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Electric dipole antennas with magnetic-coated PEC cores: Reaching the Chu lower bound on Q

Oleksiy S. Kim

Abstract—The radiation properties of spherical electric dipole antennas with electric current excitation and material-coated perfectly electrically conducting (PEC) cores are investigated analytically using vector spherical wave functions. Closed-form expressions for electric and magnetic stored energy as well as the radiation quality factor Q are derived. Using these, it is shown that properly selected magnetic coating and radius of the PEC core vastly reduce the internal stored energy, and thus make the Q of an electric dipole antenna approach the Chu lower bound.

Index Terms—Electrically small antennas, fundamental limitations, electric dipole, quality factor, Chu lower bound

I. INTRODUCTION

A limited frequency bandwidth of electrically small antennas is one of the fundamental problems encountered in antenna miniaturization. It is well known that a fundamental lower bound exists for the antenna radiation quality factor Q , which is inversely proportional to the bandwidth for single-mode electrically small antennas. This bound, also known as the Chu lower bound, can be written as [1]–[3]

$$Q_{\text{Chu}} = \frac{1}{(ka)^3} + \frac{1}{ka} \quad (1)$$

where k is the free-space wave number. The bound (1) is derived assuming no stored energy (except that needed to make an antenna resonant) inside the minimum sphere of radius a circumscribing the antenna. When the internal stored energy is accounted for electric and magnetic dipole antennas with an air-core and electric excitation current, the bounds become $1.5Q_{\text{Chu}}$ and $3.0Q_{\text{Chu}}$, respectively, in the limit $ka \rightarrow 0$ [4]–[6]. Numerous embodiments of both kinds of antenna have confirmed these bounds [7]–[12].

The bounds for the air-core antennas can however be overcome. In [13], [14], it is shown that a solid magnetic core reduces the lower bound for magnetic dipole antennas to

$$Q \approx \{1 + 0.366(ka)^2\} Q_{\text{Chu}}. \quad (2)$$

Furthermore, a magnetic-coated metal core allows magnetic dipole antennas to reach the Chu lower bound irrespective of their electrical size ka [15]. Recent investigations of electrically small electric dipole antennas show that the Chu lower bound can be approached for these antennas as well, using thin high-permeability magnetic shells [16], [17].

In this paper, a spherical electric dipole antenna with a material-coated perfectly electrically conducting (PEC) core is studied. In particular, the radiation problem for the electric

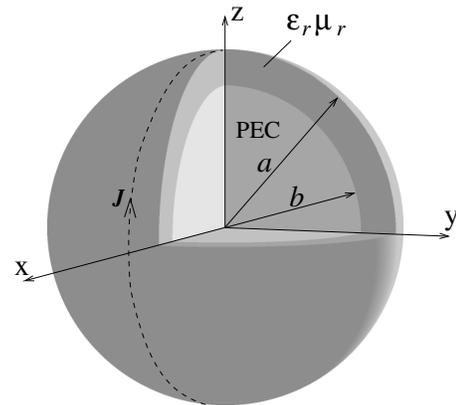


Fig. 1. Electric dipole antenna with a material-coated PEC core. The excitation is a time-harmonic electric surface current density \mathbf{J} distributed over the surface of the antenna.

current excitation is solved rigorously, and expressions for the stored energy and radiation quality factor Q are derived in closed form. It is shown that with such a core an electric dipole antenna can also exhibit the Q approaching the Chu lower bound.

The time factor $\exp(-i\omega t)$, where ω denotes the angular frequency and t is the time variable, is assumed and suppressed throughout the paper.

II. THEORY

We consider a PEC core of radius b enclosed in a concentric shell of radius a made of a magneto-dielectric material with the relative permittivity ϵ_r and relative permeability μ_r (Fig. 1). The material is assumed to be homogeneous, isotropic, and lossless. A Cartesian coordinate system and an associated spherical coordinate system (r, θ, ϕ) are defined so that their origin is located at the center the antenna core. On the outer surface of the shell, an impressed electric current density radiating the electric dipole (TM₁₀) fields is defined as

$$\mathbf{J} = \hat{\mathbf{a}}_\theta J_0 \sin \theta \quad (3)$$

where $\hat{\mathbf{a}}_\theta$ is the unit vector and J_0 is the amplitude (A/m). In practice, the ideal current excitation (3) can be closely approximated in many ways; see, for example, [7], [8], [10], [12].

