Prediction of the creep properties of discontinuous fibre composites from the matrix creep law

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Abstract

Existing theories for predicting the creep properties of discontinuous fibre composites with non-creeping fibres from matrix creep properties, originally based on a power law, are extended to include an exponential law, and in principle a general matrix law. An analysis shows that the composite creep curve can be obtained by a simple displacement of the matrix creep curve in a log $\sigma$ vs. log $t$ diagram. This principle, that each point on the matrix curve has a corresponding point on the composite curve, is given a physical interpretation. The direction of displacement is such that the transition from a power law to an exponential law occurs at a lower strain rate for the composite than for the unreinforced matrix. This emphasizes the importance of the exponential creep range in the creep of fibre composites. The combined use of matrix and composite data may allow the creep phenomenon to be studied over a larger range of strain rates than would otherwise be possible. A method for constructing generalized composite creep diagrams is suggested. Creep properties predicted from matrix data by the present analysis are compared with experimental data from the literature.
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1. INTRODUCTION

Several models have been proposed (Mileiko 1970; Kelly & Street 1972; McLean 1972; Bøcker Pedersen 1974) for the prediction of creep properties of discontinuous fibre composites from matrix creep properties. A review of the early models has been given by Lilholt (1973). These models have only been explicitly evaluated on the basis of a power law for creep of the matrix. It is, however, a well-known fact (Sherby & Burke 1967) that the power law is only a good approximation for the creep of pure materials at diffusion-compensated strain rates, $\dot{\varepsilon}/D$, lower than approximately $10^9$ cm$^{-2}$. At higher strain rates an exponential law is usually a better approximation to the experimental creep properties.

Experiments by Kelly & Tyson (1966) on the creep of silver reinforced with tungsten fibres showed stress sensitivities indicative of exponential creep. We therefore find it relevant to examine the influence of the matrix creep law - and in particular an exponential matrix creep law - on the creep law of a fibre composite. An important result of the analysis is that the transition from a power law to an exponential law occurs at a lower strain rate for the composite than for the unreinforced matrix. This emphasizes the importance of the exponential range for the creep of fibre composites.

2. THEORY

We shall base our analysis on the models by McLean (1972) and Kelly & Street (1972). McLean's model is chosen because of
its simplicity which makes a physical interpretation of the results very easy. Kelly & Street's model is chosen partly because it provides a more detailed description of the deformation, and partly because earlier experiments (Kelly & Tyson 1966) have been compared with predictions based on this model (Street 1971). We shall treat the two models separately and finally make a comparison between the results obtained from each model.

It is assumed in both models that the fibres are perfectly aligned and of identical shape and size, and that the stress is applied parallel to the fibre direction. We further confine the analysis to one of the cases considered by Kelly & Street, namely composites with non-creeping fibres and perfect interfacial bond.

A list of the symbols used in the following analysis is presented in the appendix.

2.1. Analysis based on McLean's model

McLean considered a creeping matrix, which creeps in shear only. He established a relation between the average shear strain rate in the matrix and the overall tensile strain rate of the composite. By combining this with a strain energy balance and a matrix creep law (viz. a power law) he obtained an expression for the creep strength of the composite.

McLean assumed a doubling of the shear strain rate in the matrix in order to compensate for hole formation in the matrix. We omit this doubling so as to obtain an equal basis for the comparison with the results derived from Kelly and Street's model. We note that the conversion relation between shear strain and tensile strain is \( \gamma = \frac{3}{2} \varepsilon \) for simple extension and \( \gamma = 2 \varepsilon \) for simple shear. Stresses are in both cases related by \( \sigma = 2 \tau \).

The relations for simple extension are used for conversion between shear values and tensile values obtained from creep tests on pure matrix. The relations for simple shear are used for conversion between the (local) shear values in the matrix of the composite and their corresponding (local) tensile values.

