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da Providência, J.; Jalkanen, Karl J.; Bohr, Henrik

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J. da Providência, K.J. Jalkanen, and H.G. Bohr

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Abstract

Superconductivity is described by the well-known Bardeen-Cooper-Schrieffer (BCS) theory, which is a symmetry breaking approximation. Color superconductivity shows up in extremely high density matter and temperature, which is here investigated and compared to the other end of the scale of low energy/temperature of organic superconductors. An approach to color superconductivity conciliating the BCS theory with the color SU(3) symmetry, the cornerstone of the rigorous theory of the strong interaction, Quantum Chromo-Dynamics (QCD), is used to describe the superconducting phase.

The magnetization of a high density relativistic fluid of elementary particles is studied. We find that the magnetic field of spin polarized matter with densities of 2 to $3\rho_0$, where $\rho_0$ is the equilibrium density of nuclear matter, is rather huge, of the order of $10^{17}$ Gauss.

Finally we look at the chiral nature of nuclear forces and interactions as they possibly relate to chirality of nuclei (atoms) in molecules as a source of chirality in amino acids and hence in life. Previous works have not investigated the nuclear forces as a possible bias which initiated the bias towards L-amino acids as the building blocks on proteins, and later life.

1 Introduction

In this article we shall discuss symmetry breaking and various phases of superconductivity at high and low densities. The phenomenon is treated in an exact quantization scheme that is useful when analyzing symmetries. The formalism applies to all kinds of interactions, including applications to biophysical phenomena, but here we will mostly deal with the high energy scales. The

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*CFT, Department of Physics, University of Coimbra, 3004-516 Coimbra, Portugal. Email: providencia@teor.fis.uc.pt.
†DTU Nanotech, Department of Micro- and Nanotechnology, Technical University of Denmark, Building 345E, DK-2800 Kgs. Lyngby, Denmark. Email: karljalkanen@gmail.com
‡Quantum Protein (QuP) Center, Department of Physics, Technical University of Denmark, Building 309, DK-2800 Kgs. Lyngby, Denmark. Email: hbohr@fysik.dtu.dk
techniques of exact quantization by constructing annihilation and creation operators has also been shown to be useful in electronic quantum calculations of molecular structures. Ordinarily, organic matter with biomolecules is not usually associated with solid state concepts like ferromagnetism and superconductivity, which are related to the bulk properties of matter, and not the properties of individual molecules, which have been the focus of both theoretical and experimental chemists. Only with the recent development of material chemistry and nanoscience have chemists really appreciated that the properties of bulk matter, both theoretically and experimentally, versus those of individual molecules are fundamentally different. This has resulted from theoretical chemists first studying individual molecules in the gas phase, then in non-polar, noninteracting solvents, and finally more recently in polar, strongly interacting, hydrogen bonding solvents like water, the solvent (environment) of life [1]. However, the concepts of ferromagnetism and superconductivity are of great relevance to molecular biophysics as seen in the following.

Ferromagnetism and especially the Ising model are of great importance for modeling, e.g. protein folding and denaturation as well as isomerisation of biomolecules and analysis of protein phases and phase transitions. Actually some of the most successful models for folding have been spin-glass type models [3]. In addition, recently nucleic acids and proteins have been investigated for use as molecular wires, and then molecular electronic and spintronic devices [4]. Later we shall be mentioning briefly how nuclear chirality, coming from the different masses of the quarks making up the nucleons, could induce chirality in molecular orbital structure and also mention the organic superconductors in relation to inorganic superconductors.

The phenomena of superconductivity is certainly also relevant for biomolecules. Recent experiments by Kasumov et al. [5] have demonstrated that DNA also can exhibit properties like superconductivity in a similar way as that of nanotubes. There are also organic superconductors. In the following we shall focus more on the mathematical aspects of ferromagnetism and superconductivity and directly apply them (these concepts) to hadronic matter.

