



## **Selection of the optimum combination of responses for Wave Buoy Analogy - An approach based on local sensitivity analysis**

**Montazeri, Najmeh; Nielsen, Ulrik Dam; Jensen, Jørgen Juncher**

*Published in:*

Proceedings of the 13th International Symposium on PRACTical Design of Ships and Other Floating Structures (PRADS' 2016)

*Publication date:*

2016

*Document Version*

Peer reviewed version

[Link back to DTU Orbit](#)

*Citation (APA):*

Montazeri, N., Nielsen, U. D., & Jensen, J. J. (2016). Selection of the optimum combination of responses for Wave Buoy Analogy - An approach based on local sensitivity analysis. In U. Dam Nielsen, & J. Juncher Jensen (Eds.), Proceedings of the 13th International Symposium on PRACTical Design of Ships and Other Floating Structures (PRADS' 2016) Technical University of Denmark (DTU).

---

### **General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

# Selection of the optimum combination of responses for Wave Buoy Analogy- An approach based on local sensitivity analysis

Najmeh Montazeri<sup>1)</sup>, Ulrik Dam Nielsen<sup>2,3)</sup>, and Jørgen Juncher Jensen<sup>2)</sup>

<sup>1)</sup>Vessel Performance Solutions, SCION DTU, Denmark

<sup>2)</sup>DTU Mechanical Engineering, Technical University of Denmark

<sup>3)</sup>Centre for Autonomous Marine Operations (AMOS), NTNU, Trondheim, Norway

## Abstract

*One method to estimate the wave spectrum onboard ships is to use measured ship responses. In this method, known also as Wave Buoy Analogy, amongst various responses that are available from sensor measurements, a couple of responses (at least three) are usually utilized. Selection of the best combination of ship responses is important. Optimally, this selection should not be implemented manually in onboard applications. Therefore, availability of an automatic response selection procedure would be a great advantage for decision support. In this paper, a local sensitivity analysis is applied to evaluate the importance of individual responses in sea state estimation. The sensitivity factor is defined by calculation of the partial derivatives of wave parameters with respect to the variance of individual responses.*

## Keywords

Wave Buoy Analogy; Response selection; Sensitivity analysis; Derivatives; Sensitivity Factors.

## Introduction

Operational performance management of ships is one of the main concerns in the shipping industry. Many newly-built vessels and marine structures are equipped with data collection systems for monitoring purpose. Decision support systems are nowadays available to provide optimal ship operational performance and route optimisation.

The effect of waves degrade a ship's operational efficiency to some extent. Added resistance due to waves enforce additional trust to achieve the desired speed. In order to implement performance analysis, this wave induced added resistance should be estimated. Moreover, accidents occur due to unexpected and dangerous sea states, which can make the crew unable to keep the ship under proper control. For these reasons, estimation of sea state onboard ships is very important for performance analysis and operational guidance. On the other hand, the

wave estimates in those applications deal with uncertainties. Therefore, it is beneficial to develop more reliable methods for onboard wave estimations.

Different methods have been used so far for onboard wave estimation. One method, which is called Wave Buoy Analogy, WBA, considers the ship as a wave buoy. So, the measured responses of the ship and their transfer functions are used to estimate the encounter waves. Different approaches in this field can be found in the literature e.g. (Tannuri et al., 2003; Pascoal and Guedes Soares, 2009; Nielsen and Stredulinsky, 2012; Nielsen, 2008). Selection of the optimum combination of ship responses for wave estimation is an important issue in the WBA. Depending on the dimensions and also the loading conditions of a ship, the values of individual wave-induced responses and, consequently, the usefulness of them in wave estimation varies from one case to another. Therefore, this choice should be made based on prior knowledge about the transfer functions of a particular ship in different operational conditions.

In (Andersen and Storhaug, 2012) and (Lajic, 2010) studies on automatic selection of responses have been initiated, where sea state estimation is carried out using individual responses separately. Then, a proper combination of responses are those, for which the wave parameters or the wave spectral moments are closest to each other. In this paper, a more systematic and mathematically advanced method based on a local sensitivity analysis is applied to quantify the importance of different responses for wave estimation.

## Basic formulation

The theoretical relationship between the cross-spectral density of the  $i^{th}$  and  $j^{th}$  responses,  $\Phi_{ij}(\omega)$ , and the directional wave spectrum is given by:

$$\Phi_{ij}(\omega_e) = \int_{-\pi}^{\pi} H_i(\omega, \theta) H_j^*(\omega, \theta) S(\omega, \theta) d\theta, \quad (1)$$

where  $\omega$  is frequency,  $\omega_e$  is encounter frequency and  $\theta$  is relative wave direction.  $S$  is the directional wave spectrum,  $H$  denotes the complex-valued transfer function

and  $*$  is the conjugate notation. The responses are measured with respect to the moving reference frame of the ship and, hence, they are considered in the encounter-frequency domain. In other words, the left- and right-hand sides of Eq. (1) are the measured and the calculated cross-spectral density of responses, respectively.

