Influence of High Axial Tension on the Shear Strength of non-shear RC Beams

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Summary

This paper deals with the influence of high axial tension on the shear strength of beams without shear reinforcement. An experimental program with shear-tension tests was carried out. The experimental results have been used to evaluate the applicability of the Eurocode 2 (EC2) design formula in cases with large normal forces. In addition, the experiments have been used to evaluate an extension of the plasticity based Crack Sliding Model (CSM) to cover cases with large normal forces. The test results show, that even in the present of very high axial tensile stresses and strains, the member is still able to carry significant shear stresses. The analysis reveals that the EC2 formula is over conservative in this regard.

Keywords: Shear tests, Concrete beams, Axial tension, Eurocode 2, Crack Sliding Model.

1. Introduction

It is well-known that the shear capacity of reinforced concrete beams without stirrups can be enhanced by a compressive normal force. This enhancement effect is incorporated in most design codes, including the EC2 [1]. Even though the EC2-formula was developed to account for axial compression, it is also used for axial tension. This is apparent when reading the accompanying code text and the background document, see [2]. The formula predicts a linear reduction of the shear strength for members with axial tension. Very little experimental work has been available for verification of this approach. Further, in the few existing tests, the applied axial tension was rather low, [3], [4], [5], [6].

Some studies have indicated that the shear strength is not dramatically reduced as predicted by the codes provided that the member is properly reinforced for the applied tension, see e.g. [5] and [6]. The question is whether this also holds for high axial tension, measured both in term of stress and strain.

The issue of members subjected to shear in combination with high axial tension is relevant in many situations in practice. This could for instance be in a concrete slab which is a part of a continuous steel-concrete composite bridge deck. At the intermediate support, the negative moment from the global actions may result in large tension in the slab, which at the same time could be subjected to shear from the local action of traffic loads.

In this paper the influence of high axial tension on the shear strength is investigated. A test series has been conducted and the results are compared with the EC2-design formula. Further, the test results have also been used to evaluate, whether the plasticity based CSM could be applicable to model the effect of axial tension.
2. Experiments

The experimental program consisted of 23 beams without stirrups tested in combined shear-tension, [7]. The beams were tested for axial tension $\sigma_t = \frac{N}{bh}$ ranging from 0 to 50\% of the concrete compressive strength. Maximum $N$ corresponded to 83\% of the yield force of the longitudinal reinforcement. This is equivalent to a maximum average tensile strain of $\sim 0.40\%$.

The beam geometry is shown in Figure 1 (top). The widths of the support and load plates were 50mm. The beams, together with 27 cylinders with a diameter x height measure of 100mm x 200mm, were cast from one single concrete batch. The concrete was made from a Rapid Portland Cement and aggregate with maximum size of 16 mm. According to [8] a conversion factor equal to 0.97 were used to convert the strength to a standard cylinder with diameter x height = 150mm x 300mm.

The specimens were longitudinally reinforced with four $\Omega 15\text{mm}$ DYWIDAG thread bars with measured yield stress $f_y = 1027$ MPa and ultimate strength $f_u = 1227$ MPa. The c-c distance of the longitudinal reinforcement was in both directions 129mm. The high strength made it possible to conduct tests with high axial tensile strains.

The beams were tested in symmetric three point bending with shear span to depth ratio, $\frac{a}{h}$, equalling either 2.25 or 2.75. The tensile normal force was first applied and then kept constant while the specimen was subjected to transverse loading until failure. A U-shaped steel frame was used to induce tension into the beam. The arrangement is schematically shown in figure 1 (bottom). In one end, two hydraulic jacks were placed between a thick steel plate and the steel frame. The thick steel plate functioned as anchorage plate for two DYWIDAG $\Omega 26.5\text{mm}$ thread bars, which were positioned horizontally through oversized holes in the flanges of the frame leg. These bars were then - via two smaller steel plates – connected to the four thread bars sticking out of the concrete beam end (the $\Omega 15\text{mm}$ reinforcement bars were extended 250 mm beyond the beam ends). At the other end, a similar system for force exchange was used. Here, however, the DYWIDAG bars were anchored directly to the steel frame. Pressure from the hydraulic jacks then induced tension in the beam while subjecting the U-shaped frame to a closing moment. Since the specimen was placed on rollers it was ensured that the same force was applied from both ends. The strains in the DYWIDAG bars were monitored to ensure that the axial load was applied centrally to the beam. The frame together with the specimen fitted inside the 2000 kN Amsler testing rig used to apply transverse loading.
The main test results have been summarized in Table 1. Generally, it was observed that the shear strength was not significantly affected by tensile normal forces less than about 40% of the specimen tensile yield strength. This corresponds to an average normal strain of ~0.20%. Beyond this level of tension, a decrease in the shear strength was generally observed for increasing axial tension.

