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Abstract: The focus in this paper is on active fault diagnosis (AFD) in closed-loop sampled-data systems. Applying the same AFD architecture as for continuous-time systems does not directly result in the same set of closed-loop matrix transfer functions. For continuous-time systems, the LFT (linear fractional transformation) structure in the connection between the parametric faults and the matrix transfer function (also known as the fault signature matrix) applied for AFD is not directly preserved for sampled-data system. As a consequence of this, the AFD methods cannot directly be applied for sampled-data systems. Two methods are considered in this paper to handle the fault signature matrix for sampled-data systems such that standard AFD methods can be applied. The first method is based on a discretization of the system such that the LFT structure is preserved resulting in the same LFT structure in the fault signature matrix as obtained for continuous-time systems. The other method is an approximation method, where the same structure is obtained for small parametric faults.

Keywords: Active fault diagnosis, sampled-data systems, parametric faults, controller parameterization.

1. INTRODUCTION

There exist two groups of fault diagnosis methods, passive based methods and active based methods. In the first group, the fault diagnosis is based on passive observations of the systems. There exist various passive based methods for both deterministic or stochastic based diagnosis, see e.g. Basseville and Nikiforov [1993], Blanke et al. [2006], Campbell and Nikoukhah [2004], Chen and Patton [1998], Gertler [1998], Gustafsson [2000] for mention some of the books in this area.

For active based methods, the diagnosis is also based on observations of the system, but auxiliary inputs are injected to get a faster diagnosis of faults in the system or get a diagnosis at a specified time, i.e. when an auxiliary input is injected at the system at a given time. The area of active fault diagnosis (AFD) has not been investigated so much as passive based methods. Some relevant references in the area of active fault diagnosis are e.g. Ashari et al. [2011, 2012], Campbell and Nikoukhah [2004], Kerestecioglu [1993], Niemann [2006], Simandl and Puncochar [2009], Zhang [1989].

The focus in this paper is on AFD in sampled-data systems. The papers mentioned above deals only with continuous-time or descrete-time system. Fault diagnosis for sampled-data systems has only been touched briefly and no detailed analysis had been given. The AFD approach for closed-loop systems described in Niemann [2006] and later used in Niemann [2012], Poulsen and

Niemann [2008] depends on the structure in the system. It is assumed that an LFT representation of the system with parametric faults is valid. This gives a certain structure in the closed-loop matrix transfer function that is used directly in AFD. This matrix transfer function is also called the fault signature matrix. Based on the structure in the fault signature matrix, conditions for both fault detection and fault isolation are given, Niemann and Poulsen [2014a,b].

It is possible to apply the same setup for AFD in sampled-data system as described in Niemann [2006, 2012], Poulsen and Niemann [2008]. However, an LFT structure in the continuous-time system will not in general be preserved when the system is discretized. The result is that the fault signature matrix for sampled-data systems does not have the same structure as for the continuous-time systems. As a result of this missing structure in the fault signature matrix for sampled-data system, the fault detection and fault isolation results known from the continuous-time case cannot directly be applied in the sampled-data case.

The main contribution in this paper is an analysis of the fault signature matrix for sampled-data systems. Two methods are described to transform the fault signature matrix for sampled-data system into a form that has the same structure as for the continuous-time systems. This will allows us to use the AFD results from continuous-time directly on sampled-data systems. The first method is based on a dedicated discretization of the continuous-time system that will preserve the LFT structure. An

overview of methods that preserve an LFT structure in the discretization has been described in Toth et al. [2012]. The other method is an approximation method. Here, an approximation of the continuous-time system for small parametric faults is derived followed by a discretization. This follow the lines in the approach described in Niemann and Poulsen [2014a,b].

Another result of the analysis of AFD for sampled-data systems, is that it is not a necessary condition, that the system has an LFT structure. It turns out, that it is possible to apply the AFD approach on systems that do not have an LFT structure.

The rest of this paper is organized as follows. In Section 2, the system set-up is given. The YJBK parameterization is introduced in Section 3 followed by a description of the fault signature matrix in Section 4. Based on the fault signature matrix for sampled-data system, an analysis of the matrix is given in Section 5. In Section 6 include a discussion of the results including a discussion of relaxing the condition of an LFT structure in the system. The paper is closed with a conclusion in Section 7.

