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A simplified model predicting the weight of the load carrying beam in a wind turbine blade

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Abstract. Based on a simplified beam model, the loads, stresses and deflections experienced by a wind turbine blade of a given length is estimated. Due to the simplicity of the model used, the model is well suited for work investigating scaling effects of wind turbine blades. Presently, the model is used to predict the weight of the load carrying beam when using glass fibre reinforced polymers, carbon fibre reinforced polymers or an aluminium alloy as the construction material. Thereby, it is found that the weight of a glass fibre wind turbine blade is increased from 0.5 to 33 tons when the blade length grows from 20 to 90 m. In addition, it can be seen that for a blade using glass fibre reinforced polymers, the design is controlled by the deflection and thereby the material stiffness in order to avoid the blade to hit the tower. On the other hand if using aluminium, the design will be controlled by the fatigue resistance in order to making the material survive the 100 to 500 million load cycles experience of the wind turbine blade throughout the lifetime. The aluminium blade is also found to be considerably heavier compared with the composite blades.

1. Introduction

A wind turbine blade is a long slender structure where the dominating loads are given by the aerodynamics and the gravitation. Throughout the years, the turbines have grown larger requiring longer blades. The longest blade at the moment is the 88.4 m long and 33.7 tons heavy blade from LM Wind Power [1]. The growth of the turbine size has resulted in cost of energy which for onshore installation pricewise can compete with conventional energy sources. Off-shore, this is still not the case and properly even larger turbines are required before this will be the case. During this upscaling, the material in a wind turbine blade is pushed to the limits, where mainly the stiffness and the fatigue resistance are the key material parameters. Wind turbine blades have until now been based on glass fibre composites and only by improving the fibre properties, the matrix fibre interface and the fibre architecture this has been made possible. A pure carbon fibre based solution is still too costly, while aluminium only is relevant for small blades.

Advanced and specialised models and material characterization methods are used in the effort to optimize the material used in the blade. Nevertheless, seen on the large scale, a wind turbine blade is a simple beam structure with well-defined load and boundary conditions. The beam theory describing the blade can be made at different levels of complexity depending on its use. An example of a more complex version is the BECAS model (BEam Cross section Analysis Software), see e.g.[2]. Even though the presented model is much simpler, it is still bridging the gap from the aerodynamic and

gravitational loads to an estimate of the material stresses, deflection and blade weight. These weights are then compared with existing wind turbine blades. This approach has been used to estimate the weight of a glass fibre reinforced wind turbine blade for the DTU 10 MW reference turbine [3] with a blade length of 86 m, and it is found that replacing the material with aluminum will increase the blade weight drastically. On the other hand, using a pure carbon fiber based blade will increase the price of the blade significantly.

2. Loads on a wind turbine blade

The wind flow around a turbine rotor can simplified be considered following a 1D momentum theory as illustrated in figure 1. Going from upstream $()_u$ to downstream $()_d$ the wind speed is slowed down from the upstream wind speed u_∞ to the downstream wind speed u_d . Due to the decrease of the wind speed, the corresponding cross sectional area of the wind flow will increase from the upstream area A_u over the known turbine rotor area, A_t to the downstream cross sectional area A_d . Based on this information, we can estimate the aerodynamic loads working on the wind turbine blade, see e.g.[4]. The procedure will be described briefly below.
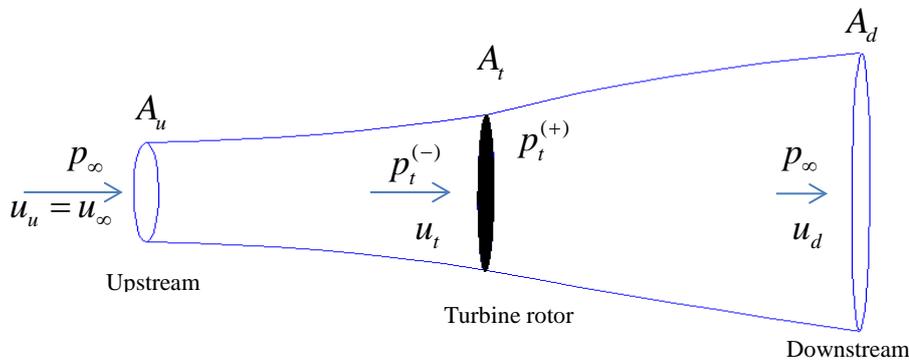

