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Soliton collision and Raman gain regimes in continuous-wave pumped supercontinuum generation

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Abstract: We numerically investigate supercontinuum generation using continuous-wave pumping. It is found that energy transfer during collision of solitons plays an important role. The relative influence of Raman gain on spectral broadening is shown to depend on the width of the calculation time window. Our results indicate that increasing the spectral linewidth of the pump can decrease the supercontinuum spectral width. Using a fiber with smaller dispersion at the pump wavelength reduces the required fiber length by decreasing the temporal width of the solitons formed from modulation instability. This also reduces the sensitivity to the pump spectral linewidth.

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1. Introduction

Spatially coherent light with an extremely broad optical spectrum (> 100 nm) can be produced by pumping light with a narrow spectrum and high intensity through a nonlinear medium [1]. The interaction between the high intensity light and the nonlinear medium initiates several mechanisms, which broaden the spectrum of the light into a so-called supercontinuum (SC) as it propagates through the medium. Photonic crystal fibers (PCFs) are often used as the nonlinear medium, not only because the silica core confining the light can be made very small, thus increasing the intensity of the light, but also because the dispersion properties of these fibers are highly controllable. The zero dispersion wavelength (ZDW) can, e.g., be shifted to the visible region of the optical spectrum [2], or there can even be two ZDWs in the optical spectrum [3–5]. To some degree, similar dispersion engineering can be performed by tapering standard telecommunications fiber [6]. By choosing a PCF with a suitable dispersion-profile, the broadening mechanisms can be made more efficient for generating light in particular wavelength regions [2].

Pump light with high intensity has traditionally been achieved using femtosecond lasers, offering pulses with several kilowatts of peak power [2, 4, 7]. Femtosecond lasers are typically complex and bulky; therefore more compact picosecond [8] and even nanosecond [7] pump sources have also been used. Nanosecond pumped SC generation technology has reached a point of maturity where commercial sources are available (see, e.g., www.koheras.com, SuperK source).

Advances in fiber laser technology in recent years have also led to continuous-wave (CW) fiber lasers with sufficiently high power (~ 0.1–15 W) to be used for SC generation [9–11]. It has been demonstrated that a CW-pumped SC source provides sufficiently high output power and broad bandwidth to be used for ultrahigh resolution optical coherence tomography (OCT) [12]. Experimental characterization of the noise of the CW pumped SC showed that the relative intensity noise (RIN) can be 30-50 dB lower than that achieved using a femtosecond pump [13]. Other experiments with a CW pumped SC found RIN values comparable to those with a femtosecond pump [14].

CW pumping for SC generation has been investigated both by pumping in the anomalous-dispersion region (ADR) [10,11,13,15–17] and the normal-dispersion region (NDR) [18,19]. In the present work we focus on pumping in the ADR. The spectral broadening achieved using CW pumping in the ADR has previously been explained as being seeded by modulation instability (MI), which causes breakup of the CW into solitons. The solitons are then red-shifted due to the soliton self-frequency shift (SSFS) [13, 15–17]. During the red-shift the solitons may also transfer energy to blue-shifted dispersive waves. Thus, after MI-induced breakup into solitons, the spectral broadening is essentially caused by the same mechanisms as when using fs pumping [4].

Solitons generated directly from MI can be too temporally broad to undergo any significant red-shift [20]. It was shown by Islam et al. that a significant red-shift can still occur because MI-
generated solitons undergo collisions, in which they transfer energy between each other [21]. It is then possible for one of the solitons to acquire sufficient energy to undergo a large red-shift. In this work, we examine the conditions for proper modeling of the soliton collisions and their effect on the spectral broadening. The influence of the CW pump spectral linewidth and the fiber dispersion on the spectral broadening is also investigated.

The paper is organized as follows. In Section 2 we provide an overview of previous numerical modeling of CW pumped SC generation, and justify our choice of how to model the CW input. Section 3 then describes how we simulate the propagation of the partially coherent CW beam along the fiber, and the implemented phase noise model. In Section 4 we describe the primary mechanisms involved in the spectral broadening during propagation along the fiber. It is shown that soliton collisions play an important role in CW-pumped supercontinuum generation for the parameters used here. It is also shown how the influence of soliton collisions is practically absent, if the time window of the calculations is too narrow. We then present results showing that the output spectrum can differ significantly depending on the spectral linewidth of the CW pump laser. Finally, we show that SC generation in a fiber with smaller dispersion at the pump wavelength is less sensitive to the spectral linewidth of the pump laser.

2. Previous numerical modeling

One important aspect of modeling the nonlinear propagation of a CW beam in an optical fiber is how to model the CW input.