According to Sir Rudolf Peierls [6], two types of spontaneously broken symmetries are possible, namely, *spontaneously broken symmetry of the first kind*, where a symmetric ground state is degenerate with an asymmetric one, and *spontaneously broken symmetry of the second kind*, where the ground state belongs to a representation of the relevant symmetry group other than the identical one (identity). When spontaneous symmetry breaking of first kind occurs, linear combinations of states with different symmetries arise as a consequence of small perturbations inherent to the processes of preparation or measurement. Such perturbations may have their origin in the chiral nature of nuclear forces.

This is the case of optically active molecules, such as the chiral sugars like α-D-glucose and α-L-glucose, and of course Pasteur’s molecules, D- and L-tartaric acid. Pasteur first recognized differences in the chiral (left and right) objects (crystal shapes) under a light microscope, separated them, redissolved them in solution, and made measurements on the two solutions [2]. The two solutions had equal and opposite rotations of plane polarized light. This was a major breakthrough in chirality in chemistry and also showed how a property at a molecular level (chirality) can manifest
itself at the bulk crystalline and property level, that is, chiral, mirror image crystals, and then equal and opposite optical rotations of plane polarized light when dissolved in equal concentrations in nonchiral solvents. In solution it is nearly impossible to separate the individual chiral molecules from one another. But in the crystalline solid, in many cases it is, as performed by Pasteur.

To date no such easy experiment has been performed which has allowed one to separate chiral nuclei or nucleons. Note that the chiral objects of Pasteur had lifetimes and interactions with each other and their mirror images (enantiomers) which were such, that the chiral objects preferred their like partners versus their mirror image enantiomers. This made experiment possible, as Pasteur could perform the separation under a magnifying lens. For both nuclear, atomic and molecular cases, then one experimentally does not observe the (possibly degenerate) ground state(s) of the system, but one observes two non-stationary asymmetric states (forms) which will very slowly transform into one another in the course of time. Chemically this is called recemization. In many cases there is an energy barrier which makes the two enanriomers (mirror images) stable, and hence one can separate them. Because for nuclear states the excitation energy of the antisymmetric state is an extremely tiny fraction of the eV, the lifetime of each form (state) is extremely long. Again, a simple experiment like Pasteur’s which allowed the separation of the mirror image chiral crystalline macroscopic objects composed of chiral molecules has yet to be made for bare nuclei or nucleons.

An example of spontaneous symmetry breaking of second kind will be discussed in Section 3. A symmetry breaking approximation is also possible, but, in this case, it is the approximation method used to describe the symmetric ground state of a quantal system, which violates the symmetry, in order to take into account in a simple manner important correlations between the particles of the system. Color superconductivity is an example of such a case which is here discussed. A novel Bardeen-Cooper-Schrieffer (BCS)-type formalism has been constructed [7] in the framework of a schematic QCD inspired quark model [8], having in mind the description of color symmetrical superconducting states. In the usual approach to color superconductivity, the pairing correlations affect only the quasi-particle states of two colors (and two flavors), the single particle states of the third color (and the third flavor, if it plays a role) remain unaffected by the pairing correlations. This is the so-called 2SC phase [9, 10]. As a consequence, the average numbers of quarks depend on color and this is an unphysical and undesirable feature. The color $SU(3)$ symmetry should be preserved, at least on average. In [7] a theory of color symmetrical superconductivity has been proposed in which the pairing correlations affect symmetrically the quasi-particle states of the three colors so that a vanishing net color-charge is automatically insured. It has been found that the ground state energy of the color symmetrical sector of the so-called Bonn model [8] is well approximated by the average energy of the color symmetrical superconducting state proposed in [7].
2 Color superconductivity