In the wave buoy analogy, this equation can be considered as a cost function relating the measured and calculated responses. Another cost function that could be implemented is formed using the equivalence of the amount of variance between the measured and the theoretical responses. This equation can be derived by integration of the two sides of Eq. (1) with respect to frequency. Thus, the variance of individual responses,  $R_i$ , can be written as:

$$R_i = \int_0^\infty \int_{-\pi}^\pi H_i^2(\omega, \theta) S(\omega, \theta) d\theta d\omega, \quad (2)$$

Eq. (2) is used in (Montazeri et al., 2016) for estimation of parametric wave spectrum.

## Sensitivity measure

The method used here is adopted from (Brun et al., 2002) where a sensitivity analysis is used for importance ranking of the parameters to be estimated. Assuming that the estimated quantity,  $f$ , is a function of  $n$  input variables, i.e.,  $f = f(x_1, x_2, \dots, x_n)$ , the sensitivity of  $f$  to individual variables can be calculated using the derivatives:

$$sf_j = \frac{\partial f}{\partial x_j}, \quad j = 1, 2, \dots, n \quad (3)$$

where the derivative of  $f$  with respect to the parameter  $x_j$  is evaluated at a point in the parameter space, where the sensitivity analysis is carried out.  $sf_j$  represent the sensitivity factors, which form the sensitivity vector  $SF = \{sf_j\}$ .

In the parametric estimation, the wave spectrum is represented in terms of integrated wave parameters,  $p$ . Those parameters are considered as  $f$  in Eq. (3). The variance of the  $j^{th}$  response,  $R_j$  in Eq. (2), is used as  $x_j$ . Therefore, Eq. (3) can be rewritten as

$$sf_{pj} = \frac{\partial p}{\partial R_j}. \quad (4)$$

For the sake of comparability between the different responses, the sensitivity factor should be scaled and non-dimensionalised. Therefore, this quantity is multiplied by  $\frac{R_j}{p}$  so that the scaled sensitivity factor is given by

$$\overline{sf_{pj}} = \frac{\partial p}{\partial R_j} \cdot \frac{R_j}{p}. \quad (5)$$

Eq. (5) expresses the sensitivity of a wave parameter to a change in the variance of the measured  $j^{th}$  response. A high  $\overline{sf_{pj}}$  means that the magnitude of the  $j^{th}$  response has an important influence on the wave parameter estimate and vice versa.  $\frac{\partial p}{\partial R_j}$  in Eq. (5) can be evaluated as outlined in the following section.

**Table 1. Ship characteristics**

Properties	Values
Overall Length [m]	349.0
Beam [m]	42.8
Design Draft [m]	14.5
Speed [kn]	20

## Influence of wave parameters on the response spectra

The transfer function of a particular response exhibits properties that typically change as the wave parameters (particularly the wave period and the wave direction) change. This means that spectral calculations, where transfer functions are combined with a wave spectrum, in general, lead to different outcomes depending on the wave parameters. In other words, the impact of varying one parameter of the wave spectrum, keeping the other parameters fixed, on the standard deviation of individual responses of a ship is usually notable (Nielsen, 2010).

A 9400 TEU container ship is studied in this paper as a case study. The main characteristics of the vessel are given in Table 1. The amplitudes of the transfer functions for the particular operational condition in Table 1 are shown in the appendix.

As an example, Figures 1 and 2 show the influence of the peak period and the mean wave direction, respectively, on the response spectra,  $\Phi$ , of pitch motion for this ship. A long-crested JONSWAP spectrum is used for these illustrations. Similar plots have been presented in (Nielsen, 2010). It can be seen in Figure 1 that the standard deviation of pitch motion, i.e. the square root of the area under the spectrum, has a very small magnitude if the wave energy is concentrated at high frequencies (low peak periods). This is due to the large ship size relative to the wave length. As the peak period increases, this motion becomes considerable. Figure 2 shows that a shift of 90 degrees in wave direction from head/following sea towards beam sea results in a very small amplitudes of the pitch transfer function and, consequently, a small standard deviation in this motion.