Kobtsev and Smirnov used the one photon per mode approach [16]. This method has previously been used to model SC generation using picosecond pulses [8] and consists of phenomenologically adding one photon with random phase to each frequency bin of the input field. It was shown by Smith in 1972 that this fictitious injection of photons to the input leads to the same output power at the Raman Stokes wavelength (downshifted from the pump by 13.2 THz) as Raman amplification of spontaneous emission along the fiber length [22]. However, as we show in Fig. 1(right) this approach suffers mainly from two drawbacks: (1) There is no inherent spectral linewidth, and (2) the power spectral density at the Raman Stokes wavelength can be significantly underestimated. As mentioned in Subsection 3.2 it is reasonable to assume a Lorentzian spectrum for a partially coherent CW field. We show in Fig. 1(right) that a Lorentzian spectrum for a CW field with an average power of 10 W, center wavelength of 1064 nm and a full width at half-maximum of 30 GHz (~0.1 nm) has significantly more spectral power at the Raman Stokes wavelength 1116 nm, than a CW field containing one photon per mode. The one photon per mode method is therefore unsuited to investigate the influence of the pump laser spectral linewidth and may underestimate the Raman Stokes power spectral density at the input. This is critical when Raman scattering is dominant because the input Raman Stokes power acts as a seed for the exponential gain along the fiber.

Vanholsbeeck et al. also take a phenomenological approach by taking the input spectrum of the field to be $\tilde{E}(v) = \sqrt{S_m(v)} \times \phi(v)$, where $\phi(v)$ is a random spectral phase and $S_m(v)$ is the measured power spectral density of the pump laser to be modelled [17]. This approach mathematically assures that the power spectrum of the input field is as desired, $|\tilde{E}(v)|^2 = S_m(v)$. But the choice of randomly imposing a spectral phase has no physical justification. The strong temporal fluctuations of the input field $E(t)$ obtained by inverse Fourier transform of $\tilde{E}(v)$ are therefore not necessarily present in a real partially coherent CW beam, as also pointed out by the authors in Ref. [17]. Recently, it was pointed out by Barvian et al. that modifying the statistics of the spectral phase $\phi(v)$ has a significant influence on the spectral evolution along the fiber [19]. One must therefore be careful when using arbitrary statistical properties for the spectral phase.

Cavalcanti et al. used a time-independent average field on which noise appears as small fluc-
tuations in amplitude and phase to study the influence of MI on the statistical properties of a partially coherent beam [23]. Mussot et al. disregarded amplitude fluctuations and included only phase noise to investigate the spectral broadening mechanism [15]. As shown in Subsection 3.2, the phase noise can be directly related to the measurable linewidth of a CW laser. Furthermore, the underlying phase-diffusion model is widely accepted as an appropriate model for lasers operating far above threshold (see e.g. Refs. [15, 23], and Refs. therein). Since this method involves both a physically justified choice of phase noise statistics and includes a finite spectral linewidth, it is physically reasonable and used in the work presented here. We can therefore investigate the influence of the spectral linewidth of the CW pump on the SC generation.

3. Theory

3.1. Propagation equation

We model the propagation of the CW field using the generalized nonlinear Schrödinger equation (NLSE) [24, 25]

$$\frac{\partial E}{\partial z} = i \sum_{m \geq 2} \frac{\beta_m}{m!} \frac{\partial^m E}{\partial T^m} - \frac{\alpha}{2} E + i \gamma(\omega) \left[ 1 + \frac{i}{\omega_0} \frac{\partial}{\partial T} \right] \left[ E(z,T) \int_{-\infty}^{\infty} R(T') |E(z,T-T')|^2 \,dT' \right],$$

(1)

where $E(z,T)$ is the field envelope in a retarded time frame $T = t - \beta_1 z$ moving with the group velocity $1/\beta_1$ of the pump, along the fiber axis $z$. $\omega_0$ is the carrier angular frequency of the CW input. $\gamma(\omega) = n_2 \omega_0 / [c A_{\text{eff}}(\omega)]$ is the nonlinear parameter, where $n_2 = 2.6 \times 10^{-20} \text{ m}^2/\text{W}$ is the nonlinear-index coefficient for silica, $c$ is the speed of light in vacuum, and $A_{\text{eff}}$ is the effective core area [24]. The dispersion parameters $\beta_m$ are obtained from a polynomial fit to the dispersion profile calculated using fully-vectorial plane-wave expansions [26].

Dispersion profiles for the PCFs considered in this paper are shown in Fig. 1(left). The PCF with pitch $\Lambda = 1.72 \mu m$ and relative hole size $d/\Lambda = 0.65$ corresponds to a fiber made by Crystal Fiber A/S. It was used in Refs. [11, 12] (note that both papers state that $d = 0.65 \mu m$, but this does not correspond with the reported core size of $d_{\text{core}} = 2.3 \mu m$).
and is believed to be a misprint [27]). To investigate the influence of the dispersion profile, we have also considered a PCF with a relative hole size $d/\Lambda = 0.378$. The transverse field distribution $E(x, y, \omega)$ of the fundamental mode was also calculated with fully-vectorial plane-wave expansions. From this $A_{\text{eff}}(\omega)$ was calculated using the more general definition suitable for fibers, where some of the field energy may reside in the air-holes [28]. $R(t)$ is the Raman response function [24,29]

$$R(t) = (1 - f_R)\delta(t) + f_R h(t) = (1 - f_R)\delta(t) + f_R \frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2} \exp(-t/\tau_2) \sin(t/\tau_1) \Theta(t)$$  (2)

where $f_R = 0.18$ is the fractional contribution of the delayed Raman response, $\tau_1 = 12.2$ fs, and $\tau_2 = 32$ fs. $\Theta(t)$ is the Heaviside step function.