It is generally accepted that quark and gluon fields are the building blocks of nuclear structure, in the framework of Quantum Chromo-Dynamics (QCD) as the rigorous theory of the strong interaction. The color quantum numbers are associated with the color $SU(3)$ symmetry which characterizes this non-Abelian gauge theory. The phase equilibrium of hadronic matter is a topic of great current interest of particular relevance to astrophysics and astrobiology. When the density of hadronic matter reaches values which are several times higher than the nuclear density at equilibrium, a great diversity of phases is expected, namely: restoration of chiral symmetry, deconfinement, and color superconductivity. Due to the complexity of the theory, it is extremely difficult, if not impossible, to obtain reliable results directly from QCD. It is therefore usual to resort to effective models such as the Nambu-Jona-Lasinio (NJL) model [11, 12, 13]. These models assume that the gluonic degrees of freedom produce an attractive two-body force between the quarks, which, effectively, replaces the gluons. Already in 1985, an interaction of this type was considered by Henrik Bohr and collaborators [8]. Later, Henrik Bohr was involved in the derivation of such an interaction from QCD by path integral techniques [14]. Since this force is attractive, it originates (or contributes to/is responsible for) the phenomenon of color superconductivity, which is analogous to the similar superconducting effect familiar from condensed matter physics and is described, in the framework of the BCS theory, as the formation of Cooper pairs. It is generally accepted that in hadronic matter, at several times the normal nuclear density, QCD leads to color superconductivity induced by the phenomenon of Cooper instability. It is supposed (hypothesized) that this phase may be found in the interior of neutron stars. The color superconducting phase of quark matter breaks color symmetry. Since this symmetry is a gauge symmetry, it is not actually broken by the physical state itself [16] and so, in the standard approach, the symmetry is preserved in the average as an artificial constraint insuring that the quark number does not depend on color. This constraint reduces inevitably the stability of the phase. Following [15], where strange matter was considered, we discuss here hadronic matter with flavor $SU(2)$ symmetry and show that this undesirable feature is easily avoided if the BCS approach is properly formulated, as proposed in [7].

2.1 Hamiltonian and thermodynamical potential

The thermodynamical potential is obtained from the Hamiltonian by imposing a constraint on the quark number. The chemical potential $\mu$ is the Lagrange multiplier which fixes the number of quarks in the nuclei as it does the number of electrons for atoms and molecules for the chemical potential in both wave function theory (WFT) and Kohn-Sham density functional theory (KS-DFT). Here we focus on 2-flavor quark matter. In the mean-field approximation, the thermodynamical potential operator for quark matter is given by
\[
\hat{K}_{MFA} = \int d^3x \left[ \bar{\psi}(p \cdot \gamma + M - \mu \gamma_0)\psi + \frac{1}{2} \sum_{j \in \{2, 5, 7\}} \left( (\Delta_j^* \bar{\psi}^C \gamma_5 \lambda_j \tau_2 \psi + h.c.) + \frac{|\Delta_j|^2}{2G_c} \right) \right],
\]

where
\[
\Delta_j = -G_c (\bar{\psi}^C \gamma_5 \lambda_j \tau_2 \psi).
\]

Here, \((\bar{\psi}^C \gamma_5 \lambda_j \tau_2 \psi)\) denotes the expectation value of the diquark condensate \((\bar{\psi}^C \gamma_5 \lambda_j \tau_2 \psi)\) in the BCS vacuum, \(\lambda_j, j \in \{2, 5, 7\}\), are the antisymmetric Gell-Mann matrices, and \(\tau_2\) is the antisymmetric Pauli matrix.

The current quark mass is \(M\). We wish to work in momentum space. Therefore, we will assume that, in the mean-field approximation, the thermodynamical potential is given simply by

\[
\hat{K}_{MFA} = \sum_{iap} \epsilon_p c_{iap}^\dagger c_{iap} + \sum_{ijkab} \Delta_k c_{iap}^\dagger c_{i\eta_{ab}}^\dagger c_{i\eta_{ab}} + \frac{V}{2G_c} \sum_j \Delta_j \Delta_j^* ,
\]

\[
\Delta_k = \frac{G_c}{V} \sum_{ijab} (c_{i\eta_{ab}} c_{i\eta_{ab}}^\dagger) \epsilon_{ijk} \epsilon_{ab} \eta_p,\tag{1}
\]

where \(\epsilon_p = \sqrt{p^2 + M^2 - \mu}\).