Using Eq. 2, the partial derivative of the variance with respect to a wave parameter,  $p$ , is achieved by

$$\frac{\partial R}{\partial p} = \int_0^\infty \int_{-\pi}^\pi \frac{\partial S}{\partial p} H^2(\omega, \theta) d\theta d\omega, \quad (6)$$

where  $\frac{\partial S}{\partial p}$  can be obtained analytically or numerically.  $S$  is considered as a single short-crested unimodal wave modelled by the JONSWAP and a "cos" model spreading factor. The significant wave height,  $H_s$ , the peak period,  $T_p$ , and the mean wave direction,  $\mu$ , are considered as the main parameters of the wave spectrum. Substituting  $H_s$  for  $p$  and calculating the derivative, Eq. (6) can be written

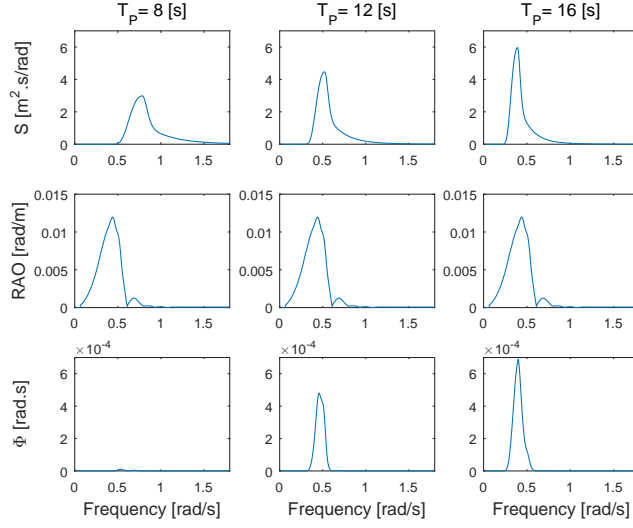


Figure 1. Effect of changing the peak period,  $T_p$ , on pitch motion at  $V=20$  kn,  $H_s=4$  m,  $\mu=135$  deg

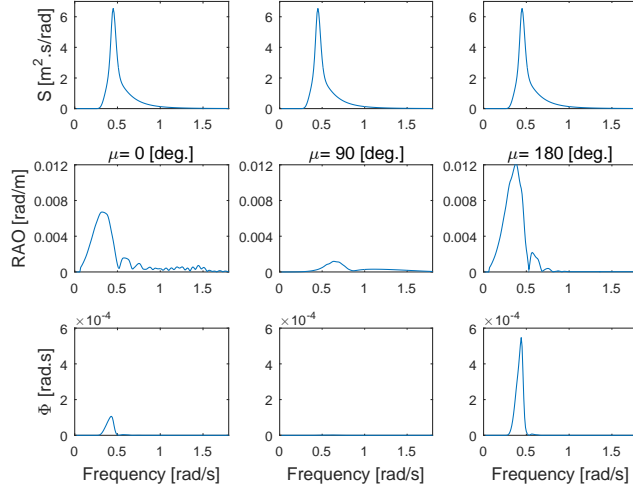


Figure 2. Effect of changing the relative wave direction,  $\mu$ , on pitch motion at  $V=20$  kn,  $H_s=4$  m,  $T_p=14$  s

as

$$\frac{\partial R}{\partial H_s} = 2 \times 5.061 \frac{H_s}{T_p^4} [1 - 0.287 \ln(\gamma)] \times \int_0^\infty \int_{-\pi}^\pi \frac{g^2}{\omega^5} \exp\left[-\frac{5}{4} \left(\frac{2\pi}{\omega T_p}\right)^4\right] \gamma^{\exp\left[-\frac{(\frac{\omega T_p}{2\pi} - 1)^2}{2\sigma^2}\right]} \times H^2(\omega, \theta) N(s) \cos^2 s \left(\frac{\theta - \mu}{2}\right) d\theta d\omega. \quad (7)$$

Consequently, the sensitivity factor, Eq. (5), for  $H_s$  is obtained using the inverse of Eq. (7)

$$\overline{s f_{H_s}} = \left(\frac{\partial R}{\partial H_s}\right)^{-1} \frac{R}{H_s} = \frac{1}{2}. \quad (8)$$

This constant value of  $\overline{s f_{H_s}}$  would be observed for any wave spectral description that is proportional to the square of the significant wave height. It means that regardless of the value of the significant wave height, all

responses have the same importance in estimation of the significant wave height. This is quite reasonable because the transfer function, by definition, does not depend on the wave height. Therefore,  $H_s$  is neglected in the sensitivity analysis here.

