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Investigation of spherical aberration effects on coherent lidar performance

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Abstract: In this paper we demonstrate experimentally the performance of a monostatic coherent lidar system under the influence of phase aberrations, especially the typically predominant spherical aberration (SA). The performance is evaluated by probing the spatial weighting function of the lidar system with different telescope configurations using a hard target. It is experimentally and numerically proven that the SA has a significant impact on lidar antenna efficiency and optimal beam truncation ratio. Furthermore, we demonstrate that both effective probing range and spatial resolution of the system are substantially influenced by SA and beam truncation.

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References and links
1. The final report of the EU FP6 project UPWIND, http://www.upwind.eu/

1. Introduction

One of the main considerations in the development of the wind industry is the metrology issues. Essentially, more cost efficient and accurate wind velocity and turbulence mapping systems are highly desired [1]. Since the traditional cup and sonic anemometers require meteorological
masts, a detailed turbulence mapping will require a tremendous amount of masts at different locations and heights. For this particular task the laser remote sensing (lidar) technology offers an attractive alternative [2]. In these systems, precise control of multiple lidar units are required in order to acquire the full 3D wind vectors with high a spatial resolution, which is determined by the overlap between the individual lidar weighting function that describes the spatial sensitivity and confinement along each beam direction.

Descriptions of various lidar system designs are well documented in the literature [3–7]. However, those are all theoretical treatments and focus mainly on diffraction limited system designs. A previous theoretical analysis did include the aberration effects [8], but only for a fixed degree of beam truncation at the exit aperture that is optimal for an aberration-free system. This work is dedicated to investigating the weighting function change under the influence of SA to provide an experimental counterpart of the theory proposed by Rye [8], and we will expand the analysis to include optimization of the beam truncation in the presence of SA. In practice, most lidar designs involve lens selection for the telescope or optical transceiver, which predominantly suffers from SA due to the difficulty in manufacturing lenses with SA-corrected surfaces especially for larger aperture sizes with short focal lengths. Furthermore, Rye showed numerically [8] that for equal degrees of coma, SA and astigmatism, the SA is found as the dominant contributor to the lidar signal reduction.

2. Experimental setup

![Fig. 1. Schematic layout of the system setup. The size of the exit aperture is the diameter of $L_2$. During the experiments several diffraction limited aspherical lenses, $L_1$ with different focal lengths are used in order to probe different $\rho$ values; while two different $L_2$ are used to introduce different degree of aberrations. The rotating belt is used to generated the Doppler signal for our measurements.](image)

In this work the impact of the SA is measured by probing the lidar weighting function of the system with a rotating belt as the hard target using a 1550 nm CW beam output as shown in the schematic layout in Fig. 1. Both the simulation and the experiment follows the geometry in Fig 1. The distance between $L_1$ and $L_2$ is adjusted such that the lidar signal is optimized with the hard target (rotating belt) placed at a range of 80 m. Around 0.5 mW of the diode laser output is tapped within the optical circulator and is used as the local oscillator (LO) for the heterodyne detection. Both the signal from the rotating belt and the LO is focused onto the detector through the optical circulator. The "virtual" back propagated local oscillator (BPLO) from the detector plane matches the transmit beam with a Gaussian field amplitude profile of radius, $w$, at the plane of lens $L_2$. This configuration is commonly referred to as the Wang design [7]. Different truncation ratios, $\rho = w/r_{L_2}$ where $r_{L_2}$ is the radius of $L_2$ aperture, can be probed by changing the focal length ($f_1$) of lens $L_1$, since the imaging magnification of the...
beam is dependent on the focal length ratio between $L_1$ and $L_2$. Two different $L_2$, both with exit aperture radius of $r_{L2} = 35.65$ mm (3 inch optics), are used in our experiments in order to evaluate the system under different degrees of SA. The lenses ($L_2$) are respectively a singlet lens ($f_2 = 200$ mm) that is not corrected for SA and a doublet lens ($f_2 = 216$ mm) designed for reduced SA. The correlation overview between the $L_1$ focal lengths and $\rho$ can be found in Table 1. A quantitative illustration of the SA introduced by the $L_2$ lenses is shown in Fig. 2(a),