The propagation Eq. (1) is solved using the split-step Fourier method [24]. The relative change in photon number (a measure of the numerical error that is ideally zero in the absence of loss [29]) was less than 5% for all results presented here.

3.2. Noise model

We model the input field envelope as [15,23]

$$E(0, T) = \sqrt{P_0} \exp[i\delta \phi(T)]$$  (3)

where $\delta \phi$ is a small fluctuation with zero ensemble average, $\langle \delta \phi \rangle = 0$. $P_0$ is the power of the CW input field. We will model $\delta \phi$ as a Gaussian random process.

In the following, we determine how to properly relate $\delta \phi$ to a measurable quantity, namely the full width at half-maximum (FWHM) power spectral linewidth $\Delta \nu_{\text{FWHM}}$ of the CW input.

The random phase fluctuations can be seen as arising from a random fluctuation $\nu_R$ of the CW frequency $\nu_0$, so that the instantaneous frequency $\nu_i$ is [30]

$$\nu_i = \nu_0 + \frac{1}{2\pi} \frac{d(\delta \phi)}{dt} = \nu_0 + \nu_R(t).$$  (4)

It can be assumed in most cases that $\nu_R(t)$ is a zero mean, statistically stationary fluctuation [30]. Here, as in Ref. [23], we model $\nu_R(t)$ as Gaussian white noise with zero mean and variance $\sigma^2_{\nu_R}$. The phase fluctuation $\delta \phi(t)$ is then found from [30]

$$\delta \phi(t) = 2\pi \int_{-\infty}^{t} \nu_R(\xi) d\xi,$$  (5)

and is a statistically nonstationary random process. In the following, we determine the relation between $\sigma^2_{\nu_R}$ and $\Delta \nu_{\text{FWHM}}$.

It can be shown that the structure function $D_{\delta \phi}(\tau) = \langle (\delta \phi(t + \tau) - \delta \phi(t))^2 \rangle$ is given by [30]

$$D_{\delta \phi}(\tau) = 8\pi^2 \tau \int_{0}^{\infty} S_{\nu_R}(\eta) d\eta,$$  (6)

where $\Gamma_{\nu_R}$ is the autocorrelation function of $\nu_R(t)$. Using the Wiener-Khinchin theorem [30]

$$\Gamma_{\nu_R}(\tau) = \int_{-\infty}^{\infty} S_{\nu_R}(\nu) \exp(-i2\pi\nu\tau) d\nu,$$  (7)

where $S_{\nu_R}(\nu)$ is the power spectral density of $\nu_R(t)$, we obtain for white noise (i.e. $S_{\nu_R}$ frequency independent) over a limited bandwidth $B$:

$$\Gamma_{\nu_R}(\tau) = S_{\nu_R} \int_{-B/2}^{B/2} \exp(-i2\pi\nu\tau) d\nu.$$  (8)
The bandwidth limit is introduced because the split-step Fourier method implies both a finite time- and frequency-domain in the calculations. Since \( \sigma_{\nu R}^2 = \Gamma_{\nu R}(0) \) [30], we have

\[
\sigma_{\nu R}^2 = \Gamma_{\nu R}(0) = S_{\nu R} \int_{-B/2}^{B/2} d\nu = S_{\nu R} B \Rightarrow S_{\nu R} = \frac{\sigma_{\nu R}^2}{B} \tag{9}
\]

Inserting this in Eqs. (6) and (8), we obtain

\[
D_{\delta \phi}(\tau) = 8\pi^2 \tau \frac{\sigma_{\nu R}^2}{B} \int_0^{\infty} \int_{-B/2}^{B/2} \exp(-i2\pi \nu \eta) d\nu d\eta = 4\pi^2 \tau \frac{\sigma_{\nu R}^2}{B} . \tag{10}
\]

The structure function \( D_{\delta \phi}(\tau) \) is related to \( \Delta \nu_{\text{FWHM}} \) by [23] \( D_{\delta \phi}(\tau) = 2\pi \Delta \nu_{\text{FWHM}} \tau \), which together with Eq. (10) finally gives the following relation between \( \sigma_{\nu R}^2 \) and \( \Delta \nu_{\text{FWHM}} \):

\[
\sigma_{\nu R}^2 = \frac{\Delta \nu_{\text{FWHM}} B}{2\pi} \tag{11}
\]

In summary, we model \( \nu_R(t) \) as Gaussian white noise, with zero mean and variance \( \sigma_{\nu R}^2 \). Equation (5) then gives the phase fluctuation \( \delta \phi(t) \), which is inserted in Eq. (3) to obtain the input field for the propagation simulation. Due to the assumptions of the statistical properties of the phase noise, the spectrum of the input field has a Lorentzian line shape [23], with a FWHM given by \( \Delta \nu_{\text{FWHM}} \).