On this basis the geometrical relation between the average shear strain rate in the matrix and the overall tensile strain rate of the composite is given by

\[
\dot{\varepsilon}_c = \frac{3}{2} \dot{\gamma}
\]

or

\[
\dot{\varepsilon}_c = \log(2 \dot{\gamma} + \log(\frac{3}{2} \dot{\gamma}))
\]

The rate of work done on the composite equals the rate of (average) energy dissipation in the matrix. Since the matrix is assumed to deform in simple shear, the expression is (McLean's eq. 2):

\[
\sigma_c \cdot \dot{\varepsilon}_c = \bar{T} \cdot \dot{\gamma} \cdot v_m
\]

Using eq. (1) we find

\[
\sigma_c = (2 \bar{T}) \cdot v_m \cdot \frac{1}{4} \varepsilon
\]

or

\[
\log \sigma_c = \log(2 \bar{T}) + \log(\frac{\varepsilon}{4})
\]
The equations (3) and (6) suggest a principle of corresponding points: in a log \( \varepsilon \)- vs. log \( \sigma \) diagram a point on the matrix creep curve corresponds to a point on the composite creep curve obtained by displacement of the matrix point by \( \log(\frac{\varepsilon}{\bar{\varepsilon}}) \) along the log \( \varepsilon \)-axis and by \( \log(\frac{\sigma}{\bar{\sigma}}) \) along the log \( \sigma \)-axis. The composite creep curve can thus be generated from a known matrix creep curve by a simple displacement of the latter. This is illustrated in figure 1, where the tensile values \( \sigma_0, \varepsilon \) and \( \sigma_1, \varepsilon_1 \) (equivalent to the shear values of the matrix) are indicated.

The physical interpretation of the principle of corresponding points is obvious in McLean's model. McLean assumed an average shear strain rate in the matrix (eq. 1) and thereby also an average shear stress. The corresponding points are thus points which represent the same (average) shear values in the composite and in the unreinforced matrix. The stress sensitivity is governed by the shear values, and is therefore also the same in corresponding points.

We note that the components of the displacement vector \( \left( \log(\frac{\varepsilon}{\bar{\varepsilon}}), \log(\frac{\sigma}{\bar{\sigma}}) \right) \) are independent of \( \varepsilon \) and \( \sigma \) and that they are derived without any reference to the form of the matrix creep law. The derivation is therefore valid for any matrix creep law, and thus also for an exponential creep law.

The matrix creep law may consequently be introduced in the general form

\[ \sigma = f(\varepsilon) \]
From eqs. (2) and (5) it is clear that the creep law for the composite is obtained by replacing \( \sigma \) with \( \sigma_c \), \( \varepsilon \) with \( \varepsilon_c \), and \( \frac{h_0}{h} \). This leads to the composite creep law

\[
\sigma_c \frac{h_0}{h^2} = f(\varepsilon_c \cdot \frac{h}{h_0})
\]  

(8)

In the above treatment we have followed McLean in omitting an additive matrix term, which should be included in order to make the composite creep law converge to the matrix creep law for \( V_f \) approaching zero. Such a term is included in the model by Kelly & Street. Numerical calculations show that this term contributes less than about 20% when \( \rho V_f \) is larger than about \( \rho \). We therefore take \( \rho V_f = \rho \) as the limiting condition for our analysis of McLean's model. This condition is fulfilled in most practical composites.

2.2. Analysis based on Kelly & Street's model

Kelly & Street considered a creeping matrix which creeps in shear and in tension. They included the tensile creep of the matrix through the additive matrix term. We shall omit this term, and as discussed in § 2.1., this is acceptable for \( \rho V_f > \rho \). We further note that it is implied in their analysis that \( \rho \) cannot be too small.

Kelly & Street expressed the shear strain rate in the matrix as a linear function of distance, \( z \), along the fibres. They obtained the shear stress in the matrix (and at the interface) by combining the expression for \( \dot{\gamma} \) with a matrix creep law. By integration they found the fibre stress as a function of \( z \). Finally they evaluated the average fibre stress and used it to establish the creep strength of the composite.

In an analysis based on their model, it is necessary to introduce a specific creep law for the matrix. In order to obtain simple analytical solutions to the integrals involved, we employ the approximate creep laws - the power law at intermediate stresses and the exponential law at high stresses - rather than any of the several creep equations, containing a hyperbolic sine function, which have been proposed to cover both regions (Sherby & Burke 1967).