2. SYSTEM SETUP

Consider the following generalized 2×2 system,

$$\Sigma_\theta : \begin{cases} e(t) = G_{ed}(\theta)d(t) + G_{eu}(\theta)u(t) \\ y(t) = G_{yd}(\theta)d(t) + G_{yu}(\theta)u(t) \end{cases} \quad (1)$$

where $t \in \mathcal{R}$, $d \in \mathcal{R}^r$ is a disturbance input vector, $u \in \mathcal{R}^m$ the control input signal vector, $e \in \mathcal{R}^q$ is the external output signal vector to be controlled, and $y \in \mathcal{R}^p$ is the measurement vector. Further, the vector θ, $\theta^T = [\theta_1, \cdots, \theta_l]$ describes the parametric/multiplicative faults in the system. The nominal system is given by $\theta = 0$.

Further, let the dynamical system in (1) be controlled by the following stabilizing sampled-data feedback controller

$$u_k = K(z)y_k \qquad k \in \mathcal{Z} \qquad (2)$$

where the connections between the continuous-time signals $u(t), y(t)$ and the discrete-time signals u_k, y_k are given by

$$y_k = \mathcal{S}_\tau y(t)$$
$$u(t) = \mathcal{H}_\tau u_k$$

where $\mathcal{S}_\tau$ is a sampler and $\mathcal{H}_\tau$ is a zero order hold with τ as the sampling period.

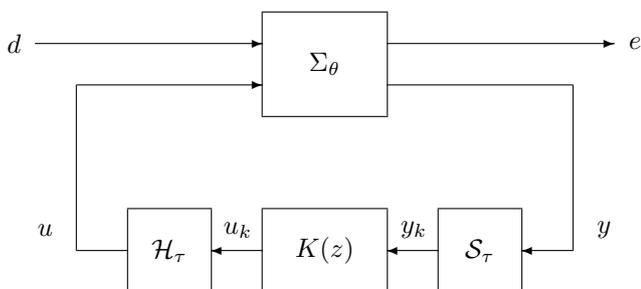A block diagram of the system is shown in Figure 1.

Fig. 1. Standard sampled-data system setup.

A more explicit description of the system setup for systems with parameter faults can be given by including an extra input and output vector in the system. The above system is then given by

$$\Sigma_\theta : \begin{cases} z = G_{zw}w + G_{zd}d + G_{zu}u \\ e = G_{ew}w + G_{ed}d + G_{eu}u \\ y = G_{yw}w + G_{yd}d + G_{yu}u \end{cases} \quad (3)$$

where the connection between the two external vectors w and z is given by

$$w = \theta z \quad (4)$$

This description is equivalent with the general description of system with model uncertainties, see e.g. Zhou et al. [1995].

3. THE YJBK PARAMETERIZATION

The YJBK and the dual YJBK parameterization are shortly introduced in this section.

The YJBK parameterization was first derived by Youla et al. [1976a,b] and independently by Kucera [1975]. It has later been applied in numerous cases in connection with feedback control, see e.g. Anderson [1998], Boyd and Barratt [1991], Boyd et al. [1988], Dahleh and Diaz-Bobillo [1995], Tay et al. [1997], Zhou et al. [1995]. The YJBK parameterization for sampled-data system has been considered in Toivonen and Medvedev [2003].

3.1 The YJBK Parameterization

Consider a generalized nominal 2×2 system given by (1) controlled by a sampled-data controller $K(z)$ given by (2). The discrete-time transfer function from u_k to y_k be defined by

$$G_{yu}(z) = \mathcal{S}_\tau G_{yu}(s)\mathcal{H}_\tau$$

i.e. the transfer function that the controller look into. A coprime factorization of the system $G_{yu}(z)$ and the controller $K(z)$ are given by:

$$\begin{aligned} G_{yu}(z) &= NM^{-1} = \tilde{M}^{-1}\tilde{N}, \ N, M, \tilde{N}, \tilde{M} \in \mathcal{RH}_\infty \\ K(z) &= UV^{-1} = \tilde{V}^{-1}\tilde{U}, \qquad U, V, \tilde{U}, \tilde{V} \in \mathcal{RH}_\infty \end{aligned} \quad (5)$$

where the eight matrices in (5) must satisfy the double Bezout equation given by, see Zhou et al. [1995]:

$$I = \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \begin{pmatrix} M & U \\ N & V \end{pmatrix} = \begin{pmatrix} M & U \\ N & V \end{pmatrix} \begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} \quad (6)$$

