



Full-Scale Spectrum of Boundary-Layer Winds

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1 Full scale spectrum of boundary-layer winds

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4

5

6 **Abstract** Extensive mean meteorological data and high frequency sonic anemome-
7 ter data from two sites in Denmark, one coastal onshore and one offshore, have
8 been used to study the full-scale spectrum of boundary-layer winds, over frequen-
9 cies f from about 1 yr^{-1} to 10 Hz . 10-min cup anemometer data are used to
10 estimate the spectrum from about 1 yr^{-1} to 0.05 min^{-1} ; in addition, using 20-
11 Hz sonic anemometer data, an ensemble of 1-day spectra covering the range 1
12 day^{-1} to 10 Hz has been calculated. The overlapping region in these two mea-
13 sured spectra is in good agreement. Classical topics regarding the various spectral
14 ranges, including the spectral gap, are revisited. Following the seasonal peak at
15 1 yr^{-1} , the frequency spectrum $fS(f)$ increases with f^{+1} and gradually reaches
16 a peak at about 0.2 day^{-1} . From this peak to about 1 h^{-1} , the spectrum $fS(f)$
17 decreases with frequency with a -2 slope, followed by a $-2/3$ slope, which can be
18 described by $fS(f) = a_1 f^{-2/3} + a_2 f^{-2}$, ending in the frequency range for which
19 the debate on the spectral gap is ongoing. It is shown here that the spectral gap
20 exists and can be modelled. The linear composition of the horizontal wind vari-
21 ation from the mesoscale and microscale gives the observed spectrum in the gap
22 range, leading to a suggestion that mesoscale and microscale processes are uncor-
23 related. Depending on the relative strength of the two processes, the gap may be

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24 deep or shallow, visible or invisible. Generally, the depth of the gap decreases with
25 height. In the low frequency region of the gap, the mesoscale spectrum shows a
26 two-dimensional isotropic nature; in the high frequency region, the classical three-
27 dimensional boundary-layer turbulence is evident. We also provide the cospectrum
28 of the horizontal and vertical components, and the power spectra of the three ve-
29 locity components over a wide range from 1 day^{-1} to 10 Hz , which is useful in
30 determining the necessary sample duration when measuring turbulence statistics
31 in the boundary layer.

32 **Keywords** Atmospheric turbulence, Mesoscale velocity spectra, Microscale
33 velocity spectra, Planetary boundary layer, Spectral gap

34 1 Introduction

35 Spectral analysis is a classical theme. It is central to the study of a number of issues
36 in atmospheric boundary-layer dynamics, among them equations of motions, prob-
37 ability functions of first-order and higher-order moments, structure functions and
38 exceedance statistics. Analysis of wind variation in the frequency or wavenumber
39 domain provides important information for various applications, such as dynamic
40 wind loading and turbulence diffusion of constituents, where turbulent energy in
41 specific frequency bands is directly relevant.

42 The velocity spectrum has been studied and applied in the context of wind
43 engineering for more than half a century. There remain a series of outstanding
44 questions, including:

- 45 – Is there a gap between microscale and mesoscale, centered around 1 hr^{-1} ?
- 46 – How do microscale and mesoscale motions interact? Can these motions be
47 considered correlated, uncorrelated, or just weakly correlated?
- 48 – In the mesoscale range, how do the spectra vary with height, how do the
49 spectra and cross-spectra of the three wind components (the longitudinal, the
50 transverse and the vertical) vary with height and stability?

51 In Lumley and Panofsky (1964), the Van der Hoven wind spectrum close to the
52 ground (Van der Hoven 1957) was shown as evidence of the existence of a “gap” at
53 a period of about 1 hr, separating the three-dimensional (3D) turbulence from the

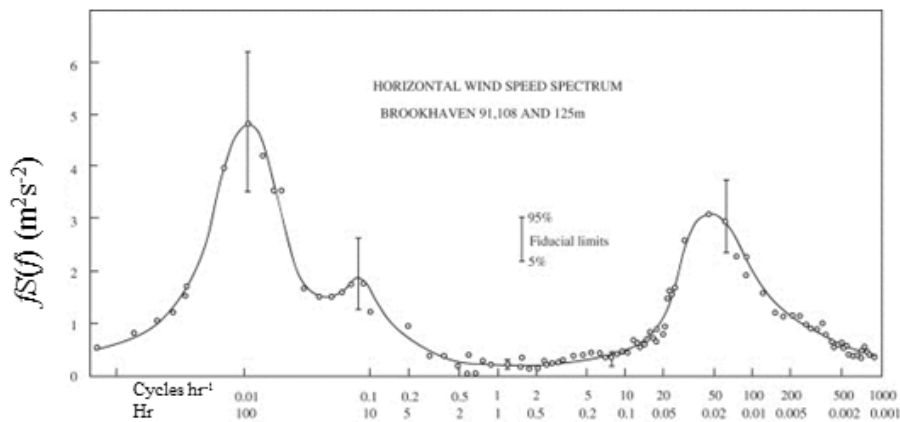


Fig. 1 Horizontal wind-speed spectrum at Brookhaven National Laboratory at about 100-m height, $fS(f)$ against f . Reproduced from Van der Hoven (1957).

54 two-dimensional (2D), mesoscale to macroscale motions. The authors stated: “It
 55 can be seen that such a gap exists, which makes the prospect of applying our ideas
 56 in meteorology seem attractive”. Kaimal and Finnigan (1994) stated: “Implicitly in
 57 the development of spectral forms in the energy-containing range is the assumption
 58 that a spectral gap exists, separating boundary-layer turbulence from external
 59 fluctuations.” They also cited the Van der Hoven spectrum as evidence, shown in
 60 Fig. 1. When analyzing measured turbulence, the basic assumption on ergodicity
 61 and stationarity requires existence of the gap. It is because the assumption of
 62 stationarity implies the existence of a correlation integral time scale, which in
 63 turn implies that the frequency spectrum $fS(f)$ of e.g. boundary-layer turbulence
 64 must be limited at low frequencies by a f^{+1} dependence, according to the Wiener-
 65 Khintchine theorem for a stationary, random process. Here, f is the frequency. In
 66 the atmospheric boundary layer, the existence of a f^{+1} spectral range of 30 - 60
 67 min lends credibility to considering the fast fluctuations as stationary processes
 68 and hence to analyze the time series through statistically stationary theories and
 69 models. In terms of processes, the spectral gap therefore constitutes the lower
 70 frequency limit for boundary-layer turbulence, being responsible for the fluxes
 71 between the surface and the atmosphere.

72 The search for the gap historically goes back to the early fifties (Panofsky
 73 and McCormick 1954; Panofsky and Van der Hoven 1955; Griffith et al. 1956).
 74 Over the years there have been numerous discussions on wind velocity spectra,
 75 the shape of the spectrum, and the division into microscale and mesoscale due to
 76 the gap (Fiedler 1971; Fiedler and Panofsky 1970; Goldman 1968; Oort and Taylor
 77 1969; Vinnichenko and Dutton 1969; Vinnichenko 1970; Smedman-Högström and
 78 Högström 1974; Vickers and Mahrt 2002; Weinstock 1980; Atkinson 1981; Heggem
 79 et al. 1998; Courtney and Troen 1990; Troen and Petersen 1989). Most studies
 80 describe a near-surface gap observed at periods of the order of 1 hr. Sometimes
 81 the gap was not observed and some studies interpreted this as due to features such
 82 as longitudinal vortices, rolls, jets and convective cells. (LeMone 1976; Smedman
 83 1991; Smedman et al. 1995; Heggem et al. 1998).

84 It has been questioned by many whether the study by Van der Hoven really
 85 proves that the gap exists, e.g. Goldman (1968). His spectrum was patched to-
 86 gether from spectral analysis of eight time series covering periods from 1 yr to 1
 87 hr. What really has set a question mark over the proof of a gap comes from the fact
 88 that the high frequency region was measured during the passage of a hurricane.
 89 This issue reflects one of the major challenges in finding the gap: lack of time series
 90 long enough to cover the range from high frequency turbulence and the transition
 91 into the mesoscale. Courtney and Troen (1990) and Troen and Petersen (1989), in
 92 the Lammefjord experiment (LAMEX), overcame this difficulty by applying the
 93 latest optical disk storage devices. A 1-yr record of 8-Hz data was analyzed and
 94 they found that a spectral gap can be identified but is not as deep as expected.

95 Högström et al. (2002) divided a measured horizontal velocity spectrum from
 96 4×10^{-5} Hz to 10 Hz into four ranges; their Fig. 4 shows the mean longitudinal ve-
 97 locity spectrum normalized with the friction velocity u_* , $fS(f)/u_*^2$, plotted against
 98 frequency, f in Hz. We indicate their four ranges on top of our Fig. 3d. Range *i* is
 99 the Kolmogoroff inertial subrange, where the spectrum is characterized by a $-2/3$
 100 slope; range *ii* corresponds to the so-called shear production range (Tchen et al.
 101 1985), where the spectrum is characterized by a plateau $fS(f)/u_*^2 \approx \gamma$ with $\gamma \approx 1$;
 102 range *iii* corresponds to a spectrum characterized by $fS(f)/u_*^2 \propto f$ and in range
 103 *iv* $fS(f)/u_*^2$ increases with decreasing f . Ranges *i* to *iii* represent boundary-layer
 104 turbulence, while ranges *iii* and *iv* are separated by a spectral gap.