<table>
<thead>
<tr>
<th>Focal length of $L_1$</th>
<th>4.6 mm</th>
<th>8.1 mm</th>
<th>11.3 mm</th>
<th>15.6 mm</th>
<th>18.8 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$, singlet lens</td>
<td>1.28</td>
<td>0.73</td>
<td>0.52</td>
<td>0.38</td>
<td>0.31</td>
</tr>
<tr>
<td>$\rho$, doublet lens</td>
<td>1.38</td>
<td>0.79</td>
<td>0.56</td>
<td>0.41</td>
<td>0.34</td>
</tr>
</tbody>
</table>

where the optical path difference (OPD) is measured in number of waves. The curves in Fig. 2(a) are generated using a Zemax simulation with monochromatic input, zero incident angle and assuming circular symmetry in the transverse plane, i.e. only the symmetrical components of wavefront errors are presented. The first six nonzero Zernike fringe coefficients, generated in Zemax, are listed in Table 2. Those coefficients are used to generate the OPD curves and they differ slightly from the standard Zernike coefficients, which is evident from the polynomials provided in the table. A detailed description of those coefficients can be found in the user’s manual of Zemax [9]. It is evident from the table values that the SA ($Z_9$) is the dominant source of the wavefront errors.

![Fig. 2. a) The OPDs of the two different $L_2$ used in our experiments. The OPDs are generated in Zemax with zero incident angle and circular symmetry. b) The calculated transverse irradiance profile of the output beam in different axial distances from the singlet $L_2$. The focal length of $L_1$ is 15.6 mm in the simulation.]

An accurate theoretical prediction of the field or irradiance distribution on the target side of $L_2$ is possible using those OPD curves. The SA can be incorporated in the theoretical simulation by introducing an extra phase term, $\phi_{SA}$ to the truncated Gaussian field at $L_2$ according to Eq. (1) [8]

$$E(r) = \begin{cases} E_0 \exp \left\{ -\frac{r^2}{w^2} + i\phi(r) + i\phi_{SA}(r) \right\} & \text{for } 0 \leq r \leq r_{L2}, \\ 0 & \text{for } r > r_{L2} \end{cases}, \quad \phi_{SA} = 2\pi \ast \text{OPD}(r) \quad (1)$$

where $E_0$ is the peak amplitude, $r$ is the radial coordinate, $\phi(r)$ is the phase of the beam due
to field curvature, $\phi_{SA}$ is the phase term induced by the SA and the OPD($r$) is either curves shown in Fig. 2(a). Due to the circular symmetry, the field on the target plane can be calculated numerically by a simple Fourier-Bessel transformation of Eq. (1) with an appropriate field curvature [8].

<table>
<thead>
<tr>
<th>Term $Z_j$</th>
<th>Singlet $L_2$</th>
<th>Doublet $L_2$</th>
<th>Zernike fringe polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$</td>
<td>-3.27339159</td>
<td>0.03707405</td>
<td>1</td>
</tr>
<tr>
<td>$Z_4$</td>
<td>1.50844787</td>
<td>-0.05266555</td>
<td>$2p^2 - 1$</td>
</tr>
<tr>
<td>$Z_9$</td>
<td>4.87996531</td>
<td>-0.18513369</td>
<td>$6p^4 - 6p^2 + 1$</td>
</tr>
<tr>
<td>$Z_{16}$</td>
<td>0.10038714</td>
<td>-0.10019848</td>
<td>$20p^6 - 30p^4 + 12p^2 - 1$</td>
</tr>
<tr>
<td>$Z_{25}$</td>
<td>0.00231510</td>
<td>-0.00501370</td>
<td>$70p^8 - 140p^6 + 90p^4 - 20p^2 + 1$</td>
</tr>
<tr>
<td>$Z_{36}$</td>
<td>0.00005509</td>
<td>-0.00021833</td>
<td>$252p^{10} - 630p^8 + 560p^6 - 210p^4 + 30p^2 - 1$</td>
</tr>
</tbody>
</table>

