Oscillators - a simple introduction

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Abstract—Oscillators are kernel components of electrical and electronic circuits. Discussion of history, mechanisms and design based on Barkhausens observation. Discussion of a Wien Bridge oscillator based on the question: Why does this circuit oscillate?

I. INTRODUCTION

Oscillators are observed everywhere in the universe on all levels. Oscillators are defined as subsystems of the universe which exhibit an oscillating behavior. In connection with an oscillator there is an energy field which goes from the center to infinite. Oscillators couple by means of exchange of energy. All kinds of oscillators may couple. The coupling may take place over enormous relative distances.

Galaxies are oscillators. They are composed of solar systems which couple. Solar systems are oscillators composed of stars and planets. Gravity is the field mechanism for exchange of energy between oscillators of this kind. Other fields for exchange of energy are electromagnetism, the strong and the weak nuclear forces [1]. Apparently the coupling of oscillators is the basic principle of the universe [2].

Electrical circuits are man-made systems for handling and transport of energy. The electrical world may be coupled to the mechanical world by means of flux (generators, motors). The electrical world may be coupled to the chemical world by means of charge (batteries). Electrical circuits are nonlinear systems.

Very often design of electrical circuits is based on the assumption of linear lumped models for the elements in order to be able to setup analytic expressions for the behavior. An electrical circuit is a fractal pattern of coupled oscillators e.g. a resistor may be modeled as a linear damped oscillator if the parasitic components are taken into account.

Electronic circuits are electrical circuits for handling of information. Oscillators are kernel components of electronic circuits. Oscillators create sine waves as carriers of signals (Radio, TV) or square waves as clock control in digital systems. Steady state oscillators are considered nonlinear circuits having a time-varying DC bias point. They may be investigated as time-varying linear systems. Apparently the steady state chaotic behavior is more common than the steady state limit cycle behavior wanted in electrical oscillators.

The aim of this contribution is to give a simple introduction to electronic oscillators. It may be seen as an addendum to [3].

II. ELECTRONIC OSCILLATORS

In the following the topics of history, mechanisms and design of electronic oscillators are discussed.
(1949) together with oscillators for which the inventor is unknown like the harmonic LC oscillator, the RC phase-shift oscillators, the negative resistance oscillators and the multivibrators.

The Meacham [9], [10] and the Hewlett oscillators are bridge oscillators. The Hewlett oscillator is the well-known Wien bridge oscillator [11], [12]. Bridge oscillators seem to be candidates for realizing good approximations to linear oscillators. The Clapp and the Vackar oscillators are modified Colpitts oscillators.

Today most off-the-shelf oscillators are crystal oscillators (Pierce, Meacham) where a coupling between a mechanical oscillator (piezo-electrical) and an electronic oscillator is made use of in order to obtain a very stable and clean oscillator.

B. Mechanisms

Linear oscillators are either damped oscillators or unstable oscillators. In both cases they end-up in a DC bias point with no oscillations. You can not create a linear real world steady state oscillator with a complex pole pair on the imaginary axis. You can not balance on the razors edge. Real world steady state oscillators must be non-linear circuits. They may be treated as time varying linear circuits so it make sense to study the poles (eigenvalues) of the small signal model as function of time.

The bias point of the circuit vary with time so it is important to study the power source current in order to understand the mechanism behind the behavior. The startup phase begins with switching on the power source. You may "stop" time, calculate a DC bias point and derive the small signal model from the linearized Jacobian of the differential equations. If the poles are in RHP (the right half of the complex frequency plane) the signals will increase and either steady state oscillations or a new DC bias point will occur. If the poles are in LHP (the left half of the complex frequency plane) you may change some elements so the poles come to RHP and the situation with the unstable small signal model may result in a self starting oscillator. Often you observe that a complex pole pair goes to the real axis and split-up into two real poles.

So far the regenerative principle with linear positive feed-back around a non-linear amplifier where a small output signal is fed back and amplified until some steady state is obtained is the answer to the question:

Why does an oscillator oscillate?

Other mechanisms may be the introduction of a negative resistance to compensate the losses of a damped linear oscillator or the introduction of an impulse to compensate the losses similar to the escape mechanism of the mechanical pendulum clock.

If we introduce a nonlinearity in the feed-back circuit and assume an ideal operational amplifier or a perfect linear amplifier we may have a mechanism for minimizing the phase noise and the harmonics.