Based on the above coprime factorization of the system $G_{yu}(z)$ and the controller $K(z)$, we can give a parameterization of all controllers that stabilize the system in terms of a stable transfer matrix function $Q(z)$, i.e. all stabilizing controllers are given by Tay et al. [1997] (left factored form):

$$K(Q) = \tilde{V}(Q)^{-1}\tilde{U}(Q) \quad (7)$$

where

$$\tilde{U}(Q) = \tilde{U} + Q\tilde{M}, \ \tilde{V}(Q) = \tilde{V} + Q\tilde{N}, \ Q \in \mathcal{RH}_\infty$$

Using the Bezout equation, the controller given either by (7) can be realized as an lower LFT in the parameter Q,

$$K(Q) = \mathcal{F}_l(J_K, Q) \quad (8)$$

where J_K is given by

$$J_K = \begin{pmatrix} \tilde{V}^{-1}\tilde{U} & \tilde{V}^{-1} \\ V^{-1} & -V^{-1}N \end{pmatrix} \quad (9)$$
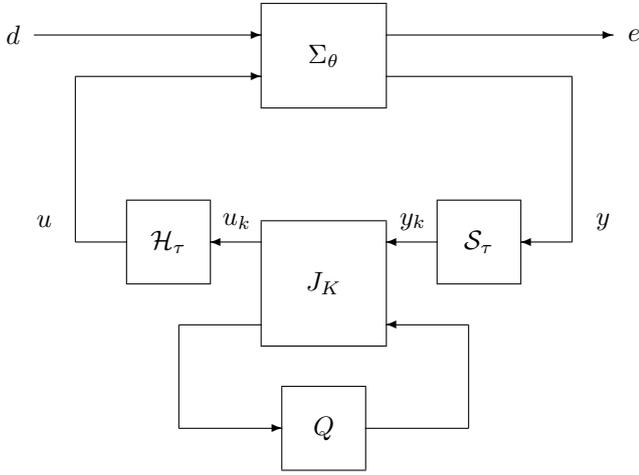

Fig. 2. Controller structure with parameterization for a sampled data system.

The closed-loop operator describing the sampled-data closed-loop system from input d to external output e had been considered in Niemann and Stoustrup [2004]. It was shown that the closed-loop operator is an affine operator of Q as it is in the continuous-time case. The connection between d and e is given by:

$$e = G_{ed}(s)d$$
$$+ G_{eu}(s)\mathcal{H}_\tau(U(z) + M(z)Q(z))\tilde{M}(z)\mathcal{S}_\tau G_{yd}(s)d \quad (10)$$

From (10), it is clear that the closed loop operator between the external input d and the external output e is an affine operator in Q. Therefore, the closed loop stability will not be affected as long as Q is selected as a stable transfer function.

3.2 The Dual YJBK Parameterization

Equivalent, there exist also a parametrization of all systems stabilized by one controller, i.e. the dual YJBK parametrization. The parametrization is given by Tay et al. [1997] (the left factored form):

$$G_{yu}(S) = \tilde{M}(S)^{-1}\tilde{N}(S) \quad (11)$$

where

$$\tilde{N}(S) = \tilde{N} + S\tilde{V}, \ \tilde{M}(S) = \tilde{M} + S\tilde{U}, \ S \in \mathcal{RH}_\infty$$

An LFT representation of (11) is given by:

$$G_{yu}(S) = \mathcal{F}_l(J_G, S) \quad (12)$$

where J_G is given by

$$J_G = \begin{pmatrix} NM^{-1} & \tilde{M}^{-1} \\ M^{-1} & -M^{-1}U \end{pmatrix} \quad (13)$$

It can be shown that the dual YJBK matrix transfer function S can be calculated by using the primary YJBK parameterization. It turns out that S is given by a LFT:

$$S = \mathcal{F}_u(J_K, G_{yu}(S)) \quad (14)$$

The dual YJBK parameterization is the basis for the fault diagnosis considered in the rest of this paper.