When an exponential crepe law

\[
\dot{\varepsilon} = \dot{\varepsilon}_0 \exp(\frac{\sigma}{\rho_0})
\]

(9)

is used instead of a power law, Kelly & Street's eq. (7) modifies to

\[
\dot{\gamma} = \frac{3}{2} \dot{\varepsilon}_0 \exp(\frac{2\gamma}{\rho_0})
\]

(10)

The shear strain rate is given by

\[
\dot{\gamma} = \frac{\dot{\varepsilon}_0}{h}
\]

(11)

Here \( h \) is the thickness of the zone of constant shear strain rate. We shall express \( h \) as

\[
h = a \cdot d
\]

(12)

Kelly & Street set \( h \) equal to one half of the minimum spacing between surfaces of fibres in a hexagonal array, which corresponds to

\[
a = \left[ \frac{2\pi r}{\sqrt{3}} V_f \right]^{\frac{3}{2}} - 1
\]

(13)

By combining eqs. (10), (11) and (12) we find
The stress distribution in the fibres is found from the integral
\[ \sigma_f = \frac{\rho}{2} \int_0^{1/2} \sigma_0' \ln \left( \frac{\rho \cdot \varepsilon_C}{3\sigma \varepsilon_0} \right) \, dz = \sigma_0' \frac{2}{3} \ln \left( \frac{2\rho \cdot \varepsilon_C}{3\sigma \varepsilon_0} \right) \] (15)
and the average stress in the fibres becomes
\[ \bar{\sigma}_f = \frac{2}{3} \int_0^{1/2} \sigma_0' \frac{2}{3} \ln \left( \frac{\rho \cdot \varepsilon_C}{3\sigma \varepsilon_0} \right) \, dz \] (16)

The composite strength is calculated from the law of mixtures, and ignoring the additive matrix term we obtain
\[ \sigma_c = \bar{\sigma}_f V_f = \frac{\sigma_0' V_f}{2} \ln \left( \frac{\rho \cdot \varepsilon_C}{3\sigma \varepsilon_0} \right) \] (17)
which may be rewritten
\[ \frac{2\sigma_0'}{\sigma V_f} = \ln \left( \frac{\rho \cdot \varepsilon_C}{3\sigma \varepsilon_0} \right) \] (18)

On the basis of a power law, Kelly & Street obtained the following expression for \( \sigma_c = \bar{\sigma}_f V_f \)
\[ \sigma_c = \left( \frac{2}{3} \right)^{n/n} \left( \frac{\varepsilon_C}{3\varepsilon_0} \right)^{1/n} \] (19)
which may be recast
\[ \frac{(2n+1) \sigma_c}{n(\varepsilon_C)^{n/n} \bar{\sigma}_f \varepsilon_0} = \left( \frac{\rho \cdot \varepsilon_C}{3\sigma \varepsilon_0} \right)^{1/n} \] (20)

The factor \( \frac{2n}{n(\varepsilon_C)^{n/n}} \) varies from 1.975 for \( n=3 \) to 1.898 for \( n=7 \), and can thus to a good approximation be set equal to 2, so that the left hand sides of eqs. (18) and (20) become identical.

This indicates that a matrix law, \( \sigma = f(\varepsilon) \), which can be approximated by a power law at intermediate stresses and by an exponential law at high stresses, in Kelly & Street's model leads to a composite creep law
\[ \sigma_c = \frac{2}{\sigma V_f} = f(\varepsilon_c) \] (21)

This formulation is analogous to that presented in eqs. (7) and (8) and therefore leads to the logarithmic expressions
\[ \log \sigma_c = \log \sigma + \log(\frac{\sigma V_f}{2}) \] (22)
\[ \log \varepsilon_c = \log \varepsilon + \log \left( \frac{3 \sigma \varepsilon_0}{\rho} \right) \] (23)

In the model by Kelly & Street the components of the displacement vector are thus \( (\log(\sigma V_f), \log(3 \sigma \varepsilon_0)) \).

The physical interpretation of the displacement vector is not so immediately obvious in this model as in the model by McLean, because Kelly & Street assume that \( \gamma \) varies linearly with \( z \) along the fibre length (eq. 11). Eq. (18) does show, however, that in the exponential range there is one and only one displacement vector which is of constant magnitude and direction. This is also the only vector which connects points with the same stress sensitivity. A combination of eqs. (11) and (23) shows that the displacement vector connects a composite point with the matrix point which has the same value of \( \gamma \) as the composite has at \( z = \frac{1}{2\varepsilon_0} \). In other words, for an exponential matrix creep law, the stress sensitivity varies along the fibre length, but the overall stress sensitivity of the composite is the same as that found at \( z = \frac{1}{2\varepsilon_0} \). The value of \( \gamma \)
at this point is therefore the reference value which causes identical stress sensitivities of the matrix and the composite.