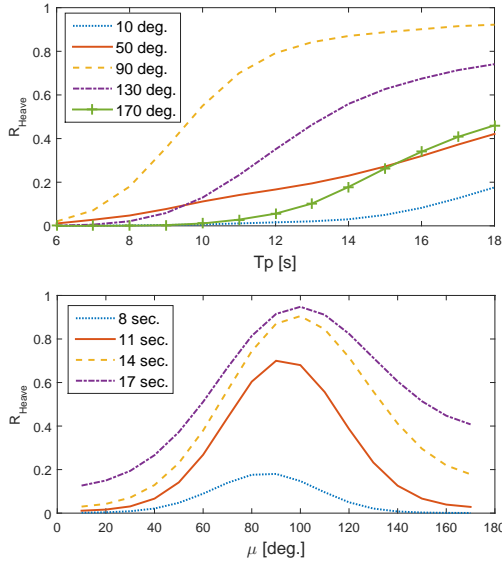
The derivative of the response variance with respect to the peak period is calculated by

$$\frac{\partial R}{\partial T_p} = \int_0^\infty \int_{-\pi}^\pi \left[ \frac{-4}{T_p} + \frac{5}{T_p^5} \left(\frac{2\pi}{\omega}\right)^4 - \frac{\omega}{2\pi\sigma^2\gamma} \left(\frac{\omega T_p}{2\pi} - 1\right) \times \exp\left[-\frac{(\frac{\omega T_p}{2\pi} - 1)^2}{2\sigma^2}\right] \right] S(\omega, \theta) H^2(\omega, \theta) d\theta d\omega, \quad (9)$$

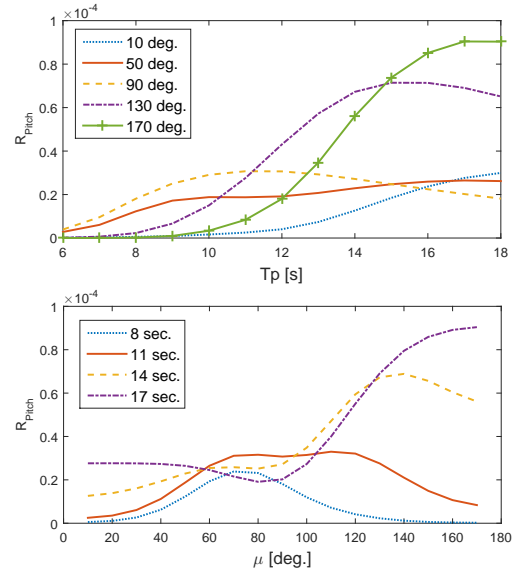
and, the sensitivity factor for  $T_p$  is:

$$\overline{s f_{T_p}} = \left(\frac{\partial R}{\partial T_p}\right)^{-1} \frac{R}{T_p}. \quad (10)$$

Finally, the derivative of the response variance with re-



**Figure 3.** The variance of heave motion at different wave parameters.  $H_s=4$  m



**Figure 4.** The variance of pitch motion for different wave parameters.  $H_s=4$  m

spect to the mean wave direction is

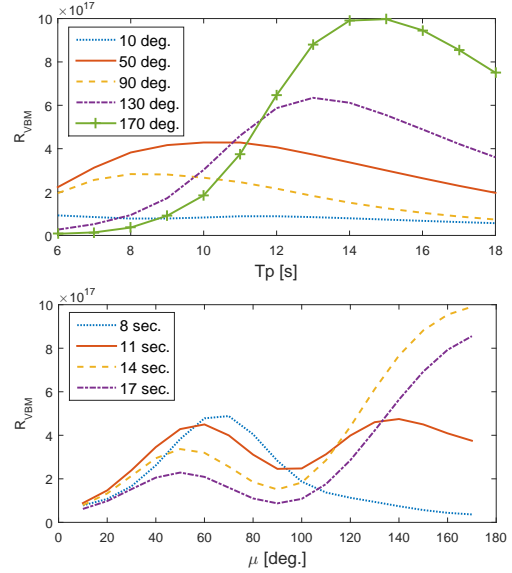
$$\frac{\partial R}{\partial \mu} = 2s \times 5.061 \frac{H_s}{T_p^4} [1 - 0.287 \ln(\gamma)] \times \int_0^\infty \int_{-\pi}^\pi \frac{g^2}{\omega^5} \exp\left[-\frac{5}{4} \left(\frac{2\pi}{\omega T_p}\right)^4\right] \gamma^{\exp\left[-\frac{(\omega T_p - 1)^2}{2\sigma^2}\right]} \times H^2(\omega, \theta) N(s) \cos^2 s \left(\frac{\theta - \mu}{2}\right) \tan\left(\frac{\theta - \mu}{2}\right) d\theta d\omega, \quad (11)$$

where  $\mu$  is in *rad*. The sensitivity factor for  $\mu$  is normalised by  $2\pi$ :

$$\overline{sf_\mu} = \left(\frac{\partial R}{\partial \mu}\right)^{-1} \frac{R}{2\pi}. \quad (12)$$

In order to get a visual understanding of the importance of individual responses, the variances of responses can be plotted for different wave periods and directions. Figures 3 to 7 show the variations of heave, pitch, vertical bending moment at midship section, sway and roll at  $V=20$  kn and  $Draft=14.5$  m. The steepness of these curves with respect to periods and directions represents the quantities of Eqs. (9) and (11), respectively.