### 3. Results and discussions

To illustrate the degree of SA for the singlet $L_2$ case, the transverse irradiance profiles in different distances, $z$ after the singlet $L_2$ are calculated and shown in Fig. 2(b). A side by side comparison between the numerical simulation and the experimental counterpart of the beam profiles is displayed in Fig. 3. The observed beam profiles are recorded with aid of an IR-detection card. An intensity clipping level was introduced in the presentation of the numerical results in order to simulate the saturation effect of the IR-detection card. In Fig. 3 the consistency between the simulations and the measurements is quite evident, qualitatively validating the accuracy of our theoretical simulations.

In order to maximize the signal in diffraction limited monostatic lidar systems, $\rho \approx 0.8$ through the $L_2$ is required [3]. However, in the presence of SA, optimal $\rho$ will differ from...
0.8 and the overall antenna efficiency will decrease compared to the aberration-free case. It is easiest to calculate the antenna efficiency, $\eta_a$, using the target-plane formalism based on Siegman’s antenna theorem \[3, 10\].

$$
\eta_a(z) = \frac{\lambda^2 R^2}{A_r} \iint I_{\text{target}}(x,y,z) dxdy
$$

where $\lambda$ is the wavelength, $R$ is the intended imaging or probing range (80 m in our case), $A_r$ is the area of $L_2$ and $I_{\text{target}}$ is the irradiance distribution of the output beam at the target plane position, $z$ and is normalized by the total beam power before truncation.

In Fig. 4(a) the calculated antenna efficiency at $z = R$ as a function of $\rho$ is shown for both $L_2$ lenses and compared with the aberration-free case (the dashed lines). The simulation is constructed such, that the algorithm is imitating the same distance optimization procedure as in the experiments. It is evident that not only the overall antenna efficiency has decreased as the consequence of SA, but the optimum $\rho$ also shifts with different degrees of aberrations. Even for the doublet case where the SA is minimal, there is still a clear shift of the optimal $\rho$ and a quite noticeable drop in the maximum antenna efficiency. It demonstrates the importance of considering the aberration effects in designing a lidar system. The result in Fig. 4(a) indicates that optimal $\rho$ decreases with increasing degree of SA.

The measured hard target lidar signal as a function of $\rho$, normalized to the maximum data point of the doublet case, is also shown in Fig. 4(a). The experimental data shows the same tendencies as the numerical calculations. While the singlet data coincides quite well with the simulations, there is a slight deviation for the doublet case. Since the simulation only includes the SA effect, it is reasonable that the doublet data can deviate from the simulation due to other aberration effects in the rest of the system (we do observe small degree of astigmatism from the output of the optical circulator). The singlet data is, on the other hand, dominated by the SA, which explains the good agreement with the simulation.

So far we have shown how the SA affects the optimal $\rho$ with respect to the maximum antenna efficiency at fixed target distance equal to the intended probing range $R$. However, the graph in Fig. 4(a) only provides an optimal $\rho$ for hard target case. For aerosol target, it is necessary
to analyze the lidar weighting function, which provides the effective probing range (may differ from $R$ due to aberration), the spatial resolution and the total signal strength of the system. In a monostatic CW lidar the weighting function is commonly described by a Lorentzian function, $F$ under the assumption of an ideal and untruncated Gaussian beam [11],

$$F = A \left( \frac{1}{(z - R)^2 + z_0^2} \right)$$

(3)

where $A$ is a normalization constant, $z$ is the distance from $L_2$ and $z_0$ is the Rayleigh length of the output beam. However, we just demonstrated in Fig. 4(a) that a rather large $\rho$ of around 0.8 is required in order to obtain the optimal antenna efficiency for the aberration-free system. In that case the output will suffer from significant diffraction effects due to the truncation and therefore no longer be considered as an ideal Gaussian beam. In Fig. 4(b) the antenna efficiency is calculated as a function of the distance for two different cases: 1) including the truncation effect by numerical integration of Eq. (2) and numerical Fourier-Bessel transform of the field in Eq. (1) to obtain $I_{\text{target}}$. 2) Using Eq. (3) based on the untruncated Gaussian beam assumption. The result demonstrates a broadening of the weighting function by 56% when including the truncation effect, which is quite significant. Since the width of this Lorentzian distribution is directly related to the spatial resolution of the lidar system, the truncation effect should not be underestimated in the system design.