Figure 1. 1D momentum flow

Assuming that the air speed changes continuously across the turbine blades and that the pressure far upstream and far downstream are equal to the pressure of the undisturbed airflow the velocity and the pressure drop at the turbine $()_t$ with the swept rotor area A_t can be found to

$$u_t = \frac{1}{2}(u_\infty + u_d) \tag{1}$$

and

$$\Delta p_t = p_t^{(-)} - p_t^{(+)} \tag{2}$$

where the relation between the air speed and the pressure in the upstream and downstream part is given by the Bernoulli equation

$$p_\infty + \frac{1}{2}\rho_{air}u_\infty^2 = p_t^{(-)} + \frac{1}{2}\rho_{air}u_t^2$$
$$p_\infty + \frac{1}{2}\rho_{air}u_d^2 = p_t^{(+)} + \frac{1}{2}\rho_{air}u_t^2 \tag{3}$$

Introducing (3) into (2) , the pressure drop over the turbine can be written as

$$\Delta p_t = \frac{1}{2}\rho_{air}(u_\infty^2 - u_d^2) \tag{4}$$

The thrust force working over the turbine rotor, Q_t, can now be found as the swept rotor area, A_t multiplied with the pressure drop Δp_t where equation (4) determine the pressure drop.

$$Q_t = \frac{1}{2}\rho_{air}A_t(u_\infty^2 - u_d^2) \tag{5}$$

The only unknown parameter in equation (4) is the downstream wind speed u_d describing how much the turbine is slowing down the wind and can be determined optimizing the power P_t taken out of the wind. The power is given by the difference between the kinetic energy contained in the wind on the

upstream to and on the downstream side: $P_t = \frac{1}{2}\dot{m}(u_\infty^2 - u_d^2)$. With the mass flow through the turbine given by $\dot{m} = \rho_{air} A_t u_t$ this can be written as

$$P_t = \frac{1}{2}\rho_{air} A_t u_t (u_\infty^2 - u_d^2). \tag{6}$$

which corresponds to $P_t = Q_t u_t$. Now introducing the parameter

$$a = (u_\infty - u_t)/u_\infty, \tag{7}$$

a combination of the equations (1), (4) and (6) will results in the following relations

$$P_t = \frac{1}{2}\rho_{air} A_t u_\infty^3 4a(1-a)^2 \tag{8}$$

$$Q_t = \frac{1}{2}\rho_{air} A_t u_\infty^2 4a(1-a) \tag{9}$$

From equation (8), the so-called Lanchester-Betz limit, $a = 1/3$, describes the maximum obtainable power which corresponds to 59.3% of the reference wind power $P_w = \frac{1}{2}\rho_{air} A_t u_\infty^3$. The factor $a = 1/3$ corresponds to a wind speed at the turbine rotor of $u_t = 2/3 u_\infty$ and a downstream wind speed of $u_d = 1/3 u_\infty$. Most modern wind turbines are pitch regulated. Thereby, it is possible to regulate the position of the wind turbine blades in order to achieve the optimal power output. When the wind turbine power is reaching the generator power $P_t = P_G$, the blade position will be chosen in order to maintain this production and not exceeding it. Examples of wind turbine power curves can be found in [4], [5]. The incoming wind speed, u_∞, where the optimal power, P_t, just reaches the generator power, P_G, is called the rated wind speed, u_R and is a design parameter determining the ratio between the swept rotor area, $A_t = \pi L^2$ where L is the blade length, and the generator power P_G based on the knowledge about the dominating wind speed at a given location. Thus, the parameter a can be written as

$$a = \begin{cases} 1/3 & for \quad u_\infty \leq u_R \\ f(P_G, u_\infty) & for \quad u_\infty > u_R \end{cases} \tag{10}$$

