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GENERATING THE OPTIMAL MAGNETIC FIELD FOR MAGNETIC REFRIGERATION

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ABSTRACT

In a magnetic refrigeration device the magnet is the single most expensive component, and therefore it is crucially important to ensure that an effective magnetic field as possible is generated using the least amount of permanent magnets. Here we present a method for calculating the optimal remanence distribution for any desired magnetic field. The method is based on the reciprocity theorem, which through the use of virtual magnets can be used to calculate the optimal remanence distribution. Furthermore, we present a method for segmenting a given magnet design that always results in the optimal segmentation, for any number of segments specified. These two methods are used to determine the optimal magnet design of a 12-piece, two-pole concentric cylindrical magnet for use in a continuously rotating magnetic refrigeration device.

Keywords: Permanent magnet, Design, Remanence, Optimization, Segmentation, Virtual

1. INTRODUCTION

Generating a strong magnetic field is of great importance in magnetic refrigeration. Most magnetic refrigeration devices use permanent magnets to accomplish this task, as these do not require any energy input to generate a magnetic field. However, permanent magnets are expensive and thus it is important to utilize them most efficiently. This means that the permanent magnet structure must generate the high magnetic field using the least amount of magnet material possible; indeed the magnet is the most expensive component in a magnetic refrigeration device [1,2].
Previously, design of permanent magnet systems for magnetic refrigeration have relied on adapting existing well-known geometries, such as the Halbach cylinder or the “C”-shaped magnet, to a given regenerator geometry [3,4]. An optimization method exists that can be used to optimize a given magnet design [5], but it requires an existing geometry and remanence distribution. The large spread in efficiency of published magnet designs [6] indicates the very diverse methods used to design magnets for magnetic refrigeration, but also the potential for improving performance.
Here we present a method that can determine the optimal distribution of remanence and the border of the magnet for any desired magnetic field. Furthermore, we present a method for segmenting a given magnet design that always results in the optimal segmentation, for any number of segments specified.

2. THE RECIPROCITY THEOREM AND THE OPTIMAL MAGNET

The optimization method presented here is not applied to a predefined magnet design but is based solely on the desired magnetic field. The foundation for determining the optimal distribution of permanent magnet material is the so-called reciprocity theorem in magnetostatics, which can be expressed as [7]:

$$\int \boldsymbol{B}_{\boldsymbol{r},1}(\boldsymbol{x}) \cdot \boldsymbol{H}_2(\boldsymbol{x})\, d\boldsymbol{V} = \int \boldsymbol{B}_{\boldsymbol{r},2}(\boldsymbol{x}) \cdot \boldsymbol{H}_1(\boldsymbol{x})\, d\boldsymbol{V} \tag{1}$$

This equation is an energy equivalence between two magnetic systems, 1 and 2. The equation states that the magnetic energy in system 1 in virtue of its remanence, $\boldsymbol{B}_{\boldsymbol{r},1}$, when placed in the field generated by system 2, $\boldsymbol{H}_2$, is equal to the energy in system 2 with remanence $\boldsymbol{B}_{\boldsymbol{r},2}$, when placed in the field generated by system 1, $\boldsymbol{H}_1$. The integration is performed over all space. However, since $\boldsymbol{B}_{\boldsymbol{r},1}$ is only non-zero in places where permanent magnet material is present in system 1, and likewise for system 2, we can limit the integrals in Eq. (1) to these regions. In deriving Eq. (1) we have assumed that there are no free currents in the system and that the materials in the system obeys a linear $\boldsymbol{B}-\boldsymbol{H}$ relation, $\boldsymbol{B}=\mu\boldsymbol{H}+\boldsymbol{B}_{\boldsymbol{r}}$, where the permeability, μ, must be the same for both systems [8].