4. THE FAULT SIGNATURE MATRIX

The connection between the dual YJBK matrix transfer function S and the fault parameters θ has been considered in details for continuous-time systems, see e.g. Niemann [2003, 2006]. This connection is given by:

$$S(\theta) = T_{3,\theta}\theta(I - T_{1,\theta}\theta)^{-1}T_{2,\theta} \quad (15)$$

where $T_{i,\theta} \in \mathcal{RH}_\infty, \ i = 1, 2, 3$ are given by

$$T_{1,\theta} = G_{zw} + G_{zu}U\tilde{M}G_{yw}$$
$$T_{2,\theta} = G_{zu}M$$
$$T_{3,\theta} = \tilde{M}G_{yw}$$

The strong connection between the parametric faults θ and the dual YJBK matrix transfer function $S(\theta)$ given by (15) and the importance in active fault diagnosis is evident. The dual YJBK matrix transfer function $S(\theta)$ is in this context also named as the fault signature matrix.

(15) is valid for continuous-time systems and discrete-time systems but not for sampled-data system. The reason is that the derivation involves both continuous-time as well as discrete-time elements. For continuous-time systems (and also for discrete-time systems), the derivation of the fault signature matrix $S(\theta)$ is based on the following relation:

$$G_{yu}(S) = G_{yu}(\theta) \quad (16)$$

or

$$\tilde{M}^{-1}\tilde{N} + \tilde{M}^{-1}S(I + M^{-1}US)^{-1}M^{-1}$$
$$= G_{yu} + G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zu}$$

where $G_{yu}(S)$ is given by (11). In the sample-data case, first we need to consider the system $G_{yu}(\theta)$ given by:

$$G_{yu}(\theta, z) = \mathcal{S}_\tau G_{yu}(\theta)\mathcal{H}_\tau \quad (17)$$

or

$$G_{yu}(\theta, z) = \mathcal{S}_\tau(G_{yu} + G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zu})\mathcal{H}_\tau$$
$$= \mathcal{S}_\tau G_{yu}\mathcal{H}_\tau + \mathcal{S}_\tau G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zu}\mathcal{H}_\tau \quad (18)$$

For sampled-data systems, (11) is given by:

$$\mathcal{S}_\tau G_{yu}(S)\mathcal{H}_\tau = G_{yu}(z) + \tilde{M}^{-1}S(I + M^{-1}US)^{-1}M^{-1} \quad (19)$$

where $G_{yu}(S)$ is considered as the real system, i.e. $\mathcal{S}_\tau G_{yu}(S)\mathcal{H}_\tau = G_{yu}(\theta, z)$. Rewriting (19) gives:

$$S = \tilde{M}\tilde{G}(\theta, z)(M + US) \quad (20)$$

or

$$S = \tilde{M}\tilde{G}(\theta, z)(I - U\tilde{M}\tilde{G}(\theta, z))^{-1}M \quad (21)$$

where

$$\tilde{G}(\theta, z) = \mathcal{S}_\tau G_{yu}(S)\mathcal{H}_\tau - G_{yu}(z)$$
$$= \mathcal{S}_\tau G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zu}\mathcal{H}_\tau \quad (22)$$

or

$$\tilde{G}(\theta, z) = \tilde{M}^{-1}S(I + M^{-1}US)^{-1}M^{-1} \quad (23)$$

Using (22) in S given by (21) gives the following fault signature matrix for sampled-data systems:

$$S(\theta) = \tilde{M}\mathcal{S}_\tau G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zu}\mathcal{H}_\tau$$
$$\times (I - U\tilde{M}\mathcal{S}_\tau G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zu}\mathcal{H}_\tau)^{-1}M \quad (24)$$

The fault diagnosis matrix for sampled-data system $S(\theta)$ given by (24) is a non-linear function of the parametric faults θ as in the continuous-time case. However, the fault signature matrix for sampled-data systems is not as simple as it is in the continuous-time case.

5. ACTIVE FAULT DIAGNOSIS

The fault signature matrix given above for the sampled-data case will now be analyzed with respect to active fault detection as well as active fault isolation in MIMO system.

Before an analysis of the fault signature matrix $S(\theta)$ is given, let's consider the closed-loop system shown in Fig. 2. The block diagram without Q is shown in Fig. 3.
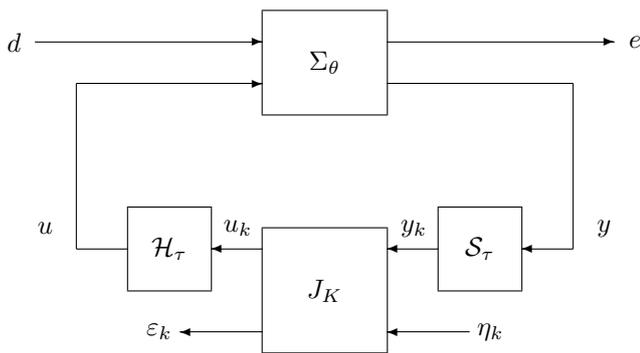

Fig. 3. Controller structure with the YJBK parameterization architecture for a sampled data system.