The electric and magnetic fields inside the shell (superscript “-”) and in free space (superscript “+”) are represented in terms of vector spherical wave functions $\mathbf{F}(\mathbf{r})$ [18] as

$$\mathbf{E}^-(\mathbf{r}) = \frac{k_s}{\sqrt{\eta_s}} C^- \mathbf{F}_{201}^{(6)}(\mathbf{r}) \quad (4a)$$

$$\mathbf{H}^-(\mathbf{r}) = -ik_s \sqrt{\eta_s} C^- \mathbf{F}_{101}^{(6)}(\mathbf{r}) \quad (4b)$$

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$$\mathbf{E}^+(\mathbf{r}) = \frac{k}{\sqrt{\eta}} C^+ \mathbf{F}_{201}^{(3)}(\mathbf{r}) \quad (4c)$$

$$\mathbf{H}^+(\mathbf{r}) = -ik\sqrt{\eta} C^+ \mathbf{F}_{101}^{(3)}(\mathbf{r}) \quad (4d)$$

where η_s and η are the intrinsic admittances of the shell and free space, respectively, and $k_s = \sqrt{\varepsilon_r \mu_r} k$. The $\mathbf{F}^{(3)}(\mathbf{r})$ functions are as defined in [18] (also provided in [13], [15]). The $\mathbf{F}^{(6)}(\mathbf{r})$ functions can be obtained from, for example, $\mathbf{F}^{(5)}(\mathbf{r})$ in [15, eq.(5)] by replacing $z_1^{(5)}(k_s r)$ with a combination $z_1^{(6)}(k_s r)$ of the spherical Bessel j_1 and Neumann y_1 functions, so that the boundary condition on the surface of the PEC core is satisfied

$$z_1^{(6)}(k_s r) = y_1(k_s r) \{(k_s b) j_1(k_s b)\}' - j_1(k_s r) \{(k_s b) y_1(k_s b)\}' \quad (5)$$

In (5) and from now on, $\{f(x)\}'$ denotes the derivative of the function $f(x)$ with respect to its argument x .

The coefficients C^\pm are determined from the boundary condition on the outer surface of the shell $r = a$, subject to (3), as

$$C^- = J_0 i a \frac{1}{\sqrt{\eta_s}} \frac{4\sqrt{\pi}}{\sqrt{6}} \{(ka) h_1^{(1)}(ka)\}' I^{-1} \quad (6a)$$

$$C^+ = J_0 i a \frac{\sqrt{\eta}}{\eta_s} \frac{4\sqrt{\pi}}{\sqrt{6}} \{(k_s a) z_1^{(6)}(k_s a)\}' I^{-1} \quad (6b)$$

where

$$I = \frac{\eta}{\eta_s} k a h_1^{(1)}(ka) \{(k_s a) z_1^{(6)}(k_s a)\}' - k_s a z_1^{(6)}(k_s a) \{(ka) h_1^{(1)}(ka)\}'$$

To determine the stored energy, the energy density of the propagating field in free space is subtracted from the total energy density [3]. Then, by a direct spatial integration the internal electric W_E^- , internal magnetic W_H^- , external electric W_E^+ , and external magnetic W_H^+ stored energy are obtained as

$$W_{E/H}^\pm = \frac{1}{4\omega} |C^\pm|^2 \hat{W}_{E/H}^\pm \quad \text{with} \quad (7)$$

$$\hat{W}_E^- = \frac{1}{2} \left\{ L(k_s a) - L(k_s b) + (k_s a) \{R_1(k_s a)\}'^2 \right\}$$

$$\hat{W}_H^- = \frac{1}{2} \left\{ L(k_s a) - L(k_s b) - (k_s a) R_2(k_s a) \{R_1(k_s a)\}' \right\}$$

$$\hat{W}_E^+ = \frac{1}{(ka)^3} + \frac{1}{ka}$$

$$\hat{W}_H^+ = \frac{1}{ka}$$

where

$$L(k_s r) = R_1(k_s r) [(k_s r) R_1(k_s r) - R_2(k_s r)]$$

$$R_l(k_s r) = (k_s r) y_l(k_s r) \{(k_s b) j_l(k_s b)\}' - (k_s r) j_l(k_s r) \{(k_s b) y_l(k_s b)\}'$$