It is convenient to define the effective gap as

\[
\Delta_{eff} = \sqrt{\frac{1}{3} \sum_j \Delta_j \Delta_j^*}.\]

We denote the color indices as 1, 2, 3, meaning red, green, blue, and the flavor indices as \(a, b\). Upon a color rotation the gaps \(\Delta_j, j \in \{1, 2, 3\}\), change, but \(\Delta_{eff}\) and the equation of state do not change.

We postulate that the BCS vacuum of the so-called 2SC phase is given by

\[
|\Phi⟩ = e^S|\Phi_0⟩, \quad |\Phi_0⟩ = \prod_{iap(\epsilon_p \leq 0)} c_{iap}^\dagger |0⟩,
\]

where

\[
S = -\sum_{p(\epsilon_p > 0)} K_p \sum_{ijkab} c_{iap}^\dagger c_{i\eta_{ab}}^\dagger c_{i\eta_{ab}} + \sum_{p(\epsilon_p \leq 0)} \tilde{K}_p \sum_{ijkab} c_{iap} c_{i\eta_{ab}}^\dagger c_{i\eta_{ab}}.\]

It follows that the BCS vacuum satisfies

\[
d_{iap} |\Phi⟩ = d_{iap} |\Phi⟩ = 0, \quad i \in \{1, 2, 3\}, a \in \{u, d\}\]

being, for \(\epsilon_p = \sqrt{p^2 + M^2 - \mu} > 0\),

\[
d_{1up} = c_{1up} + K_p (c_{2up}^\dagger - c_{3up}^\dagger) \eta_p, \quad \eta_p = -\eta_p.
\]
The remaining operators are obtained by cyclic permutations of the indices 1, 2, 3 and $u, d$. For $\epsilon_p = \sqrt{p^2 + M^2} - \mu \leq 0$ we have,

$$d_{1ap} = c_{1ap}^\dagger - \tilde{K}_p(c_{2ap} - c_{3ap})\eta_p,$$

plus cyclic permutations of the indices 1, 2, 3 and $u, d$. The operators $d_{iap}$ and $d_{jbp}^\dagger$ do not obey canonical anti-commutation relations, since, $\{d_{iap}, d_{jbp}^\dagger\} \neq \delta_{ij}\delta_{ab}\delta_{pq}$, although $\{d_{iap}, d_{jbp}\} = 0$.

From the postulated ansatz it follows that the gaps $\Delta_k$ are real and $k$-independent, that is, $\Delta_k = \Delta, \; k = 1, 2, 3$. It is easily found that the optimal BCS vacuum leads to the gap equation

$$1 = \frac{2G}{V} \left( \sum_{p, \epsilon_p \leq 0} + \sum_{p, \epsilon_p > 0} \right) \frac{1}{\sqrt{\epsilon_p^2 + 3\Delta^2}},$$

and to the expression for the average thermodynamical potential (1), which is given by

$$\langle \tilde{K}_{MFA} \rangle = \sum_{p, \epsilon_p \leq 0} \left( 7\epsilon_p - 2\sqrt{\epsilon_p^2 + 3\Delta^2} \right) + \sum_{p, \epsilon_p > 0} \left( 2\epsilon_p - 2\sqrt{\epsilon_p^2 + 3\Delta^2} \right) + 3V\frac{\Delta^2}{2Gc}.$$

Next, we give the occupation numbers. With $i \in \{1, 2, 3\}, \; a \in \{u, d\}$, we find:

$$\langle \phi | c_{iap}^\dagger c_{iap} | \phi \rangle = \frac{2}{3} \left( \frac{1}{3} \frac{\epsilon_p}{\sqrt{\epsilon_p^2 + 3\Delta^2}} \right) \quad \text{for} \quad \epsilon_p \leq 0,$$

$$\langle \phi | c_{iap}^\dagger c_{iap} | \phi \rangle = \frac{1}{3} \left( \frac{1}{3} \frac{\epsilon_p}{\sqrt{\epsilon_p^2 + 3\Delta^2}} \right) \quad \text{for} \quad \epsilon_p > 0,$$

showing that color symmetry is indeed preserved on average by the postulated ansatz. That it is equivalent to the 2SC phase (with the important advantage that the constraint imposing color symmetry on average has been avoided) follows from the observation that by an appropriate color rotation it is possible to transform the gaps $\Delta_1 = \Delta, \; \Delta_2 = \Delta_3 = 0$, into $\Delta_1 = \Delta_2 = \Delta_3 = \Delta/\sqrt{3}$.