The weighting function can be acquired experimentally by measuring the lidar signal with a moving hard target. The experimental data of six different combinations of $L_1$ and $L_2$ are presented in Fig. 5 along with their numerical counterparts. In general the numerical results coincide quite well with the experimental data, but the singlet case gives a much better match than the doublet case. As we discussed earlier the simulations only include the SA effect, since the measurement with singlet $L_2$ is dominated by SA while the doublet $L_2$ case is not, it is expected that singlet lens case will provide a better fit. The simulation results do suggest that the full system (not only $L_2$) potentially suffers from other aberration effects like astigmatism and coma, which are comparable with the SA introduced by the doublet $L_2$, since the experimental
data shows a visible broadening of the weighting function compared with their numerical counterparts. Comparing the simulation with the experimental data there is a broadening of 21% for the green dash line \((f_1 = 8.1\, \text{mm})\) while the broadening is 70% for the blue dash line case \((f_1 = 18.8\, \text{mm})\). The residual broadening is likely due to the astigmatism of the beam from the optical circulator, which also explains the observed increase in the degree of broadening with the focal length of \(L_1\).

From Fig. 5 it is quite obvious that in general one should reduce SA in the system, since both the spatial resolution/confinement and the maximum signal strength of the optimal doublet case (green dashed line) are much better than the optimal case for the singlet lens (blue solid line). However, the area under the weighting function (estimating the total lidar signal strength for aerosol target) has only increased by around 30% from the optimal singlet case to the doublet one. So for measurements of laminar air flow (i.e. negligible spatial dependence of wind vector), the benefit gained from using the more expensive doublet \(L_2\) is minor in terms of signal strength enhancement but more on improved spatial resolution. For the more general turbulent air flow in the probing volume, higher signal strength enhancement due to tighter spatial confinement is of course expected.

From the previous theoretical treatments [8] we know that for a fixed \(\rho\) the weighting function will suffer from both peak shift and broadening effects under the influence of SA. Recalling Eq. (2) and the transverse irradiance profiles in Fig. 2(b) it is expected that the weighting function for the singlet \(L_1\) case will have a peak around 60 m, since the effective beam confinement is tightest there and not at the intended imaging range, 80 m. The data shown in Fig. 5 provides, to our knowledge, the first experimental confirmation of these tendencies. But our numerical and experimental results also show that both the peak shift and the broadening effect can be compensated to certain degree by selecting a \(\rho\) appropriate for a particular degree of SA, which is not described in the theoretical work by Rye [8]. From the OPD curve in Fig. 2(a) we know that the doublet \(L_2\) only suffers from minor SA, nevertheless, we still observe a weighting function peak shift of 2.4 m (for \(L_1\) focal length of 8.1 mm), indicating the sensitivity of the lidar system to the SA effect. We also note in Fig. 5 that the peak shift is larger for lower \(\rho\). Thus, in applications where delicate probing range control is necessary, it is advisable to calibrate the lidar system with a hard target by mapping the weighting function.

### 4. Conclusion

In this paper we have shown both numerically and experimentally that SA has a significant impact on antenna efficiency, optimal truncation ratio and the shape of the weighting function of a CW coherent lidar. If the system suffers from strong SA effect only very limited spatial confinement can be obtained as shown in Fig. 5. It is also evident that the degradation of spatial confinement or broadening of the lidar weighting function due to SA can be reduced by tuning the beam truncation through \(L_2\). This corrective measure results from the novel finding in this work that the optimal truncation ratio depends on the degree of SA. Furthermore we have shown that both SA and truncation ratio influence the peak shift and width of the weighting function. In applications where precise probing range and spatial resolution are essential, a weighting function calibration of the lidar system using a hard target might be necessary. It is worth to stress that this study can also be applied to accurately model the weighting function of pulsed coherent lidar systems [2].

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