Compared with Fig. 2, ε_k is the input vector to the YJBK matrix transfer function Q and η_k is the output vector from Q. Further, using the relation between the system and S given by (14), we have directly that:

$$\varepsilon_k = S(\theta)\eta_k \qquad (25)$$

This connection is used directly in active fault diagnosis for closed-loop systems, where η_k is applied as an auxiliary input vector. Further, it can be shown that the output vector ε_k is also a residual vector for the system. The fault detection and isolation is then done by an investigation of the signature from the auxiliary input vector in the output vector or residual vector ε_k. This approach has been considered in details in Niemann [2006] for continuous-time systems. In Niemann and Poulsen [2014a,b], the active fault diagnosis problem for MIMO systems has been considered with respect to design the optimal auxiliary inputs and residuals. It should also be noted that the output vector ε_k will also include signals from the other external input d in real systems. However, the influence of the disturbance d on ε_k will not be considered in this paper.

Let's start the analysis with considering the fault free case. From (24), we have directly the following simple relation between parametric faults and the fault signature matrix:

$$S(\theta) = 0, \text{ for } \theta = 0 \qquad (26)$$

This is the same simple condition as in the continuous-time case, Niemann [2006]. Condition (26) gives a very simple way to detect parametric faults in the system.

Detection of faults using the active approach is very easy by using the condition given above. However, detection of faults in MIMO systems or isolation of faults is more complicated. To be able to select the correct auxiliary input η_k, a more detailed analysis is needed. The analysis will be done with respect to small faults. The reason is that large faults are reasonable simple to detect and some cases also possible to isolate. For small faults, the selection of auxiliary input as well as also the output direction for MIMO system to be able to both detect and isolate the faults.

The fault signature matrix given by (24) will be analyzed in two ways. The first method is based on a discretization of the continuous time system and then analyzing the fault signature matrix in discrete-time. The second method is to make an approximation of the continuous-time system and then discretize it. Both methods will give a fault signature matrix with the same structure as known from continuous-time systems.

5.1 Discretization of $\tilde{G}(\theta)$

The first approach is based on a discrete time description of $\tilde{G}(\theta)$ given by (23). The continuous-time part of system has an LFT description given by:

$$\tilde{\Sigma} : \begin{cases} z(t) = G_{zw}w(t) + G_{zu}u(t) \\ y(t) = G_{yw}w(t) \end{cases} \qquad (27)$$

where the connection between the two external vectors w and z is given by

$$w = \theta z$$

One approach to preserve the LFT structure in the system through the discretization is describe in Toth et al. [2012]. Here, the discretization is done in every single element in the system. For doing this, it is assumed that the parametric fault matrix θ in our case is constant between the samplings. The discrete-time version of (27) is given by:

$$\tilde{\Sigma} : \begin{cases} z_k = \mathcal{S}_\tau G_{zw}\mathcal{H}_\tau w_k + \mathcal{S}_\tau G_{zu}\mathcal{H}_\tau u_k \\ y_k = \mathcal{S}_\tau G_{yw}\mathcal{H}_\tau w_k \end{cases} \qquad (28)$$

or

$$\tilde{\Sigma} : \begin{cases} z_k = \bar{G}_{zw}w_k + \bar{G}_{zu}u_k \\ y_k = \bar{G}_{yw}w_k \end{cases} \qquad (29)$$

Using (29) in $\tilde{G}(\theta, z)$ gives:

$$\tilde{G}(\theta, z) = \bar{G}_{yw}\theta(I - \bar{G}_{zw}\theta)^{-1}\bar{G}_{zu} \qquad (30)$$

The fault signature matrix given by (24) is the given by:

$$S(\theta) = \tilde{M}\bar{G}_{yw}\theta(I - \bar{G}_{zw}\theta)^{-1}\bar{G}_{zu}$$
$$(I - U\tilde{M}\bar{G}_{yw}\theta(I - \bar{G}_{zw}\theta)^{-1}\bar{G}_{zu})^{-1}M \qquad (31)$$

Rewriting (31) gives

$$S(\theta) = \tilde{M}\bar{G}_{yw}\theta(I - (\bar{G}_{zw} + \bar{G}_{zu}U\tilde{M}\bar{G}_{yw})\theta)^{-1}\bar{G}_{zu}M \qquad (32)$$

The fault signature matrix given by (32) has now the same structure as for the continuous-time case. The results from Niemann and Poulsen [2014a,b] can now directly be applied on the fault signature matrix for the sampled-data systems given in (32).

