On polarization metrology (estimation) of the degree of coherence of optical waves

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Abstract: A new approach is proposed for estimating the degree of coherence of optical waves. The possibility of transformation of the spatial polarization distribution in the measured spatial intensity distribution for determining the degree of correlation of superposing waves, linearly polarized in the plane of incidence, is shown.

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1. Introduction

Coherence of superposing vector optical waves can manifest itself in spatial intensity modulation due to interference, as well as in spatial modulation of the polarization of the resulting distribution. Limiting cases, i.e. intensity modulation of a field caused by interference of equally polarized components and polarization modulation without intensity modulation for superposition of orthogonally polarized waves of equal intensity, have been comprehensively studied [1]. The possibility of using the data contained in such distributions for estimation of the degree of coherence of the corresponding waves has been shown. Interference techniques are widely used for this purpose. Discussion of the feasibilities for estimating the degree of coherence based on polarimetry has been started in [2–4], although mainly within a heuristic aspect. The paraxial approximation has shown that the visibility of interference patterns of the Stokes parameters contains data on the degree of coherence of vector optical fields [3,4]. If the longitudinal (z-) component of the vector optical field is considered, new experimental solutions are needed. One of the first experimental attempts to take advantage of the spatial polarization modulation of a field at the plane of incidence for illustrating the effect of coherence for superimposed optical waves has previously been...

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presented [2]. But the cases when both spatial intensity modulation and polarization modulation take place simultaneously with superposition of waves are advancing for practical use. In these cases, estimating the degree of coherence of waves presumes accounting for the contributions of both these factors for modulation of the resulting distribution, using the ideas formulated in Refs [2,5,6]. These ideas deal with additivity of correlations in the field and, mainly, searching for the experimental means for obtaining quantitative correlation parameters of the superposing waves contained in the spatial intensity and the polarization distributions. This problem is addressed in this paper.

The practical importance of this problem is increasing owing to the development of techniques based on confocal microscopy of isolated molecules, including long molecules oriented along the direction of beam propagation, and due to systems for 3D imaging of such molecules [7], where knowledge about the \( z \)-component of the field is absolutely necessary [8]. In general, such situations occur in problems like:

- transmission of radiation through optically anisotropic crystals;
- multiple light scattering of coherent radiation in turbid media, as well as transmission of optical radiation through optical waveguides;
- heterodyning (nonlinear mixing) of optical waves of different states of polarization, as well as analysis in the near zone of a field scattered by random phase objects.

2. Experimental study

Let us consider a superposition of optical waves polarized in the plane of incidence. We consider superposition of two waves of equal intensities, which are linearly polarized in this plane and converging at an angle of 90° (cf. Figure 1). The field spatially modulated with respect to polarization, but with constant intensity results from superposition of such mutually coherent waves, as it is seen in Fig. 2. The associated video shows that the resulting field is modulated with respect to polarization in the plane of incidence. Depending on the phase difference of the interfering waves, i.e. depending on the position of the observation plane, the state of polarization is changed from linear, with orientation of the resulting vector along 0°-0° direction, through elliptical and circular, and back to linear, with orientation of the resulting vector along the 90°-90° direction. This takes place when the interfering waves are of equal intensity. A distribution that is homogeneous in intensity in the registration plane results from superposition of two interference distributions corresponding to the interference of the \( E_x \) and \( E_z \) components shifted half a spatial period (see Fig. 2, upper right part). This case corresponds to zero visibility, i.e. \( V = 0 \).
Fig. 1. Scheme of superposition of plane waves linearly polarized at the incidence plane and converging at angle of 90°. Magnitudes $|\eta^{(1,3)}| = 1$, $|\eta^{(2,3)}| = 1$ and $|\eta^{(3)}| = 1$ determine the correlation degree between the corresponding waves; $E^{(i)}$.  

