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Appendix to Integration of different CHP steam extraction modes in the stochastic unit commitment problem

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I. INTRODUCTION TO THE IMPROVED HYBRID DECOMPOSITION

In this document we explain in detail how the suggested improvements for the scenario partition and decomposition method, variant 1 (SPDA1) proposed in [1] are carried out. The improvements consist in applying heuristics to find a suitable number of scenario partitions or clusters for the specific problem and find a partly fixed first stage-decision to initialize the problem solution. These heuristics are based in the Progressive Hedging algorithm and rounding techniques. The Progressive Hedging algorithm was first introduced by [2] and has been applied to solve large-scale stochastic programming problems in different applications such as forest planning [3], resource allocation problems [4] and unit commitments problems [5]. The Progressive Hedging is an iterative process in which first the problem is solved for each scenario individually and the solutions obtained for the first-stage decisions are averaged for all scenarios. From these solutions a multiplier is created and afterwards, the problem is solved again for each scenario including this multiplier as a penalty in the objective function. Using a squared proximal term to form Progressive Hedging the way of averaging the first-in-order to determine a suitable number of partitions, we take use these values to initialize the solution of the two-stage commitment problems [5]. The Progressive Hedging is an improved variant 1 (SPDA1) scenario partition and decomposition improvements for the work done in [1]. The finite set of scenarios Ω is divided into different subsets named Ωp, which is comprised of all the scenarios ω ∈ Ω that belong to partition p ∈ P. The hybrid unit commitment writes as follows.

\[ \min x_{\omega}, y_{\omega}, p \sum_{t \in T} \sum_{g \in \mathcal{G}} (a_g x_{g,t} + C_{SU} g y_{g,t} + C_{SD} g z_{g,t}) + \sum_{p \in P} \rho_p \gamma_p \]  
(6a)

s.t. \( \gamma_p \geq \sum_{t \in T} \sum_{g \in \mathcal{G}} b_g y_{g,t} + \sum_{t \in T} \sum_{m \in \mathcal{M}} c_{L}^{\text{hed}}(n,t,\omega) \)  
(6b)

+ \[ \sum_{t \in T} \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} c_{CHP}^{\text{shed}}(g,m) \]  
(6c)

\[ (\forall p \in P, \forall \omega \in \Omega_p) \]

where \( \rho_p \) represents the probability attached to each partition that is calculated as follows.

\[ \rho_p = \sum_{\omega \in \Omega_p} \pi_{\omega} \quad (\forall p \in P) \]

The auxiliary variable \( \gamma_p \) equals the worst-case system cost for partition \( p \) and therefore the second term in the objective function (6a) represents the expected value of the worst-case scenarios at each partition \( p \in P \). To formulate the decomposition algorithm, we need to distinguish between the master problem and the subproblems. Both are formulated as in [1]. The master problem (MP) is formed by both first-stage and second-stage decisions. It solves one per partition \( p \in P \) and for iteration \( i \) it writes as follows.

\[ \min x^i, y^i, p^i \sum_{t \in T} \sum_{g \in \mathcal{G}} (a_g x_{g,t}^i + C_{SU} g y_{g,t}^i + C_{SD} g z_{g,t}^i) + \gamma^i \]  
(7a)

s.t. \( \gamma^i \geq \sum_{t \in T} \sum_{g \in \mathcal{G}} b_g y_{g,t}^i + \sum_{t \in T} \sum_{m \in \mathcal{M}} c_{L}^{\text{hed}}(n,t,\omega) \)  
(7b)

+ \[ \sum_{t \in T} \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} c_{CHP}^{\text{shed}}(g,m) \]  
(7c)

\[ (\forall \omega \in \Omega^i_p) \]

Where \( X^i = \{ x_{g,t}^i, y_{g,t}^i, z_{g,t}^i \} \) and \( Y^i = \{ u_{g,m,t,\omega}^i, v_{g,m,t,\omega}^i, w_{g,m,t,\omega}^i \} \). One subproblem (SP) per scenario \( \omega \in \Omega_p \) is...
solved determining the second-stage decision variables.

\[
\min \sum_{t \in T} \sum_{g \in G} b_{g, t} p_{g, t, \omega} + \sum_{t \in T} \sum_{m \in M} C_{t} L^{reg}_{m, t, \omega} \\
+ \sum_{t \in T} \sum_{g \in G} a_{g, m} t_{g, m, t, \omega} \\
+ \sum_{t \in T} \sum_{g \in G} \sum_{m' \in M} \sum_{t' \in T} h_{t, t'} (p_{g, m, t, \omega} + \varphi_{g, m} g_{g, m, t, \omega}) \\
\text{s.t. (1f) } - (1n), (2a), (2v) 
\tag{8b}
\]

Where \( y_{g, t} = \{u_{g, m, t, \omega}, v_{g, m', t, \omega}, i_{g, m, t, \omega}, p_{g, t, \omega}, L_{t, \omega}, W_{f, t, \omega}, P_{t, \omega}, S_{i, \omega}, \phi_{h, t, \omega} \} \).

### III. SOLUTION APPROACH

The solution algorithm is described in the following. Note that the master problems (7a)-(7c) and subproblems (8a)-(8b) for each partition \( p \in P \) are solved in parallel and that they are called instances of the SPDA1 algorithm.