Our starting point for the oscillator circuit is an unstable small signal model for the closed loop circuit: The modified Barkhausen criterion.

An electronic circuit is normally a circuit with 3 ports: 2 input ports and 1 output port. One input port is the power source (battery) the other input port is the signal input port. The output port is the signal output port. If the circuit is linear superposition gives that the output signal is the sum of the input signals transferred and modified through the circuit.

An oscillator is a circuit with only 2 ports: the battery input port and the signal output port. The signal observed on the output port at time "zero plus" is the step response of the initial small signal model of the circuit.

When we switch on the battery at time "zero minus" all coils are open-circuits and all capacitors are short-circuits. At time "zero plus" we have the start-up of the DC bias point where all coils are short-circuits and all capacitors are charged based on time constants of their equivalent parallel resistors. If no steady state oscillations are observed we have a DC bias point with constant currents in the coils and constant voltage on the capacitors. If oscillations are observed we have a time varying DC bias point.

When we switch on the constant voltage power supply to the nonlinear closed loop circuit we may observe two situations: (1) steady state oscillations or (2) a transient from the zero bias point to a bias point different from zero. In situation (2) we may add an impulse and observe steady state oscillations or a transient to an other bias point (2 rails).

The time varying DC point is a picture of the movements of energy (charge and flux) in the circuit. It might be the starting point for a search of a sufficient criteria for oscillations. So far we have succeeded for more than 100 years with assuming design of a linear oscillator by "hit and try".

We want to understand the mechanisms behind steady state oscillations in the time domain. We want to find a sufficient criterion for steady state oscillations.

Bild 1. Allgemeines Schema einer Rückkopplung

\[ u_o = -R u_i; \quad R = \frac{u_o}{u_i}\text{"Verstärkungsfaktor"} \]

\[ \mathbb{R} = -\frac{R}{R} = \text{"Rückkopplungsfaktor"} \]

\[ \mathbb{R} = \frac{1}{\mathbb{R}} \text{ oder } \mathbb{R} = 1 \text{(Allgemeine Selberregungsformel.)} \]

Fig. 2. Barkhausen's original observation

C. Design

In 1921 Barkhausen [13], [14] pointed out that an oscillator may be described as a linear inverting amplifier (a vacuum
tube) with a linear feedback circuit (Fig. 2) which determine the frequency. The amplifier is a two-port with a static gain-factor equal to the ratio between the signals at the ports. The linear feedback circuit is a two-port with a feed-back-factor equal to the ratio between the port signals. It is obvious that the product of the two factors becomes equal to one. This observation is called the Barkhausen criterion or the Allgemeine Selbsterregungsformel in German language.

This criterion has been used for design of oscillators assuming a linear amplifier with gain $A$. The loop is opened and the circuit is designed with gain 1 and phase-shift $\neq 0$. The loop is closed and regeneration is assumed to start-up oscillations.

Very often you observe oscillations but unfortunately you have no guarantee of steady state oscillations. When you open the loop you study a circuit closely related to the oscillator circuit but it is the closed loop circuit which you want to be an oscillator. The open loop circuit is just an active filter with a time invariant bias point.

When you close the loop you have a linear circuit with a complex pole pair on the imaginary axis. Often the criterion is modified so that the complex pole pair is moved to the RHP. The linear circuit then becomes unstable and steady state oscillations are supposed to occur because the nonlinearities will limit the growth of the amplitude in some way. Unfortunately this might not always be the case [15]. Steady state oscillators are nonlinear circuits.

Fig. 3. Barkhausens observation. Characteristic polynomial

\begin{align*}
V(3,5) &= A \cdot V(1,2) = -A \cdot V(2,1) \\
H(s) &= V(6,7) / V(4,5) = V(2,1) / V(3,5) \\
V(2,1) &= V(3,5) \cdot H(s) = -A \cdot V(2,1) \cdot H(s) \\
1 &= -A \cdot H(s) = -A \cdot N(s) / D(s) \\
D(s) + A \cdot N(s) &= 0
\end{align*}

Fig. 3. Barkhausen's observation. Characteristic polynomial

Barkhausens figure may be redrawn as shown in fig. 3 where the non-linear amplifier is assumed to be a perfect amplifier with infinite input impedance, zero output impedance and linear time-varying gain $A$. The linear feedback circuit is assumed to be a linear, lumped element, time-invariant passive two-port with a rational transfer function $H(s)$. It is obvious that the loop-gain is always equal to one (1) and the phase-shift is equal to a multiple of $360^\circ$ ($2\pi$). Furthermore it is seen that the Barkhausen criterion is just an expression for the characteristic polynomial of the circuit as function of the amplifier gain. For zero gain the characteristic polynomial becomes equal to the denominator of $H(s)$. For infinite gain the characteristic polynomial becomes equal to the numerator of $H(s)$. The circuits may be divided into two groups: four-terminal- and three-terminal- coupled two-ports.

