Compressive Online Robust Principal Component Analysis with Multiple Prior Information

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1. Motivation

- Applications: Computer vision, web data analysis, anomaly detection, and data visualization, etc.
- Robust Principal Component Analysis (RPCA): Batch-based, decomposes all data samples (matrix M) into low-rank (L) and sparse (S), e.g., all frames in a video, high computational and memory requirements

\[
\min_{L,S} \|L\|_2 + \lambda \|S\|_1 \quad \text{subject to} \quad M = L + S
\]

Challenges:

- Online method processing a sequence of signals per time instance from a small set of measurements \(y_t = \Phi(x_t + v_t)\)
- Minimizing at time instance \(t\)

\[
\min_{\{L_t, v_t\}} \|L_t - x_t\|_2 + \lambda \|x_t - v_t\|_1 \quad \text{subject to} \quad y_t = \Phi(x_t + v_t)
\]

where \(\lambda > 0\) and \(\beta > 0\) are weights across the side information signals, and \(W_j\) is a diagonal matrix with weights for each element in the side information signal \(x_j\); namely, \(W_j = \text{diag}(w_j(1), w_j(2), \ldots, w_j(n))\) with \(w_j > 0\) being the weight for the \(j\)-th element in the \(x_j\) vector.

The CORPCA algorithm:

- Solving \(\ell_1\)-minimization via the soft thresholding operator and the single value thresholding operator, at iteration \(k + 1\)

\[
\begin{align*}
\hat{x}_t^{k+1} &= \arg \min_{x_t} \frac{1}{2} \|\Phi(x_t + v_t) - y_t\|^2 + \lambda \|x_t - v_t\|_1 \\
\hat{v}_t^{k+1} &= \arg \min_{v_t} \frac{1}{2} \|\Phi(x_t + v_t) - y_t\|^2 + \lambda \|x_t - v_t\|_1
\end{align*}
\]

where \(f(v_t) = \frac{1}{2} \|\Phi(x_t + v_t) - y_t\|^2\), \(g(v_t) = \sum_{j=1}^m \|W_j(x_j - v_j)\|^2_2\), and \(h(v_t) = \|B_{t-1} - v_t\|^2_2\).

- Updating weights \(\beta_j\) and \(W_j\)

- After solving for time instance \(t\): Prior updates

2. Compressive Online RPCA (CORPCA) With Multiple Prior Information

Problem formulation:

- Incorporating multiple prior information: at time instance \(t\) we observe \(y_t = \Phi(x_t + v_t)\) with \(y_t \in \mathbb{R}^n\) given priors \(Z_{l_t} = \{z_{l_t,1}, \ldots, z_{l_t,n}\}\) and \(B_{l_t} = \{b_{l_t,1}, \ldots, b_{l_t,n}\}\) from \([x_{l_t,1}, \ldots, x_{l_t,n}]\) and \([v_{l_t,1}, \ldots, v_{l_t,n}]\), respectively.

- Solving the \(\ell_1\)-minimization problem

3. Experimental Results

- Generating low-rank components: \(n = 500, d = 100\) (training), \(n = 100\) (testing), \(r = 5\) (rank)
- \(L = UV^T\), where \(U \in \mathbb{R}^{n \times r}\) and \(V \in \mathbb{R}^{r \times r}\) yields \(L = [v_1 \ldots v_r]\)
- Generating sparse components with \(\|x_0\|_0 = 80\) and \(\|x_2 - x_1\|_0 = 50/2\) obtaining \(S = [x_1 \ldots x_2]\)
- Testing on \(M = [x_0_1 \ldots x_0_{100}] + \{x_1_1 \ldots x_1_{100}\} + \{x_2_1 \ldots x_2_{100}\}\)
- Measuring probabilities of successful decomposition, \(P\{\text{success}\}\), success if \(\|x_t - x_0\|_2^2 \leq 10^{-2}\)

4. Summary

- Solution for an \(\ell_1\)-minimization
  - Incorporating efficiently multiple prior information
  - Updating iteratively weights

The proposed CORPCA algorithm:

- Processing a data vector per time instance using compressive measurements
- Solving the \(\ell_1\)-minimization and updating priors for the next instance

Evaluation of CORPCA on synthetic data and actual video data

- Outperforming classical compressive sensing (CS) \(\ell_1\) minimization and CS with single prior information \(\ell_1\)-minimization
- The superior performance improvement compared to the existing methods

CORPCA source code, test sequences, and the corresponding outcomes. [Online]: Available: https://github.com/huynhluvd/huynhlvd