It can be realized from Figures 3 and 4 that in head sea and following sea conditions, the energies of heave and pitch are negligible when the peak period falls in the wind sea range (e.g.  $T_p < 10$  s). Therefore, in such conditions, other responses should be used for wave estimation. As seen in Figure 5, the wave bending moment responds to almost all wave conditions and since the inverse of steepness of variations are relatively high, the sensitivity of  $T_p$  and  $\mu$  to this response is considerable for a large range of  $T_p$  and  $\mu$ . The non-zero variances of pitch and bending moment in beam sea condition are due to the asymmetric geometry of the ship with respect to midship section. Sway motion is also useful in many wave conditions as

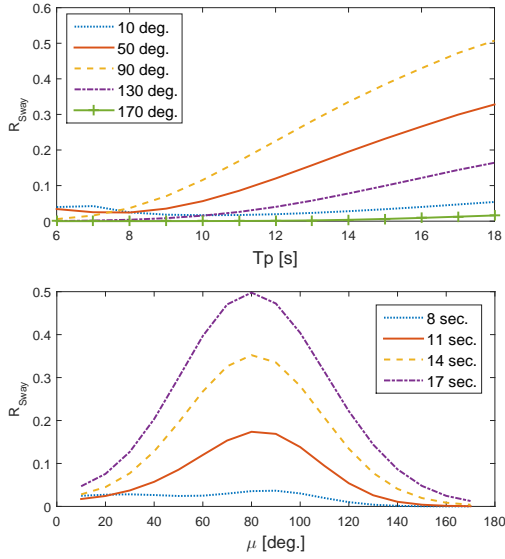


**Figure 5.** The variance of vertical bending moment for different wave parameters.  $H_s=4$  m

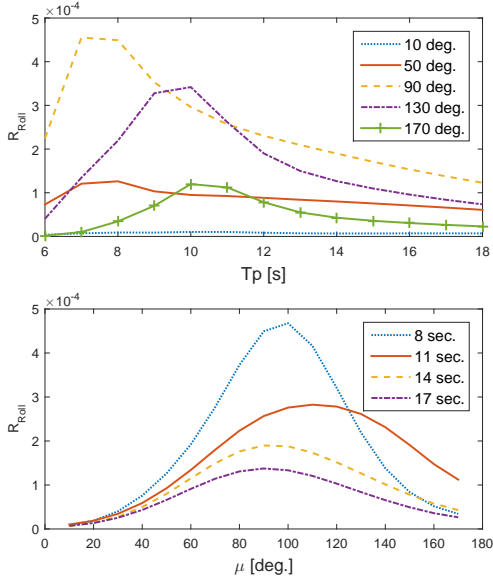
shown in Figure 6. However, as expected, the variance of this response is very small in head sea and following sea. It can be seen in Figure 7 that the energy of roll motion is highly dependent on the wave condition. As inferred from Figures 3 to 7, the most critical condition for wave estimation is following sea condition, where all responses have small magnitudes.

## Results and discussion

In the following, the sensitivity analysis is implemented for various waves with peak periods between 7 and 17 seconds, and mean wave directions from following sea to