There are examples where the poles of the initial small signal model are in LHP so it becomes necessary to apply initial conditions (energy) e.g. an impulse somewhere in order to start-up steady state oscillations. The size of this impulse is crucial for the behavior. A nonlinear circuit may have several stable DC bias points but some of these points may be potentially unstable. Some times an average DC bias point is introduced.

If the amplifier is a real world operational amplifier i.e. an active nonlinear circuit then the transfer function ($gain \ A(t)$) of the amplifier $A(t) = V_{out}/V_{in}$ as function of time should be investigated in order to obtain insight in the behavior. Also the introduction of nonlinearities in the feedback circuit should be considered in order to minimize the influence of the amplifier nonlinearities [15].

The Barkhausen criterion is a starting point for the design of an oscillator. The Nyquist criterion may be looked upon as a generalization of the Barkhausen criterion. Both criteria are necessary but not sufficient criteria for an oscillator based on a nonlinear amplifier with positive linear feedback [16]. In order to find a sufficient criterion for steady state oscillations you should investigate the nonlinear dynamics of electronic oscillators [17], [18], [19].

It is an open question if it is possible to find a sufficient criterion for steady state oscillations. So far you have to use a "hit and try" approach.

Fig. 4. Wien Bridge Oscillator

III. DISCUSSION OF A WIEN BRIDGE OSCILLATOR

The Wien Bridge oscillator in Fig. 4 is investigated in [15] at the frequency 0.7958 Hz well below the 12 Hz dominating pole frequency of the $AD712$ operational amplifier in order to obtain a linear oscillator. Here we design the oscillator to 3.333 kHz (period 300$\mu$s). The start-up phase and the steady state phase are investigated.
Assuming ideal operational amplifier the component values corresponding to a complex pole pair at the imaginary axis becomes: \( RA = RB = 20k \Omega, CA = CB = 2.387324147\text{nF}, RC = 6.00k \Omega, RD = 3k \Omega. \) For \( RC = 6.010k \Omega \) the pole pair is in \( RHP. \) In order to make the analysis more close to the real world conditions a 1Ω internal resistance is assumed for the power sources \( V \) and \( V \). Also a rise time of 1\( \mu s \) is assumed. \( V \) is switched on 200\( \mu s \) before \( V \). The results depends on course of the operational amplifiers used. We may investigate the Wien Bridge oscillator as a modified multi-vibrator i.e. the capacitor \( CA \) may be removed [20].

Figure 5 shows the startup phase. Figure 6 shows the steady state behavior. Without the two diodes and resistor \( RCL \) a large pulse is seen in the current of \( V \) and the amplitude of \( V(3) \) is close to the power source voltage.

![Fig. 5. Start-up of oscillator with \( V \) delayed 200\( \mu s \) after \( V \). Cadence PSpice Lite 16.5 is used for simulation](image1)

![Fig. 6. Steady state without and with diodes and \( RCL \)](image2)

![Fig. 7. Transfer characteristic of the amplifier \( V(3) \) as function of \( V(1,2) \) without and with the diodes and \( RCL \)](image3)

With the diodes and \( RCL \) the pulse disappear and the amplitude of \( V(3) \) is about 0.6V. Figure 7 shows the transfer characteristic of the amplifier i.e. the output voltage \( V(3) \) as function of the input voltage \( V(1,2) \).

The answer to the question: Why does a Wien Bridge oscillator oscillate may be given as follows.

Due to the unstable small signal model the amplitude of the output voltage \( V(3) \) will increase until the power source level is reached. The mechanism seems to be the common multi-vibrator mechanism. When the two diodes and the resistor \( RCL \) are introduced in the feed-back we have the time varying small signal model mechanism for creating an energy balance with a complex pole pair moving between \( RHP \) and \( LHP \).

**REFERENCES**


