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Publication date:
2014

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):

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An Equivalent Source Method for Modelling the Global Lithospheric Magnetic Field

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Summary
We produce a new model of the global lithospheric magnetic field based on 3-component vector field observations at all latitudes from the CHAMP satellite using an equivalent source technique.

Method
A regularized iteratively reweighted least squares algorithm is applied. Data error covariance matrices are implemented, including both the latitude dependence of data error variances \( \sigma^2 \) (Fig. 1) and covariances \( \mathbf{C} \) between the vector field components due to unmodelled sources. The regularization norm \( \mathbf{R} \) is defined to be the Euclidean length of the model solution. Our scheme iteratively minimizes:

\[
\Theta(m_i) = \mathbf{d} - \mathbf{G} m_i \mathbf{W}_i^{-1} (\mathbf{d} - \mathbf{G} m_i) + \lambda \mathbf{R}(m_i)
\]

Huber weighting ensures a robust solution in the presence of non-Gaussian data errors

\[
\mathbf{H}_i = \min \left( \frac{1.5}{d - \mathbf{G} m_i} / \sigma^2 \right)
\]

An initial unregularized (\( \lambda = 0 \)) model is derived using 10 iterations. The final model is obtained with 5 further iterations using quadratic regularization and \( \lambda = 3 \times 10^{-13} \).

Equivalent Source Method
The equivalent potential field sources \( \mathbf{m} \) (monopoles) are arranged in an icosahedron grid (Fig. 2), consisting of \( K = 30722 \) vertices and midpoints, placed at a depth of 100km below the Earth’s surface. The derived model can be transformed into a spherical harmonic representation by:

\[
g_\ell^m = \sum_{k=1}^{K} \frac{\rho_i}{\omega_i^2 + \rho_i^2} m_k P_\ell^m(\cos \theta) \cos (\phi_k)
\]

\[
h_\ell^m = \sum_{k=1}^{K} \frac{\rho_i}{\omega_i^2 + \rho_i^2} m_k P_\ell^m(\cos \theta) \sin (\phi_k)
\]

Results and Outlook
The presented model has a power spectrum that compares well to CHAOS-4, MF7 and CMS5 (cf. Poster EGU2014-6883) models to degree \( n = 100 \) (Fig. 4). Ongoing investigations concern non-quadratic regularization using maximum entropy. Looking forward, we plan to explore local grid refinement options in order to incorporate aeromagnetic survey data.

Fig 1: Latitude dependent standard deviations of the assigned errors due to unmodelled sources.

Fig 2: Right: Icosahedron grid with 483 of vertices (red) and midpoints (green). Left: The corresponding \( K \) equivalent potential field sources \( \mathbf{m} \) can be directly transformed into spherical harmonics of order \( \ell \) and degree \( n \). \( \phi \) = Earth mean radius, \( (\theta, \phi) \) = source location.

Fig 3 left: Final model degree correlation with CHAOS-4, MF7, GRIMM-L120 and CMS. right: Sensitivity matrix between final model and CHAOS-4. The scale saturates at 100 nT.

Fig 4 left: Modeled lithospheric radial magnetic field at the Earth’s surface for degree \( n = 16-180 \). The scale saturates at 200 nT. right: Power spectrum for MF7, CMS, CHAOS-4 models in comparison to model results with different regularization damping values. The chosen model is represented by the red line.