$$V' = 2 \sum_x \sqrt{\text{tr}[W(Q_x, Q_x, 0)] \text{tr}[W(Q_x, Q_x, 0)]} \left| p^{(i)}_x \right|^2$$  

$$+ 2 \sum_x \sqrt{\text{tr}[W(Q_x, Q_x, 0)] \text{tr}[W(Q_x, Q_x, 0)]} \left| p^{(i)}_x \right|^2 \cos[\phi_x] + \text{ and } E^{(3)}$$ are amplitudes of the superimposed waves.
Fig. 2. Polarization distribution in the plane of superposition of two plane, linearly polarized waves of equal intensities along axis OX shown by red. Red fragment at the left illustrates homogeneous intensity distribution. The dependence of the field intensity shown in relative units from coordinate (right upper part of a figure) shows the mechanism for obtaining homogeneous intensity in case of a half-period shift of the interference distributions of equal visibility for the $E_x$ and $E_y$ components. The resulting visibility $V=0$. Negative magnitudes of $x$-coordinate are explained by the fact that the introduced coordinate $<0$ corresponds to the initial point of convergence of waves, where the resulting polarization is linear having a phase difference is zero. Media 1.

An experimental study of such distribution has been carried out using a linearly polarized plane reference wave, correlated at least with one of two superposing waves, perpendicular to the registration plane. In such a way, transformation of the spatial polarization distribution of the resulting field into a periodic spatial intensity distribution has been implemented. In other words, we consider the result of three-beam superposition for the waves polarized at the incidence plane, $E^{(i)}(Q_1, t)$, $E^{(2)}(Q_2, t)$ and $E^{(3)}(Q_3, t)$; here $E^{(3)}(Q_3, t)$ is the reference wave. $Q_1, Q_2, Q_3$ - are the coordinates of the pinhole centers.

In general, a random electromagnetic field formed at instant $t$ by the sources $Q_1, Q_2, Q_3$ and observed at point $r$ at the screen can be represented as,

$$ E(r, t) = E(Q_1, t) \frac{\exp(ikR_1)}{R_1} + E(Q_2, t) \frac{\exp(ikR_2)}{R_2} + E(Q_3, t) \frac{\exp(ikR_3)}{R_3} $$  \hspace{1cm} (1)

where $k$ is the wavenumber, $R_1 = |r - Q_1|$, $R_2 = |r - Q_2|$, $R_3 = |r - Q_3|$ are distances of point $r$ from the pinhole centers. The point $r$ is the point of analysis in the registration plane.

The time-averaged intensity distribution in the observation plane is given by [6]
\[ I(\mathbf{r}) = \sum_{ij} \left[ \psi_{ij}^{(m)}(\mathbf{r}) + \psi_{ij}^{(n)}(\mathbf{r}) + \phi_{ij}^{(m,n)}(\mathbf{r}) + 2 \sqrt{[\text{tr}[W(Q_x, Q_z, 0)] \text{tr}[W(Q_x, Q_z, 0)]} \eta_{ij}^{(m,n)} \cos[\phi_{ij}^{(m,n)}] \cos[\delta_{ij}] + 
\]
\]
\[ + 2 \sqrt{[\text{tr}[W(Q_x, Q_z, 0)] \text{tr}[W(Q_x, Q_z, 0)]} \eta_{ij}^{(m,n)} \cos[\phi_{ij}^{(m,n)}] \cos[\delta_{ij}] + 
\]
\[ + 2 \sqrt{[\text{tr}[W(Q_x, Q_z, 0)] \text{tr}[W(Q_x, Q_z, 0)]} \eta_{ij}^{(m,n)} \cos[\phi_{ij}^{(m,n)}] \cos[\delta_{ij}] \right] \] 

Here \( \text{tr}[W(Q_x, Q_z, 0)] = \sum_i W_{ii}(Q_x, Q_z, 0) \), where \( W_{ii}(Q_x, Q_z, 0) \) are the diagonal elements of the 2 \times 2 matrix of mutual coherence \( W(Q_x, Q_z, t) \), describing correlation of the corresponding sources; \( \psi_{ij}^{(m)}(\mathbf{r}) = \langle E_i^{(m)}(\mathbf{r}, t) E_j^{(m)}(\mathbf{r}, t) \rangle \), \( m = 1, 2, 3; i, j = x, z \); and \( \eta_{ij}^{(m,n)} \) determines the degree of correlation of the field components \( m, n = 1, 2, 3; \alpha_{ij}^{(m,n)} \) is the argument of the complex value of \( \eta_{ij}^{(m,n)} \) and determines the phase difference between the \( i \)- and \( j \)-components of the field, and \( \delta_{ij} = k(R_i - R_j) \) are the phase differences of the corresponding fields in the plane of registration.