1. Initialize iteration \( j = 0 \). Select the initial number of partitions \( k^0 \) applying hierarchical clustering to the set of scenarios \( \Omega \).
2. Create \( k^0 \) parallel instances of the SPDA1 algorithm.
3. Initialize iteration \( i \) and set \( \Omega_i^0 = 0 \).
4. Solve the master problem and return the optimal solution found for the vector of first stage decisions \( \chi_i^{0} \). Obtain the Lower Bound (LB) as \( \sum_{t \in T} \sum_{g \in G} \{a_{g, t} x_{g, t}^{0} + C_{t} y_{g, t}^{0} + C_{g} y_{g, t}^{0} + \gamma_{t}^{0} \} \).
5. Solve the subproblems (SP) with the first-stage decision variables fixed at \( \chi_i^{0} \). Once all the subproblems are solved, obtain the scenario \( \omega' \) that yields the highest system cost. Include this scenario in the reduced set of worst-case scenarios (\( \Omega_i^{j} \)) such that \( \Omega_i^{j+1} = \Omega_i^{j} \cup \{\omega'\} \) and obtain the Upper Bound (UB) as \( \sum_{t \in T} \sum_{g \in G} \{a_{g, t} x_{g, t}^{0} + C_{t} y_{g, t}^{0} + C_{g} y_{g, t}^{0} + \gamma_{t}^{0} \} + \sum_{g \in G} b_{g, t} p_{g, t, \omega} + \sum_{t' \in T} \sum_{m \in M} C_{t} L^{reg}_{m, t, \omega} + \sum_{g \in G} \sum_{m \in M} \sum_{m' \in M} \sum_{t' \in T} h_{t, t'} (p_{g, m, t, \omega} + \varphi_{g, m} g_{g, m, t, \omega}) + \varphi_{g, m} g_{g, m, t, \omega} \).
6. Check convergence. If \( |UB - LB| \leq \xi \), where \( \xi \) is the tolerance value, the iterative process \( i \) stops. If \( |UB - LB| > \xi \) then \( i := i + 1 \) and go to step 4.
7. Once all partitions have converged, we obtain the first-stage decision vector for each partition \( \chi_i^{0} \).
8. Increase iteration number \( j := j + 1 \). Calculate the average value for the first-stage commitment decisions over all partitions \( \bar{X} = \sum_{p \in P} p \chi_i^{j-1} \). Obtain squared distance \( \sigma^2 = ||\bar{X} - \bar{X}^0||^2 \) (where \( \bar{X} = 0 \)). If \( \sigma^2 \leq \epsilon \) we stop the iteration process for \( j \) and move a step forward. If \( \sigma^2 > \epsilon \), we increase the number of partitions \( k^j := k^{j-1} + 1 \) and go step 2.
9. Obtain the partly fixed commitment decisions using the rounding technique:
\[
\bar{X} = \begin{cases} 
1 & \text{if } \bar{X} \geq 1 - \alpha \\
0 & \text{if } \bar{X} \leq \beta \\
\bar{X} \in \{0, 1\} & \text{if } \beta < \bar{X} < 1 - \alpha
\end{cases}
\]

10. Solve (6a)-(6c) for the scenarios finally retained in the set of worst-cases scenarios \( \Omega_i^{j} \) using \( \bar{X} \) as partly fixed commitment decisions.

The pseudocode for the proposed improved SPDA1 algorithm is provided in Algorithm 1.

### Algorithm 1 Improved Scenario Partition and Decomposition Algorithm: Variant 1 (Improved SPDA1)

1. Set \( j := 0 \).
2. Choose initial \( k^0 \) and apply hierarchical clustering to \( \Omega \) and obtain \( \Omega_0^0 \).
3. Repeat
4. For all \( p \in P \) do
5. Set \( i := 0 \) and \( \Omega_i^0 = \emptyset \).
6. Repeat
7. Solve Master Problem
8. Compute optimal solution \( \Omega_i^0 \).
9. Compute Lower Bound (LB)
10. Set \( \Omega_i^1 := \Omega_i^0 \) and solve SP \( \forall \omega \in \Omega_i^0 \).
11. Compute Upper Bound (UB)
12. Identify worst-case scenario \( \omega' \)
13. Set \( \Omega_i^{j+1} := \Omega_i^j \cup \{\omega'\} \).
14. Set \( i := i + 1 \).
15. Until \( \Omega_i^j \) is fixed
16. End for
17. Obtain \( \Omega_i^j \) \( \forall p \in P \).
18. Set \( j := j + 1 \).
19. Repeat
20. Compute average value \( \bar{X} \).
21. Obtain the quadratic distance value \( \sigma^2 \).
22. Increase number of partitions \( k^j := k^{j-1} + 1 \).
23. Apply hierarchical clustering to \( \Omega \) and obtain \( P^{j} \).
24. Until \( \sigma^2 < \epsilon \).
25. Calculate \( \bar{X} \).
26. Solve (6a)-(6c) replacing \( \Omega_p \) with \( \Omega_p^j \) \( \forall p \) and using \( \bar{X} \) as partly fixed commitment decisions.

### REFERENCES