The theorem in Eq. (1) can be used to determine the optimal remanence, $\boldsymbol{B}_{\boldsymbol{r},1}$, that produces a given desired magnetic field, $\boldsymbol{H}_1$. We consider a *virtual* magnet system that has a virtual remanence, $\boldsymbol{B}_{\boldsymbol{r},2}$, parallel to the desired magnetic field, $\boldsymbol{H}_1$, in every point in the air gap [9,10]. In this case the right hand side of equation Eq. (1) will be maximized as $\boldsymbol{B}_{\boldsymbol{r},2} \parallel \boldsymbol{H}_1$. The left hand side will thus also be maximized if the real remanence is aligned to the field produced by the virtual remanence everywhere. An illustration of this concept is shown in Fig. 1. Here we consider a system where we want to generate a magnetic field as shown on the figure to the left, i.e. a field that is radial in two air gaps and zero between these. Firstly, virtual magnets are 'placed' in the air gap, with a remanence identical to the desired magnetic field. These virtual magnets generate a magnetic field, shown in the right hand side of Fig. 1. The optimal remanence in the design area is then aligned everywhere to this virtual magnetic field, to maximize the averaged projection of the produced field onto the desired virtual remanence.

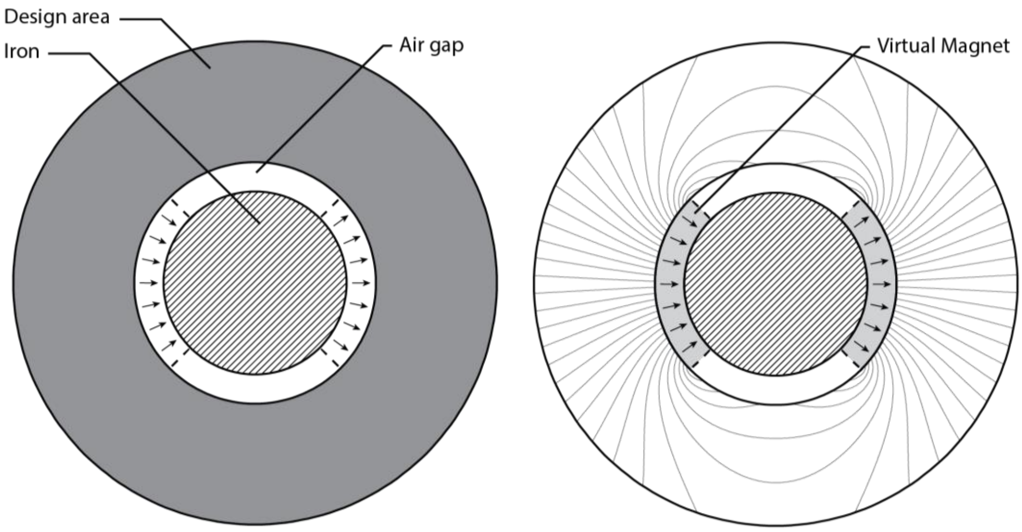

Figure 1. An illustration of the reciprocity theorem. The left-hand figure show the design area as well as the desired magnetic field in the air gap. The right-hand figure show the corresponding virtual system, with virtual magnets with a remanence identical to the desired magnetic field of the real system and the field produced by these.

It is also seen from Eq. (1) that once the real remanence is aligned to the virtual field $\boldsymbol{H}_2$ in a given point $\boldsymbol{x}$, the contribution of that site to the integral in Eq. (1) is proportional to the norm H_2. This means that if the magnet is surrounded by air ($\mu_r = 1$) the optimal border between magnet and air will be a contour level of H_2, as points inside a contour level of H_2 will contribute more to the integral in Eq. (1) than points outside. If the magnet design area is surrounded by a high permeability material, e.g. iron, the optimal border can also be calculated. Assuming an infinite permeability for iron, the virtual field $\boldsymbol{H}_2$ will be normal to the border of the areas with iron present. Now note that the magnetic field can also be written as the gradient of a magnetic scalar potential, $\boldsymbol{H}_2 = -\nabla\Phi_2$. Here the magnetic field is also normal to contours of Φ_2. Thus, there is an analogy between these two cases. By choosing a contour of Φ_2 as the border of the magnet, we can ensure that the energy efficiency of the system is maximized [11]. If the relative permeability of the surrounding matter is neither 1 nor infinite, an iterative approach can be used to determine the border between magnet and the surroundings.