### 3.3. Numerical considerations

An ideal CW field has an infinite extent in time, but with the split-step Fourier method we can only model the propagation of an input field with finite temporal width. Furthermore, the method implies that the input field is periodic [24]. This means that energy going out through one side of the time window will reappear from the other side of the time window.

Due to the phase noise, the phase \( \delta \phi(0) \) of the input field at one edge of the time window, will differ from the phase \( \delta \phi(T_{\text{max}}) \) at the other edge of the time window. There will therefore be a large phase discontinuity in the periodic input field (Fig. 2, bottom). Unfortunately, this phase discontinuity leads to large intensity fluctuations at the edges of the time window, since phase noise is converted to intensity noise upon propagation in a dispersive medium (see, e.g., Ref. [31]).

To avoid this problem in the numerical simulations, one can artificially replace the CW field with a broad Gaussian [23] or super-Gaussian [10] pulse. Here, we multiply our input field [Eq. (3)] with a super-Gaussian pulse and obtain

\[
E(0,T) = \sqrt{P_0} \exp[i\delta \phi(T)] \exp \left[ -\frac{1}{2} \left( \frac{T}{T_{\text{IG}}} \right)^{2m} \right] , \tag{12}
\]

where we use \( m = 10 \), and \( T_{\text{IG}} \) is the 1/e intensity half-width of the super-Gaussian pulse, see Fig. 2(top). We show in Fig. 1(right) that multiplication with the super-Gaussian pulse practically does not affect the spectrum of the CW input field.

There are now two important deviations of the model from a real CW field: (1) \( T_{\text{IG}} \) is finite, meaning that due to dispersion some of the pulse energy, e.g. in the form of a soliton with a carrier frequency slightly offset from the center frequency of the CW-field, can move beyond the edges of the super-Gaussian pulse. This soliton will then no longer interact with the remaining super-Gaussian pulse, unless it exits one side of the time window and meets with the other edge of the super-Gaussian pulse. (2) The time window is finite; as explained before this means that...
Fig. 2. An illustration of the modeled periodic super-Gaussian input pulse. Top: power variation in time, bottom: phase fluctuation in time. In this example the width of the time window is 59.0 ps, the width of the super-Gaussian pulse is $2T_sG = 30$ ps, and $\Delta \nu_{FWHM} = 265$ GHz.

pulse energy leaving one side of the time window reappears from the other side of the time window, as was also described in Ref. [17].

Instead of an infinite CW field, we are thus actually modeling a CW laser which is turned on and off periodically. Each time the laser is turned on, the phase noise fluctuations are the same as before (see Fig. 2). The influence of these deviations decreases as the widths of the super-Gaussian pulse and the time window are increased. In the following Section, we examine the effects of these limitations closer.

It should be noted that an optical spectrum analyzer typically has a minimum integration time on the order of milliseconds. One should therefore in principle simulate a quasi-CW pulse with this duration to obtain sufficient statistical information to compare with an experimental measurement. Unfortunately, the calculation time of the numerical simulations based on the fast Fourier transform scales with $N \log_2(N)$ (see e.g. [32]), where $N$ is the number of computational points and is proportional to the width of the time window for a fixed temporal resolution. To decrease calculation time it is therefore common to simulate several shorter pulses, each only differing by the random initial conditions, and then average over the ensembles [16, 17, 23].

4. Numerical results

In the following Subsection we first show that there are mainly three physical mechanisms responsible for the spectral broadening observed in this work. Although the mechanisms are well known, the role of soliton collisions in CW pumped SC generation, first described by Islam et al. for a quasi-CW pump [21], is often overlooked (e.g. Refs. [16, 17]). In Subsection 4.2 we consider pumping in the fiber with relative hole size $d/\Lambda = 0.65$ and find that, depending on pump spectral linewidth and the width of the time window, the red-shift of solitons can be suppressed. Subsection 4.3 then considers pumping in the fiber with relative hole size $d/\Lambda = 0.378$. In this case it turns out that many solitons are formed, and that the spectral broadening is not so sensitive to the pump spectral linewidth.
4.1. Physical mechanisms

For the cases investigated in this work we have found three dominant physical mechanisms responsible for spectral broadening. One mechanism is Raman gain, which occurs when some of the power of a strong optical field (the pump) is converted to a longer wavelength (the Raman Stokes wave) [24]. The energy difference between a pump photon and a Raman Stokes photon is transferred to molecular vibrations in the propagation medium. In silica, the material typically used for PCFs, most of the Raman Stokes photons are downshifted from the pump frequency by 13.2 THz. A weak optical field propagating at a frequency upshifted from the pump by 13.2 THz (the anti-Raman Stokes wave) is exponentially damped due to energy transfer to the pump.

The second mechanism is MI which causes a CW pump to break up into a periodic pulse train [24]. The pulse train can then evolve into a series of solitons.