**Figure 6. The variance of sway motion for different wave parameters.  $H_s=4$  m**

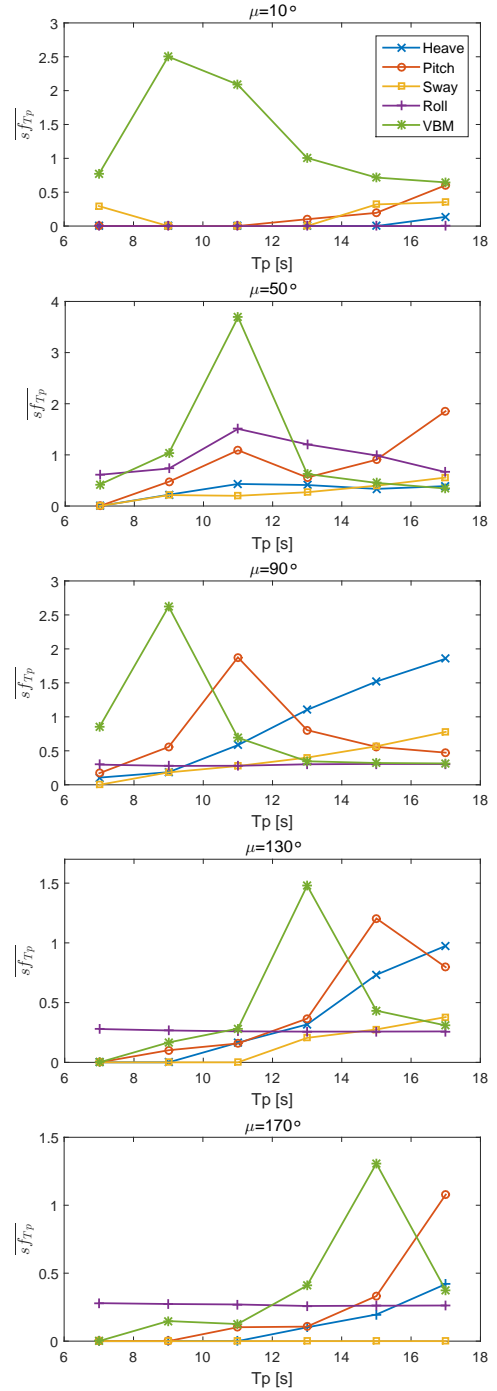


**Figure 7. The variance of roll motion for different wave parameters.  $H_s=4$  m**

head sea. The loading condition of the ship is the same as the previous section. The significant wave height is fixed at 4 meters.  $\gamma$  and  $s$  are also fixed at 3.3 and 25, respectively.

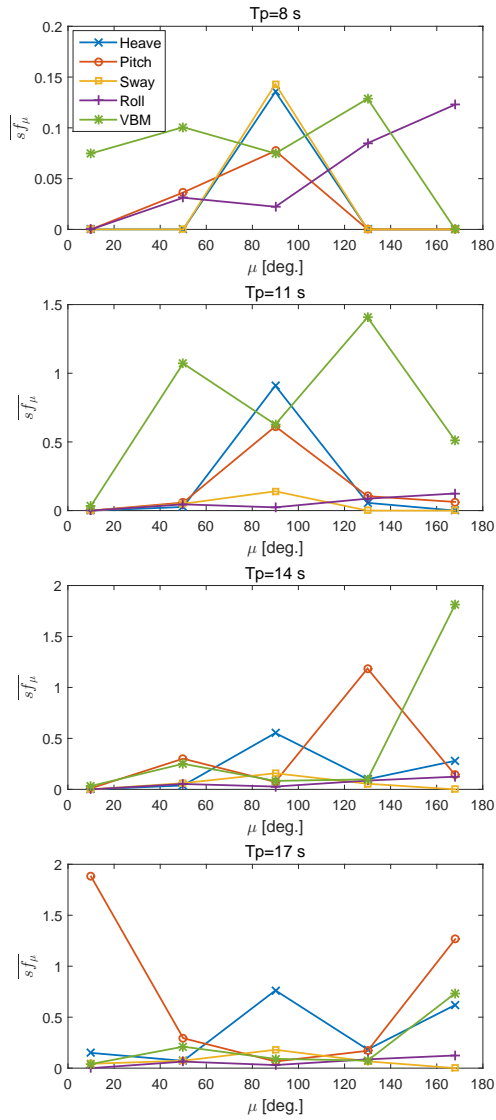
The sensitivity factors based on Eqs. (10) and (12) are calculated for heave, pitch, sway, roll and vertical bending moment at the midship section. It is assumed for calculation of the derivatives that the wave parameters are known. In practice, in order to perform the sensitivity analysis, the predicted wave parameters obtained, for instance, from the trend analysis in (Montazeri et al., 2015) can be used. The sensitivity factors can be calculated in real-time. As an alternative, for a specific ship, the procedure can be also pre-analysed at different loading and

wave conditions so that the optimum combination of responses are determined over a range of probable conditions, and can be utilized during the operations. The latter approach is beneficial in the interest of time saving for real-time decision support applications.



**Figure 8. Normalised sensitivity factor for  $T_p$ , (The legends are identical in all plots)**

Figures 8 and 9 show the values of  $\overline{s_f T_p}$  and  $\overline{s_f \mu}$ , respectively, at different wave conditions. Both quantities should be considered simultaneously for response selection. In the case of  $\mu = 10^\circ$ , since the energy amounts of the most responses are very small as mentioned in the



**Figure 9. Normalised sensitivity factor for  $\mu$ , (The legends are identical in all plots)**

previous section, the sensitivity factors are very small or zero. However, the wave bending moment is an effective response for estimation of all peak periods in this case. When the peak period is long, e.g.  $T_p > 16$  s, pitch could be useful for estimation of direction according to Figure 9. As seen in Figure 4, at  $T_p = 17$  s, the differentiation of the variance of pitch with respect to direction is close to zero at both head sea and following sea, so the magnitude of  $\overline{s_{f_{\mu}}}$  becomes very large. When  $\mu = 50^\circ$ , the bending moment, roll and pitch can have a big role in estimation of peak periods. This combination is also influential in estimation of wave directions in the stern-quartering sea condition.