5.2 Approximation of the fault signature matrix

Instead of calculating $\tilde{G}(\theta, z)$ such that the LFT structure is preserved, a direct approximation of $S(\theta)$ can be done. This approximation is done by making a Taylor expansion of $S(\theta)$ around the nominal value of the fault vector θ. Using the nominal value as $\theta = 0$, we get the following linear function of the parametric faults θ_i, i.e. $S(\theta)$ is given by:

$$S(\theta) \approx \sum_{i=1}^{k} \left(\frac{\partial}{\partial \theta_i} S(\theta)|_{\theta=0} \right) \theta_i \qquad (33)$$

For calculation the derivative of S, the following matrix rules are used, Petersen and Pedersen [2008]:

$$\partial(XY) = (\partial X)Y + X(\partial Y)$$

$$\partial X^{-1} = -X^{-1}(\partial X)X^{-1}$$

$$\frac{\partial X_{kl}}{\partial X_{ij}} = \delta_{ik}\delta_{lj}$$

Using these rules, the derivative of $S(\theta)$ is given by:

$$\frac{\partial S(\theta)}{\partial \theta_i} = \tilde{M}\frac{\partial \tilde{G}(\theta, z)}{\partial \theta_i}(I - U\tilde{M}\tilde{G}(\theta, z))^{-1}M$$
$$+ \tilde{M}\tilde{G}(\theta, z)(I - U\tilde{M}\tilde{G}(\theta, z))^{-1} \qquad (34)$$
$$U\tilde{M}\frac{\partial \tilde{G}(\theta, z)}{\partial \theta_i}(I - U\tilde{M}\tilde{G}(\theta, z))^{-1}M$$

$\frac{\partial S(\theta)}{\partial \theta_i}$ needs to be evaluated in the nominal point, i.e. for $\theta = 0$. Evaluating $\tilde{G}(\theta, z)$ in $\theta = 0$ gives directly

$$\tilde{G}(0) = 0$$

Applying this in (34) reduce $\frac{\partial S(\theta)}{\partial \theta_i}$ to the following simple equation:

$$\frac{\partial S(\theta)}{\partial \theta_i}|_{\theta=0} = \tilde{M}\frac{\partial \tilde{G}(\theta, z)}{\partial \theta_i}M \qquad (35)$$

The Taylor expansion in (33) is then given by:

$$S(\theta) \approx \tilde{M}\sum_{i=1}^{k} \frac{\partial \tilde{G}(\theta, z)}{\partial \theta_i}\theta_i M$$
$$= \sum_{i=1}^{k} \bar{S}_i \theta_i \qquad (36)$$

where $\bar{S}_i = \tilde{M}\frac{\partial \tilde{G}(\theta,z)}{\partial \theta_i}M$.

The Taylor expansion given by (36) is a general result and does not require that $G_{yu}(\theta)$ has a special structure. It requires only that $\frac{\partial \tilde{G}(\theta,z)}{\partial \theta_i}$ can be calculated.

Now, assuming that $G_{yu}(\theta)$ has an LFT structure, then $\frac{\partial \tilde{G}(\theta,z)}{\partial \theta_i}$ can be calculated. $\frac{\partial \tilde{G}(\theta,z)}{\partial \theta_i}$ is given by:

$$\frac{\partial \tilde{G}(\theta, z)}{\partial \theta_i} = \mathcal{S}_\tau G_{yw}\frac{\partial \theta}{\partial \theta_i}(I - G_{zw}\theta)^{-1}G_{zu}\mathcal{H}_\tau$$
$$+ \mathcal{S}_\tau G_{yw}\theta(I - G_{zw}\theta)^{-1}G_{zw}\frac{\partial \theta}{\partial \theta_i} \qquad (37)$$
$$(I - G_{zw}\theta)^{-1}G_{zu}\mathcal{H}_\tau$$