Imposing a reference wave results in redistribution of the states of polarization of the field in the registration plane, Fig. 3. Therefore, besides spatial polarization modulation, intensity modulation occurs, cf. Media 2. Only this parameter is measured.

Fig. 3. Video illustrating changes of modulation depth of the resulting interference distribution for change in time of a phase of the reference wave synchronized with changes of the resulting state of polarization along the axis OX (red). Red and black correspond to the distribution caused by interference of \( E_x \)– components and \( E_z \)– components, respectively. The VMD \( M = \max[V^+] - \min[V^+] \) is determined by the difference of maximal and minimal magnitudes of visibility, which in this case equals unity (i.e. \( \max[V^+] = 1 \) and \( \min[V^+] = 0 \)).

Changing the phase of the reference wave within the interval \( [0, 2\pi] \) results in a periodic change in visibility of the registered interference pattern following the harmonic law:

\[ \text{Media 2.} \]
\[
V^\phi = 2\sum_{ij} \sqrt{\text{tr}[W(Q_i, Q_j, 0)] \text{tr}[W(Q_j, Q_i, 0)]} |\eta_{ij}^{(1,2)}| + \\
2\sum_{ij} \sqrt{\text{tr}[W(Q_i, Q_i, 0)] \text{tr}[W(Q_j, Q_j, 0)]} |\eta_{ij}^{(1,3)}| \cos|\varphi_j| + \\
2\sum_{ij} \sqrt{\text{tr}[W(Q_j, Q_j, 0)] \text{tr}[W(Q_i, Q_i, 0)]} |\eta_{ij}^{(2,3)}| \cos|\varphi_j| + 
\]

(3)

where \(\varphi_1\) and \(\varphi_2\) are the phase differences between the two initial superposing waves and the reference wave.

One observes the change (from unity to zero) in the visibility of the periodic spatial interference distribution for superposition of three mutually coherent waves in Fig. 3. The visibility modulation depth (VMD) is determined by:

\[
M = \max[V^\phi] - \min[V^\phi] = \\
2\sum_{m} \sum_{ij} \sqrt{\text{tr}[W(Q_m, Q_m, 0)] \text{tr}[W(Q_j, Q_j, 0)]} |\eta_{ij}^{(m,3)}| \\
\phi_{ij}^{(m)}(r) + \phi_{ij}^{(3)}(r) |\eta_{ij}^{(m,3)}| 
\]

(4)

for \(m = 1, 2; i, j = x, Z\)

Choosing the reference wave to be completely correlated with one of the initial waves, to say \(\eta^{(1,3)} = 1\), one can see that the VMD of the interference pattern, \(M\), characterizes, within a constant depending on the intensity ratio, the degree of mutual coherence of the reference wave and the second of the initial waves, i.e. \(M = K |\eta^{(2,3)}|\). Taking into account that \(\eta^{(1,3)} = 1\), one concludes that \(|\eta^{(2,3)}| = |\eta^{(1,2)}|\). Thus, by a proper choice of intensities of the interfering waves one can obtain \(K \to 1\), and thus \(|\eta^{(1,2)}|\) will be determined by the VMD of the interference pattern, i.e. \(M = |\eta^{(1,2)}|\). In this case, the VMD is unity, which strictly corresponds to the degree of coherence of the initial superposing waves. Figure 4 demonstrates that polarization modulation, really, takes place in the plane of incidence.
The initial attempt of holographic estimation of the degree of coherence of such waves has previously been described [2], where for providing “purity” of an experiment an immersion liquid is used as shown in Fig. 5, so that the angle between the two plane waves is 90°, and only the spatial polarization modulation of the resulting distribution is present. Usage of an immersion liquid also facilitates an effective reconstruction of the beam at the readout stage. The use of a holographic technique is called for due to the necessity of registration of spatial intensity distribution at very high spatial frequencies.