3. OPTIMAL SEGMENTATION

Once a remanence vector field has been selected, be it optimal or not, the magnetic structure must be segmented before it can be realized. When segmenting a magnet design, the system is split into uniformly magnetized segments. Previously segmentation of magnet designs was done by numerically determining the optimization

direction of the remanence on a predefined segmented geometry. However, it has recently been shown that it is possible to determine the globally optimal segmentation of a 2D magnetic system [8].
Because of superposition, the field generated by each point in the magnet is independent from the other points. One can show that this implies that it is never optimal to split a region over which the direction of the virtual field $\boldsymbol{H}_2$ is uniform [8], which for 2D systems can be shown to be equivalent with the fact that the optimal border between two adjacent segments must lie on a contour of $\psi = \arctan\left(H_{2,y} / H_{2,x}\right)$. We are thus left with choosing the optimal contours that gives the desired number of segments. This problem can be shown to be equivalent with maximizing the perimeter of a piecewise linear approximation of a continuous curve $\mathrm{H}_{\mathrm{int}}(\psi_2)$ defined as the integral of $\boldsymbol{H}_2$ between two contours ψ_1 and ψ_2 [8]. It is possible to determine the globally optimal solution to the curve approximation problem up to the desired precision by dynamic programming using numerical techniques described elsewhere [8].

4. EXAMPLE

We will consider an example to illustrate the applicability of the techniques presented above for use in designing magnets for magnetic refrigeration. We consider a rotating magnet design with an outer cylindrical magnet, an inner iron cylinder and two high field regions and two low field regions in the air gap between the cylinders. This is a geometry well known from literature [12]. We assume an inner radius of the air gap of 0.125 m, and an outer radius of 0.165 m. The total high field cross-sectional area is thus 182 cm^2. The magnet is assumed to have a maximum outer radius of 0.33 m. We consider a design where the part of the outer cylinder that is not permanent magnet is iron, which here is assumed to have a permeability of $\mu_r = 1000$. The remanence of the permanent magnets is 1.4 T.
We desire a magnetic field in the high field air gaps similar to the field illustrated in Fig. 1. In order to design the magnet, we follow the procedure described above. First, virtual magnets with a remanence equal to the desired magnetic field are 'placed' in the high field air gaps. The remanence in the design area is then aligned to the field produced by these virtual magnets. Finally a contour of the magnetic scalar potential of the virtual field, Φ_2, is chosen as the border between the magnet and the surrounding iron. The choice of contour of Φ_2 is a choice of the area of the permanent magnet. Following this, segmentation into 12 pieces is chosen, following the optimal segmentation technique described above.
The difference in average field between the high and the low field regions as function of the fraction of the outer cylinder that is permanent magnet material is shown in Fig. 2a. Also shown in this figure is the Λ_{cool} figure of merit parameter for the magnet design [6]. As can be seen from the figure, at a difference in average