Figure 2 shows two examples of the crack patterns observed at the stage of failure. Typically, a system of tensile crack appeared during the application of the tensile normal force. Then, during transverse loading, diagonal cracks crossing the tensile cracks started to develop. This took place already in the beginning of the transverse loading process. Finally, shear failure would take place in one of the diagonal cracks. For specimen ST-7 shown in Fig. 2 (bottom), development of diagonal cracks initiated when the transverse load reached ~16 kN (corresponding to a shear force of 8 kN), which was about 20% of the ultimate load.

![Fig. 2: Typical crack patterns: (top) specimen ST-3 with \( N/A_s f_y \sim 0.27 \), (bottom) specimen ST-7 with \( N/A_s f_y \sim 0.55 \).](image)

Indications of flexural failure, for instance in the form of wide open flexural cracks at mid span, were not observed even though some of the tests with very high axial tension theoretically had a bending strength slightly smaller than the shear strength (see e.g. Table 1 ST-23).

### 3. Calculations

Calculations by use of the EC2-formula have been conducted and compared with test results. This investigation is interesting from a practical point of view since this formula at the present moment forms the basis for shear strength verification in many European countries. Moreover, the tests are also compared with calculations by use of the plasticity based CSM, see [9] and [10]. The CSM is interesting in this context because it allows a direct modelling of the influence of normal forces. Correlation of the model with tests on members with axial compression has been investigated by Zhang [9] and Jensen and Hoang [11]. In this paper, the model is extended to cover the case of large axial tension.

#### 3.1 Calculation using Eurocode 2

According to EC2, the shear strength of a beam without shear reinforcement should be taken as:

\[
V_u = \max \left( \left(0.18k \left(100\rho_l f_y \right)^{1/3} + 0.15\sigma_{cp}\right)bd, \left(0.035k^{3/2} \sqrt{\frac{f_c}{f_y}} + 0.15\sigma_{cp}\right)bd \right)
\]

Here \( k = 1 + \left[ \frac{200}{d} \right]^{0.5} \leq 2 \) (\( d \) in mm), \( \rho_l = A_s/bd \leq 0.02 \) and \( N_E/A_c < 0.2f_c \) (in MPa). As formulated in the explanatory text in EC2, “\( N_E \) is the axial force in the cross-section due to loading or
prestressing \( (N_E > 0 \text{ for compression}) \). Hence, although intended to primarily cover the effect of compressive normal force, the formula is also be used when a tensile normal force is present.

### Table 1: Summary of main test results

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>( f_c ) [MPa]</th>
<th>( a/h )</th>
<th>( N ) [kN]</th>
<th>( V_{u,\text{test}} ) [kN]</th>
<th>( V_{u,\text{CSM}} ) [kN]</th>
<th>( V_{u,\text{EC2}} ) [kN]</th>
<th>( V_{u,\text{flexural}} ) [kN]</th>
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<tr>
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<td>2.25</td>
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<td>43.9</td>
<td>0</td>
<td>55.6</td>
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<td>43.5</td>
<td>45.1</td>
<td>29.7</td>
<td>96.4</td>
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<tr>
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<td>45.1</td>
<td>14.2</td>
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<tr>
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<td>2.25</td>
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<td>49.0</td>
<td>44.3</td>
<td>108.6</td>
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<tr>
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<td>49.7</td>
<td>44.6</td>
<td>108.6</td>
</tr>
<tr>
<td>ST-6</td>
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<td>2.25</td>
<td>300.0</td>
<td>43.0</td>
<td>45.6</td>
<td>0</td>
<td>70.8</td>
</tr>
<tr>
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<td>600.4</td>
<td>35.2</td>
<td>32.3</td>
<td>0</td>
<td>28.0</td>
</tr>
</tbody>
</table>

