Dual Decomposition for Large-Scale Power Balancing

Halvgaard, Rasmus Fogtmann; Jørgensen, John Bagterp; Vandenberghe, Lieven

Published in:
Proceedings of the 18th Nordic Process Control Workshop

Publication date:
2013

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Dual Decomposition for Large-Scale Power Balancing

Rasmus Halvgaard ∗ John B. Jørgensen ∗
Lieven Vandenberghe ∗∗

∗ Technical University of Denmark,
Kgs. Lyngby, Denmark (e-mail: {rhal,jbjo}@dtu.dk).
∗∗ UCLA, Los Angeles, CA 90095 USA
(e-mail: lieven.vandenberghe@ucla.edu)

Abstract: Dual decomposition is applied to power balancing of flexible thermal storage units. The centralized large-scale problem is decomposed into smaller subproblems and solved locally by each unit in the Smart Grid. Convergence is achieved by coordinating the units consumption through a negotiation procedure with the dual variables.

Keywords: Decomposition methods, Decentralized Control, Model Predictive Control, Smart Grid, Smart power applications

1. INTRODUCTION

A large number of flexible thermal storage units, e.g. electrical heating in buildings or cooling in refrigeration systems, will soon part of the Danish power system. These units could potentially provide a large flexible consumption by aggregating or pooling them together. This will enable them to be coordinated and help follow the fluctuating energy production from renewables such as wind power. We formulate an optimization problem of tracking a power reference. To solve this large-scale control problem in real-time, we decompose the original problem into smaller subproblems and solve them locally by each unit. Each unit has its own model and variables and can make decisions based on its own local control strategy. The need for system level flexibility is communicated through a negotiation procedure with the dual variables. The units could potentially provide a large flexible electrical heating in buildings or cooling in refrigeration systems, will soon part of the Danish power system.

We solve the problem (2) by solving its unconstrained dual problem with the subgradient descent method Vandenberghe (2011); Bertsekas (1999). The dual is obtained via the Lagrangian L

2. PROBLEM FORMULATION

The centralized large-scale problem to be solved at every time instant t is

$$
\text{minimize} \quad g(p(t), q(t)) \\
\text{subject to} \quad p(t) = \sum_{k=1}^{n} u_k(t) \\
\quad x_k(t+1) = A_k x_k(t) + B_k u_k(t) \quad (1) \\
\quad y_k(t) = C_k x_k(t) \\
\quad y_k^{\min} \leq y_k(t) \leq y_k^{\max} \\
\quad u_k^{\min} \leq u_k(t) \leq u_k^{\max} .
$$

$q(t), t = 1, ..., N$ represents a desired power consumption profile over a period of length $N$. $p(t)$ is the actual power demand and is a sum of the power demands $p_k(t), k = 1, ..., n$ for each of the $n$ units. A power capacity limitation can also be included by adding the inequality constraint $p(t) \leq p^{\max}(t)$. $y_k(t)$ is the output of a linear system with input $u_k(t)$. The variables are $p(t), p_k(t), x_k(t), d_k(t), u_k(t)$. We define the set $F_k$ as a bounded polyhedron containing the linear state space system and its constraints in (1). To lighten notation further the time argument will be omitted from here on. The unit constraints in $F_k$ can be moved to the objective by expressing them as an indicator function

$$
f_k(u_k) = \begin{cases} 0 & \text{if } u_k \in F_k \\ +\infty & \text{otherwise} \end{cases}
$$

Finally, the optimization problem to be solved by the receding horizon controller at every time instant is

$$
\text{minimize} \quad g(p) + \sum_k f_k(u_k) \\
\text{subject to} \quad p = \sum_k u_k 
$$

3. DUAL DECOMPOSITION

We solve the problem (2) by solving its unconstrained dual problem with the subgradient descent method Vandenberghe (2011); Bertsekas (1999). The dual is obtained via the Lagrangian $L$
\[ L = \frac{1}{2} ||p - q||^2 + \sum f_k(u_k) + z^T \left( p - \sum_k u_k \right) \]

where \( z \) is the dual variable. The dual function is

\[ \inf_p L = \frac{1}{2} ||z||^2 + q^T z - \frac{1}{2} \sum_k \inf_{u_k} \left( f_k(u_k) - z^T u_k \right) \]

\[ = -\frac{1}{2} ||z||^2 + q^T z - \sum_k \sup_{u_k} \left( z^T u_k - f_k(u_k) \right) \]

Finally, the dual problem is

\[ \text{maximize} \quad -\frac{1}{2} ||z||^2 + q^T z - \sum_k S_k(z) \quad (3) \]

with

\[ S_k(z) = \sup_{u_k \in F_k} z^T u_k. \]

\( S_k(z) \) is the support function of \( F_k \). If \( F_k \) is a bounded polyhedron, we can evaluate \( S_k \) at \( z \).

Solving (3) with the subgradient projection method gives us the updates

\[ z^+ = z + t^+ \left( \sum_k u_k^+ - (z + q) \right). \quad (5) \]

The step size \( t^+ \) must be decreasing at each iteration \( j \), i.e. \( t^+ = \frac{1}{j} \rightarrow 0 \), for \( j \rightarrow \infty \). If it doesn’t decrease the subgradient method will not converge to the minimum.

With the chosen LP subproblems the dual gradient method converges but the primal solution is not easily recoverable from the dual. An extra strictly convex term can be added to the subproblems, e.g. a temperature reference on the output

\[ \text{minimize} \quad \frac{1}{2} || \sum_k u_k - q ||^2 + \frac{1}{2} || y_k - r_k ||^2 \]

subject to \( u_k \in F_k \).

The LP subproblems from (4) are now QPs on the form

\[ u_k^+ = \text{argmin}_{u_k \in F_k} \left( \frac{1}{2} || y_k - r_k ||^2 + z^T u_k \right). \]

This problem formulation is equivalent to the ordinary optimal control problem with an added linear term, that can be solved efficiently by methods based on the Riccati recursion Jorgensen et al. (2004, 2012).

If an upper bound on the power \( p \) is added, the dual variable \( z \) can be clipped in (5) by keeping \( 0 \leq p \leq p^{\text{max}} \).

4. NUMERICAL EXAMPLE

An example with two different first order thermal storage systems is was simulated. The models have unity gain, time constants 5 and 10, and both a temperature reference equal to \( r_k = y_k^\text{ref} \). The results for step size \( t = 0.3 \) after 100 iterations is shown in Fig. 1. The power tracking profile is seen match most of the time, but it is not possible to control the consumption amplitude of each unit very accurate through the dual variables, since each unit has its own objective leaving the tracking at some compromise.

\[ \text{REFERENCES} \]