The third mechanism is collision of solitons followed by soliton red-shift [21]. In the context of optical fibers, a soliton is a pulse representing a solution to the unperturbed NLSE [24, 33],

$$\frac{\partial E}{\partial z} = \frac{-i\beta_2}{2} \frac{\partial^2 E}{\partial T^2} + i\gamma |E|^2. \quad (13)$$

The unperturbed soliton propagates without changing its shape, amplitude or width even after collision with a soliton of different carrier frequency (and therefore different group velocity) [34]. Perturbations such as third-order dispersion (TOD), self-steepening, and delayed Raman response [all included in Eq. (1)] lead to, in the case of TOD, emission of radiation and position shifts during collision [35]. Two solitons colliding in the presence of delayed Raman response transfer energy from the soliton with the highest carrier frequency to the other soliton; this has been shown by both numerical simulations [21, 36] and theoretical analysis [21, 37, 38]. In general, solitons can transfer energy during collisions in non-integrable models; there is some indication that energy is preferentially transferred to the soliton with largest amplitude [39]. A soliton propagating in a medium with delayed Raman response is downshifted in frequency (red-shift) during propagation due to the SSFS [24, 40]. The red-shift is enhanced during collision with another soliton [36]. It is expected that delayed Raman response is the perturbation with the most significant effect on soliton collisions [38, 41].

All of these physical mechanisms are studied in the spectrogram movie shown in Fig. 3. Initially, the CW pump breaks up into a periodic pulse train after just a few meters of propagation due to MI. To verify that the breakup is caused by MI we have in Fig. 4(left) plotted $|\tilde{P}(\nu)|^2$, where $\tilde{P}(\nu)$ is the Fourier transform of the pulse power $P(t) = |E(t)|^2$. MI provides maximum gain for oscillations with a frequency given by [24]

$$\nu_{\text{max}} = \frac{\sqrt{\gamma P_0}}{2\pi^2 |\beta_2|} \quad (14)$$

which we have indicated in Fig. 4(left) by vertical lines for the two fibers considered here. It is seen that oscillations at the MI frequency are significant for both fibers and are therefore the main cause of pulse train formation.

As seen from the spectrogram movie, Fig. 3, the periodic pulse train evolves into solitons. The number of solitons can be estimated as the ratio between the super-Gaussian pulse width and the period $T_m$ of the periodic pulse train: $2T_{\text{sG}}/T_m = 2T_{\text{sG}} \times \nu_{\text{max}} \approx 30 \text{ ps} \times 0.75 \text{ THz} \approx 23$. This corresponds well with the number of solitons in the top right of Fig. 3. The solitons have slightly different group velocities for two reasons. First, the carrier frequency of the CW pump fluctuates in time due to the phase noise. Second, they have slightly different temporal widths, which means that the rate of SSFS is also different. As mentioned previously, SSFS causes the carrier frequency $\nu_0$ of a soliton to red-shift as it propagates along the fiber. The rate of red-shift
Fig. 3. Spectrogram movie for one simulation of propagation in the \( d/\Lambda = 0.65 \) fiber. Input super-Gaussian pulse width \( 2T_{CG} = 30 \) ps, \( \Delta \nu_{FWHM} = 30 \) GHz, time window \( T_{\text{max}} = 59 \) ps. Note that the color scale changes during propagation, as some of the solitons acquire higher peak power. The horizontal white line on the bottom Figs. indicates the Raman Stokes wavelength at \( 1116 \) nm. pitch1p72_dL0p65_30GHz.avi (1.93 MB).

Fig. 4. Left: Frequency spectrum of pulse power \( P(t) \) at \( z = 9 \) m calculated for one simulation for the \( d/\Lambda = 0.65 \) fiber (blue, solid) and one simulation for the \( d/\Lambda = 0.378 \) fiber (green, dashed). The vertical lines indicate the frequency with maximum MI gain in the corresponding fibers. Right: Maximum peak power along the fiber length for the same simulation as shown in Fig. 3. The horizontal black line at \( 152 \) W indicates the minimum peak power required for a soliton to red-shift to \( 1116 \) nm after \( 74 \) m of propagation.
\[ \frac{dv_0}{dz} = -\frac{8|\beta_2|T_R}{2\pi 15 T_0^4}, \quad T_0 \gg 76 \text{ fs}, \quad (15) \]

where \( T_R \approx 3 \text{ fs} \) is related to the slope of the Raman gain spectrum [24]. For a soliton duration shorter than 76 fs the red-shift rate is given by [43]

\[ \frac{dv_0}{dz} = -\frac{0.09|\beta_2|\Omega_R^2}{2\pi T_0}, \quad T_0 \lesssim 76 \text{ fs}. \quad (16) \]

Because of the slightly different group velocities, and perhaps to some degree also by soliton attraction/repulsion [24], the solitons undergo collisions as they propagate. As stated above, each collision leads to a fractional amount of energy transfer to the most red-shifted soliton. A soliton that gains energy from a collision increases its peak power and decreases its temporal width to maintain its fundamental soliton shape [24]. The decrease in temporal width leads to an increased red-shift rate, which in turn modifies the group velocity [21]. This soliton will therefore also have a greater probability of colliding with other solitons. The mechanism is therefore self-amplifying. It is seen in Fig. 3 how one particular soliton gradually gains energy and obtains such a short temporal width that it red-shifts significantly faster than the other solitons. Figure 4(right) shows the maximum peak power of the propagated field along the fiber, for the same simulation as in Fig. 3. It is known from Fig. 3 that the soliton formation from MI is complete after less than \( \sim 10 \text{ m} \) of propagation. It is seen from Fig. 4(right) that the maximum peak power continues to increase along the fibre after a propagation length of \( z = 10 \text{ m} \) because of the energy transfer during soliton collisions.