In beam sea condition,  $\mu = 90^\circ$ , for estimation of low peak periods, pitch and bending moment are proper responses. As the period increases, the importance of bending moment decreases whereas the importance of heave and sway motions increases based on Figure 8. Although the variance of roll motion is relatively large in beam sea

condition (as seen in Figure 7), this response has the lowest sensitivity factor in terms of both wave parameters. Thus, it makes more sense to use sway motion instead of roll in this condition. In the case of  $\mu = 130^\circ$ , both wave parameters have rather high sensitivities to wave bending moment and roll while  $T_p < 14$  s. As the peak period increases, heave and pitch become more important than bending moment and roll.

When  $\mu = 170^\circ$ , as seen in Figure 8, the impact of different responses on the peak period is similar to  $\mu = 130^\circ$ . The sensitivity factor of the wave direction, on the other hand, depends very much on the value of the peak period (Figure 9). Bending moment, pitch, roll and heave could be useful in this case. As the sensitivity factor and the amount of energy of sway motion is almost zero, using this response in head sea condition is inefficient.

All in all, it can be inferred from this paper that the wave bending moment is generally the most effective response for estimation of both the wave period and the wave direction for the specific ship. It is notable that the sensitivity analysis should be provided in different loading conditions in terms of speeds and drafts. However, the sensitivity factors are not expected to be subject to major changes since the magnitudes and the trends of transfer functions do not differ considerably. It can also be expected that the importance of the responses are somewhat similar for other vessels of similar size and with a similar range of operational conditions.

It is noteworthy that the method in this paper assumes the accuracy of different transfer functions to be in the same level. However, hydrodynamic models and numerical calculations of transfer functions are exposed to uncertainties, which may vary from one response to another. Therefore, beside the sensitivity analysis, the responses can be weighted based on knowledge about the accuracy of transfer functions calculations. For example roll would be given less weight than heave and pitch, since the uncertainty of this motion is believed to be higher.

## Conclusion

In this paper, identification of an optimum combination of ship responses for sea state estimation is carried out. The method is based on the sensitivity of the major wave parameters in a standard spectrum (JONSWAP) to the magnitudes of individual response variances. Those sensitivity calculations are simply obtained using first order derivatives of response variances with respect to the wave parameters. It is believed that choosing the responses with higher importance at typical conditions can make the outcome of the WBA more efficient and reliable. It should be noted that this method can be used as a prior input to any response-based wave estimation method including parametric and non-parametric methods such as (Nielsen, 2007; Tannuri et al., 2003; Pascoal and Guedes Soares, 2008). The method helps to make on-board wave estimates more accurate, which can improve the efficiency of performance management and decision

support.

## Acknowledgment

The authors sincerely acknowledge Class NK for supporting the research project in cooperation with the Technical University of Denmark. Thanks are also given to Vessel Performance Solutions for financial support for participation in the conference.

## References

- Andersen, I. M. and Storhaug, G. (2012). Dynamic selection of ship responses for estimation of on-site directional wave spectra. In *Proceedings of the 31th International Conference on Offshore Mechanics and Arctic Engineering*.
- Brun, R., Kühni, M., Siegrist, H., Gujer, W., and Reichert, P. (2002). Practical identifiability of asm2d parameters systematic selection and tuning of parameter subsets. *Water Research*, 36(16):4113–4127.
- Lajic, Z. (2010). *Fault-tolerant Onboard Monitoring and Decision Support Systems*. PhD thesis, Department of Mechanical Engineering, Technical University of Denmark.
- Montazeri, N., Nielsen, U. D., and Jensen, J. J. (2015). Trend modelling of wave parameters and application in onboard prediction of ship responses. In *Proc. of MTS/IEEE OCEANS 15*.
- Montazeri, N., Nielsen, U. D., and Jensen, J. J. (2016). Estimation of wind sea and swell using shipboard measurements a refined parametric modelling approach. *Applied Ocean Research*, 54:73–86.
- Nielsen, U. D. (2007). Response-based estimation of sea state parameters- Influence of filtering. *Ocean Engineering*, 34(13):1797–1810.
- Nielsen, U. D. (2008). The wave buoy analogy estimating high-frequency wave excitations. *Applied Ocean Research*, 30(2):100–106.
- Nielsen, U. D. (2010). *Ship Operations -Engineering Analyses and Guidance*. Lecture notes, Technical University of Denmark.
- Nielsen, U. D. and Stredulinsky, D. C. (2012). Sea state estimation from an advancing ship A comparative study using sea trial data. *Applied Ocean Research*, 34:33–44.
- Pascoal, R. and Guedes Soares, C. (2008). Non-parametric wave spectral estimation using vessel motions. *Applied Ocean Research*, 30(1):46–53.
- Pascoal, R. and Guedes Soares, C. (2009). Kalman filtering of vessel motions for ocean wave directional spectrum estimation. *Ocean Engineering*, 36(6-7):477–488.

