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On the electron to proton mass ratio and the proton structure

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Abstract – We derive an expression for the electron to nucleon mass ratio from a reinterpreted lattice gauge theory Hamiltonian to describe interior baryon dynamics. We use the classical electron radius as our fundamental length scale. Based on expansions on trigonometric Slater determinants for a neutral state, a specific numerical result is found to be less than three percent off the experimental value for the neutron. Via the exterior derivative on the Lie group configuration space we derive approximate parameter-free parton distribution functions that compare rather well with those for the u and d valence quarks of the proton.

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The mass ratio and the model. – The ratio we get between the electron mass $m_e$ and the proton mass $m_p$ is

$$\frac{m_e}{m_p} = \frac{1}{\alpha} \frac{1}{\pi E},$$

(1)

where $\alpha = e^2/(\hbar 4\pi \varepsilon_0)$ is the fine-structure constant [1] and $E = E/\Lambda$ is the dimensionless ground-state eigenvalue of a reinterpreted lattice gauge theory Kogut-Susskind Hamiltonian [2],

$$\frac{\hbar c}{a} \left[-\frac{1}{2} \Delta + \frac{1}{2} \text{Tr} \chi^2 \right] \Psi(u) = E \Psi(u),$$

(2)

with Manton’s action [3] used now as a potential for a configuration variable $u = e^{i\chi}$ in the Lie group $u(3)$ instead of a link variable $U$ in the $SU(3)$ algebra [4]. The energy scale $\Lambda \equiv \hbar c/a$ corresponds to a fundamental length scale $a$, which we shall relate to the classical electron radius $R_0$ by

$$a \pi = R_0,$$

(3)

where $R_0$ is determined by the electron self-potential energy [5]

$$\frac{1}{4 \pi \varepsilon_0} \frac{e^2}{R_0^2} = m_e c^2.$$

(4)

We assume (2) to describe the baryon spectrum and identify the ground state with the proton. With $E = m_p c^2 = E\Lambda = E\hbar c/a$ and (4) applied in (3), eq. (1) follows directly. Our configuration space is “orthogonal” to the laboratory space wherefor (2) describes a truly interior dynamics which may be projected on laboratory space parameters through the eigenangles $\theta_j$ parametrizing the eigenvalues $e^{i\theta_j}$ of $u$. The projection introduces the dimensionful scale $a$, thus

$$x_j = a \theta_j.$$

(5)

Now a shortest geodesic [6] to track along the full extension of the $u(3)$ maximal torus runs from the origin $u = I$, where all eigenvalues are $1 = e^{i0}$, to $u = -I$, where all eigenvalues are $-1 = e^{i\pi}$, see also fig. 1. When, for instance, the neutron decays to the proton — and the electron is created as a “peel off” — the topological change in the interior baryon state maps by projection to laboratory space. It is not a new idea to suggest the classical electron radius as a fundamental length in elementary particle physics [7,8]. Here we specify the introduction of the scale (3) via the projection (5). Conjugate to the space (angle) parameters in (5) are canonical momentum (action) operators

$$p_j = -i\hbar \frac{1}{a} \frac{\partial}{\partial \theta_j} = \frac{\hbar}{a} T_j,$$

(6)

The toroidal generators $T_j$ induce coordinate fields $\partial_j$ according to the above-mentioned eigenangle parametrization of the $u(3)$ torus. In general the nine generators $T_k$ of $u(3)$ namely induce coordinate fields as follows:

$$\partial_k = \frac{\partial}{\partial \theta} u e^{i\theta T_k} |_{\theta = 0} = u T_k.$$

(7)
The remaining six off-toroidal generators are important in the baryon spectroscopy phenomenology resulting from (2) since they take care of spin and flavour degrees of freedom. With these parametrizations the Laplacian from (2) become those which take care of spin and flavour in the baryon spectroscopy phenomenology resulting from (2) since they take care of spin and flavour degrees of freedom. With these parametrizations the Laplacian in a polar decomposition reads [9,10]

\[ \Delta = \sum_{j=1}^{3} \frac{1}{J^2} \frac{\partial}{\partial \theta_j} J^2 \frac{\partial}{\partial \theta_j} - \sum_{i<j}^{3} \frac{K_k^2 + M_k^2}{8 \sin^2 \frac{1}{2}(\theta_j - \theta_i)}. \]  

Here the Van de Monde determinant, the “Jacobian” of the parametrization is [11]

\[ J = \prod_{i<j}^{3} 2 \sin \left( \frac{1}{2}(\theta_i - \theta_j) \right). \]