### 3 Polarized hadronic matter

Following refs. [17, 18], we consider a possible mechanism for magnetization and for spin-polarization of matter at very high densities, which may be considered a spontaneously broken symmetry of the second kind. In those works, it was reported that, due to the use of non-relativistic Skyrme forces, permanent magnetization may arise at quite low densities. A relativistic counterpart of these forces involving a 4-fermion tensor-tensor interaction that is expected to lead to a similar behavior may easily be found. Let us consider the relativistic chiral invariant analog of the Skyrme interaction, appropriate for an effective QCD approach, which is given by

$$L_{TT} = G_{TT}\left[ (\bar{\psi}\gamma^\mu\gamma^\nu\gamma_5\tau_k\psi)(\bar{\psi}\gamma_\mu\gamma_\nu\gamma_5\tau_k\psi) - (\bar{\psi}\gamma^\mu\gamma^\nu\gamma_5\psi)(\bar{\psi}\gamma_\mu\gamma_\nu\gamma_5\psi) \right].$$

If we postulate this interaction, the mean field approximation leads to the Lagrangian

$$L_{MFA} = (\bar{\psi}\gamma^\mu\partial_\mu\psi) - F_k(\bar{\psi}\Sigma_\Delta\tau_k\psi) - \frac{F_k^2}{2G}, \quad F_k = G(\bar{\psi}\Sigma_\Delta\tau_k\psi),$$
which in turn leads to the Dirac equation

$$(-i\alpha \cdot \nabla + \tau_3 F \beta \Sigma_3)\psi = \varepsilon \psi,$$

where $\varepsilon = 1$ for quarks $u$ and $\varepsilon = -1$ for quarks $d$ denote the eigenvalues of $\tau_3$. At very high density we may assume that chiral symmetry has been restored, so that the current quark mass may be neglected. The eigenvalues of the Dirac equation then are given by

$$\varepsilon_p = \pm \sqrt{|F| \pm \sqrt{p_1^2 + p_2^2} + p_3^2}.$$

Let us now assume, for simplicity, that $|F| > \mu$. Then the system is fully polarized and only the levels with energy $((|F| - \sqrt{p_1^2 + p_2^2})^2 + p_3^2)^{1/2}$ are filled up so that the Fermi surface is not spherical but doughnut shaped and, taking into account that $6$ is the color-flavor degeneracy, the particle number and the energy are given by

$$N = \frac{12V}{(2\pi)^3} \int d^3p \theta(\varepsilon_p - \mu) = \frac{12V}{(2\pi)^3} \pi^2 |F| \mu^2,$$

$$E = \frac{12V}{(2\pi)^3} \int d^3p \theta(\varepsilon_p - \mu) \varepsilon_p + \frac{VF^2}{2G} = \frac{8V}{(2\pi)^3} \pi^2 |F| \mu^3 + \frac{VF^2}{2G},$$

respectively, where the term $VF^2/(2G)$ comes from the standard mean field approximation. Notice that $-\mu \leq p_3 \leq \mu$ and $F - \mu \leq \sqrt{p_1^2 + p_2^2} \leq F + \mu$. The thermodynamical potential then becomes

$$\Omega = E - N\mu = -\frac{4V}{(2\pi)^3} \pi^2 |F| \mu^3 + \frac{VF^2}{2G},$$

implying that

$$F = \frac{4G \pi^2 \mu^3}{(2\pi)^3}.$$

The condition $|F| > \mu$ implies $\frac{4\pi^2 G \mu^2}{(2\pi)^3} > 1$ $(G \mu^2 > 6.28)$ which may occur, depending on the $G$ value, if $\mu$ is high enough. Obviously, quarks $u$ and $d$ are oppositely polarized.