Again, using $\theta = 0$ in (37) gives:

$$\frac{\partial \tilde{G}(\theta, z)}{\partial \theta_i}|_{\theta=0} = \mathcal{S}_\tau G_{yw}\frac{\partial \theta}{\partial \theta_i}G_{zu}\mathcal{H}_\tau \qquad (38)$$

or

$$\frac{\partial \tilde{G}(\theta, z)}{\partial \theta_i}|_{\theta=0} = \mathcal{S}_\tau G_{yw}J_{ii}G_{zu}\mathcal{H}_\tau \qquad (39)$$

where J_{ii} is a quadratic matrix with "1" at (i, i) and zero elsewhere. Let G_{yw} and G_{zu} be partitioned into k columns and k rows, respectively, given by:

$$G_{yw} = [(G_{yw})_{:,1} \cdots (G_{yw})_{:,k}]$$

$$G_{zu} = \begin{bmatrix} (G_{zu})_{1,:} \\ \vdots \\ (G_{zu})_{k,:} \end{bmatrix} \qquad (40)$$

Using (40) in (39) gives the following equation for the derivative of $\tilde{G}(\theta, z)$ with respect to θ_i:

$$\frac{\partial \tilde{G}(\theta, z)}{\partial \theta_i}|_{\theta=0} = \mathcal{S}_\tau (G_{yw})_{:,i}(G_{zu})_{i,:}\mathcal{H}_\tau \qquad (41)$$

Using (41) in (35) gives:

$$\frac{\partial S(\theta)}{\partial \theta_i}|_{\theta_0=0} = \tilde{M}\mathcal{S}_\tau (G_{yw})_{:,i}(G_{zu})_{i,:}\mathcal{H}_\tau M \qquad (42)$$

The Taylor expansion in (33) is then given by:

$$S(\theta) \approx \tilde{M}\sum_{i=1}^{k} \mathcal{S}_\tau (G_{yw})_{:,i}(G_{zu})_{i,:}\mathcal{H}_\tau \theta_i M$$
$$= \sum_{i=1}^{k} \bar{S}_i \theta_i \qquad (43)$$

where $\bar{S}_i = \tilde{M}\mathcal{S}_\tau (G_{yw})_{:,i}(G_{zu})_{i,:}\mathcal{H}_\tau M$.

The above results are discussed in next section.

6. DISCUSSION OF THE RESULTS

The above section, it has been shown that it is possible to obtain a fault signature matrix for sampled-data systems with the same structure as for continuous-time systems. It is then possible to use the AFD results from the continuous-time case directly in the sampled-data case.

In the first method described in Section 5.1, a discretization that will preserve the LFT structure from the continuous-time system in the discrete-time system. Here, it is assumed that the fault matrix θ is constant between the samplings. This condition will in general be satisfied as long as the changes in the parametric faults are reasonable. Abrupt fault changes can also be handled, there might be a sample or two where the discrete-time model does not be exact correct.

The second method described in Section 5.2 is based on a direct approximation of the continuous-time system for small faults. This is done by using a Taylor expansion of the sampled-data fault signature matrix given by (24). By doing this, we get the same structure for the fault signature matrix as considered in Niemann and Poulsen [2014a,b] for continuous-time systems. These fault signature matrices for both the continuous-time case as well as for the sampled-data case are only valid for small parametric faults. However, detection and isolation of small parametric faults can be difficult. Therefore, it is important to be able to design optimal auxiliary inputs as well as residual vectors. This is done by using the simplified fault signature matrices.

At last, it need to be pointed out, that the fault signature matrix given by (21) is quite general. Here, there is no assumption about the structure in the system. It is not required that it has an LFT structure. The calculation followed in (35) gives the general partial derivative of the fault matrix with respect to a parametric fault θ_i. This is a general result and is not only valid for sampled-data systems.

7. CONCLUSION

Active fault diagnosis of parametric faults in closed-loop sampled-data system has been considered in this paper. The main focus has been on the fault signature matrix. It is shown that this central matrix in AFD for closed-loop systems does not have the same structure as for continuous-time systems. The missing LFT structure in the system through discretization has a direct influence on the structure of the fault signature matrix. Two methods have been described to obtain the same structure in the fault signature matrix known from the continuous-time case. Using dedicated discretization of the system, the structure in the fault signature matrix can be preserved for sampled-data systems. The other method is to make a Taylor expansion of the system and then discretize it. This will result in a structure in the fault signature matrix that can be applied directly for both fault detection and isolation.

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