Finally, the radiation quality factor Q is found as

$$Q = 2\omega \frac{\max(W_E^- + W_E^+, W_H^- + W_H^+)}{P_{\text{rad}}} \quad (8a)$$

$$= \max \left(\frac{|C^-|^2}{|C^+|^2} \hat{W}_E^- + \hat{W}_E^+, \frac{|C^-|^2}{|C^+|^2} \hat{W}_H^- + \hat{W}_H^+ \right) \quad (8b)$$

using (6), (7), and the radiated power $P_{\text{rad}} = \frac{1}{2} |C^+|^2$.

III. MINIMIZING Q

First, it is instructive to examine the characteristics of the antenna with a pure magnetic shell ($\varepsilon_r = 1$) and a fixed electrical size ka . The ratio of the magnetic $W_H = W_H^+ + W_H^-$ to electric $W_E = W_E^+ + W_E^-$ stored energy and the quality factor normalized to the Chu lower bound Q/Q_{Chu} are shown in Fig. 2a and 2b, respectively, versus the shell permeability μ_r for $ka = 0.5$ and three values of the relative radius of the PEC core $b/a = 0.0, 0.6, \text{ and } 0.9$. For the antenna with $b/a = 0$, the stored energy is predominantly electric, as it should be for an electrically small electric dipole antenna, in which the stored magnetic energy is further suppressed by a solid magnetic core. The magnetic stored energy dominates only in very narrow regions between internal and radiating resonances, which appear interchangeably as μ_r increases (Fig. 2). The first type of resonance differs from the second one by the absence of the radiated power, so that $Q \rightarrow \infty$. As the relative radius of the PEC core increases, the separation between resonances becomes larger – particularly, for the first pair of resonances – and in these regions the magnetic stored energy dominates. An important consequence of the increased separation between the resonances is that the radiating resonance moves away from the region, where the Q is very high due to the internal resonance (Fig. 2b). As we show later, this opens a possibility for a self-resonant pure electric dipole antenna with a Q close to the Chu lower bound.

Meanwhile, we focus on determining the optimal geometry and material parameters minimizing the antenna Q . Inspecting Fig. 2b, we note that the increasing b/a leads to higher Q in the air-core case¹ ($\mu_r = 1$), whereas the minima between the resonances becomes deeper. Obviously, there is an optimum permeability μ_r providing the lowest Q for given ka , b/a , and ε_r . On the other hand, if both material parameters (μ_r and ε_r) are fixed, an optimum b/a can be determined for a given ka . Here, we consider the latter case, because it seems somewhat more practical to design geometrical rather than material parameters of an antenna.

The optimum radius of the PEC core is obtained by taking the derivative of (8) with respect to b/a as

$$\begin{aligned} \frac{\partial Q}{\partial(b/a)} &= \varepsilon_r \frac{1 - (ka)^2 + (ka)^4}{(ka)^3} \left\{ \frac{\partial}{\partial(b/a)} \left\{ \frac{\hat{W}_E^-}{(k_s a) \{R_1(k_s a)\}'^2} \right\} \right. \\ &\quad \left. \frac{\partial}{\partial(b/a)} \left\{ \frac{\hat{W}_H^-}{(k_s a) \{R_1(k_s a)\}'^2} \right\} \right\} \\ &\equiv \varepsilon_r f(ka) \begin{cases} \frac{\partial \Psi_E(k_s a, b/a)}{\partial(b/a)} & \text{for } W_E > W_H \\ \frac{\partial \Psi_H(k_s a, b/a)}{\partial(b/a)} & \text{for } W_E < W_H \end{cases} \quad (9) \end{aligned}$$

and then setting the result equal to zero

$$0 = \begin{cases} \frac{\partial \Psi_E(k_s a, b/a)}{\partial(b/a)} & \text{for } W_E > W_H \quad (10a) \\ \frac{\partial \Psi_H(k_s a, b/a)}{\partial(b/a)} & \text{for } W_E < W_H \quad (10b) \end{cases}$$