In the power law range the stress sensitivity is the same all along the fibre length, and the reference value of $\gamma$ could therefore be chosen arbitrarily at any $z$ along the fibre length. But since it is practical to operate with the same displacement vector in the power law range and in the exponential law range, we choose $\gamma$ at $z = \frac{1}{2\alpha}$ as the reference value also in the power law range. This choice is made by introducing the factor $(\alpha)^{-1/n}$ in going from eq. (19) to eq. (20).

2.3. Comparison between the displacement vectors obtained from the two models

The displacement vectors obtained from the two models can be compared by setting $s = 2h = 2\alpha d$ in the vector obtained from McLean’s model. This makes the geometrical arrangements essentially identical. Since $\frac{1}{\alpha} = \rho$, we find for the two components of the vector

$$\log(\frac{V_{f}}{\eta_{f}}) = \log(\frac{1}{\rho} \phi_{f} - \frac{V_{m}}{\eta_{f}})$$

$$\log(\frac{V_{e}}{\eta_{f}}) = \log(\frac{\phi_{f}}{\rho})$$

The displacement vectors derived from the two models differ slightly. This arises partly because McLean calculated $\sigma_0$ from a strain energy balance, whereas Kelly & Street calculated $\sigma_0$ from the average fibre stress, and partly because McLean’s average value of $\gamma$ corresponds to a reference value at $z = \frac{1}{\alpha}$, whereas the treatment of Kelly & Street’s model leads to a reference value of $\gamma$ at $z = \frac{1}{2\alpha}$.

Actually, the two methods of calculating $\sigma_0$ are not so basically different as they may initially seem, since eq. (16) can be rewritten

$$\sigma_{f} = \frac{1}{2} \int_{0}^{L/2} \sigma_{f} dz = \frac{8h}{\varepsilon_{c}} \int_{0}^{L/2} \gamma(z) \gamma(z) dz .$$

3. DISCUSSION

The above analysis is valid for $\rho \phi_{f} > 1$. It is easily shown, that the stress component of the displacement vector is always positive and the strain rate component always negative in this range. The transition from a power law to an exponential law will therefore occur at a lower strain rate (and a higher stress) for the composite than for the matrix. This accentuates the relevance of exponential creep for fibre composites.

The change of the transition region has important consequences for the prediction of creep properties of the composite on the basis of known properties of the matrix. Previous predictions of composite creep strength (Mileiko 1970; Street 1971) have been made from matrix creep data at the same strain rate as in the composite - although this condition has not been specifically mentioned. With such an approach, matrix data in the $\varepsilon$-range marked A in figure 2 would predict composite creep data marked A'. The correct corresponding composite creep is marked $A''$, which is placed at lower strain rates and lower stresses than $A'$. The correct composite data in the selected $\varepsilon$-range are marked $A'''$ and would be predicted from $A''$. 
The range $A_c'$ differs from the $A_c^*$ range in two important respects: the stress is lower and the stress sensitivity is higher. Thus the prediction $A_c'$ is an overestimate of the composite strength. At very low stresses and strain rates, where both the unreinforced matrix and the composite are creeping according to a power law, predictions based on the same strain rate would be safe, but it should be noted, that the break-down of the power law occurs at much lower strain rates for the composite than for the matrix (e.g. approximately 25 times lower for a composite with $V_f = 0.4$ and $p = 30$).

An interesting implication of the principle of corresponding points is that experiments in the range of reliably measurable strain rates contain information on creep behaviour below and above this range. Matrix creep data at low strain rates correspond to composite creep data at even lower strain rates. Conversely, composite creep data at high strain rates correspond to matrix creep data at even higher strain rates.

Finally, we wish to point out that the form of the creep equations (8) and (21) suggests a method for constructing generalized diagrams for composite creep data. If we choose eq. (21) as the basic relationship, such diagrams are obtained by plotting $\log(c^{*p})$ vs. $\log(c^{*p}_{f})$.  

**FIGURE 2.** Diagram showing the prediction of the composite strength from the matrix creep curve. The regions marked with $A'$s are explained in the text.
FIGURE 1. Comparison of experimental data from creep of Ag with W-fibres at 400°C with predicted composite creep curves. The matrix creep data are: •, Ag with low O\textsubscript{2} content, (Price 1966); △, Ag with equilibrium O\textsubscript{2} content, (Price 1966); ■, Ag, (Kelly & Tyson 1966). The composite creep data are: ◆, Ag with W-fibres, $V_f = 0.4$, $\rho = 30$; ▲, Ag with W-fibres, $V_f = 0.4$, $\rho = 60$, (Kelly & Tyson 1966). Full drawn composite creep curves are predicted from the full drawn matrix creep curve on the basis of the present analysis with $V_f = \log 6$, $\log 0.0412$ and $V_f = \log 12$, $\log 0.0206$. Dashed composite creep curves are predicted from the dashed matrix creep curve on the basis of the treatment by Street (1971).