105 In comparison to boundary-layer turbulence, the spectrum at frequencies lower
 106 than the spectral gap has been relatively less studied and understood. Measure-
 107 ments and theory have suggested the following spectral behaviour for scales be-
 108 tween several hundreds of km to a few km,

$$109 \quad E(k) = d_1 k^{-5/3} + d_2 k^{-3} \quad (1)$$

110 where k is the wavenumber in m^{-1} , E is the power spectrum in the k domain,
 111 with coefficients $d_1 \approx 9.1 \times 10^{-4} \text{ m}^{4/3} \text{ s}^{-2}$ and $d_2 \approx 3.0 \times 10^{-10} \text{ s}^{-2}$, e.g. Gage
 112 and Nastrom (1986) and Lindborg (1999). Similar spectral behaviour was observed
 113 from climatological, long term time series in the frequency domain in Larsén et al.
 114 (2013) for scales from several days to about 10 min,

$$115 \quad S(f) = a_1 f^{-5/3} + a_2 f^{-3} \quad (2)$$

116 where $a_1 = 3 \times 10^{-4} \text{ m}^2 \text{ s}^{-8/3}$ and $a_2 = 3 \times 10^{-11} \text{ m}^2 \text{ s}^{-4}$. The range iv as in
 117 Högström et al. (2002) is seemingly located in the region where the spectral slope
 118 is $-5/3$ as in Eqs 1 and 2. The -3 slope part of the spectrum corresponds to
 119 geostrophic turbulence, interpreted generally to be related to the baroclinic insta-
 120 bility.

121 For improved clarity, we address the various frequency ranges as follows: in
 122 macroscale: $1 \text{ yr}^{-1} \lesssim f \lesssim 1 \text{ day}^{-1}$, with the high frequency bound where the
 123 spectral slope, $S(f)$ versus f in the log-log scale, of -3 meets the spectral slope
 124 $-5/3$; in mesoscale: $1 \text{ day}^{-1} \lesssim f \lesssim 1 \text{ hr}^{-1}$, where the spectral slope is $-5/3$; in
 125 microscale: scales smaller than the gap range where the spectrum corresponds to
 126 3D boundary-layer turbulence.

127 The purpose of our study is to improve our knowledge of the full scale velocity
 128 spectrum from the order of 1 yr^{-1} to the microscale inertial subrange, by using a
 129 consistent calculation approach to extensive datasets of both 10-min mean wind
 130 data and sonic turbulence measurements.

131 The measurements are introduced in Sect. 2, and the method by which the
 132 spectra are calculated, analyzed and presented is given in Sect. 3. Results are
 133 presented in Sect. 4, followed by discussions and conclusions in Sects. 5 and 6.

134 2 Observations

135 The long term observations were obtained from two sites in Denmark, Høvsøre
136 and Horns Rev (Fig. 2). Høvsøre is a coastal site on land, less than 2 km from the
137 coast and Horns Rev is offshore, with the shortest distance about 20 km from the
138 shore.

139 For Høvsøre, we analyzed both 10-min time series and sonic anemometer data
140 from the years 2012 and 2013. Coverage of the 10-min averaged wind data is 98.4%
141 for 2012 and 97.9% for 2013. The 10-min averaged winds are available at heights
142 10 m, 40 m, 60 m, 80 m, 100 m and 116.5 m and all data are used in the analysis.
143 The 20-Hz wind components were obtained at heights 10 m, 20 m, 40 m, 60 m,
144 80 m and 100 m. Here, we use turbulence measurements from heights 10 m, 20 m,
145 80 m and 100 m because data quality and data coverage at 40 m and 60 m are
146 problematic. Standard meteorological and turbulence measurements were made at
147 the same mast, which is 116.5 m tall. Horizontal wind profiles were measured with
148 cup anemometers and wind vanes, and the 3D turbulence properties were measured
149 with sonic anemometers. Details of the instrumentation, including influences on
150 the measurements from the mast and the row of wind turbines, can be found in
151 Peña et al. (2015). The mast was at the southern end of a row of turbines in the
152 north-south direction; on average, measurements at the mast suggest that about
153 2% of the time, the wind direction is from the sector $0^\circ - 25^\circ$ where wakes may be
154 present. Unfortunately there are no records of turbine operation and generation of
155 wakes that would affect the turbulence signals obtained from the mast. Therefore
156 it is not possible for us to quantify the uncertainty caused by the wakes in the
157 current analysis. In connection with the spectrum calculation using the sonic data,
158 we require that for each day, data coverage $> 99.95\%$, which leads to fewer useful
159 days from each year. Statistics regarding the chosen days and the data availability
160 are presented in Table 1 for each year and at the four levels. Note that the data
161 at 100 m are less dense than at the other levels. For the spectral analysis for low
162 frequencies, the 10-min mean wind-speed time series from 2005 to 2014 are used.

163 For the offshore site Horns Rev (Fig. 2), we use both 10-min mean cup data
164 (1999 to 2006) and turbulence measurements (1999 to 2005) from Mast 2 (Larsén
165 et al. 2013). Data are available from 1999 to 2006. The wind farm (www.hornsrev.dk)

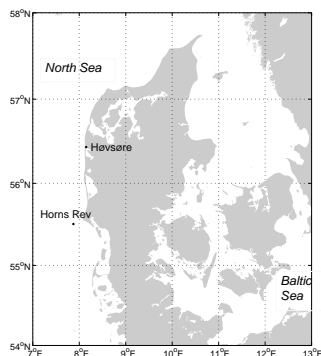


Fig. 2 Map of Denmark and the locations of the two sites Høvsøre and Horns Rev.

166 was in operation from December 2002. Before this, instruments are not influenced
167 by the wind farm, but since 2003, the data could have been affected by the wakes
168 from the wind farm if the wind direction is from the sector $130^\circ - 160^\circ$. Out of
169 498 days with turbulence measurements (Table 2), for 19 days the wind direc-
170 tion is from that sector and these days were not included in the calculation. The
171 turbulence was available from sonics at 20 Hz for years 1999 and 2000, but at
172 12 Hz since 2001. Wind speeds were measured with cup anemometers and they
173 are available at 15 m, 30 m, 45 m and 62 m while turbulence measurements were
174 obtained from one single level, 50 m. Yearly data coverage for wind speed varies
175 from 60% to 99.9% (Table 1 in Larsén et al. (2013)). Table 2 lists the number of
176 days in each month where turbulence data are available.

177 3 Method

178 We aim at obtaining a “climatologically representative” spectrum with frequency
179 from the order of 1 yr^{-1} to 10 Hz. The sonic data at Høvsøre have shown question-
180 able quality for frequencies higher than about 3 Hz, likely caused by flow distortion
181 of the mast when winds are from the north (Peña et al. 2015). All calculation has
182 been conducted to the Nyquist frequency of the time series but the presentation
183 sometimes is limited to a few Hz, which is usually in the inertial subrange where
184 spectral properties are well known.

Table 1 Turbulence data availability (number of days) at Høvsøre for each month in 2012 and 2013, where for each day the data coverage is greater than 99.95%. Also shown are sonic anemometer sampling frequency and measurement height h .

Year	Sampling frequency (Hz)	h (m)	Total days	J	F	M	A	M	J	J	A	S	O	N	D
2012	20	10	181	9	8	2	10	24	22	29	23	13	11	15	15
		20	170	9	3	1	10	24	21	29	20	13	11	14	15
		80	174	9	4	2	10	24	22	29	23	13	11	14	13
		100	38	6	4	1	9	11	4	0	3	0	0	0	0
2013	20	10	200	27	20	20	27	25	8	23	8	6	10	13	13
		20	186	25	20	18	27	25	8	23	8	5	5	9	13
		80	190	22	20	19	25	25	8	23	8	6	9	13	12
		100	24	0	0	5	9	0	0	0	0	0	0	3	7

Table 2 Turbulence data availability (in days) at Horns Rev at 50 m for each month for all years. Also shown are measurement height h and sonic sampling frequency at different periods.

Year	Sampling frequency (Hz)	h (m)	J	F	M	A	M	J	J	A	S	O	N	D
1999-2000	20													
2001-2005	12													
1999-2005		50	110	55	85	65	27	0	0	1	5	14	64	72

185 Climatological 10-min mean cup anemometer data are used to calculate the
 186 power spectra for frequencies up to the Nyquist frequency, 0.05 min^{-1} . The spec-
 187 trum was calculated using the Fourier transform, with a linear detrending applied
 188 to the yearly time series. The few missing data in the time series were filled in
 189 using linear interpolation with data before and after the gaps. A smoother spec-
 190 trum is obtained afterwards by averaging the values of the spectral power in bins
 191 of $\log_{10} f$, with 25 bins used for each decade. We refer to this as log-smoothing.
 192 Good data coverage is important for a reliable calculation of the Fourier spectrum
 193 and all time series used herein have a data coverage better than 97%.

194 In order to obtain the spectral information around the expected gap frequen-
 195 cies, we use the time series from the high frequency sonic measurements with a
 196 length of one day, thus covering the spectral gap range and merging with the
 197 spectrum from the 10-min measurements at a frequency of about 1 day^{-1} .