where $f(P_G, u_\infty)$ is determined by the condition

$$a(1-a)^2 = \frac{P_G}{2\rho_{air} u_\infty^3 A_t} \tag{11}$$

From (10) and (11), the turbine load (9) can be determined as a function of the remote wind speed u_∞. Thereby, it can be found that the maximum turbine load is found for $u_\infty = u_R$

$$Q_{max} = \frac{4}{9}\rho_{air} u_R^2 \pi L^2 \tag{12}$$

This load will be distributed with 1/3 to each blade. Along the individual blade, the load intensity (load per unit length), q_x^{flap}, can be approximated by linear varying distribution from a vanishing value at the root to the maximum value at the blade tip given by

$$q_x^{flap} = \frac{8}{27}\rho_{air} u_R^2 \pi x \tag{13}$$

In (13) and later, the terms $(\ )_x$ denote a variable dependent on the axial blade coordinate x measured from the root of the blade. In addition to the aerodynamic load, a gravity load will be present in the wind turbine blade. Assuming now a constant blade material cross section area, A_{blade}, the load intensity in the edge-wise direction is given by

$$q_x^{edge} = \rho_{blade} A_{blade} g \tag{14}$$

with the material density, ρ_{blade}, and the gravitational constant $g \approx 9.81 m/s^2$.
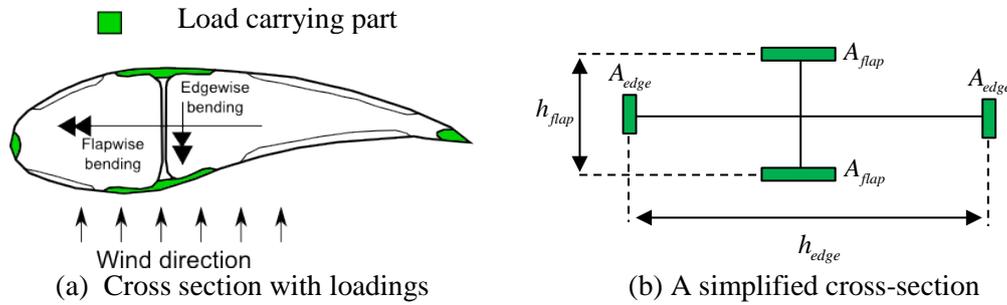
3. Stresses in the blade material

Figure 2. Typical wind turbine cross section

In order to calculate the stress values in the blade material, a simplification of the cross section of the blade is used. A typical and the simplified cross section are shown in figure 2. The part marked with green is the load carrying part contributing to the overall stiffness of the blade. Using the simplified cross-section shown in figure 2b together with the load intensities defined earlier in equation (13) and (14), the flap and edge-wise moment in the blade, see e.g. [5] can be found as

$$M_x^{flap} = q_x^{flap}(L-x)\frac{L-x}{2} + \frac{q_L^{flap} - q_x^{flap}}{2}(L-x)\frac{2(L-x)}{3} \tag{15}$$

$$M_x^{edge} = \rho_{blade} A_{blade} g(L-x)\frac{L-x}{2} \tag{16}$$

which after substitution of the load intensity can be written in stresses as

$$\sigma_x^{flap} = \frac{8}{81}\frac{\rho_{air}u_R^2\pi L^3}{A_{flap}h_x^{flap}}\left(1+\frac{x}{2L}\right)\left(1-\frac{x}{L}\right)^2 \tag{17}$$

$$\sigma_x^{edge} = \frac{1}{2}\frac{\rho_{blade}A_{blade}gL^2}{A_{edge}h_x^{edge}}\left(1-\frac{x}{L}\right)^2 \tag{18}$$

with $A_{blade} = 2(A_{flap} + A_{edge})$. In order to achieve a constant stress level, $\sigma_x^{flap} = \sigma_x^{edge} = \sigma_0$, in the blade material, it can be seen from (17) and (18) that the blade height, h_x^{flap}, and blade width, h_x^{edge}, should vary in the following way

$$\frac{h_x^{flap}}{h_0^{flap}} = \left(1+\frac{x}{2L}\right)\left(1-\frac{x}{L}\right)^2 \quad ; \quad \frac{h_x^{edge}}{h_0^{edge}} = \left(1-\frac{x}{L}\right)^2 \tag{19}$$

