$ then $i := i+1$ and go to step 4.
7) Once all partitions have converged, we obtain the first-stage decision vector for each partition $\mathcal{X}^j_p$.
8) Increase iteration number $j := j+1$. Calculate the average value for the first-stage commitment decisions over all partitions $\overline{\mathcal{X}}^j$=$\sum_{p\in P}\rho_p \mathcal{X}^{j-1}_p$. Obtain squared distance $\sigma^j = ||\overline{\mathcal{X}}^j - \overline{\mathcal{X}}^{j-1}||$ (where $\overline{\mathcal{X}}^0 = 0$). If $\sigma^j \le \varepsilon$ we stop the iteration process for j and move a step forward. If $\sigma^j > \varepsilon$, we increase the number of partitions $k^j := k^{j-1}+1$ and go step 2.
9) Obtain the partly fixed commitment decisions using the rounding technique:

$$\overline{\mathcal{X}}^{\mathrm{round}} = \begin{cases} 1 & \text{if } \overline{\mathcal{X}}^j \ge 1-\alpha \\ 0 & \text{if } \overline{\mathcal{X}}^j \le \beta \\ \mathcal{X}\in\{0,1\} & \text{if } \beta < \overline{\mathcal{X}}^j < 1-\alpha \end{cases}$$

10) Solve (6a)-(6c) for the scenarios finally retained in the set of worst-cases scenarios $\Omega^{'}_p$ using $\overline{\mathcal{X}}^{\mathrm{round}}$ as partly fixed commitment decisions.

The pseudocode for the proposed improved SPDA1 algorithm is provided in Algorithm 1.

Algorithm 1 Improved Scenario Partition and Decomposition Algorithm: Variant 1 (**Improved SPDA1**)

```
Set j := 0.
Choose initial k^0 and apply hierarchical clustering to Ω and obtain P^0.
repeat
    for all p ∈ P^j do
        Set i := 0 and Ω'^0_p = ∅
        repeat
            Solve Master Problem
            Return optimal solution X^i_p
            Compute Lower Bound (LB)
            Set X_p := X^i_p and solve SP ∀ω ∈ Ω_p
            Compute Upper Bound (UB)
            Identify worst-case scenario ω'
            Set Ω'^{i+1}_p := Ω'^i_p ∪ {ω'}
            Set i := i + 1
        until | UB − LB | ≤ ξ
        Set Ω'_p := Ω'^{i−1}_p
    end for
    Obtain X^j_p ∀p ∈ P
    Set j := j + 1.
    Compute average value X̄^j
    Obtain the quadratic distance value σ^j
    Increase number of partitions k^j := k^{j−1} + 1
    Apply hierarchical clustering to Ω and obtain P^j
until σ_j ≤ ε
Calculate X̄^round
Solve (6a)-(6c) replacing Ω_p with Ω'_p, ∀p and using X̄^round as partly fixed commitment decisions.
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