Tannuri, E. A., Sparano, J. V., Simos, A. N., and Da Cruz, J. J. (2003). Estimating directional wave spectrum based on stationary ship motion measurements. *Applied Ocean Research*, 25(5):243–261.

## Appendix

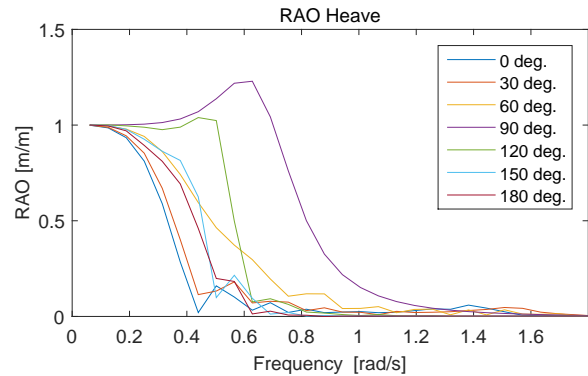


Figure 10. Amplitudes of RAO for heave

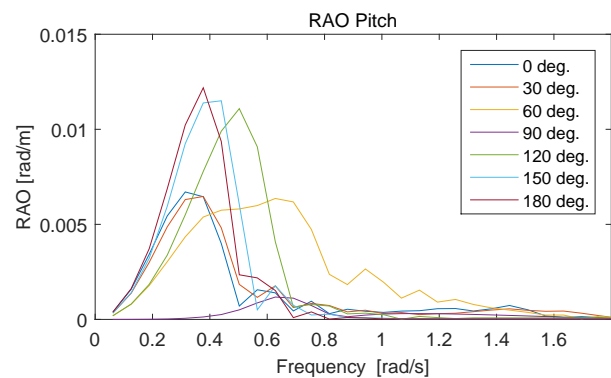


Figure 11. Amplitudes of RAO for pitch

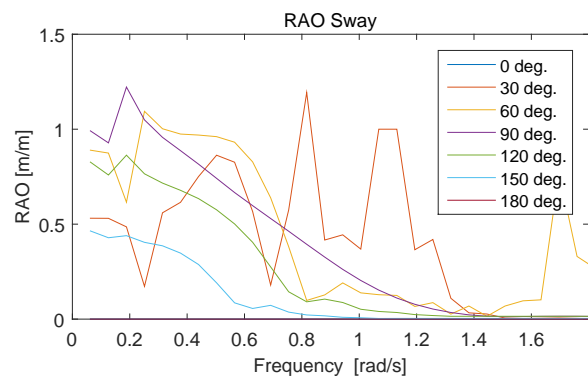
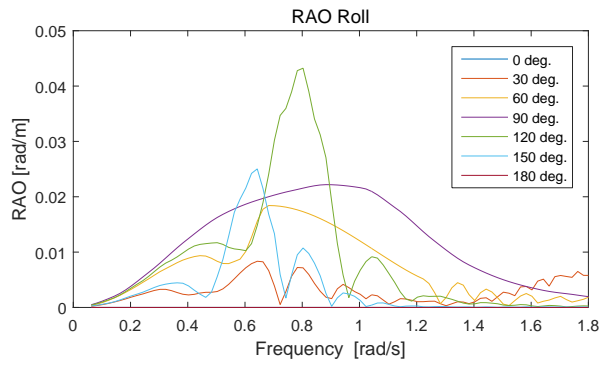
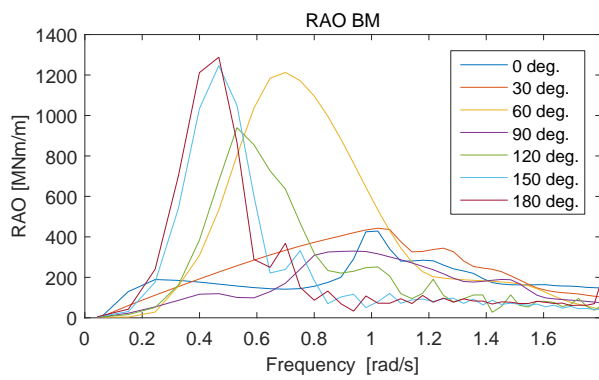


Figure 12. Amplitudes of RAO for sway





**Figure 13. Amplitudes of RAO for roll**



**Figure 14. Amplitudes of RAO for vertical bending moment at midship section**