Following (19) will result in blade geometries which are solely material strength given. In reality, the blade geometry is given by a combination of structural and aerodynamic requirements. In figure 3, equation (19) is compared with typical blade cross-section variations, where the case $L = 86m$ is taken from [3], while the other two cases are representative blade geometries averaged from a number of known blade geometries. The blade height h_0^{flap} and width h_0^{edge} is specified such that (19) is crossing the representative blade geometry in the largest value points which for the thickness direction is at the blade root while it for the width is at a point located a little away from that, see figure 3. During this, it turns out that h_0^{flap} and h_0^{edge} as a good approximation can be assumed to follow linear functions, with L as the blade length

$$h_0^{flap} = 0.066L - 0.369m \quad ; \quad h_0^{edge} = 0.130L + 0.082m \tag{20}$$
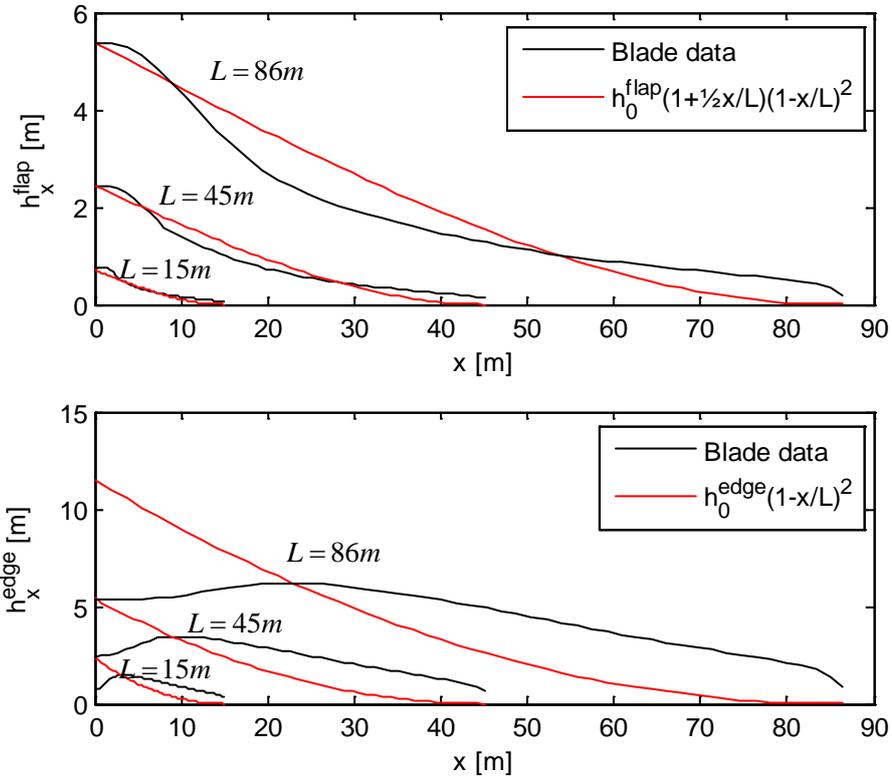
Figure 3 Typical blade geometries compared with the strength determined geometries defined in equation (19).

Table 1. Blade parameters

Type	Parameters
Blade geometry	L ; h_0^{flap} ; h_0^{edge}
Blade material	ρ_{blade} ; σ_0
Rated wind speed	u_R
Density of air	$\rho_{air} = 1.2 kg/m^3$
Gravitation	$g = 9.81 m/s^2$

The weight of the load carrying laminates in the blade can be estimated as

$$M_{blade} = 2(A_{flap} + A_{edge})L\rho_{blade} \qquad (21)$$

with the cross-sections found by combining equation (17), (18) and (19)

$$A_{flap} = \frac{8}{81}\frac{\rho_{air}u_R^2\pi L^3}{\sigma_0 h_0^{flap}} \quad ; \quad A_{edge} = \frac{\rho_{blade}gL^2}{\sigma_0 h_0^{edge} - \rho_{blade}gL^2}A_{flap} \qquad (22)$$

