On the influence of crack closure on strength estimates of wood

Nielsen, Lauge Fuglsang

Published in:
European Journal of Wood and Wood Industries (Print)

Link to article, DOI:
10.1007/s00107-003-0446-x

Publication date:
2004

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
On the influence of crack closure on strength estimates of wood

L. F. Nielsen

Published online: 6 February 2004
© Springer-Verlag 2004

Abstract Three well-known duration of load models (Gerhard, Barrett/Foschi, DVM) are considered in this note with respect to their ability to predict lifetime of wood subjected to harmonically varying loads. The result obtained is that they practically predict the same lifetime—which for low frequency loading can be considered approximately true. For higher frequencies, however, this result can be far too overestimated. The reason is that the models considered do not take into account the effect of the crack closure phenomenon (which are the main mechanisms of energy dissipation causing fatigue failure in metals).

It is suggested that any of the simple models can be used in practice when low frequency load variations are considered. The DVM model, however, should be preferred because of its ability to predict residual strength, and because of its ‘build in’ flexibility with respect to wood quality and ambient climatic conditions.

For high frequency load histories more refined models are required. The extended DVM model, recently developed by the author, is suggested as such a model—especially because it has the potentials of being further developed to consider arbitrary load variations (such as earthquakes).

Finally, the widely spread concept of estimating long-term strength by multiplying short time strength with a codified factor (so-called \( k_{MOD} \) factor) is discussed. It is concluded that the \( k_{MOD} \)-method can be justified in practice with low frequency load variations. When high frequency load histories or unexpected peak loads are considered, the \( k_{MOD} \)-method may cause considerably overestimated lifetimes.

Notations

Load and strength

- Theoretical strength (no damage): \( \sigma_1 \)
- Real strength at \( t=0 \): \( \sigma_{CR} \)
- Strength level (wood quality): \( FL = \sigma_{CR}/\sigma_1 \)
- Load: \( \sigma \)
- Load level: \( SL = \sigma/\sigma_{CR} \)
Max load: $\sigma_{\text{MAX}}$ · Min load: $\sigma_{\text{MIN}}$ · Load ratio: $p=\sigma_{\text{MIN}}/\sigma_{\text{MAX}}$ · Cyclic time: $T$ · Frequency: $f=1/T$.
Fractional time under max load: $\beta$ · Real strength at $t$: $\sigma_{\text{CR}}(t)$ · Residual strength: $S_F=\sigma_{\text{CR}}(t)/\sigma_{\text{CR}}$

**Damage**
Initial crack length: $l_o$ · Immediate crack length: $l$ · Damage ratio (or just damage): $\kappa=ll_o$ · Fatigue parameters in extended DVM model: $C, M$ · Damage degree in the models of Gerhard’s and Barrett/Foschi’s: $\alpha$ · Fit parameters in Gerhard’s model: $A, B, C$ · Fit parameters in Barrett/Foschi: $C, D, E, F$

**Time and creep**
Time in general: $t$ · Creep function: $c=(1+(t/T)^{1/b})/E$ · Young’s modulus: $E$ · Relaxation time in creep: $\tau$ · Creep power: $b$ · Time shift parameter: $q=(0.5(1+b)(2+b))^{1/b}$

---

**1 Introduction**

Three lifetime models for wood are currently discussed\(^1\) with respect to their ability to act as a duration of load basis in ‘Reliability-based design of timber structures’: The theory of Gerhard’s (Gerhard 1979), the theory of Barrett/Foschi (1978a, b), and the theory of Fuglsang’s (Nielsen 1982, 1991; Madsen 1992), the so-called DVM theory (Damaged Viscoelastic Material).

Basically, all three models are developed for constant loads. The DVM model, however, is prepared also to consider some continuous load variations. In spite of any restrictions on the three models—but because of their simplicity, there is a demand from the wood community for using one of them as a basis for ‘rules’ in design of wood structures subjected to variable loads.