**Notations:**
- \( f_c \): Tested concrete compressive strength.
- \( a/h \): Shear span to depth ratio.
- \( N \): Applied axial tension in test.
- \( V_{u,\text{test}} \): Tested shear strength.
- \( V_{u,\text{CSM}} \): Calculated shear strength, CSM.
- \( V_{u,\text{EC2}} \): Calculated shear strength, EC2.
- \( V_{u,\text{flexural}} \) = \( M_u / a \): Calculated bending strength.
3.2 Calculation by the Crack Sliding Model

The CSM developed by Zhang [9] is a refinement of the classical upper bound plasticity method for beams without stirrups, see [10]. Similar to the original plasticity formulation, CSM operates with concrete as a rigid plastic material obeying the Modified Coulomb failure criterion and the associate flow rule of plastic theory. The mentioned refinement lies in the fact that a cracking criterion (see below) is introduced in order to determine the position of the critical shear crack. This criterion supplements the original concept of energy minimization when looking for the most critical shear yield line.

In CSM, a shear failure is assumed to take place as sliding in a linear diagonal crack characterised by its horizontal projection $x$, see Figure 3. The considered shear mechanism involves a displacement $u$ of part I relative to part II. Depending on the magnitude of the axial tension $N$, the rate of displacement $u$ may be vertically directed or it may have a component in the direction of $N$ as indicated. By following the procedure for calculation of the rates of internal energy dissipation and external work, see [10], the upper bound solution below is obtained from the considered failure mechanism:

$$
V_u(x) = \left\{ \begin{array}{ll}
\frac{1}{2} \nu f_c bh \left( \sqrt{\frac{x^2}{h^2}} + \frac{4}{\nu} \left( \frac{\Phi}{\nu} - \frac{\sigma_t}{\nu f_c} \right) \left( 1 - \left( \frac{\Phi}{\nu} - \frac{\sigma_t}{\nu f_c} \right) \frac{x}{h} \right) \right) , & \frac{\Phi}{\nu} - \frac{\sigma_t}{\nu f_c} \leq \frac{1}{2} (a) \\
\frac{1}{2} \nu f_c bh \left( 1 + \frac{x^2}{h^2} - \frac{x}{h} \right) , & \frac{\Phi}{\nu} - \frac{\sigma_t}{\nu f_c} \geq \frac{1}{2} (b)
\end{array} \right.
$$ (2)

Here, $b$ and $h$ are the cross sectional dimensions, $\Phi = A_s/\rho f_c$ is the mechanical degree of longitudinal reinforcement including both top and bottom rebars, $\sigma_t = N/bh$ is the average applied tensile stress. The parameter $\nu$ is the so-called effectiveness factor, which is required when applying rigid plastic theory to structural concrete. For crack sliding, Zhang [9] assumed:

$$
\nu = \frac{0.44}{\sqrt{f_c}} \left( 1 + \frac{1}{\sqrt{h}} \right) (1 + 26 \rho) \ (1 \text{ in meters and } f_c \text{ in MPa})
$$ (3)

Here the longitudinal reinforcement ratio $\rho = A_s/bh$ is calculated on the basis of both top and bottom rebars. According to CSM, the horizontal projection $x$ of the critical shear crack must be determined by introducing a cracking criterion. Basically, the criterion states that formula (2) provides a valid crack sliding solution if the crack considered exists. This means that the load required to develop the crack (here termed the cracking load $V_{cr}(x)$) must be less or equal to the load that is needed to cause sliding failure in the crack. Hence, to determine the shear strength from this upper bound model, $V_u(x)$ must be minimized with respect to $x$ and at the same time be subjected to two conditions: $x \leq a$ and $V_{cr}(x) \leq V_u(x)$. The first condition is an obvious geometrical restriction and the second condition ensures that the crack exists. For a more comprehensive explanation as well as detailed discussions of the physical aspects of the model, see [9] and [10].