Therefore, based on the parameters listed in table 1, the mass of a specific wind turbine blade can be estimated using equation (21) and (22). Keeping in mind the assumption of a constant cross-section along the blade, the height and width variation is obtained aiming for a constant stress value throughout the blade. In addition to a blade design given by the material strength, the blade should also be stiff enough in order to avoid that the blade hits the tower. In [3], the distance from the blade tip to the tower is given as 18m, this is therefore the maximum allowable deflection $w_{max} = 18$m for a $L = 86$m turbine. The distance will scale with the turbine size. In the following the transverse deflection, w, of the blade is predicted using Bernoulli-Euler equations

$$\frac{d^2 w}{\partial x^2} = \frac{M_x}{EI_x} \quad \text{with} \quad I_x = \frac{1}{2} A^{flap} \left(h_x^{flap} \right)^2 \tag{23}$$

where M_x is the moment in the beam at the location x and E the material stiffness given later in table 2. Equation (23) is solved using a simple numerical difference scheme shown below where the blade is split up into n parts where, $\theta_0 = 0$; $w_0 = 0$, is the rotation and deflection at the root section.

$$\theta_i = \theta_{i-1} + \frac{1}{2} \left(\frac{M_i}{EI_i} + \frac{M_{i-1}}{EI_{i-1}} \right) (x_i - x_{i-1}) \quad ; \quad i = 1,....n$$

$$w_i = w_{i-1} + \frac{\theta_i + \theta_{i-1}}{2} (x_i - x_{i-1}) \quad ; \quad i = 1,....n \tag{24}$$

4. Comparing materials selections for wind turbine blades

Three materials system are considered in the following, see table 2. Due to the high number of load cycles a blade experiences throughout its life time, the strength values are taken as the fatigue failure stress after $5 \cdot 10^8$ cycles.

Table 2. Representative materials properties for glass fibre composites (GFRP), carbon fibre composites (CFRP) and aluminium.

Material	Stiffness	Fatigue strength	Density
GFRP	$E = 44GPa$	$\sigma_0 = 160MPa$	$\rho = 1900kg / m^3$
CFRP	$E = 120GPa$	$\sigma_0 = 300MPa$	$\rho = 1600kg / m^3$
Aluminium	$E = 70GPa$	$\sigma_0 = 100MPa$	$\rho = 2700kg / m^3$

The curves called "fatigue design" in figure 4 show the resulting blade weight as a function of the blade length in the case the material overall is loaded to the fatigue strength, σ_0. In figure 4, this weight variation is compared with the weight of existing blade designs and the 10 MW DTU wind energy reference turbine [3]. In figure 5, the maximum deflections found using the difference scheme (24) are shown. It can be seen that the fatigue given blade design for the glass fibre composite case exceeds the maximum allowed tip deflections given in [3]. Therefore, an additional case is shown in figure 4 and 5 called "deflection design". In that case, the flap area A_{flap} is increase with 50% making it possible to stay below $w_{max} = 18$m for $L = 86$m. As nearly all the existing wind turbine blades are made of glass fibre composite, this case should be compared with the existing blades. The estimation is found to be 20-40% below the existing turbine blade, which may be considered realistic as only the weight of the load carrying beam in the blade is taken into consideration. In all the estimates, the material cross section, A_{blade}, is for simplicity assumed constant. Nevertheless, in realty, the blade