On this background the three models are compared in this note with respect to their ability to predict lifetime of wood subjected to harmonically varying block loads as defined in Fig. 2. The predictions obtained are evaluated against ‘experimental data’ represented by an extended version of the DVM theory (Nielsen 1993, 2000) which has proven its ability to describe very well measured lifetime (‘real experimental data’) of wood subjected to load histories similar to the kind just defined.

---

### 1.1 Model presentation

The basic expressions of the three theories mentioned above are the following:

$$\frac{da}{dt} = \frac{1}{A} \cdot \exp(B \cdot SL) \quad \text{Gerhard}$$

$$(A, B) = (1.58 \times 10^{14} \text{ hours}, 36.2) \quad (1)$$

\(^1\) For example in the European Commission COST-E24 project where preliminary studies are made with respect to code preparation for reliability based design of timber structures.

---

\[ \frac{d\alpha}{dt} = C \cdot (SL - F)^D + E \cdot \alpha \quad \text{Barrett/Foschi} \]

$$C, D, E, F = (10/\text{hour}, 25, 10^{-8.5}/\text{hour}, 0) \quad (2)$$

\[ \frac{dk}{dt} = \frac{(\pi FL)^2}{8\tau} \cdot \frac{\kappa SL^2}{(1/(\kappa SL^2) - 1)^{1/b}} \]

$$q = (0.5(1+b)(2+b))^{1/b} \quad \text{Simple DVM}$$

$$(FL, b, \tau) = (0.3, 0.2, 10 \text{ days}) \quad (3)$$

Basiclly the theories of Gerhard’s and Barrett/Foschi’s are empirical where the parameters $A–F$ have to be determined by calibration to experimental data. The so-called degree of damage $\alpha$ develops from 0 at loading time ($t=0$) to 1 at time ($t_{\text{CAF}}$) where catastrophic failure occurs. SL is load level as defined in the list of notations.

The DVM theory is based on a crack mechanical analysis of wood. The damage ratio $\kappa$, defined in the list of notations, develops from 1 at start of loading to 1/$SL^2$ at failure, where the rate of the damage ratio becomes infinitely high. FL is wood quality as defined in the list of notations. The relaxation time, $\tau$, and the creep power, $b$, define the creep experienced by the wood in damage areas (see Fig. 1 and list of notations). Equation 3 is subsequently referred to as the simple DVM theory. The other version of the DVM theory previously mentioned (Nielsen 1993, 2000) will be referred to by the adjectives ‘extended’ or ‘generalized’.

**Remark.** Contrary to Eqs. 1, 2 and 3, the extended DVM theory operates with two dissipative mechanisms being responsible for the duration of load effects in wood: One is the creep mechanism (already taken care of in Eq. 3). The other one is the so-called crack closure mechanism well known from fatigue studies of metals. The generalized theory applies for any damaged viscoelastic material (with Power law creep). As special cases metals are included with no creep, $\tau=\infty$.

---

**Fig. 1** Creep function for materials with so-called Power-law creep

**Abb. 1** Kriechfunktion für Materialien, deren Kriechen nach dem sog. Potenzgesetz verläuft
2 Preparation of analysis

In the following the Gerhard’s, the Barrett/Foschi’s, and the simple DVM model will be analyzed with respect to their ability to describe duration of load effects for variable loading as defined in Fig. 2 with fractional time under max load, \( b = 0.5 \).

As previously mentioned the ‘experimental’ data used in the evaluation process will be simulated by the extended DVM theory. To get a common basis in the analysis we calibrate all three theories to the BM-trend line established by Borg Madsen in (Madsen 1992, Fig. 6.1) for the lifetime \( t_{\text{CAT}} \) of clear wood specimens subjected to long term loading,

\[
t_{\text{CAT}} = \frac{1.58 \times 10^{14} \times 10^{-15.7 \times SL}}{} \text{hours BM - trend} \quad (4)
\]

The fit parameters \((A-F)\) in Eqs. 1 and 2 and the material parameters \((FL, b, \tau)\) in Eq. 3 have the order of magnitudes indicated. The Gerhard fit is exact. The calibration quality of the Barrett/Foschi expression can be studied in Fig. 3, (in the present context the Barrett/Foschi is fitted sufficiently well introducing \( F=0 \)). The DVM theory is calibrated to the BM-trendline data using a computer program reported in (Nielsen 2003). The results are \((FL, b, \tau) = (0.3, 0.2, 10 \text{ days})\) which are within the range of reliable OM-estimates (order of magnitudes) declared in (Nielsen 2000). The calibration quality can be studied in Fig. 4.