198 When calculating the climatological spectrum for high frequencies, only days
 199 when missing sonic data $< 0.05\%$ are chosen. For Høvsøre, the number of such

200 days in each month is given in Table 1 for each height for the two years considered.
201 At the Horns Rev site, there are 498 days in total. The incomplete turbulence data
202 coverage and uneven distribution throughout the year may question the climato-
203 logical representativity in some aspects of the results from these data. This will be
204 discussed further in Sect. 4. The spectrum is calculated from the sonic data using a
205 Fourier transform after applying a linear detrending. We have chosen not to use a
206 Hanning window to the day-long time series, see the Appendix. The log-smoothing
207 is applied. The spectrum for the horizontal and vertical wind speeds, U and w ,
208 respectively, as well as the cospectrum of U and w , were first calculated from each
209 day and averaged afterwards to obtain the mean spectrum and mean cospectrum.
210 In the Appendix we discuss the stationarity conditions required for the calcula-
211 tion of the Fourier tranform using 1-day long time series. To control the possible
212 spectral energy leakage associated with strongly non-stationary conditions, i.e.,
213 to avoid the “outliers”, when calculating the mean spectrum we used all data at
214 each frequency that satisfy $\langle S(f) \rangle - 2\sigma(S(f)) < S(f) < \langle S(f) \rangle + 2\sigma(S(f))$, where
215 $\langle S(f) \rangle$ and $\sigma(S(f))$ are the mean value and standard deviation of S at f . This
216 process removed about 5% of data.

217 Three wind components over the range from 1 day^{-1} to the Nyquist frequency
218 of the time series, the longitudinal wind (u), the lateral wind (v) and the vertical
219 (w) were also investigated. In order to decompose the horizontal winds into u and
220 v , both the wind speed and direction need to be stationary. Days where the wind
221 direction does not vary more than 50 degrees are selected for this purpose. At
222 Horns Rev, the realignment of the sonic with the horizontal wind every 10 min
223 done in the original preprocessing of the data results in missing information about
224 the lateral wind (v) for scales larger than 10 min. For this reason, the results for
225 the wind components are only from Høvsøre.

226 4 Results

227 The availability of both standard wind and turbulence measurements at Høvsøre
228 at several levels from 10 m to 100 m provides a unique opportunity to examine how
229 the full range velocity spectrum varies with height, including the power spectrum,
230 cross spectrum and the spectral gap. The measurements from Høvsøre and Horns

231 Rev together are expected to contribute to our understanding of the spectral
232 behaviour for onshore and offshore conditions.

233 4.1 The full-scale power spectrum

234 The full range spectra of wind speed from Høvsøre, with f from about 1 yr^{-1}
235 to 10 Hz , are calculated and presented as $fS(f)$ versus f on a log-log scale in
236 Figs. 3 and 4, where the data from 2012 and 2013 are used, respectively. The
237 results from the two years are presented separately in order to show that, even
238 though the data distribution over the year is not the same (Table 1), the results are
239 consistent, suggesting that data from either of the two years are climatologically
240 representative for certain spectral aspects. The spectra are shown at three heights
241 where both the mean wind speed and turbulence data are available, namely 10 m,
242 80 m and 100 m. At each height, there are three portions of spectra presented.
243 For the macroscale to mesoscale, the grey curve is from the 1-year long time series
244 of 10-min averaged wind speed. Here we see the large fluctuations caused by the
245 availability of only few data points at the lowest frequencies. To obtain a more
246 climatologically representative spectrum, 10 years of data are used here, which
247 is shown as the black dashed curve. This is consistent with the one-year data,
248 and as expected, the fluctuation of $fS(f)$ is greatly reduced at the frequencies of
249 about 1 yr^{-1} when more data are used. The thin black curves are the average of
250 the spectra calculated from 1-day long time series of 20-Hz wind speed for days
251 as listed in Table 1. The mesoscale to microscale spectra from the turbulence
252 data from four heights are shown together, for 2012 (Fig. 3d) and 2013 (Fig.
253 4d), respectively. From Figs. 3 and 4, we observe: (1) In general, the spectra
254 based on sonic data converge rather smoothly to the spectra made from the 10-
255 min time series, suggesting the consistency between the two data series in their
256 overlapping range. (2) The spectral transition of $fS(f)$ from the microscale to
257 mesoscale suggests a “gap” at 10 m when f is close to 10^{-3} Hz , but is shown
258 as a plateau at 80 m and 100 m. Figures 3d and 4d suggest a gradual decrease
259 in the gap depth with height. (3) For frequencies higher than the 10-m gap, the
260 turbulence intensity decreases with height, and it is the opposite for frequencies
261 lower than the 10-m gap. (4) There is a narrow peak at 1 yr^{-1} from the 10-year

262 time series. (5) There is a maximum value of $fS(f)$ at about 0.2 day^{-1} for the
263 energy-containing range from 1 yr^{-1} to 1 hr^{-1} . (6) There is a narrow peak at 1
264 day^{-1} at 10 m height, but not at 80 m and 100 m.

265 As in Figs. 3 and 4, the full range spectrum of wind speed is presented in
266 Fig. 5 for the offshore site Horns Rev. To examine the “climatological represen-
267 tativity” of a one-year time series (from 2002) in representing the spectrum, we
268 also selected from the Horns Rev measurements all months from 1999 to 2006
269 where the data coverage is greater than 99% and the mean spectrum from all
270 these months (referred to as “the good months” in Fig. 5) is compared to that
271 from 2002. The agreement is good for their overlapping frequency range. For the
272 mesoscale to macroscale, the 10-min wind speed at 45 m was used; the turbulence
273 properties were measured at 50 m. Note that in Fig. 5, the increasing spectral
274 energy with frequency at the tail is the folding of higher frequencies, which should
275 be disregarded. In Fig. 6a and b, the spectra from the 10-min time series at all
276 measurement heights are shown, for Høvsøre and Horns Rev, respectively. Accord-
277 ing to Fig. 6b, for the mesoscale range, the difference is negligible between 45 m
278 and 62 m; we therefore expect negligible difference between 45 m and 50 m. No
279 “gap” is observed here.

280 The increase of the power spectrum with height at mesoscales shown by the
281 sonic data is shown again in the 10-min data from Høvsøre, see Fig. 6a. Here we
282 see the land influence in the power spectrum at 10 m indicated by the clear diurnal
283 peak. This peak becomes weaker with increasing height and indistinguishable at
284 80 m. Recall that in the Van der Hoven spectrum, there is a missing diurnal peak,
285 a property that has been much discussed. The diurnal peak is documented to be
286 absent for a water site (e.g. for Horns Rev in Fig. 6b and for a number of lightships
287 and small island stations in Troen and Petersen (1989)), but for land sites, the
288 missing diurnal peak was also reported at a height of the order of 70 to 100 m in
289 a number of studies, e.g. Oort and Taylor (1969) and Petersen (1975). Troen and
290 Petersen (1989) explained that the height at which the diurnal peak becomes small
291 is where the first-order effect of the surface heat flux modulations vanishes and
292 they used this observation to construct the stability correction model. It can be
293 noted that the height at which the minimum amplitude of the diurnal cycle occurs
294 varies with season, being lowest in winter and highest in summer. The amplitude

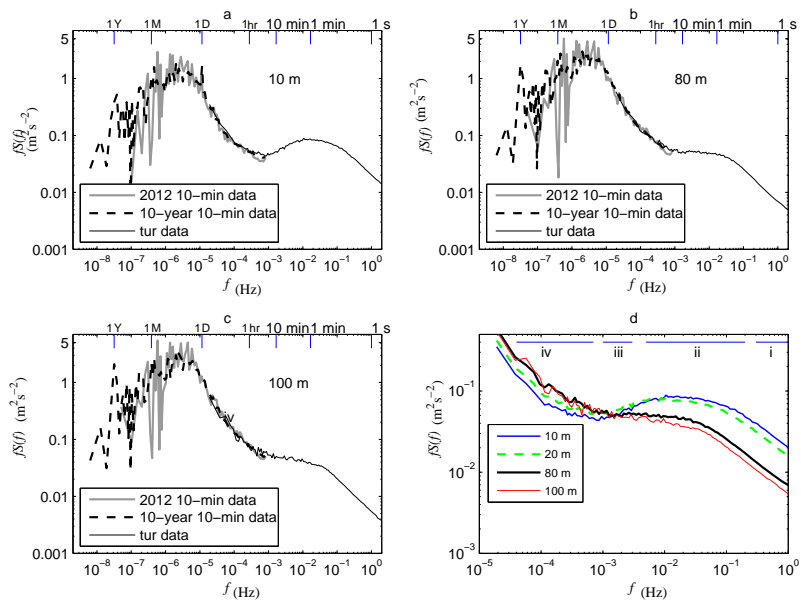


Fig. 3 The spectra of wind speed $fS(f)$ as a function of frequency f (a): at 10 m, (b) 80 m and (c) 100 m, (d) at four levels from 10 to 100 m where data are a 1-day long, 20-Hz time series. All data are from Høvsøre, 2012. (a) to (c) each consists of the spectrum calculated from the 10-min wind-speed time series from the entire year and the spectrum calculated from the sonic time series at the same height of 1-day length. The spectrum from 10 years of 10-min data (2005 - 2014) is also plotted. The scales corresponding to 1 year, 1 month, 1 day, 1 hr, 10 min, 1 min and 1 s are marked. The four ranges of Fig.4 in Höglström et al. (2002) are shown in (d).