The temporal width of the MI-generated solitons can be estimated from the MI oscillation period \( T_{\text{m}} = 1/v_{\text{max}} \). Thus, the duration of the generated solitons is \( 1/(0.75 \text{ THz}) \approx 1.3 \text{ ps} \) and \( 1/(2.5 \text{ THz}) \approx 0.4 \text{ ps} \), in the \( d/\Lambda = 0.65 \) and the \( d/\Lambda = 0.378 \) fiber, respectively. Since the red-shift rate for such soliton durations is proportional to \( T_0^{-4} \) [Eq. (15)], even a small decrease in pulse width could lead to a significant increase in red-shift rate.

To clarify that energy transfer during soliton collisions is necessary for the large observed red-shift, and that the resulting spectrum can not be explained by MI and SSFS alone, the following analysis is performed. The most red-shifted soliton in Fig. 3 has shifted to 1161 nm at \( z = 74 \text{ m} \). This corresponds to a shift of \( \Delta v_0 = -23.5 \text{ THz} \) from the input pump over \( \Delta z = 74 \text{ m} \). An estimate for the red-shift using Eq. (15) gives \( \Delta v_0 \approx -8|\beta_2|T_R\Delta z/(2\pi 15 T_0^4) \approx -3 \times 10^{-4} \text{ THz} \), where the soliton width \( T_0 \) was approximated by the pulse train period \( T_{\text{m}} = 1.3 \text{ ps} \). \( \beta_2(\lambda) \) changes only slightly from \(-4.38 \times 10^{-26} \text{ s}^2/\text{m} \) at \( \lambda_0 = 1064 \text{ nm} \) to \(-6.14 \times 10^{-26} \text{ s}^2/\text{m} \) at \( \lambda = 1161 \text{ nm} \); \( \beta_2(\lambda) \) is therefore approximated by \( \beta_2(\lambda_0) = \beta_2 \). If there is no energy transfer during soliton collisions, the change in \( T_0 \) during red-shift can also be neglected. It is thus clear that the solitons generated directly from MI, in the cases investigated here, are too temporally long to make a notable red-shift. This is in accordance with the finding by Golovchenko et al. [20], who found that for \( v_{\text{max}} \ll v_R = 13.2 \text{ THz} \) the soliton red-shift is insignificant [20].

To support this conclusion further, Fig. 5(left) shows the red-shift rate \( dv_0/dz \) in the two regimes where Eqs. (15–16) are valid. The red-shift rate has been scaled by \(-1/|\beta_2|\) for generality. The observed shift of \(-23.5 \text{ THz} \) corresponds to a mean scaled red-shift rate of \((-1/\beta_2)\Delta v_0/\Delta z \approx 7.3 \times 10^{36} \text{ s}^{-3} \). It is seen from Fig. 5(left) that the scaled red-shift rate for a soliton width \( T_0 \approx 1 \text{ ps} \) is four orders of magnitude smaller than the observed mean red-shift rate. This shows that even though there is a \( T_0^{-4} \) dependence, it is not enough for the MI-generated soliton width to change ‘slightly’, if the soliton is to make the observed red-shift.
The soliton width has to be decreased by an order of magnitude, as seen from Fig. 5(left). This is achieved through the energy transfer during soliton collisions.

We note that one can also see soliton formation from MI followed by the emergence of a few quickly red-shifting high-power solitons in the simulations performed in Refs. [16, 17]. Estimates of the red-shift rate for the parameters used in those papers indicate that solitons formed directly from MI could not explain the observed red-shift nor the short temporal width of the observed solitons. We therefore believe that these solitons were also the result of energy transfer during collisions. The authors seem to have overlooked this possibly because they did not consider in detail the formation of the high-power solitons using e.g. spectrograms.

The above conclusion on the necessity of soliton collisions to explain the observed red-shift of a few individual solitons is based on the parameters used in this work and in Refs. [16, 17]. It should be emphasized that, as shown by Eq. (14), the temporal width of MI-generated solitons can be made sufficiently small to obtain a significant red-shift without soliton collisions by increasing $\gamma$ and/or $P_0$, and/or by decreasing $|\beta_2|$. It was, e.g., shown by Golovchenko et al. that for sufficiently large pump power, the MI-generated solitons are short enough for all of them to red-shift [20].

4.2. Formation of few or no red-shifting solitons

We have found that for the fiber with $\Lambda = 1.72$ $\mu$m and $d/\Lambda = 0.65$, only few or even no red-shifting solitons are formed. We treat the cases of a CW pump with 30 GHz or 265 GHz linewidth separately.