The operators \( K_k \) commute as body fixed angular-momentum operators and \( M_k \) “connect” the algebra by commuting into the subspace of \( K_k \)

\[ [M_k, M_l] = [K_k, K_l] = -iK_m, \]

where

\[ K_1 = a\theta_2 p_3 - a\theta_3 p_2 = \hbar \lambda_7, \]
\[ K_2 = a\theta_1 p_3 - a\theta_3 p_1 = \hbar \lambda_5, \]
\[ K_3 = a\theta_1 p_2 - a\theta_2 p_1 = \hbar \lambda_2. \]  

and

\[ M_3/h = \theta_1 \theta_2 + \frac{a^2}{\hbar^2} p_1 p_2 = \lambda_1, \]
\[ M_2/h = \theta_3 \theta_1 + \frac{a^2}{\hbar^2} p_1 p_3 = \lambda_4, \]
\[ M_1/h = \theta_3 \theta_2 + \frac{a^2}{\hbar^2} p_2 p_3 = \lambda_6. \]

The lambdas are corresponding Gell-Mann generators [13]. From these and

\[ Y/h = \frac{1}{6} (\theta_1^2 + \theta_2^2 - 2\theta_3^3) + \frac{1}{3} (v_1^2 + \theta_2^2 - 2\theta_3^3) = \lambda_8/\sqrt{3}, \]
\[ 2I_3/h = \frac{1}{2} (\theta_1^2 - \theta_2^2) + \frac{1}{2} (v_1^2 - v_2^2) = \lambda_3 \]

we find by straightforward but tedious commutations the spectrum

\[ M^2 = \frac{4}{3} \left( n + \frac{3}{2} \right)^2 - K(K + 1) - \frac{1}{3} y^2 - 4i_3^2, \]

\[ n = 0, 1, 2, 3, \ldots, \]  

where \( y \) and \( i_3 \) are hypercharge and isospin three-component quantum numbers. The minimum value for the positive definite \( M^2 \) is 13/4 in the case of spin 1/2, hypercharge 1 and isospin 1/2 as for the nucleon. To solve for the eigenvalues we factorize the wave function

\[ \Psi(u) = \tau(\theta_1, \theta_2, \theta_3) \Upsilon(\alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9), \]

insert it in (2) and then integrate over the off-toroidal degrees of freedom \((\alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9)\) to get for the measure scaled wave function \( R = J\tau(\theta_1, \theta_2, \theta_3) \), for toroidal degrees of freedom

\[ \left[ -\sum_{j=1}^{3} \frac{\partial^2}{\partial \theta_j^2} + V \right] R = 2ER. \]  

Here

\[ V = -2 + \frac{1}{3} (K(K + 1) + M^2) + \frac{1}{3} \sum_{i<j} \frac{1}{8 \sin^2 \frac{1}{2}(\theta_i - \theta_j) \sin^2 \frac{1}{2}(\theta_i - \theta_j)} \]
\[ + 2v(\theta_1) + v(\theta_2) + v(\theta_3), \]

where (see fig. 2)

\[ v(\theta) = \frac{1}{2}(\theta - n\pi)^2, \quad \theta \in [(2n - 1)\pi, (2n + 1)\pi], \]

for \( n \in \mathbb{Z}. \)
By expansion on Slater determinants [10]

$$b_{pqr} = \epsilon_{ijk} \cos(p\theta_i) \sin(q\theta_j) \cos(r\theta_k)$$

(19)

with integer $p, q, r$, we can solve (16) by the Rayleigh-Ritz method [15] to yield the ground-state eigenvalue $E_0 = 4.38$ which corresponds to $m_e/m_0 = 1/1885$. This is less than three percent off the value $m_e/m_n = 1/1838.6 \ldots$ based on experimental data for the electron and neutron [16]. Note that $b_{pqr}$ is antisymmetric in the colour degrees of freedom $\theta_j$.

**Parton distribution functions.** – We can generate parton distribution functions by projections via the momentum form, i.e., the exterior derivative on the $u(3)$ manifold. For this we expand the exterior derivative $dR$ of the measure scaled toroidal wave function $R$ on the torus forms $d\theta_j$,

$$dR = \psi_j d\theta_j.$$ 

(20)

The action of the torus forms on the toroidal coordinate fields expresses the generalization to the interior configuration space manifold of the quantization inherent in the conjugate variables in (5) and (6), thus

$$d\theta_i(\theta_j) = \delta_{ij} \Leftrightarrow [\theta_i, \theta_j] = \delta_{ij}.$$  

(21)

Inspired by Bettini’s elegant treatment of parton scattering [17], we generate distribution functions via our exterior derivative. The derivation runs like this (with $\hbar = c = 1$):