295 increases with height after the minimum, but it is not well established where the
 296 increase fades out or even reverses. The reason for this observation is however
 297 unclear; Byzova (1967) found such an increase up to 300 m and Vinnichenko
 298 (1970) even found a diurnal cycle as high as 3 to 20 km. But for the planetary
 299 boundary layer it is observed that no diurnal cycle exists in the spectrum for
 300 the geostrophic wind calculated by means of surface pressure and temperature
 301 observations (Petersen et al. 1981; Larsén and Mann 2006). It can also be noted
 302 that a large yearly peak exists in the geostrophic wind spectrum and the surface
 303 wind spectra from all stations in Troen and Petersen (1989). Compared to the
 304 Høvsøre spectra, in the mesoscale the Horns Rev spectra show negligible height

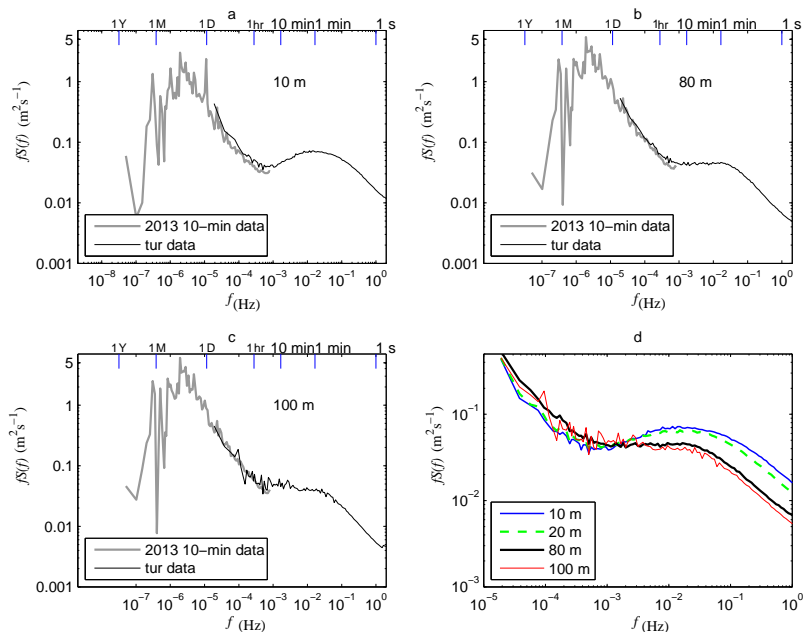


Fig. 4 The notations are the same as Fig. 3, except for 2013, excluding the 10-year spectrum in (a) to (c).

305 dependence between 15 and 62 m (Fig. 6b). This could be a result from the impact
 306 of surface fluxes becoming insignificant already at 15 m, much lower than over land
 307 because of the smooth surface and smaller stability variation. For mesoscale flow,
 308 the rougher land surface serves as a stronger sink of momentum, leading to weaker
 309 winds and accordingly smaller wind-speed variations. This is reflected as lower
 310 wind variations at levels closer to the ground.

311 We combine the full range spectra from the onshore and offshore sites together
 312 for the same height of 50 m; the spectra are shown both in the log-log and the
 313 log-linear scale in Fig. 7a and b, respectively. At Horns Rev, there are sonic data at
 314 50 m and there are 10-min data at 45 m and 62 m, but not at 50 m. As explained
 315 in producing Fig. 5, here 10-min data at 45 m were used which should give a very
 316 similar spectrum to the corresponding one at 50 m. At Høvsøre, there are 10-min
 317 data at 60 m and there are sonic data at 10 m, 20 m and 80 m, but not at 50
 318 m. We use the 10-min spectrum from 60 m, which, according to Fig. 6a, should

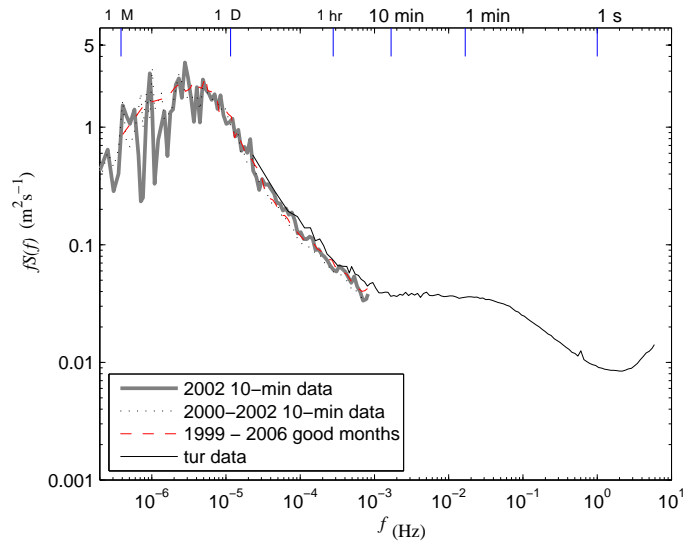


Fig. 5 The spectra of wind speed $fS(f)$ as a function of frequency f at Horns Rev, the thick gray curve is for 45 m from 10-min cup data, the thin black curve is for a height of 50 m from sonic data and the red, dashed curve is for a height of 45 m from 10-min cup data from months of with data coverage larger than 99%.

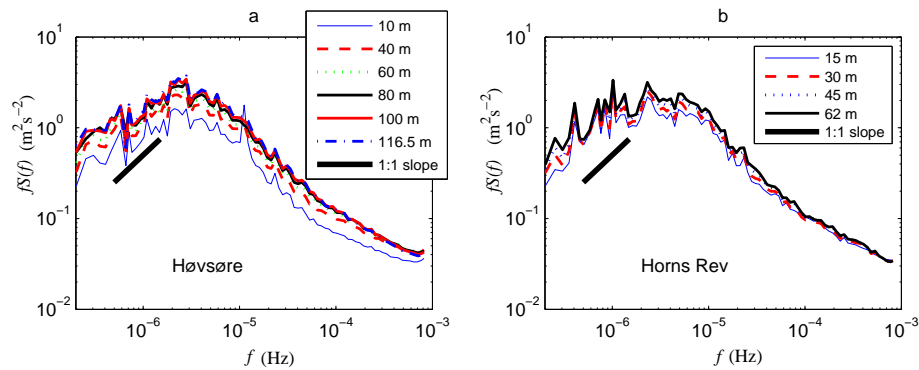


Fig. 6 The spectra of wind speed $fS(f)$ vs. frequency f from a 1-year long, 10-min time series at all measurement heights: (a) Høvsøre; (b) Horns Rev.

319 be very close to that from 50 m. In order to obtain the spectrum for 50 m from
 320 the sonic data at Høvsøre, we assumed a logarithmic vertical variation of $fS(f)$
 321 as shown in Figs. 3d and 4d. This assumption is encouraged by the test with
 322 the spectrum at 20 m. The estimated spectrum at 20 m under this assumption,

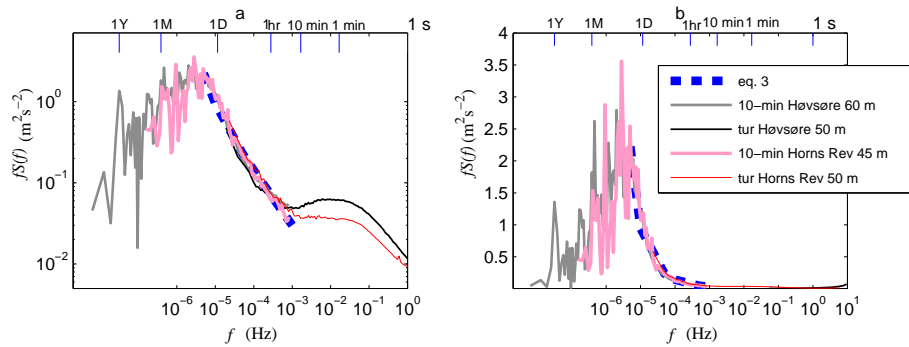


Fig. 7 The onshore (Høvsøre) and offshore (Horns Rev) spectra of wind speed plotted together: (a) in log-log scale (b) in log-linear scale.

323 using the measurements from 10 m and 80 m from both 2012 and 2013, is in
 324 good agreement with the directly measured spectrum at 20 m. Both the 10-min,
 325 mesoscale, spectra from Horns Rev and Høvsøre match satisfactorily with the
 326 spectral model Eq. 2, represented as

$$327 \quad fS(f) = a_1 f^{-2/3} + a_2 f^{-2} \quad (3)$$

328 with $a_1 = 3 \times 10^{-4} \text{ m}^2 \text{ s}^{-8/3}$ and $a_2 = 3 \times 10^{-11} \text{ m}^2 \text{ s}^{-4}$. Equation 3 is shown as the
 329 dashed (blue) curve in Fig. 7. Seemingly it is a good model for the frequency inter-
 330 val between the possible spectral gap and the macroscale spectral peak at around
 331 5 days. The turbulence spectrum merges smoothly with the corresponding low fre-
 332 quency spectrum. In comparison with the water site, at the same height over land,
 333 the microscale turbulence is significantly higher and contributes to the modifica-
 334 tion of the mesoscale $-2/3$ slope to lower frequencies. Over water, the mesoscale
 335 $-2/3$ slope extends to higher frequencies where it joins the microscale turbulence
 336 contribution. Considering the relative position of the estimated Høvsøre spectrum
 337 at 50 m to those at other heights (cf. Figs. 3d and 4d), even with uncertainties due
 338 to the assumption of vertical logarithmic variations, the above conclusion remains
 339 the same regarding the difference between land and water spectra.

340 4.2 The spectral gap

341 It is noticeable in Figures 3, 4, 5 and 7 that the turbulence spectrum undergoes
 342 transition smoothly to the 10-min data spectrum at a frequency of about 0.05
 343 min^{-1} . But how does the power spectrum from the mesoscale range merge with
 344 that from the microscale range?