4.2.1. 30 GHz linewidth

First, we consider case A illustrated in the top of Fig. 5(right): a super-Gaussian pulse width of 15 ps, and a time window of $T_{\text{max}} = 29.5$ ps. $2^{14}$ points were used with a time resolution of $\Delta t = 1.8$ fs. This results in a frequency window of $1/\Delta t \approx 556$ THz, and a frequency resolution of $1/T_{\text{max}} \approx 34$ GHz. We performed 10 simulations with different seeds for the random number generator used to simulate the phase noise. As shown in Fig. 6(left), all the resulting spectra have a peak at 1116 nm and a dip at 1013 nm. These features both correspond very well with a frequency shift from the pump by 13.2 THz, which is the peak frequency shift for Raman gain [24]. Thus, Raman gain is the dominant physical mechanism responsible for the resulting spectrum in this case.
We now consider case B illustrated in the middle of Fig. 5(right) by doubling the width of the time window to 59 ps, but keeping the width of the input super-Gaussian pulse the same as in case A. Since we keep the time resolution constant, we now use $2^{15}$ points. This means that the frequency resolution is halved from 34 GHz to 17 GHz. We assume that since the frequency resolution is already high, this has negligible effect on the results. As seen in Fig. 6(right) the peak and the dip resulting from Raman gain are now less prominent, indicating that less energy is transferred via the Raman effect. Also, in 3 of the 10 simulations a red-shifting soliton is seen with a center wavelength larger than 1100 nm.

The difference in resulting spectra between cases A and B can be explained as follows. We have calculated the difference in $\beta_1 = 1/v_g$ between the pump (1064 nm) and the Raman Stokes wave (1116.3 nm) to be 4.0 ps/m (for the $d/\Lambda = 0.65$ fiber). This means that pulse energy transferred from the pump to the Raman Stokes wave will separate from the pump in the time domain at a rate of 4.0 ps for each meter of propagation. The Raman Stokes wave is thus spread out over the time window after just a few meters of propagation. This can be seen in the bottom of Fig. 3. The transfer of energy from the pump to the Raman Stokes wave requires a temporal overlap. When we increase the time window from case A to case B, the Raman Stokes wave is spread over a larger time domain, thus decreasing the temporal overlap with the pump. The transfer of energy from the Raman anti-Stokes wave to the pump, and from the pump to the Raman Stokes wave therefore becomes less efficient for case B. Since less energy is removed from the pump wavelength there remains more energy in the solitons undergoing collisions. There is therefore a greater probability of a high-energy soliton being formed, with such a short temporal width that it red-shifts far-away from the pump.

Note that we simulated a super-Gaussian input pulse with the same width in cases A and B, and yet the resulting spectra are very different. This important result shows that the choice of time window width requires careful consideration.

To consider case C (bottom of Fig. 5, right) we keep the same time window as in case B, but now double the width of the super-Gaussian input pulse to $2T_{SG} = 30$ ps. The ratio of super-Gaussian pulse width and time window width, $2T_{SG}/T_{max}$, is now the same as in case A. From the physical explanation above, the Raman energy transfer from e.g. the pump to the Raman Stokes wave should be as effective as in case A. This is because the temporal overlap between the Raman Stokes wave and the pump is the same in both cases A and C, if we assume that the Raman Stokes wave is quickly almost uniformly spread over the entire time window. Indeed,
as seen in Fig. 7(left), the Raman anti-Stokes dip and the Raman Stokes peak have practically the same power spectral density as for case A [Fig. 6(left)]. It is also seen in Fig. 3 how the Raman Stokes wave is uniformly spread across the time window. In case A the Raman energy transfer from the pump was so severe that no red-shifting solitons were observed. However, the doubling of the super-Gaussian input pulse width, compared to case A, means that a larger number of colliding solitons will be formed from MI. This should result in a greater probability for the formation of a red-shifting soliton. As expected, it is seen in Fig. 7(left) that in 4 out of the 10 ensembles a soliton was observed to red-shift beyond 1100 nm.

4.2.2. 265 GHz linewidth

We again use the computational parameters for case C (2$T_sG = 30$ ps, $T_{max} = 59$ ps, 2$^{15}$ points) but now simulate a spectral FWHM linewidth of $\Delta\nu_{FWHM} = 265$ GHz. The resulting spectra are seen in Fig. 7(right).

The only simulation difference between Fig. 7(right) and Fig. 7(left) is the spectral linewidth of the quasi-CW input. None of the simulations with 265 GHz linewidth show distinct red-shifting solitons, whereas the simulations with 30 GHz linewidth showed a red-shifting soliton in 4 out of 10 simulations.

We suggest two possible explanations for this difference. (1) The larger pump linewidth leads to a larger spread in center frequencies of solitons as they form from the pump. Since the energy transfer between two colliding solitons decreases with increasing separation in carrier frequency [36], this leads to less overall energy transfer during soliton collisions. This could then hinder the buildup of a quickly red-shifting soliton. (2) As the linewidth of the Lorentzian power spectrum is increased, the pump power spectral density $S(\lambda = 1064 \text{ nm})$ is decreased, see Fig. 8(left). At the same time, there is more spectral power $S(\lambda = 1116 \text{ nm})$ available at the Raman Stokes wavelength. The reduction in power spectral density at the pump wavelength could also hinder the buildup of a quickly red-shifting soliton. In Fig. 8(right) we have plotted the power within a bandwidth of $\sim 4$ nm centered at the pump wavelength and the Raman Stokes wavelength, respectively, during propagation along the fiber. It is seen that for a linewidth of 265 GHz there is slightly less power at the pump wavelength, and significantly more power at the Raman Stokes wavelength, compared to the simulation with a linewidth of 30 GHz.