The equation (10) has multiple solutions; one with the highest b/a provides the lowest Q and is denoted as

$$b/a = \begin{cases} F_E(k_s a) & \text{for } W_E > W_H \quad (11a) \\ F_H(k_s a) & \text{for } W_E < W_H \quad (11b) \end{cases}$$

¹The reason for this behaviour is the same as for the TE₁₀-mode case, see [15, Section II.C]

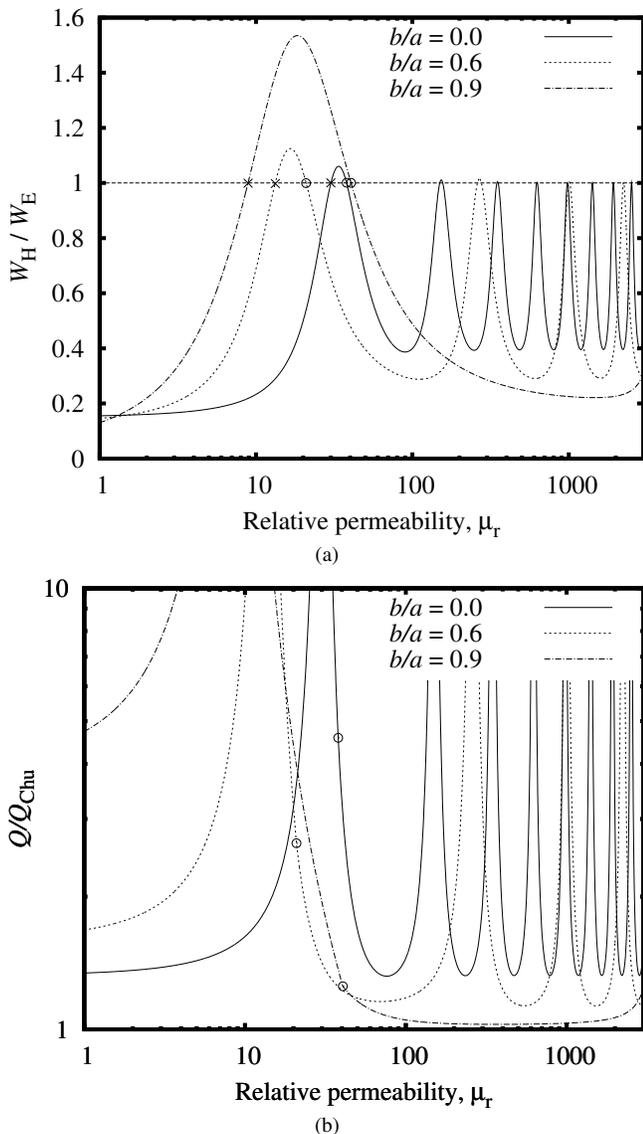


Fig. 2. Stored energy and Q as functions of μ_r ($\epsilon_r = 1$) for fixed $ka = 0.5$: (a) Ratio of magnetic to electric stored energy; (b) Q normalized to the Chu lower bound. The first internal and radiating resonances are marked with 'x' and 'o', respectively.

In other words, the function $b/a = F_E(k_s a)$ is the solution of (10a) and the function $b/a = F_H(k_s a)$ is the solution of (10b). Figure 3a shows both functions along with the parameters for the first radiating resonance. A resonant curve for given ka and ϵ_r divides the plot into two regions. The region to the left of the resonant curve corresponds to $W_E < W_H$, whereas to the right $W_E > W_H$. Consequently, the solution of (10) first follows the $F_H(k_s a)$ curve; once the resonance is reached, the solution transits to the $F_E(k_s a)$ curve. In the transition region, no solution exists for the equation (10), and therefore the transition between $F_H(k_s a)$ and $F_E(k_s a)$ occurs along the resonant curve. For some values of ka and ϵ_r , the resonant curve lies further to the right of the $F_E(k_s a)$ curve, as, for example, for $ka = 1.5$ and $\epsilon_r = 1$ in Fig. 3a. In this case, the lowest Q corresponds to the parameters that lie not at the $F_E(k_s a)$ curve, but along the resonant curve instead.