345 There have been speculations that the sum of the microscale and the mesoscale
 346 spectra results in the total power in the gap range. This idea has often been shown
 347 qualitatively, e.g. in Kim and Adrian (1999) and Höögström et al. (2002), but has
 348 not been proven true due partly to the missing of quantitative description of the
 349 mesoscale spectrum. We here follow this hypothesis: as with wave patterns, the
 350 two parts linearly superimpose. Results from this study and others (e.g. Nastrom
 351 and Gage (1985); Gage and Nastrom (1986); Larsén et al. (2013)) suggest a rather
 352 simple spectral behaviour for the mesoscale as shown in Eq. 1 or Eq. 2. For the
 353 microscale, the Kaimal spectrum is often used. Here, in line with our hypothesis,
 354 we use the Kaimal spectrum for the microscale range for frequencies lower than
 355 the peak f_p . This idea basically borrows the f^{+1} shape of the Kaimal spectrum
 356 for $f < f_p$ but still uses the measured turbulence for $f > f_p$, as demonstrated
 357 in Fig. 8a and b for 10-m height. Equation 3 is used to represent the mesoscale
 358 spectrum; here for the 10-m height. To match the measurements at 10 m that are
 359 affected by the surface (Fig. 6a), the equation is scaled using a coefficient of 0.7,
 360 shown as the dashed (green) line in Fig. 8a and b. The solid (blue) curve, in Fig.
 361 8a and b, is the average of the 10-m spectra from Figs. 3d and 4d. The Kaimal
 362 spectrum for the component u under neutral conditions reads,

$$363 \quad \frac{f S_u(f)}{u_*^2} = \frac{A f z / U}{(1 + 33 f z / U)^{5/3}}. \quad (4)$$

364 According to this expression, with the use of the logarithmic wind law for neutral
 365 conditions, $U = (u_* / \kappa (\ln(z/z_0)))$, A can be derived as a function of the roughness
 366 length z_0 , height z , the peak frequency of the 3D turbulence f_p and the corre-
 367 sponding value of $f S_u(f)|_{f=f_p}$. Conventionally, $A = 102$. For the present annually
 368 averaged spectrum, based on a measured value of $f S_u(f)$ at $f_p = 0.014$ Hz, a value
 369 of 179 is obtained for A using $z_0 = 0.03$ m. Accordingly, $f S_u(f)$ can be described

370 in terms of f , f_p and $fS_u(f)|_{f=f_p}$. For $f < f_p$, this is shown as the dotted (black)
371 line in Fig. 8a.

372 If our hypothesis, that the observed spectrum is a linear superimposition of
373 the mesoscale (dashed line) and the microscale (black dotted line) spectra, is true,
374 then their sum should match the measured total spectrum (the blue solid curve).
375 This is confirmed by the dashed-dotted (red) curve, which is the sum of the dashed
376 and dotted lines as in Fig. 8b.

377 The same is true for heights of 80 m and 100 m, as shown in Fig. 8c and d,
378 respectively. Note, for Høvsøre at 80 m and 100 m, Eq. 3 was used without being
379 scaled. When the spectral energy from the mesoscale eddies (2D turbulence) is
380 large, the microscale spectral peak can become flat or disappear, as happens at 80
381 m and 100 m. This makes it difficult to determine f_p and hence where to apply
382 the Kaimal spectral shape for the f^{+1} dependence. Here, by eye fitting, $f_p = 0.01$
383 Hz and 0.009 Hz were used for 80 m and 100 m, respectively.

384 With this theory we can explain why the most dominant gap is observed at
385 lower levels and sometimes becomes insignificant at higher levels, and why the
386 gap is clearer over land than over water at the same height. It depends on the
387 relative contribution of the mesoscale and microscale power energy. For instance,
388 at Høvsøre, compared to 80 m and 100 m, at 10 m, the 3D turbulence is stronger
389 but the 2D mesoscale variation is smaller, and this leads to a more visible gap
390 at 10 m. At the same height, e.g. 50 m, the mesoscale spectra are comparable for
391 Høvsøre and Horns Rev and they are substantial compared to the 3D turbulence in
392 the gap range, resulting in almost invisible gaps at both sites. At this height, since
393 the 3D turbulence at Horns Rev is considerably weaker than that at Høvsøre, the
394 “gap” at Høvsøre is very shallow but distinguishable (Fig. 7a), but it is invisible
395 at Horns Rev.

396 4.3 The three wind components

397 From the same datasets from Høvsøre as used in Figs. 3d and 4d, the spectra of
398 the vertical wind component, w , were calculated from 1-day long time series for
399 four heights, 10 m, 20 m, 80 m and 100 m and the results are shown in Fig. 9a
400 and b for the years 2012 and 2013, respectively. The results from the two years are

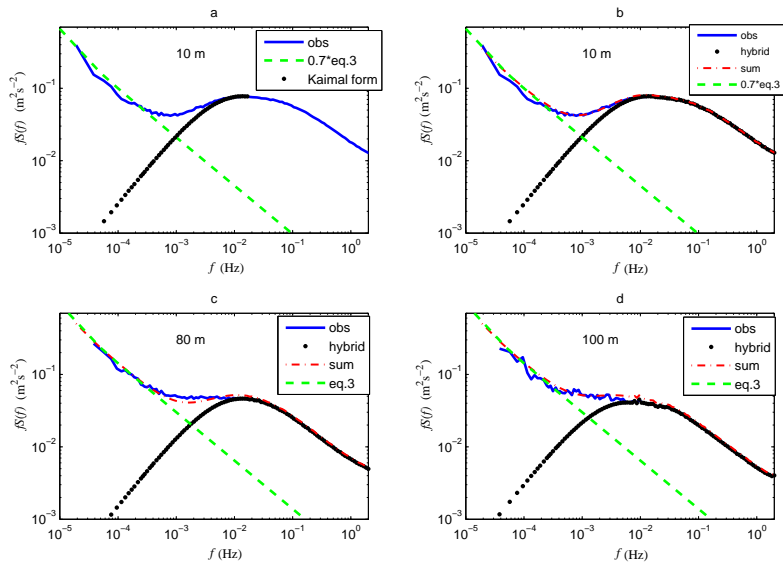


Fig. 8 Superimpose of the spectral components. (a) at 10 m, measured spectrum together with the spectral model Eq. 3 multiplied by 0.7, extending to high frequencies, and a Kaimal spectrum for frequencies lower than the peak frequency. (b) at 10 m, the sum of the mesoscale spectrum with a slope of $-5/3$ and the microscale spectrum with the shape of a Kaimal model (the red curve) gives the observed spectrum. (c) The same as (b) but for 80 m. (d) The same as (b) but for 100 m.

401 again almost the same. With increasing height, the spectral peak moves to lower
 402 frequencies, and the energy level decreases slightly with height. From Fig. 9 one
 403 could notice that towards the lowest frequencies, $fS(f)$ for w at several heights
 404 seems to increase with decreasing frequency. This could be an artifact caused by
 405 the sensitivity of the sonic measurements of w : when using such a long time series
 406 of w for Fourier transform, the slightest misalignment of the sonic could cause the
 407 contamination of w from the component u at all frequencies, but only noticeable
 408 at the low frequencies. This effect is expected to be more pronounced when the
 409 wind field is more unsteady. The longer the time series, the larger is the chance for
 410 unsteady flow. This speculation is supported by Fig. 10, for which we only selected
 411 days from Table 1 where the wind direction changes less than 50 degrees during a
 412 day. The increase of the power spectrum of w with decreasing frequency as shown
 413 in Fig. 9 is significantly reduced in Fig. 10.

414 Choosing days with relatively small direction variation ($< 50^\circ$) is for the pur-
415 pose of obtaining the longitudinal and lateral wind components, u and v , from the
416 one-day long time series of the wind speed and the daily mean direction. There
417 are 100 days from 2012 and 2013 (Table 1) satisfying this criterion for 10 m, 20 m
418 and 80 m but only 34 days at 100 m. The spectra for the three wind components
419 u , v and w are shown in four subplots in Fig. 10 for four heights, 10 m, 20 m, 80 m
420 and 100 m, respectively. In the 3D inertial subrange, we observe that v and w have
421 comparable level of energy, both greater than that of the u component, a classical
422 behaviour. Consistent with the classical 3D turbulence theory, the peak frequency
423 for w is seen to be the highest, and for u it extends to the lowest frequencies.

424 With the contribution from the mesoscale spectrum, the power spectra for both
425 u and v increase with decreasing frequency in the mesoscale range and the energy
426 level for u is comparable to that for v , consistent with Larsén et al. (2013) based on
427 1-day long 10-min mean data from several years of data at Nysted and Horns Rev
428 sites. They found that the ratio of the spectra for u and v is on average < 1 when
429 $f < 10^{-4}$ Hz but is greater than 1, reaching 1.2 when f increases and approaches
430 0.05 min^{-1} (their Fig. 7). Further, the observed zero correlation between u and
431 v supports the 2D isotropy hypothesis in this mesoscale range, and so does the
432 behaviour of u and v for $f < 10^{-4}$ Hz. However, the relatively larger u spectrum
433 than v spectrum in the range $2 \times 10^{-4} < f < 0.8 \times 10^{-3}$ Hz contradicts the isotropy
434 assumption (Frehlich and Chelton 1986). Figure 10 provides us with a wider picture
435 of the spectral behaviour in this complicated range. For $f < 2 \times 10^{-4}$ Hz, the
436 u spectrum is smaller than the v spectrum, with a ratio on average about 0.8,
437 consistent with Larsén et al. (2013). The consistency is also true for the relatively
438 larger values of the u spectrum in $2 \times 10^{-4} < f < 0.8 \times 10^{-3}$ Hz, which the
439 current study suggests is caused by the impact from 3D turbulence. As shown in
440 both Figs. 9 and 10, the w component energy approaches zero as f approaches the
441 mesoscale, suggesting the 2D nature of the relatively large-scale flow.