Imagine a proton at rest with four-momentum $P = (0, E_0)$. We boost it virtually to energy $E$ by impacting upon it a massless four-momentum $q = (q, E - E_0)$ which we assume to hit a parton $xP$. After impact the parton represents a virtual mass $xE$. Thus,

$$(xP_\mu + q_\mu) \cdot (xP^\mu + q^\mu) = x^2 E^2,$$ 

(22)

from which we get the parton momentum fraction

$$x = \frac{2E_0}{E + E_0},$$

(23)

or the boost parameter

$$\xi = \frac{E - E_0}{E} = \frac{2 - 2x}{2 - x}.$$ 

(24)

We can use the boost parameter for an angular track $\theta = \pi \xi$ on the manifold in the direction laid out by a specific toroidal generator

$$T = a_1 T_1 + a_2 T_2 + a_3 T_3.$$ 

(25)

With the toroidal generator $T$ as introtangling momentum operator we namely have the qualitative correspondence $q_\theta \sim E - E_0 \sim (1 - x)E \sim (1 - x)T$. That is, we will project along $\xi T \sim (1 - x)T$ in order to probe on $xP_\mu$.

With a probability amplitude interpretation of $R$ we project on a fixed colour base $iT_j$ and sum over the colour components for a specific generator $T$ to get the corresponding distribution function $f_T(x)$ determined by

$$f_T(x)dx = \left(\sum_{i=1}^{3} dR u = \exp(\theta i T) (i T_j) \right)^2 d\theta.$$ 

(26)

By a pull-back operation [18] to parameter space we get

$$\sum_{i=1}^{3} dR u = \exp(\theta i T) (i T_j) = dR u = \exp(\theta i T) (i (T_1 + T_2 + T_3))$$

$$= \left. \frac{d}{dt} R(u e^{t(T_1+T_2+T_3)}) \right|_{t=0}$$


$$= \left. \frac{d}{dt} R(a_1 \theta + t, a_2 \theta + t, a_3 \theta + t) \right|_{t=0}$$

$$= \sum_{j=1}^{3} \frac{\partial R}{\partial \theta_j} \mid_{(\theta_1, \theta_2, \theta_3) = (a_1 \theta, a_2 \theta, a_3 \theta)} \frac{\partial (a_j \theta + t)}{\partial t} \mid_{t=0}$$

$$= \left( \frac{\partial R}{\partial \theta_1} + \frac{\partial R}{\partial \theta_2} + \frac{\partial R}{\partial \theta_3} \right) \mid_{(\theta_1, \theta_2, \theta_3) = (a_1 \theta, a_2 \theta, a_3 \theta)}$$

$$= D(\theta \cdot a_1, \theta \cdot a_2, \theta \cdot a_3).$$ 

(27)

Along the tracks shown in fig. 3, we generate distributions by

$$T_u = \begin{bmatrix} 2/3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

and $T_d = \begin{bmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(28)

as shown in fig. 4 for a first-order approximation

$$b_{0, \frac{1}{2}, 1}(\theta_1, \theta_2, \theta_3) = \frac{1}{N} \begin{bmatrix} 1 & 0 & 1 \\ \sin \frac{1}{2} \theta_1 & \sin \frac{1}{2} \theta_2 & \sin \frac{1}{2} \theta_3 \\ \cos \theta_1 & \cos \theta_2 & \cos \theta_3 \end{bmatrix},$$ 

(29)

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to an expansion for a protonic state with normalization constant $N$. For instance,

$$x f_{Tu}(x) = x \left[ D \left( \frac{2 - 2x}{2 - x}, \frac{2 - 2x}{2 - x}, 0, 0, \frac{2 - 2x}{2 - x}, -1 \right) \right]^2 \left( \frac{\pi}{2} \right)^2 \left( 2 - x \right)^2,$$

where in its full glory

$$N D(\theta_1, \theta_2, \theta_3) =$$

$$-\frac{1}{2} \cos \frac{\theta_1}{2} \cdot (\cos \theta_3 - \cos \theta_2) - \sin \theta_1 \cdot \left( \sin \frac{\theta_3}{2} - \sin \frac{\theta_2}{2} \right)$$

$$+ \frac{1}{2} \cos \frac{\theta_2}{2} \cdot (\cos \theta_3 - \cos \theta_1) + \sin \theta_2 \cdot \left( \sin \frac{\theta_3}{2} - \sin \frac{\theta_1}{2} \right)$$

$$- \frac{1}{2} \cos \frac{\theta_3}{2} \cdot (\cos \theta_2 - \cos \theta_1) - \sin \theta_3 \cdot \left( \sin \frac{\theta_2}{2} - \sin \frac{\theta_1}{2} \right).$$

The distribution functions shown in fig. 4 contain no fitting parameters at all. Note that the $T_u$-parton momentum content of the approximate state (29) is close to the double of the $T_d$ content,

$$\int_0^1 x f_{Tu}(x) dx = \frac{0.2722}{0.1437} = 1.89 \approx 2.$$
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