4. COMPARISON WITH EXPERIMENTAL DATA

Kelly & Tyson (1966) have carried out experiments on composites of silver reinforced with 40 vol% tungsten fibres of aspect ratios 30 and 60. They have only produced a few data points on the pure silver; therefore we have supplemented their data with those obtained by Price (1966). On the basis of these points we have drawn a creep curve for the pure matrix as shown in figure 3. Our prediction of the composite creep curves is based on the model by Kelly & Street (eq. 21) with a given by eq. (13). These curves are shown in fig. 3 together with the displacement vectors corresponding to the two aspect ratios. We have further included the composite creep curves predicted from a power law by Street (1971) on the basis of Kelly & Street’s model.

The shape of our composite curves follow the experimental data quite closely, while the linear composite curves clearly do not predict the composite data so well. The excellent agreement for our curve at $\rho = 30$ is encouraging, but must be regarded as fortuitous considering the simplifications involved in the derivation of the composite creep model. Furthermore, the possible difference between a real composite and a theoretical composite with a uniform distribution of exactly aligned fibres should be remembered. For instance, the longitudinal distribution of fibres can have a significant influence on the creep strength, as discussed by Mileiko (1970), and the possibility of weak cross
sections has been mentioned by McLean (1972). At $p = 60$ the creep strength is overestimated by a constant factor of about 1.6. The reason for this overestimate is not quite clear. Mileikic (1970) noted that the aspect ratio $p = 60$ is very close to the value, where some fibres fracture. Another possible cause could be end effects arising when the fibre length becomes comparable to the specimen length.

5. CONCLUSIONS

We have shown on the basis of existing creep models, that the fibre composite creep curve can be obtained by displacement of the matrix creep curve in a log $\varepsilon$ vs. log $\sigma$ diagram, provided that $V_f > 1$. The displacement vector is $(\log(\frac{\varepsilon_c}{\varepsilon}), \log(\frac{\sigma_c}{\sigma}))$ in McLean's formulation and $(\log(\frac{\varepsilon_c}{\varepsilon}), \log(\frac{\sigma_c}{\sigma}))$ in Kelly & Street's formulation.

The direction of displacement is such, that the transition from a power creep law to an exponential creep law occurs at a lower strain rate for the composite than for the unreinforced matrix.

This has important consequences for the prediction of composite creep strength from matrix creep data, in that the present approach leads to a safer prediction than previously obtained.

A comparison between predictions based on the present analysis and experimental data from the literature shows encouraging agreement.

The combined use of matrix and composite data may allow the creep phenomenon to be studied over a larger range of strain rates (and stresses) than would otherwise be possible.

A matrix creep law, $\sigma = f(\varepsilon)$, leads to the composite creep law

$$\sigma_c \cdot \frac{2}{\rho V_f} = f(\varepsilon_c \cdot \frac{\rho}{2\varepsilon_c})$$

(using the constants in Kelly & Street's formulation).

The present approach therefore suggests a systematic method of handling composite creep data by using generalized diagrams, where $\log(\frac{\varepsilon_c}{\varepsilon})$ is plotted vs. $\log(\frac{\sigma_c}{\rho V_f})$. 
Appendix. List of Symbols

d  diameter of fibre
D  diffusion coefficient
h  thickness of zone of constant shear strain rate
l  length of fibre
m  stress exponent in creep law
n  spacing between fibres (or plates)
V  volume fraction of fibres
f  volume fraction of matrix
s  length coordinate along fibre
r  geometrical parameter
T  shear strain rate
\bar{T}  average shear strain rate in the matrix of the composite
t  tensile strain rate of matrix
c  tensile strain rate of composite
Q  constant in power creep law
\beta  constant in exponential creep law
\phi  aspect ratio (= l/d)
\sigma  tensile stress in matrix
\sigma_t  tensile stress in composite
\sigma_f  tensile stress in fibre
f  average tensile stress in fibre
\sigma_0  constant in power creep law
\sigma_\beta  constant in exponential creep law
\sigma_s  shear stress
f  average shear stress in the matrix of the composite

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