442 4.4 The cospectrum

443 From the same datasets from Høvsøre as used in Figs. 3d, 4d and 9, the cospectra
444 of wind speed U and the vertical wind w were calculated for the four heights from

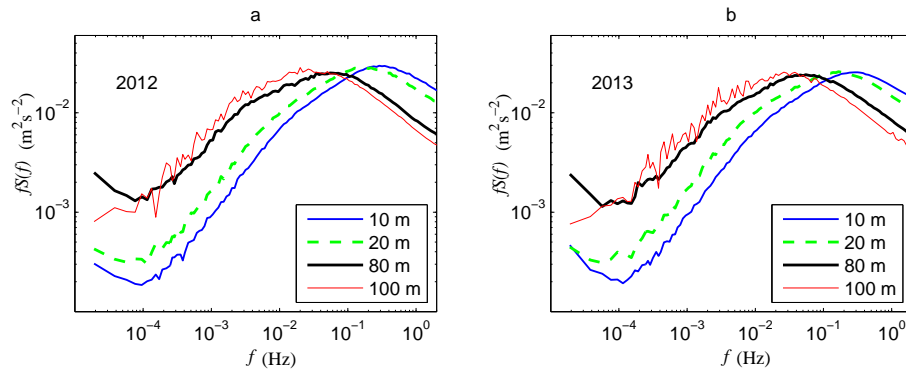


Fig. 9 The spectra of the vertical wind component (w) $fS(f)$ vs. frequency f from 1-day long, 20-Hz time series at four heights at Høvsøre: (a) from year 2012; (b) from year 2013.

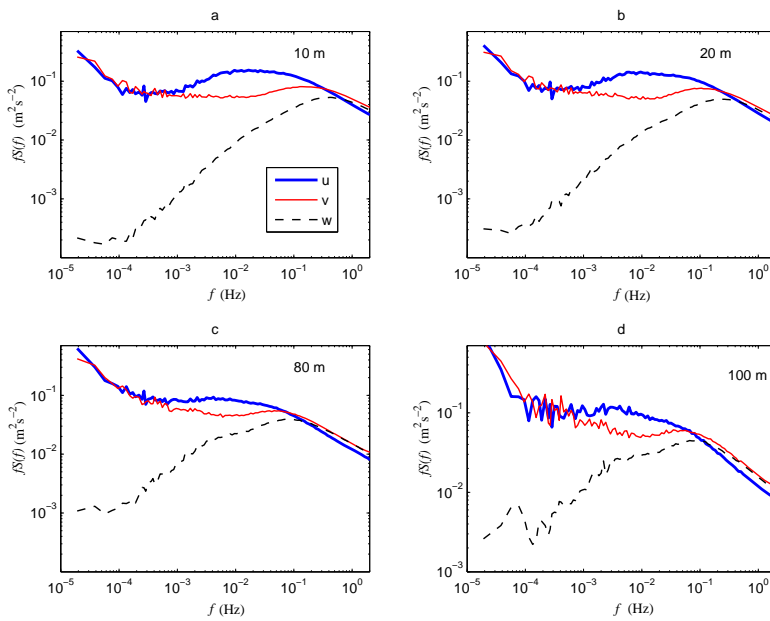


Fig. 10 The three components u , v and w from days where the daily directional change is less than 50 degrees. (a) 10 m; (b) 20 m; (c) 80 m; (d) 100 m.

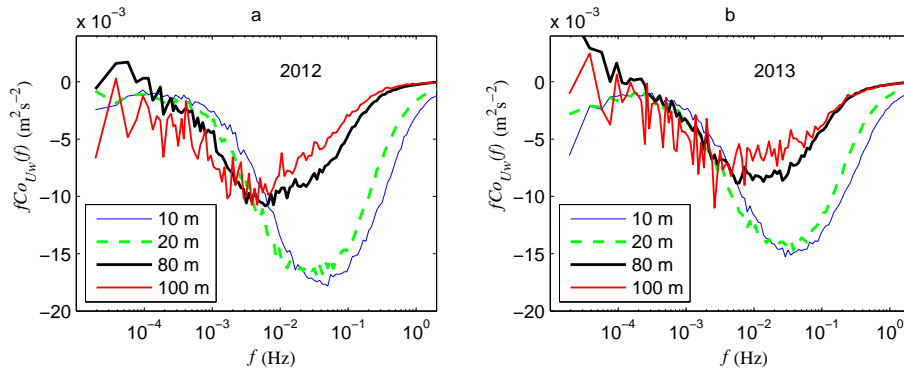


Fig. 11 The cospectra of wind speed and the vertical wind component $fCo_{Uw}(f)$ vs. f from 1-day long 20 Hz data at four heights at Høvsøre. (a) from year 2012; (b) from year 2013.

445 10 m to 100 m. The results are shown in Fig. 11a and b for the year 2012 and 2013,
 446 respectively. The wind speed U is used instead of the u component of the wind
 447 in order to make best use of the large dataset and at the same time to avoid the
 448 issue with non-stationary time series when dividing U into u and v components
 449 for a long time series.¹

450 Our day-long time series show the classic spectral behaviour of the energy-
 451 containing range where most covariance is contained. This range shifts to lower
 452 frequencies as the height increases, and so does the peak frequency. Our data show
 453 further that, at heights of 10 m and 20 m, Co_{Uw} becomes small at a frequency of
 454 10^{-3} Hz, but at 80 m and 100 m it still has a considerable magnitude. Note that
 455 the fluctuations at the lowest frequencies have the same uncertainty as the power
 456 spectrum for w (Fig. 9).

457 The cospectrum of u and w , Co_{uw} , has also been calculated using the data
 458 as in Fig. 10 (not shown). However, since the data sample size is only 1/4 of
 459 those in Fig. 11, we expect the result to be less climatologically representative and
 460 more case-sensitive. However, the spectral behaviour of Co_{uw} from 100 days of
 461 stationary wind is similar to Co_{Uw} .

¹ $U = \sqrt{(\bar{u} + u')^2 + (\bar{v} + v')^2} \approx (\bar{u} + u')(1 + \frac{v'^2}{2(\bar{u} + u')^2}) = \bar{u} + u' + \frac{v'^2}{2\bar{u}} - \frac{v'^2 u'}{2\bar{u}^2}$. Multiplying w' both sides and averaging them gives $\overline{U'w'} = \overline{u'w'} + \frac{\overline{v'^2 w'}}{2\bar{U}} - \frac{\overline{v'^2 u'w'}}{2\bar{U}^2}$. Considering the overall small values of the second and third terms on the right-hand side of the above equation, the cospectrum of U and w is considered very similar to that of u and w .

462 5 Discussion

463 Power spectra and cospectra for horizontal and the vertical wind components were
 464 computed for the frequency interval between about 1 yr^{-1} to several Hz. We have
 465 focused on two sites in the same region, one offshore site in the North Sea about
 466 20 km from the west coast of Denmark and one coastal site less than 2 km from
 467 the shore line. Data used for the calculation of the spectra include 10-min mean
 468 wind speed from cup anemometers and direction for a period of 10 years and sonic
 469 anemometer measurements at 20 Hz or 12 Hz from periods within these years. The
 470 continuous long-term 10-min wind data with good coverage are used to obtain the
 471 spectra from about 1 yr^{-1} to 0.05 min^{-1} . The sonic data are used separately
 472 for 1 day^{-1} to 6 Hz or 10 Hz (Nyquist frequency of the 12 Hz and 20 Hz data,
 473 respectively). The two datasets provide an overlapping spectral interval from 1
 474 day^{-1} to 0.05 min^{-1} .

475 There are only about half of the days during a year for which the coverage of
 476 the sonic data is more than 99.95%, which is used to qualify the days to be used for
 477 the calculation of the spectra (see Sect. 3). It is noted that the incomplete coverage
 478 as well as uneven distribution of turbulence data throughout the year make it less
 479 conclusive about the absolute level of energy regarding the power spectrum and
 480 cospectrum in a climatological sense. However, the parallel calculations, with one
 481 using data in Table 1 (results in Sects. 4.1 and 4.4) and one using stationary time
 482 series (results in Sect. 4.3), have all shown consistent spectral behaviour.

483 The new findings give rise to the following discussion topics:

484 5.1 On the power spectrum

485 In this study we defined four ranges to cover the full-range spectrum: macroscale
 486 ($f \lesssim 1 \text{ day}^{-1}$), mesoscale ($1 \text{ day}^{-1} \lesssim f \lesssim 2 \times 10^{-4} \text{ Hz}$), gap range ($2 \times 10^{-4} \lesssim$
 487 $f \lesssim 2 \times 10^{-3} \text{ Hz}$) and microscale ($f \gtrsim 10^{-3} \text{ Hz}$). For each range, the current
 488 study provides new insights into the spectral behaviour. It is noted that the ob-
 489 served, climatological spectrum of boundary-layer winds from the two Danish sites
 490 represent a mid-latitude strong wind regime, and this might limit its universality.

491 We observe a peak at $f = 1 \text{ yr}^{-1}$, which has also been shown with decades-
492 long time series from mid-latitudes, e.g., in Troen and Petersen (1989) and Larsén
493 and Mann (2006). All spectra calculated from the 10-min anemometer data show
494 another property that shapes the macroscale velocity spectrum: the broad peak
495 at a frequency of about 0.25 to 0.2 day^{-1} . This peak is also present in the Van
496 der Hoven spectrum, in Troen and Petersen (1989) and in Baker (2010), from
497 numerous measurements over Europe. Analysis of the 10-min averaged wind time
498 series from Høvsøre shows that the autocorrelation coefficient of the wind speed,
499 ρ , decreases with the time lag τ , and becomes almost zero at $\tau \approx 4$ to 5 days,
500 corresponding to the frequency where $fS(f)$ is a maximum. This suggests that
501 the peak is related to synoptic weather processes.