We have not been able to determine the relative importance of these two mechanisms in hindering the buildup of a quickly red-shifting soliton.
The simulations in Figs. 6(right) and 7(left) showed a quickly red-shifting soliton in only some of the simulations. The solitons also did not undergo the same amount of red-shift. For example, one simulation in Fig. 7(left) resulted in a soliton at \( \sim 1170 \text{ nm} \) and another simulation had a soliton at \( \sim 1270 \text{ nm} \). We show in Fig. 9 how averaging over the ensembles smooths the peaks from individual solitons. Experimental spectra were presented for similar pumping conditions in Refs. [11, 12]. The experimental spectra showed a large pump residual at 1064 nm, a Raman Stokes peak at 1116 nm about 11 dB below the pump residual, and a relatively flat continuum spanning from \( \sim 1150 \) to \( \sim 1350 \text{ nm} \) about 14 dB below the pump residual. We compare this with the ensemble average in Fig. 9(right), corresponding to case C, because this should be a better approximation to a real CW input than cases A and B. In our simulations the pump residual and Raman Stokes peak are at the same spectral power level. This could be caused by the fact that our simulations assume a polarized input pump and only consider propagation in one polarization axis, whereas the experiments were done with a non-polarized pump laser. Instead of a flat continuum from 1150 nm to 1350 nm our simulations show three distinct peaks in the same wavelength region. This occurs because we have only averaged over 10 ensembles; we know that the red-shift of each soliton is determined by the random initial conditions, so increasing the number of averaged ensembles would smooth the soliton spectra into a continuum [21]. Ideally, for comparison with the measurement of an optical spectrum analyzer with 1 ms integration time, one would need to average over \( \sim 10^7 \) ensembles, if each ensemble corresponds to a quasi-CW pulse of 30 ps duration. In practice, averaging over as few as 100 ensembles is found to give reasonable agreement with experimental measurements [16, 17].

4.3. Formation of several red-shifting solitons

We now consider the fiber with the structural parameters \( \Lambda = 1.72 \mu\text{m} \) and \( d/\Lambda = 0.378 \). This fiber has zero-dispersion wavelengths (ZDWs) at \( \sim 1025 \) and 1281 nm, as seen from Fig. 1(left). It is also seen that the dispersion at the pump wavelength is \( \sim 20 \) times smaller for this fiber, compared to the fiber with \( d/\Lambda = 0.65 \). The effective area \( A_{\text{eff}} \) is \( \sim 2 \) times larger, and using Eq. (14) this results in a MI maximum gain frequency of \( \nu_{\text{max}} = 2.5 \text{ THz} \), as was also indicated in Fig. 4(left). This means that approximately 3 times more solitons should be formed in this fiber than in the fiber with \( d/\Lambda = 0.65 \). This is also what we have observed in spectrograms. The larger number of solitons leads to a larger number of soliton collisions which in turn increases the probability of the creation of a quickly red-shifting soliton. Furthermore, the solitons are
not only more abundant, but also have a shorter temporal width giving them a larger red-shift rate (Fig. 5, right). The resulting spectra are seen in Fig. 10.

It is seen that the large number of red-shifting solitons results in a relatively smooth spectral broadening on the red side of the pump. The spectrum has achieved the large broadening at a shorter fiber distance ($z = 64$ m) than for the $d/\Lambda = 0.65$ $\mu$m fiber. We also observe the generation of dispersive waves in the normal dispersion region [4, 5] at $\sim 857$ nm and $\sim 1376$ nm.

The formation of many solitons due to the lower dispersion also means that the input linewidth is less critical than for the $d/\Lambda = 0.65$ fiber. As is seen in Fig. 10, the simulations with 30 GHz and 265 GHz linewidth show much more resemblance to each other than the corresponding simulations for the $d/\Lambda = 0.65$ fiber (Fig. 7).

5. Conclusion

Recently, it was claimed that solitons formed directly from modulation instability could red-shift far away from the CW pump [16, 17]. We found, for the cases investigated here and in Refs. [16, 17], that the solitons require a shorter temporal width to make a significant red-shift,
and showed that this can be achieved by energy transfer during soliton collisions.

It was shown how the time window width used in the calculations can affect the resulting spectrum and must be chosen carefully.

We demonstrated that increasing the spectral linewidth of the CW pump laser can hinder the formation of quickly red-shifting solitons. This indicates that narrow linewidth pump lasers should be selected for generating a broader supercontinuum. The number of solitons formed from a quasi-CW pulse and their red-shift rate can be increased by choosing a fiber with lower dispersion at the pump wavelength. This reduces the required fiber length and allows the use of a pump laser with broader linewidth while still obtaining quickly red-shifting solitons.

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