![Fig. 5. Optical arrangement for holographic experiment: Bs1 and Bs2, beam splitters; M1, M2, and M3, mirrors; P1, P2, and P3, polarizers; PR, prism; IL, immersion liquid; H, hologram.](image)
It is evident that holographic registration of the resulting interference distribution has to be carried out step-by-step, by changing the phase of the reference wave within the interval $[0, 2\pi]$ for revealing the maximal modulation depth of the distribution. Now, having high-resolution position-sensitive devices, such as CCD cameras, estimating the degree of coherence of polarized waves can be performed in real-time imposing a gradual change of the phase of a reference wave using a piezoelectric translator.

Fig. 6. Scheme of superposition of plane waves linearly polarized at the incidence plane and converging at an angle of 90°. Magnitudes $|\eta^{(1,3)}| = 0.5$, $|\eta^{(2,3)}| = 0.5$ and $|\eta^{(3,3)}| = 1$ determine the correlation degree between the corresponding waves; $E^{(1)}$, $E^{(2)}$ and $E^{(3)}$ are amplitudes of the superimposed waves.

Fig. 7. Video illustrating polarization modulation when superimposing orthogonally linearly polarized waves converging at an angle of 90° are not strictly coherent. Brown shows the lines corresponding to the contribution from incoherent component of a beam 1 (Fig. 6), green shows the lines corresponding to the coherent “residual” component of beam 1, blue shows the resulting polarization distribution. Other symbols are the same as in the previous media files. Media 4.
Changing the degree of coherence of the superimposed, orthogonally polarized waves in the same experimental arrangement results in changes of the spatial polarization modulation of the resulting distribution, Fig. 7, which also manifests itself in the VMD of an interference distribution. The following conventional signs are used for the spatial distribution of resulting polarization, obtained in the arrangement, cf. Figure 6. The contribution of an incoherent component in beam 1 is shown in brown as a line perpendicular to the direction of beam propagation. The coherent component of this beam is shown as a green vector, whose size and direction strictly coincide with the brown line of an incoherent component. That is why; it seems invisible for an observer, even though it takes part in the coherent superposition with the analogous vector of a second beam, Fig. 7. The coherent component of a second beam is shown by the same green color, but the part of its amplitude that does not take part in formation of the resulting distribution is shown in blue. Generally, for comparison of polarization distribution for superposition of completely coherent and partially coherent waves, we will represent the resulting distribution in the partially coherent case as a superposition of two polarization distributions. One of them results from the superposition of the coherent part of a beam 1 and equal-intensity part of a coherent beam 2. This results in a spatial polarization distribution corresponding to the coherent case, which here is represented by the distribution of blue lines. Adopting conventional signs and observing the spatial polarization distribution, one can qualitatively judge changes in the polarization distribution passing from the completely coherent case to the partially coherent case.

Using a reference wave polarized in the same plane of incidence makes spatial polarization modulation more complex as shown in Fig. 8. One observes the presence of two spatial polarization distributions, viz. blue (the resulting one) and a green one. In this case, our approach to formation of the resulting polarization distribution is the following: It is assumed that interaction of mutually coherent reference (3) and object wave (2) results in forming a new wave with its own amplitude and polarization properties (Fig. 3). Superposition of such a new wave with a coherent component of wave 1 leads to formation of a polarization distribution represented in blue. This results from the removal of a new wave from the initial distribution, i.e. the part of the intensity that is equal to the part of a coherent component of a wave 1. The polarization distribution shown in green corresponds to the part of intensity formed by waves 3 and 2 (a new wave), left after removing the component of equal intensity of the coherent part of wave 1. Therefore, the VMD M, Eq. (4), of the resulting intensity distribution strictly corresponds to the degree of coherence of the studied field. Figure 9 demonstrates that the polarization distributions are localized in the plane of incidence.
Fig. 8. Video illustrating changes of the modulation depth of the resulting interference distribution for temporal changing the phase of the reference wave synchronized with changes of the states of polarization along axis OX. Modulo of the correlation degree of superposing waves 1 and 2 $|\eta^{(1,1)}| = 0.5$. The VMD is 0.5. Media 5.