The fatigue parameters, \( C \) and \( M \), appearing in the extended DVM theory are chosen by their average quantities presented in (Nielsen 2000), namely \( C,M = 3.9 \), (the parameters \( FL, b, \tau \) keep their quantities from Eq. 3).

3 Analysis

Some results of an evaluation of the three theories considered will now be demonstrated. (Similar results have previously been presented in (Nielsen 1997, 2001a, 2001b, 2002a, 2002b). The results are arranged such that the three methods (Eqs. 1, 2 and 3) are mutually compared in Figs. 5, 7, and 9.

In Figs. 6, 8, and 10 the methods are evaluated against ‘experimental data’ as simulated by the extended DVM theory.

We notice that lifetime predictions by the DVM theories include predictions of ‘residual strength’ (defined as ‘strength left’ or ‘re-cycle strength). The Gerhard and Barrett/Foschi theories cannot describe this property.
Intermediate conclusions and remarks

The empirical Gerhard’s and Barrett/Foschi’s models do not consider the crack closure phenomenon as a driving mechanism in damage propagation. The simple DVM model also disregards this mechanism. These three models are compared in Figs. 5, 7, and 9. It is seen that they practically predict the same lifetime independently of load frequencies—a behavior which has clearly been demonstrated, not to be true by Bach (1979) already in 1975. The observation of Bach’s has later been confirmed by a number of other authors (e.g. Clorius 2001).

Theoretically the observation of Bach’s is in accordance with the results of the extended version of the DVM theory (Nielsen 2000) developed for lifetime studies of damaged viscoelastic materials in general.

The Gerhard’s and Barrett/Foschi’s models (and indirectly also the simple DVM model) are compared with the
It is observed that all data sets practically coincide at low load frequencies. Apparently the energy dissipation caused by creep overrides the dissipation caused by crack closure mechanisms.

This conclusion, however, cannot be maintained when rapid load variations are considered (Fig. 10). Lifetime is considerably overestimated when using the Gerhard’s, the Barrett/Foschi’s, and the simple DVM theories on rapidly varying load situations. Apparently the energy dissipation caused by crack closure mechanisms overrides the dissipation caused by creep mechanisms.

Remark. The observations just made are clearly demonstrated in Figs. 11 and 12 obtained by a software presented in (Nielsen 2003) for easy safe lifetime estimations for wood subjected to the type of load histories considered in this paper. (The horizontal line in the figures is dead load lifetime for wood loaded with $\sigma_{MAX}$. The steepest line in the figures is fatigue lifetime for wood with damage propagation totally dominated by crack closure mechanisms). Both figures refer to the wood properties considered in this paper. The load parameters in Fig. 11 are also the same as previously considered. In Fig. 12, however, the load ratio has been decreased to $p=0$ practically (complete unloading).

The transition range of frequencies between ‘low’ and ‘high’ frequency loading is suggested in (Nielsen 2000) to be $10^{10}<f<10^{5}$, which for the wood properties considered in this note means $10^{-5}<f<0.1$.

4.1 Future

Harmonic load variations have been assumed in this note. An important future research project is to develop a lifetime theory, which applies for more general variations (non-harmonic load histories). It has been demonstrated in (Nielsen 1996a, 1996b) that the extended DVM theory has the basic potentials to be further generalized for such loads. Figures 13 and 14 demonstrate some results obtained by a pilot theory in (Nielsen 1996a) being tested predicting the residual strength of wood subjected to a simulated load history caused by earthquakes.