Note, that both $F_H(k_s a)$ and $F_E(k_s a)$ start at $k_s a \approx 1.416$.

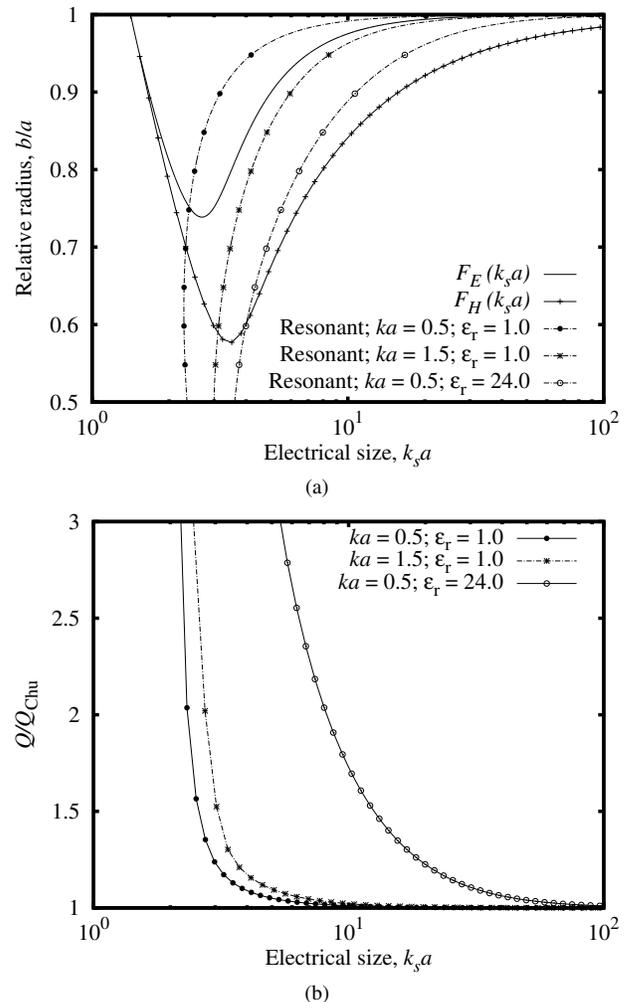


Fig. 3. Optimum and resonant relative radius of the PEC core b/a (a) and the corresponding lowest achievable ratio Q/Q_{Chu} for the optimum b/a (b).

Below this value, a solid core ($b/a = 0$) should be used. If an air-core is also an option, it is preferable to a magnetic-coated PEC core with parameters in the region $W_E < W_H$ in Fig. 3a, where the Q is generally larger than $1.5Q_{Chu}$.

By substituting the (11) into (8), an expression for the lowest possible Q corresponding to the optimum $b/a = F_{E/H}(k_s a)$ is obtained as

$$Q = \begin{cases} \epsilon_r f(ka) \Psi_E(k_s a, F_E(k_s a)) + Q_{Chu} & \text{for } W_E > W_H \\ \epsilon_r f(ka) \Psi_H(k_s a, F_H(k_s a)) + \frac{1}{ka} & \text{for } W_E < W_H \end{cases} \quad (12)$$

The resulting Q is plotted in Fig. 3b. From (12) and Fig. 3b, it is clear that the higher permittivity results in the higher Q . Nevertheless, as μ_r – and thus $k_s a$ – increases, $b/a \rightarrow 1$ and the Q approaches the Chu lower bound Q_{Chu} irrespective of the antenna electrical size ka and permittivity of the shell ϵ_r .