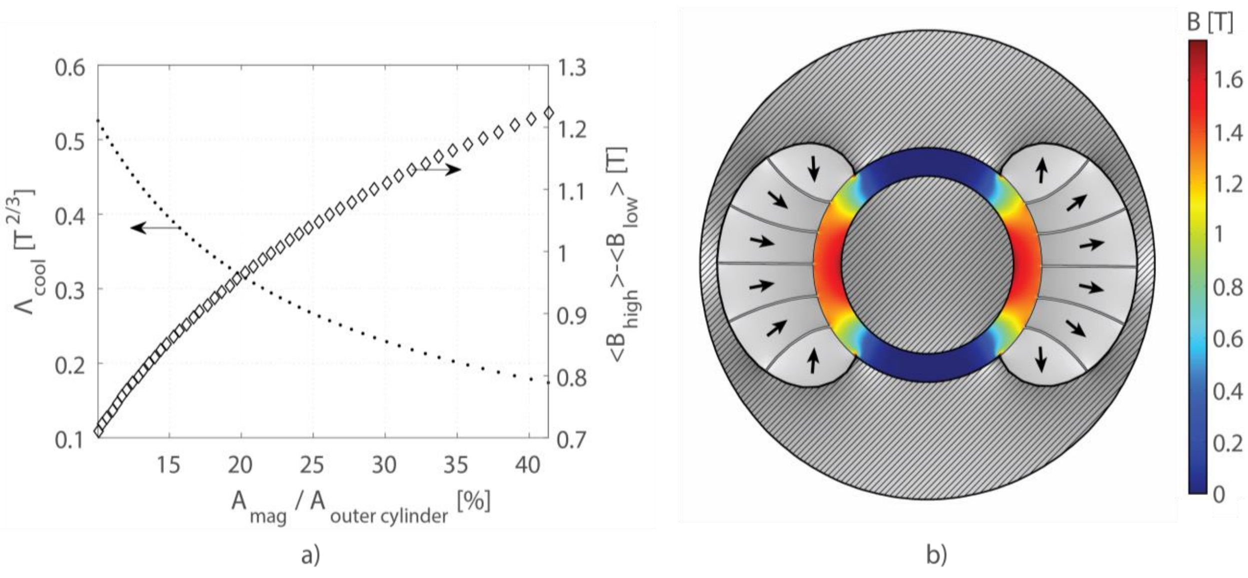

Figure 2: a) The Λ_{cool} and the difference in averaged field between the high and low field regions as function of the amount of permanent magnet material and b) the optimal magnet design for the case of $A_{mag} / A_{outer\ cylinder} = 36\%$.

field of 1 T, a value of $\Lambda_{cool}= 0.35\ \mathrm{T}^{2/3}$ can be obtained, a value significantly higher than previously reported values [6]. An illustration of the resulting design is shown in Fig. 2 below for the case of a cross-sectional magnet area of 930 cm^2, i.e. $A_{mag}/A_{outer\ cylinder} = 36\%$ for this design. The average field in the high field region is 1.25 T, while the average field in the low field region is 0.13 T.
The methods described above always produce the magnet with optimum remanence and optimum segmentation for the desired field distribution in the air gap. However, as can also be seen from Fig. 2, the shape of the individual magnet segments are not polyhedral as is usually required for cheap manufacturing. Thus, a further simplification of the determined magnet design might be needed.

5. CONCLUSIONS

We have presented two methods for designing the optimum magnet. One method can be used to determine the optimal distribution of remanence that produces a desired magnetic field. The other method can segment a given magnet design into optimally shaped segments. The methods were used to segment a magnet design typically used for rotating magnetic refrigeration devices, and the field produced was characterized. The Λ_{cool} figure of merit was found to be significantly higher than previously reported values in literature.

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REFERENCES

[1] A. Tura, A. Rowe, “Concentric Halbach cylinder magnetic refrigerator cost optimization”, *International Journal of Refrigeration* **37**, 106 (2014).
[2] R. Bjørk, C. R. H. Bahl, K. K. Nielsen, “The lifetime cost of a magnetic refrigerator”, *International Journal of Refrigeration* **63**, 48 (2016).
[3] P. V. Trevizoli, J. A. Lozano, G. F. Peixer, J. R. Barbosa Jr., “Design of nested Halbach cylinder arrays for magnetic refrigeration applications”, *Journal of Magnetism and Magnetic Materials* **395**, 109 (2015).
[4] J. Tušek, S. Zupan, A. Šarlah, I. Prebil, A. Poredoš, “Development of a rotary magnetic refrigerator”, *International Journal of Refrigeration* **33**, 294 (2010).
[5] R. Bjørk, C. R. H. Bahl, A. Smith, N. Pryds, “Improving magnet designs with high and low field regions”, *IEEE Transactions on Magnetics* **47**, 1687 (2011).
[6] R. Bjørk, C. R. H. Bahl, A. Smith N. Pryds, “Review and comparison of magnet designs for magnetic refrigeration”, *International Journal of Refrigeration* **33**, 437 (2010)
[7] W.F. Brown, “Magnetostatic principles in ferromagnetism”, North-Holland Publishing Company, Amsterdam (1962).
[8] A. R. Insinga, R. Bjørk, A. Smith, C. R. H. Bahl, “Globally optimal segmentation of permanent magnet systems”, *Physical Review Applied* **5**, 064014 (2016).
[9] N.I. Klevets, “Synthesis of magnetic systems producing field with maximal scalar characteristics”, *Journal of Magnetism and Magnetic Materials*. **285**, 401 (2005).
[10] N.I. Klevets, “Optimal design of magnetic systems”, *Journal of Magnetism and Magnetic Materials* **306**, 281 (2006).
[11] J. H. Jensen, M. G. Abele, “Maximally efficient permanent magnet structures”, *Journal of Applied Physics* **79**, 1157 (1996).
[12] D. Eriksen, K. Engelbrecht, C. R. H. Bahl, R. Bjørk, K. K. Nielsen, A. R. Insinga, N. Pryds, “Design and experimental tests of a rotary active magnetic regenerator prototype”, *International Journal of Refrigeration* **58**, 14 (2015).