material is often tapered down approaching the end of the blade. During this, a local larger blade height is required in order to make a fatigue strength given design.
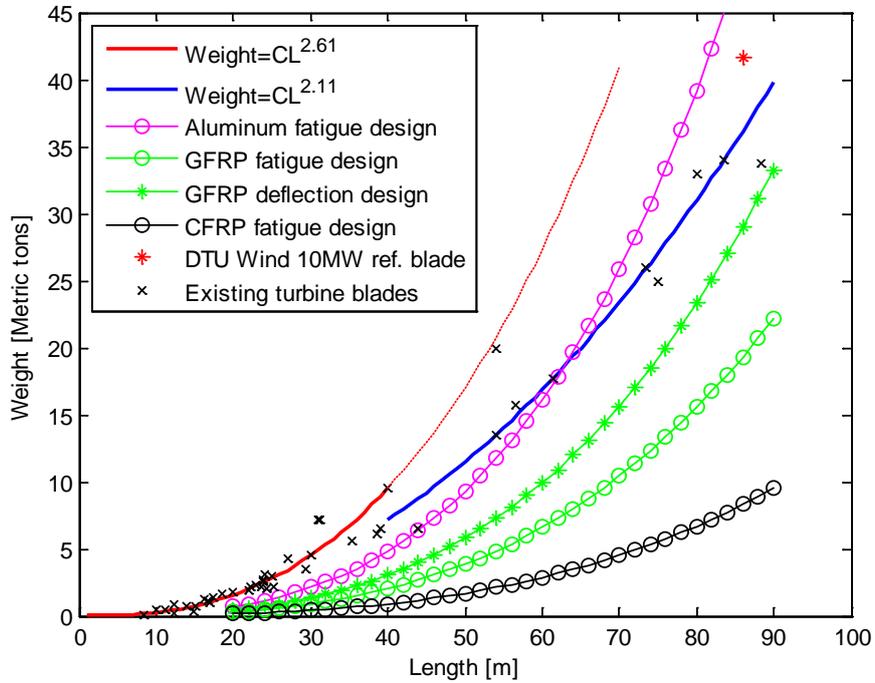

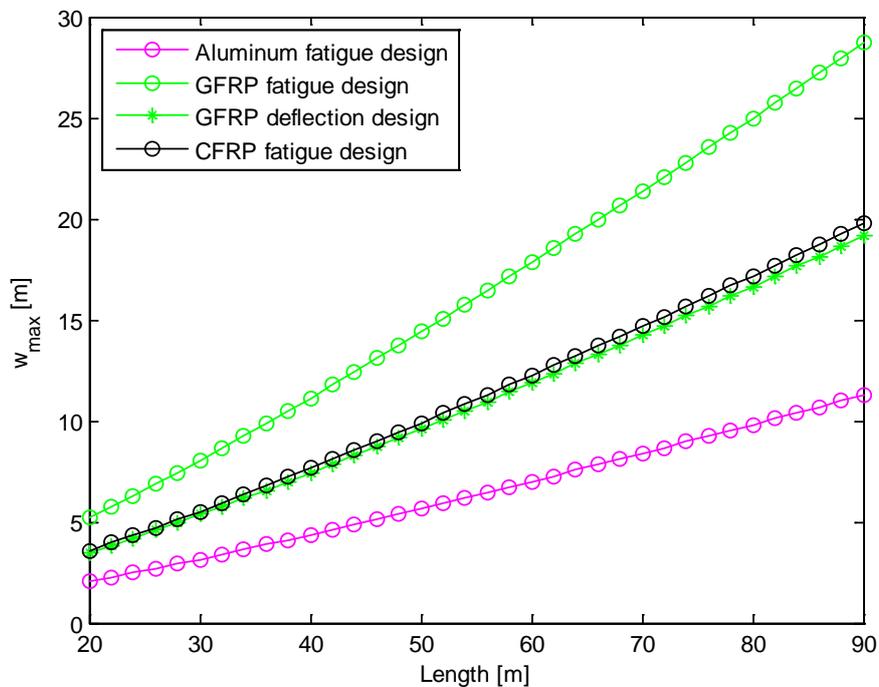**Figure 4.** Blade weight predictions and comparison with existing designs

Figure 5. Maximum deflection of the different blades

5. Conclusion

Based on a simplified beam model of a wind turbine blade, is has been shown possible to make a realistic estimate of the weight of the load carrying laminates in a wind turbine blade. Due to its simplicity, the model is well suited for course work investigating scaling effects of wind turbine blades. This approach has been used as a central element in the materials part of the cousera.org course: "Introduction to Wind Energy" [5]. Even though it turns out, that the design of a glass fibre blade is stiffness given, locally the material can still be loaded to its fatigue strength limits. Therefore, optimizing materials for wind turbine blades, large research activities are going on improving the fatigue resistance, see e.g. [6], [7].

Acknowledgments

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