Fig. 9. Illustration of the fact that the field is left polarized at the incidence plane. Media 6.
Consider the case when the degree of coherence of waves 1 and 2 is $\eta^{(1,2)} = 0.5$ and for waves 2 and 3 $\eta^{(2,3)} = 1$ (Fig. 6). The incoherent component shown in brown is mixed additively based on intensity with the spatial polarization distribution caused by the coherent components of the corresponding waves (blue and green). This leads to a decrease of the VMD of the resulting pattern to a level of 0.5, as it is seen from Fig. 8, illustrating as the graphs’ change of visibility of the measured distribution following from phase changes of the reference wave. The same can also be observed in the resulting intensity distribution. Likewise, polarization modulation takes place at the incidence plane, as shown in Fig. 9.

Similar results have been obtained for all studied magnitudes of $\eta^{(1,2)}$, namely, the VMD of the resulting interference pattern strictly corresponds to the degree of mutual coherence of the superimposed waves. Deviation of the convergence angle of fully correlated waves from 90° leads to simultaneous manifestation of both spatial polarization modulation and interference-caused intensity modulation. Figure 10 gives an impression on the resulting field distribution in the registration plane, where periodic intensity distributions caused by interference of coaxial x- and z-components of three superimposed waves are represented, as well as the resulting polarization distribution. As a matter of fact, one can state, both for the coordinate intensity distribution of the resulting field and for the spatially modulated polarization field distribution (SMP), that the maximal magnitudes at the distribution of the $E_x$ components coinciding with the minimal (zero) magnitudes at the distribution of the $E_z$ components strictly coincide with the points of linear polarization of the corresponding distribution (see Fig. 10). Thus, by proper choice of amplitude and phase of the reference wave linearly polarized in the plane of incidence, one can obtain zero magnitudes of amplitude at these positions, as well as obtain a visibility of unity of the resulting intensity distribution. This results in complete mutual coherence of the waves. Additive contribution of correlation in the resulting correlation of waves predicted in Ref [6], is obvious.

![Diagram](image_url)

Fig. 10. a) Scheme for interference of three plane waves linearly polarized in the plane of incidence, when the convergence angle differs from 90°. b) Intensity distribution in the incidence plane: interference of $E_x$-components of interacting waves; the resulting distribution. c) Polarization modulation at the incidence plane.
Besides, estimation of the degree of mutual correlation of superimposing orthogonally linearly polarized waves can be performed in another way. It can be shown that in the case of superposition of plane waves of equal intensities polarized linearly in the plane of incidence but with the angle of convergence different from 90°, their degree of mutual coherence can be estimated without using a reference wave. In this case, the intensity modulation of the resulting field manifests itself as a periodic interference pattern without reference wave [9,10], and the interconnection between the visibility of a pattern and the degree of mutual coherence is determined by

\[ \eta^{(1,2)} = \frac{V}{\cos[\Delta \theta]}, \]  

(5)

where \( \Delta \theta \) is the difference in angle of incidence of the waves. Experimental studies have proven the efficiency of both metrological approaches for estimating the degree of coherence of vector optical fields.

3. Conclusions

The feasibilities for using the data contained in the structure and peculiarities of the spatial polarization modulation of a field resulting from superposition of vector optical fields for estimation of the degree of field correlation have been investigated. This paper is a continuation of Ref [2], where manifestations of coherent interaction of polarized waves were studied both in stationary and in heterodyne regimes. The algorithm for estimation of the degree of coherence of vector optical waves consists in searching for optimal means of transformation of field polarization distributions into measurable intensity distributions, providing the necessary data.