Since CSM is based on plastic theory, Zhang [9] also adopted a plasticity approach to estimate the cracking load. In this approach, the cracking load is simply determined from a cracking mechanism involving rotation of part I about the crack tip. By including the contribution from the tensile normal force to the rate of external work in this mechanism, the original expression for the cracking load as a function of $x$ may be extended as follow:
\[ V_{cr}(x) = \frac{1}{2} \left( f_{tef} \left( h \left( 1 + \frac{x}{h} \right)^2 - N \right) \right) \text{,} \quad f_{tef} = 0,156 f_c^{2/3} \left( \frac{h}{0.1} \right)^{-0.30} \]

Here, the effective tensile strength \( f_{tef} \) of concrete is calculated by inserting \( h \) in meters and \( f_c \) in MPa. The simplicity of formulas (2) and (4) makes the process of finding the shear strength rather easy. Since formula (2) decreases monotonically with \( x \) while formula (4) increases monotonically, one just have to solve the equation \( V_{cr}(x) = V_{cr}(x) \) with respect to \( x \). This can be done numerically. If the determined \( x \)-value satisfied \( x \leq a \), then this \( x \)-value is inserted into formula (2) to calculate the shear strength. If the solution of \( V_{cr}(x) = V_{cr}(x) \) is larger than \( a \), then \( x = a \) must be inserted into (2) for calculation of the shear strength. Graphical explanations of the solution procedure have been given in [9] and [10].

4. Comparison of Test Results with Calculations

In Table 1 the calculated results using EC2 and CSM are listed together with the observed shear strength. As can be seen, the EC2-formula predicts total loss of shear capacity for \( N > 300 \) kN corresponding to \( \sigma_t = 7.5 \) MPa. This is obviously not in agreement with experimental observations. It appears that the results obtained from CSM are in better agreement with tests. The average value of test over calculation is 0.90 with a standard deviation of 0.10. The correlation has been illustrated in Figure 4.

In Table 1, the calculated flexural capacity has also been shown. These values are based on cross sectional ultimate bending moments determined by the program Response-2000 developed by Bentz [12]. It appears that four of the specimens theoretically had smaller flexural strength than shear strength. The values are however close to each other. The four tests have been emphasised in Figure 4 by coloured dots. Even though a classical flexural failure mode was not observed in the tests, it should be mentioned that a combined shear-flexural mechanism, i.e. both rotation and translation in the diagonal crack, may possibly have taken place.

Figure 5 depicts the tests with \( a/h = 2.25 \). In addition, \( \tau_c/V_f = V_A/bh \) versus \( N/A \) as determined by the CSM has also been plotted. The calculations were performed by assuming \( f_c = 27.3 \) MPa, which is the average value for the shown tests. Even though the individual test had a slightly different compressive strength than the average strength, the plot illustrates rather well the tendency of both tests and theory. The shear strength calculated from the EC2-formula has also been plotted in the figure. It is clearly seen that this formula is not applicable for high axial tension. The tests with \( a/h = 2.75 \) shows the same tendency and a plot of these results gives a very similar figure. This conclusion can also be drawn from Fig. 4; where all test results are plotted.
5. Conclusion
The influence of high axial tension on the shear strength of beams without stirrups has been studied. Shear tests combined with a high axial tensile load have been carried out. The test results indicate that the shear capacity remains almost intact even when the axial tension corresponds to ~ 0.20 % strains. The EC2-formula was not developed for cases with high axial tension even though it is used for this purpose in practice also. Comparison with tests shows that the EC2-formula is conservative for high axial tension. Comparison with tests indicates that the CSM seems to work well, even for members subjected to axial tensile strains up to 0.30 – 0.40 %.

6. References