5 Final conclusions

It seems that any of the simple duration of load models (Eqs. 1, 2 and 3) can be used in practice when low

---

**Fig. 10** Prediction of lifetime using Eqs. 1, 2 and the extended DVM theory with crack closure considered

**Fig. 11** Safe lifetime estimates of wood with properties and a load history as considered in this paper: $(FL, b, C, M)=(0.3, 0.2, 3, 9)$ and $(SL_{MAX}, p)=(0.6, 0.001)$

**Fig. 12** Safe lifetime estimates of wood with properties as considered in this paper: $(FL, b, C, M)=(0.3, 0.2, 3, 9)$. Loading is $(SL_{MAX}, p)=(0.6, 0.001)$

---

**Abb. 10** Vorhersage der Standzeit mittels Gl. 1, 2 und der erweiterten DVM-Theorie mit Berticksichtigung des Rissschlusses

**Abb. 11** Schätzen der sicheren Standzeit für Holz mit Eigenschaften und einer Belastungsgeschichte, wie sie hier betrachtet werden: $(FL, b, C, M)=(0.3, 0.2, 3, 9)$ and $(SL_{MAX}, p)=(0.6, 0.5)$

**Abb. 12** Schätzen der sicheren Standzeit für Holz mit Eigenschaften und einer Belastungsgeschichte, wie sie hier betrachtet werden: $(FL, b, C, M)=(0.3, 0.2, 3, 9)$ and $(SL_{MAX}, p)=(0.6, 0.001)$
predicted by the extended DVM model at a very high frequency loading approaches the solution, which can be predicted by a plain fatigue theory (see Figs. 11 and 12).

Finally, the widely spread concept (e.g. Svensson et al. 1999) of estimating long-term strength by multiplying short time strength with a so-called $k_{MOD}$ factor should be related to the findings in this research note:

In principles $k_{MOD}$ factors are calculated using a damage model (disregarding its original constant load assumptions) with forecasted time dependent on load histories (or load levels, $SL$). Hitherto, as in (Stang et al. 2002), the determinations of $k_{MOD}$ factors have been based exclusively on the Gerhard’s or on the Barrett/Foschi’s models. Recently, however, the simple DVM-model (Eq. 3) also has been considered by Köhler, Faber, and Svensson (Köhler 2002, Köhler and Faber 2002, Köhler and Svensson 2002) in their analysis of the influence on $k_{MOD}$ of the damage model chosen. The conclusion made by these authors is that all three models, when properly calibrated, practically imply the same $k_{MOD}$. This conclusion agrees with the intermediate conclusion previously made in this paper for lifetime predictions with models that disregard crack closure (see Figs. 5, 7 and 9).

Based on the results obtained in this paper, the following conclusion can now be made with respect to the $k_{MOD}$ method: The $k_{MOD}$-method can be justified in practice with low frequency load variations ($f^2 t < 10^2$). When higher frequency load histories or unexpected peak loads are considered, the $k_{MOD}$-method may cause considerably underestimated lifetimes.

Literature


Barrett JD, Foschi RO. (1978b) Duration of load and probability of failure in wood. Part II: constant, ramp, and cyclic loadings. Canadian J Civil Eng 4:515–532

Clorius CO (2001) Fatigue in wood—an investigation in tension perpendicular to grain. PhD Thesis, Department of Civil Engineering, Technical University of Denmark

Gerhards CC (1979) Time-related effects on wood strength: a linear cumulative damage theory. Wood Sci 3:139–144


2 The ability of Eq. 3 to consider some continuously varying loads, is utilized in these references.
Nielsen LF (2001a) Lifetime and residual strength of wood subjected to static and variable load. Contribution to the COST E24 workshop in Coimbra, Mai 2001
Nielsen LF (2001b) Lifetime and residual strength of wood subjected to various load variations. Contribution to the COST E24 workshop on ‘Reliability based design of timber structures’ held in Copenhagen, Denmark, October 19–20
Nielsen LF (2002a) On the influence of crack closure on strength estimates of wood. Paper presented at the COST E24 workshop on ‘Reliability based design of timber structures’ held in Zürich, Switzerland, October 10–11