502 Concerning the processes behind the spectra reported herein, we consider the
503 3D boundary-layer turbulence to be rather well understood, but for lower frequen-
504 cies there is not a consistent theory. However, in Larsén et al. (2013), the power
505 law that describes the velocity spectrum in the frequency range 10^{-5} Hz to 10^{-3}
506 Hz , reproduced here as Eqs. 2 and 3, was successfully derived from both the tro-
507 pospheric wavenumber spectrum of Lilly and Petersen (1983), Nastrom and Gage
508 (1985), Gage and Nastrom (1985), and Lindborg (1999), as well as from a univer-
509 sal saturation spectrum for internal tropospheric gravity waves by Fritts and van
510 Zandt (1987). The height variation of the low frequency spectrum in Fig. 6 indi-
511 cates that most of the height variation over land takes place below 50 m, implying
512 the coefficients a_1 and a_2 in Eqs. 2 and 3 are height-dependent below this height.
513 There is almost no variation with height over sea (here at Horns Rev) above the
514 first measuring height 15 m.

515 In the transition range between mesoscale and microscale, the nature of the
516 interaction of the 2D and 3D turbulence is not fully understood, and it is in this
517 range where the debate on the existence of gap is ongoing. Figure 8 shows the
518 existence of the gap. The gap is most clear at low levels where the surface impact
519 is most significant and it seems to disappear at higher levels. But the gap being
520 visible or not, the gap region can be modelled, if we assume that the 3D turbulence
521 and the 2D mesoscale variations are uncorrelated. Then the total spectrum can be
522 recovered by adding the spectra related to the two phenomena, presented in Fig. 8,
523 assuming for simplicity that the horizontal 3D turbulence follows a neutral Kaimal

524 spectrum shape for $f < f_p$ and that the mesoscale spectra can be extrapolated to
 525 higher frequencies. In a similar approach, Larsen et al. (1990) found the spectrum
 526 to be dominated by the mesoscale form for f up to 10^{-2} Hz for cases with weak
 527 3D turbulence.

528 The establishment of the mesoscale spectrum has made it possible for us to
 529 demonstrate the gap quantitatively. It has also been useful for validating mesoscale
 530 modelling (Skamarock 2004). It has been further applied for extreme wind esti-
 531 mation by introducing expected wind variability to the modelled time series that
 532 suffers from numerical smoothing effects (Larsén et al. 2012; Larsén and Kruger
 533 2014). This application so far is limited to the mesoscale. The description of the
 534 spectrum covering the mesoscale, the gap region and the microscale, as shown
 535 herein, provides a possibility for extending such an application for extreme winds
 536 to higher frequencies. Wyngaard (2004) argues that neither the ensemble mean
 537 models nor the large eddy simulation is appropriate in reproducing the significant
 538 fluxes and energy transfer in the so-called Terra Incognita between the inertial
 539 subrange and the mesoscale range. The results from our study illustrate spectral
 540 aspects of Wyngaard’s Terra Incognita.

541 5.2 On the stationary conditions for the time series

542 In the full-scale spectrum, two frequency ranges are discussed where $fS(f)$ varies
 543 with f^{+1} . The first range is at the lowest frequencies (Fig. 6) and the second range
 544 is at the high frequency end of the gap approaching the 3D turbulence (Fig. 8).

545 The first range is seen as an indication that the annual wind time series can be
 546 considered to satisfy the stationarity condition as expected for random processes
 547 to be ergodic according to the Wiener-Khintchine theorem. This then ensures the
 548 credibility of using a Fourier transformation to the year-long time series.

549 In boundary-layer studies, the often-used data length for calculating the turbu-
 550 lence variance and fluxes is 10 to 30 min. In addition, to counteracting the leakage
 551 of low frequency energy into the spectrum, a detrending process and a Hanning
 552 window are normally applied routinely to the time series. Looking at the power
 553 spectra of the wind speed and w and the cospectrum at 10-m height, Figs. 3a, 4a,
 554 9, 10a and 11, a time series length of 10 min is reasonable for including variations

555 and co-variations for this height. However, as height increases and the spectral
 556 power shifts systematically to lower frequencies, we would expect a longer time
 557 series needed to include the relevant information, as pointed out by e.g. Lenschow
 558 et al. (1994) and Vickers and Mahrt (2002). In order to understand the spectral
 559 behaviour around the “gap” and at larger scales, the time series should be taken
 560 longer than the conventionally used 10 - 30 min. In the current study, 1-day time
 561 series are used for the Fourier transformation with detrending; the reasons for not
 562 applying a Hanning window to such a long time series are given in the Appendix.
 563 Accordingly, at the low frequency end of the 1-day time series, $fS(f)$ follows the
 564 mesoscale spectrum with a $-2/3$ slope, while not as described in the standard
 565 theory (e.g. the Kaimal model) with $fS(f)$ varying with f^{+1} because implicitly
 566 a gap is assumed and with no consideration of larger scale variations. The or-
 567 ganized spectral energy adding in from the mesoscale distinguishes it from the
 568 stochastically stationary process as expected from the Wiener-Khintchine theo-
 569 rem. The good matching of the average one day-long sonic data with the 10-min
 570 mean data in the overlapping range suggests the appropriateness and robustness
 571 of our approach.

572 5.3 On the coherence

573 We have compared the coherence functions for horizontal displacement for the 3D
 574 boundary-layer wind field and the mesoscale wind field, based on 10-min averages:

$$575 \quad \text{Coh}(f, \Delta) = \exp(-a_{\Delta} \frac{f\Delta}{U}) \quad (5)$$

576 where Δ is the separation and a_{Δ} is a numerical coefficient; $\Delta = \Delta y$ corresponds to
 577 a lateral separation relative to the wind direction, while $\Delta = \Delta x$ corresponds to a
 578 longitudinal separation. For 3D boundary-layer turbulence, $a_{\Delta y} \approx 60$ and $a_{\Delta x} \approx 7$
 579 (Panofsky and Dutton 1984). The much smaller value for the longitudinal separa-
 580 tion reflects the partial validity of Taylor’s hypothesis for longitudinal coherence
 581 ($a_{\Delta x} = 0$ corresponds to Taylor’s hypothesis being ideally true). For the mesoscale
 582 fluctuations, Larsén et al. (2013) and Vincent et al. (2013) find $a_{\Delta y} \approx 7.7$ and
 583 $a_{\Delta x} \approx 5$, meaning that the mesoscale coherence functions are much less sensitive

584 to the separation direction than 3D boundary-layer turbulence and as coherent as
585 3D boundary-layer turbulence along the longitudinal direction.

586 5.4 On the two-dimensional isotropy

587 The current study (Sect. 4.3) complements the study of Larsén et al. (2013) on
588 the 2D isotropic characteristics of the mesoscale spectra. In the range from about
589 10^{-5} Hz to about 2×10^{-4} Hz, the magnitude of the spectrum of u to that of v is on
590 average 0.8 and the coherence of u and v is zero, consistent with the 2D isotropic
591 assumption. The current study suggests that the value of the ratio increases with
592 frequency in the gap range caused by a relatively larger contribution from u than
593 v from the 3D boundary-layer turbulence.

594 Larsén et al. (2013) show that the 2D isotropy characteristics disappear in the
595 presence of organized structures such as convective open cells (see their Figs. 11
596 and 13). Their Fig. 11 shows that open cells contribute significant energy in the
597 high frequency part of the mesoscale range, where the spectrum is in transition to
598 microscale turbulence. The mean spectrum corresponding to the open cells from
599 Fig. 11 in Larsén et al. (2013) is reproduced here and plotted together with the
600 full-scale spectrum from the Horns Rev site (see the thick black curve in Fig. 12).
601 Among the 18 days with open cells used for producing their Fig. 11, there are two
602 days (day 55 and 56 in 2004) where the sonic data at Horns Rev are available.
603 The two spectra, from each of the two days, are also plotted in Fig. 12. The two
604 individual cases do contribute significant wind variation in the range from about
605 10^{-4} Hz to about 3×10^{-3} Hz, especially cell case 2. Here due to the special
606 wind and stability conditions of the individual cases, the microscale spectrum is
607 enhanced relative to the climatological mean.

608 6 Conclusions

609 The analysis of long-term mean wind and turbulence data from two sites in Den-
610 mark, one onshore and one offshore, has improved our understanding of the full-
611 scale boundary-layer wind spectrum in mid-latitudes. The findings provide guide-

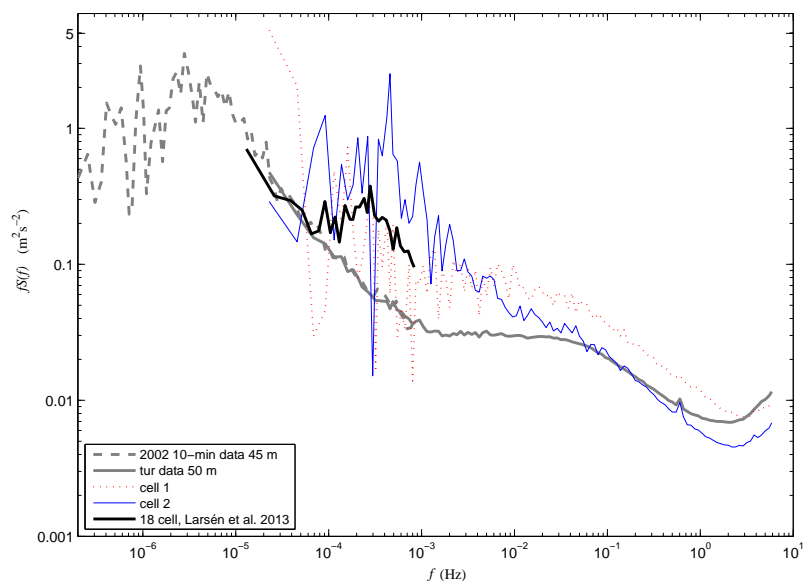


Fig. 12 The spectra of wind speed at the Horns Rev site from two cell case days, together with the mean spectrum of 18 cell case days from Larsén et al. (2013) and the climatological spectrum.