As discussed above, for some antenna sizes ka and shell permittivity ϵ_r , the lowest Q is achieved at the resonance. However, for electrically small antennas ($ka < 0.5$) and low permittivity, the optimal parameters for the lowest possible Q result in an antenna with the dominating electric stored energy (Fig. 3a). On the other hand, as seen in Fig. 3a, for large values of permeability μ_r , the resonant curve and the optimum $b/a = F_E(k_s a)$ tend to each other, which implies

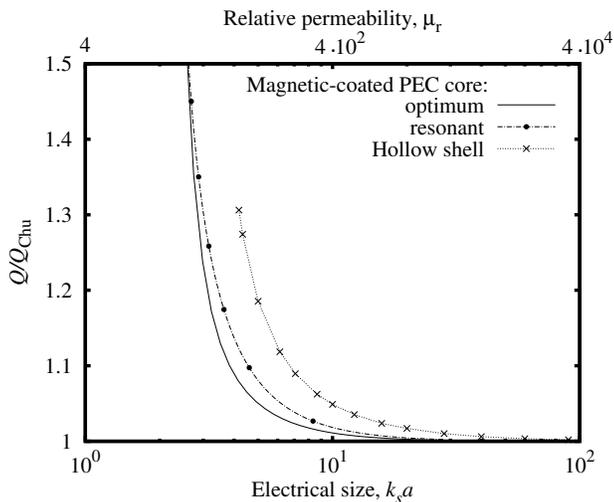


Fig. 4. The lowest achievable ratio Q/Q_{Chu} for a magnetic-coated PEC core (for the optimum and resonant b/a) and a hollow magnetic shell. In all cases, $ka = 0.5$ and $\epsilon_r = 1$.

that a self-resonant electric dipole antenna can be designed without significant sacrifice in Q . For example, for $ka = 0.5$ and $\epsilon_r = 1$, the difference between the Q 's corresponding to the optimum and resonant b/a becomes practically negligible for $\mu_r > 1000$ ($k_s a > 15.8$), as shown in Fig. 4.

The self-resonance of the TM_{10} mode is an advantage for an electric dipole antenna, since no additional reactive components are needed to compensate for the input reactance. On the other hand, if the lowest possible Q is a primary goal, a dual-mode $\text{TM}_{10} + \text{TE}_{10}$ antenna should be considered [1]. In such an antenna, the TM_{10} mode should generate an excess electric stored energy to match a corresponding amount of the excess magnetic stored energy of the TE_{10} mode, in which case the Q close to $0.5Q_{\text{Chu}}$ can be achieved, as demonstrated in [19].

A configuration with a hollow magnetic shell (without the PEC core) and the excitation electric current on the outer surface has also been investigated. COMSOL's 2D RF module for bodies of revolution was used. The antenna of $ka = 0.5$ was simulated for a number of μ_r values of the shell ($\epsilon_r = 1$); for each μ_r , the lowest Q in the range of b/a (b is the inner radius of the shell, in this case) from 0 to 1 was identified, and the results are presented in Fig. 4. In the limit of infinitely high permeability, the Q for both configurations – with and without PEC core – approaches the Chu lower bound. However, for finite μ_r , the PEC core allows the coating to have significantly lower permeability μ_r , as compared to the hollow shell, to achieve the same value of Q/Q_{Chu} . The reason is that a thin PEC-backed layer of magnetic material better approximates the perfect magnetic conductor. For instance, the hollow shell needs to have $\mu_r \approx 390$ to yield $Q/Q_{\text{Chu}} = 1.05$, whereas the magnetic coating on the PEC core requires only $\mu_r \approx 100$ to reach the same $Q/Q_{\text{Chu}} = 1.05$.

IV. CONCLUSION

The radiation problem for the TM_{10} electric current on the surface of a material-coated spherical PEC core is solved analytically using vector spherical wave functions. Closed-form expressions for the electric and magnetic stored energy

as well as for the radiation quality factor Q are presented. The optimum relative radius of the PEC core b/a providing the lowest possible Q for a given antenna electrical size ka and material parameters ϵ_r and μ_r of the coating is found. It is shown that a PEC core with an electrically thin high-permeability magnetic coating significantly reduces the internal stored energy, and thus the Q of the electric dipole antenna closely approaches the Chu lower bound. For instance, an antenna of the size $ka = 0.5$ with the relative radius of the PEC core $b/a = 0.975$ and a pure magnetic coating with $\mu_r = 400$ yields the quality factor $Q = 1.01Q_{\text{Chu}}$.

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