### 4 Comparison to superconductivity in soft organic matter

We have in the previous sections seen that superconductivity can appear in high density hadronic matter at high temperature and with a quite large energy band gap. The situation of superconductivity is quite the opposite in organic superconductors [19] and especially in the case of superconductivity in DNA. Here, the DNA molecule acts as an electric conductor at low temperature, and below 1 K even as a superconductor. As with organic, polymeric superconductors the band gap is very small. It is around 0.1 meV or about 10 times smaller than that of the case of Mercury following the rule of scaling with the critical temperature. The critical temperature for color superconductivity is about 1000 times higher than that of superconductivity in DNA and so is the energy of the band gap.
5 Induced Chirality

One of the biggest mysteries about the structure of biomolecules is their apparent chirality or breaking of chiral symmetry such as in 19 of the 20 naturally occurring amino acids that all have left-handed chirality. This sudden breaking of chiral symmetry is here thought to arise from the breaking of the chiral symmetry inside the nucleons during the formation of these building blocks of life early on. The quarks that build up the nucleons are born as chiral objects due to their different masses. This is seen in the expression for the thermodynamical potential operator both with quark fields and with creation-annihilation operators in Eq.(1). Especially in the case of isotopes, with different number of neutrons and hence different number of “down” quarks, the chirality is expected to appear, or possibly be induced in the electronic orbitals. Here, the carbon molecules are clear cases of how chirality could be induced. This is since carbon compounds always appear in a the strong mixture of isotopes in the carbon atoms (e.g. C-14), that is hypothesized to influence the orbital symmetry around the carbon nucleus and hence in the bonding they can participate in. Such chirality is induced by long-range electromagnetic, both static and dynamic forces and momentum, such as electric dipole-electric dipole, electric dipole-magnetic dipole, and electric dipole-electric quadrupole dipole interactions, and the static and dynamic electric dipole-electric dipole, electric dipole-magnetic dipole, and electric dipole-electric quadrupole dipole polarisability tensors and interactions with the other particles, atoms and/or atomic fragments during the formation of amino acids and other chiral molecules, including carbohydrates which are also inherently chiral. Such chiral dynamic forces and interactions will be expected to influence the molecular bonding that the carbon atoms take part in. Carbon is a special atom as its valence of four has possible inherent chirality when 4 different substituent groups are all added to form, for example, a D- or L- amino acid, -NHO_{\alpha}(R)CO-, where the C_{\alpha} carbon is the chiral center. Specifically, the left-right handedness of the amino acids is a matter of the orientation of the side-chains fixed to the C_{\alpha} carbon atoms in the polypeptide backbone of proteins, and also in 19 of the naturally occurring amino acids. Glycine with the side chain being hydrogen of course lacks chirality. There are other cases of breakdown of chiral symmetry especially in crystallization of melts [20, 21]. Finally, there has been recent work on trying to understand the mechanism of the Soai asymmetric autocatalytic reaction using first principles calculations where the interaction of achiral molecules with chiral surfaces results in chiral molecules [22, 23].

6 Conclusions

Here, we have investigated matter at extremely high densities, expected to exhibit the phenomenon of color superconductivity. The well-known BCS theory, which is a symmetry breaking approximation, is the standard tool for the description of the superconducting state. An approach to color superconductivity conciliating the BCS theory with the color SU(3) symmetry, the cornerstone the rigorous theory of the strong interaction, QCD, is used to describe the superconducting
phase. Finally the phenomenon of superconductivity at high density is compared to that of the other end of the scale of soft matter of biomolecules where the critical temperature is extremely low and the gap size is small and we also speculate that the colorful quarks could induce chiral effects in biomolecules.

We have also investigated the spontaneous spin-polarization of matter at extremely high densities and the ensuing magnetization. We have found that the magnetic field of spin polarized matter with densities of $2 \times 3\rho_0$, where $\rho_0$ is the equilibrium density of nuclear matter, is rather huge, of the order of $10^{17}$ Gauss.

## 7 Acknowledgements

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