612 lines for numerical modelling, turbulence analysis and wind engineering applica-
 613 tions. This can be summarized as follows,

- 614 – The spectral gap in the horizontal wind component power spectrum exists
 615 and can be modelled. The linear composite of the wind variations from the
 616 mesoscale and microscale gives the observed power spectrum in the gap range.
 617 This suggests that the turbulence from the two frequency regions are weakly
 618 correlated.
- 619 – Depending on the relative contribution to the variation from the microscale and
 620 mesoscale, the gap may be visible or invisible. The depth of the gap decreases
 621 with height, in general. The disappearance of the gap could also be caused by
 622 structured features such as open cells, which can contribute significant fluctu-
 623 ations in this frequency range. The spectral structure around the gap could be
 624 used for defining “natural” time windows for turbulence characteristics.

625 – For spatial scales larger than the gap, in the range from about 10^{-5} Hz to
 626 about 10^{-3} Hz, the turbulence is two-dimensional. The power spectra $S(f)$
 627 of the wind speed and its two components u and v increase with decreasing
 628 frequency, following a $-5/3$ dependence on frequency on a log-log scale. In
 629 this scale range, $S(f)$ increases from the ground and levels off at a height
 630 ≈ 50 m at Høvsøre, but < 15 m at Horns Rev. Our study indicates that,
 631 on average, it is possible to describe the boundary-layer turbulence (spectral
 632 range *i – iii* as in Höglström et al. (2002), Fig. 3d) as being limited by a
 633 f^{+1} behaviour at low frequency, and being statistically stationary and ergodic,
 634 at least within the surface layer. Above this layer the assumption is more
 635 uncertain, and depends on how one understands and models the “invisible
 636 gap”. However, also in the surface-layer situations, stationarity cannot always
 637 be assumed, as is well-known by meteorologists and illustrated here in Fig. 12.
 638 – Winds in the mesoscale frequency range seem more spatially coherent than
 639 winds in the 3D turbulence range, as measured by Eq. 5.

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645 **Appendix 1 The impact of using one-day sonic data to calculate the** 646 **spectrum**

647 The turbulence spectra have been calculated with 1-day long sonic data from
 648 days as listed in Table 1 and 2. This procedure ensures a detailed description of
 649 the spectral regions in and around the gap. However, it raises some issues about
 650 the statistics of the established spectra. The daily spectrum can obviously not
 651 be considered an analysis of a stationary series, because (a) the wind during the
 652 day typically undergoes a systematic diurnal variation, and (b) the low frequency
 653 region of the spectrum shows a $f^{-2/3}$ power law, which is far from the f^{+1} power
 654 law required by the Wiener-Khintchine theorem for possible stationarity. We refer
 655 to the spectrum as $fS(f)$ vs. f on a log-log scale.

656 In spite of this, we may claim that the f^{+1} spectral region of the annual
657 spectrum ($f \lesssim 2 \times 10^{-6}$ Hz, see Fig. 6) provides good reasons for expecting a
658 yearly-averaged diurnal spectrum to be determined for frequencies $> 1 \text{ day}^{-1}$, if a
659 large enough ensemble of diurnal data series are analyzed in the spectral domain
660 for the year considered.

661 However, given the spectral slope of $f^{-2/3}$ at frequencies around 1 day^{-1} , the
662 low frequency region of the spectra, from averaging spectra for the day-long time
663 series, is enhanced by leakage of energy from lower frequencies. This enhancement
664 will not disappear by ensemble averaging; hence, it has to be counteracted. This
665 is usually performed by applying windows, such as Hanning and Hamming win-
666 dows, imposing a sinusoidal window onto the time series (Kristensen et al. 1992;
667 Kristensen 1998). Unfortunately, for the present time series typically associated
668 with diurnal stability variations, the application of a window would modify the
669 relative weight of the different stability classes. Therefore the window method was
670 not used.

671 Instead, we have tried to control the leakage by excluding cases where the
672 spectral amplitude is beyond the two standard deviations of all spectra. These
673 cases have shown excessive characteristics, mostly associated with large and narrow
674 gust or strongly non-stationary conditions. All together, there are 20 days of such
675 conditions.

676 For comparison, a spectrum from a 5-day long time series is calculated and
677 shown as the blue circles in Fig. 13. At the same time, a spectrum from each of
678 the five days was also calculated and the five spectra were averaged afterwards
679 and are shown as dots in the same plot. The good agreement between the circles
680 and the dots suggest that there is no principle problem in using 1-day time series
681 for calculating the spectrum. The inertial subrange spectral values for the 5-day
682 spectrum is slightly, but systematically, larger than the similar one for the 1-
683 day spectrum. To explain this, we assume that the frequency spectrum is mainly
684 a wavenumber spectrum being advected past the sensor by a “fluctuating mean
685 wind”. The variance of this fluctuating advection flow will be larger for the 5-day
686 series than for any of the 1-day series. This would enhance the measured 5-day
687 spectrum slightly more than the 1-day spectrum, following the model of Wyngaard
688 and Clifford (1977) for Taylor’s hypothesis with a fluctuating advection wind.

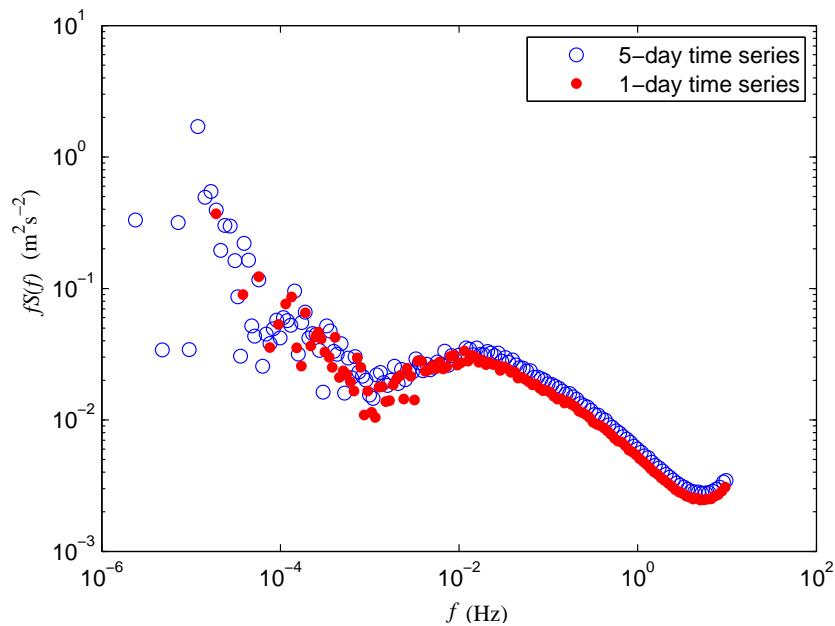


Fig. 13 The power spectrum $fS(f)$ from a five-day long time series together with the mean of five spectra calculated from each of the five days.

689 Finally in Fig. 14 we show the spectra with all sonic data in Table 1 (the dashed
 690 curves) together with those with outliers removed (solid curves, same cases as from
 691 Figs. 3d and 4d). The higher values of the dashed curved are seen as the leakage
 692 caused by days corresponding to highly non-stationary conditions. Apart from the
 693 magnitude, the distribution of the spectra with height remains the same.

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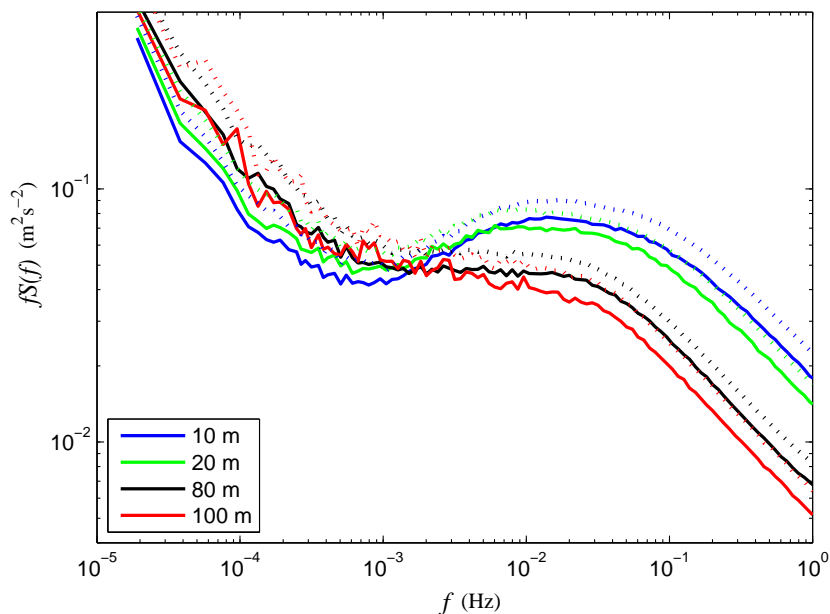


Fig. 14 Power spectra from both 2012 and 2013 (Figs. 3d and 4d together), with outliers excluded (solid curves) and included (i.e. all data from Table 1, dashed curves).

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