Testing and Modeling of Contact Problems in Resistance Welding

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Testing and Modeling of Contact Problems in Resistance Welding

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Technical University of Denmark
Preface

This thesis has been prepared as one of the requirements for the Danish Ph.D. degree at the Technical University of Denmark (DTU). The work has been carried out from April 2000 to June 2003 at the Department of Manufacturing Engineering and Management (IPL), DTU, under supervision of Prof. Dr. Techn. Niels Bay and Associate Prof. Dr. Wenqi Zhang.

I would like to express my sincere thanks to the supervisors for their great inspiration and support during the project.

My gratitude is also expressed to my colleagues who offered valuable help in the experiments. Among them Dr. Poul Henningsen gave many valuable suggestions on data acquisition, Dr. Niels Tiedje helped with operation of the Gleeble, Dr. Jan Lasson Andreasen, René Sobieski and Lars Peter Holmbæk helped with the upsetting tests, and Bjarne Clausen helped with heat treatment of the specimens. Technician Laila Leth is thanked for her aid with macrographs of the specimens.

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Lyngby, June 2003

Quanfeng Song
Abstract

As a part of the efforts towards a professional and reliable numerical tool for resistance welding engineers, this Ph.D. project is dedicated to refining the numerical models related to the interface behavior.

An FE algorithm for the contact problems in resistance welding has been developed in this work, dealing with the coupled mechanical-electrical-thermal contact problems. The penalty method is used to impose the contact conditions in the electrical and thermal contact, as well as frictionless contact and sticking contact in the mechanical model. A node-segment contact element is the basis for the formulation, and the interfaces are treated in a symmetric pattern. The frictional sliding contact is also solved employing the constant friction model.

The algorithm is incorporated into the finite element code. Verification is carried out in some numerical tests as well as experiments such as upsetting together two or three cylindrical parts as well as disc-ring pairs of dissimilar metals. The tests have demonstrated the effectiveness of the model.

A theoretical and experimental study is performed on the contact resistance aiming at a more reliable model for numerical simulation of resistance welding. The model currently employed is evaluated. It is found that the model may underestimate the constriction resistance because it is based on the assumption of continual contact area. A new model is proposed on the constriction resistance in resistance welding.

A parametric study is performed on the contact resistance with the Gleeble machine. The influence of some variables such as interface normal pressure, temperature and material properties are investigated, leading to a better understanding of the contact resistance. The models are also examined.

Finally the performance of the overall contact model is validated in some projection welding experiments. The program is also applied to solve some resistance welding operations involving contact problems, showing that numerical simulation facilitates better understand of resistance welding.
Nomenclature

\( a \) = Constriction radius
\( b_i \) = Body force
\( c, c_1, c_2 \) = Constant
\( c_a \) = Constant for the penalty parameter
\( e_j, e_1, e_2 \) = Unit vector
\( e^2 \) = R-squared value
\( f_i \) = Frictional force
\( g, g_i \) = Penetration
\( g_d \) = Gap of position in the normal direction
\( g_{d0} \) = A small positive constant
\( g_i \) = Penetration of electrical potential
\( g_n, g_{ni} \) = Penetration of velocity in the normal direction
\( g_i \) = Penetration of velocity in the tangential direction
\( g_T \) = Penetration in temperature
\( g_{d0} \) = Initial gap of velocity in the normal direction
\( k \) = Shear yield stress
\( k_{ij} \) = An element on the main diagonal of the tangential stiffness matrix
\( l, l_1, l_2 \) = Length
\( l_c \) = Thickness of contact layer
\( m \) = Friction coefficient in the constant friction model
\( q_s, q_{si} \) = Boundary traction
\( r \) = Radius
\( r_i \) = Radius of the \( i \)th spot
\( t \) = Time
\( u, u_i, u_j \) = Velocity
\( u_n^J, u_n^l \) = Normal velocity
\( u_i \) = Prescribed velocity
\( w_i \) = Generic functional
\( y \) = Coefficient
\( z \) = Coefficient
Nomenclature

A, B, S = Nodes
A_a = Apparent contact area
A_b = Load bearing area
A_c = Area of contact layer
A_d = Sectional area
A_r = Area of the boundary element
C, D = Body C and body D
E_p = Peltier coefficient
F = Load
F_n^C = Normal force on the contact boundary of body C
F_n = Force in the normal direction
F_t = Tangential force
H = Flow stress
H_b = Brinell hardness
I = Electrical current
I_p = Number of nodes in the active set
J = Vector of nodal electrical potential of a contact element
\Delta J = Vector of nodal electrical potential increment of a contact element
K = Tangent stiffness matrix
K_{cf} = Contact stiffness matrix from Coulomb frictional contact
K_{cfs} = Contact stiffness matrix from constant frictional contact
K_{cn} = Contact stiffness matrix from contact in the normal direction
K_{ct} = Contact stiffness matrix from contact in the tangential direction
K_p = Contact stiffness matrix from in the penalty method
M = A positive integer
N, N_1 = A matrix determined by geometry of the contact element
N_T = A vector determined by geometry of the contact element
P_i = Vector of penetration
Q = Heat
Q_p = Heat developed from the Peltier effect
R, R_2 = Ohmic resistance
R_c = Contact resistance
R_f = Contact resistance from surface film
R_s, R_{s1}, R_{s2} = Constriction resistance
R_{cf} = Contact force in frictional contact with Coulomb friction
R_{cfs} = Contact force in frictional contact with constant friction
R_{cn} = Contact force in the normal direction from penetration
R_{ct} = Contact force in the tangential direction from penetration
\( R_p \) = Penalty force from penetration  
\( T \) = Temperature  
\( \bar{T} \) = Vector of nodal temperature in a contact element  
\( \Delta \bar{T} \) = Vector of nodal temperature increment in a contact element  
\( T_v \) = Softening temperature  
\( T_K \) = Room temperature  
\( U \) = Vector of nodal velocity  
\( \bar{U} \) = Vector of nodal velocity of the contact element  
\( \bar{U}_0 \) = Vector of nodal initial velocity of the contact element at current time step  
\( V \) = Velocity increment  
\( V_{ol} \) = Volume of boundary element  
\( V_{th} \) = Thermal voltage  
\( \bar{V} \) = Vector of nodal velocity increment of the contact element  
\( X_S, X_A \) = Coordinate vector of point S, A  
\( \alpha \) = Penalty parameter  
\( \alpha_n \) = Penalty parameter for the normal contact  
\( \alpha_t \) = Penalty parameter for the tangential contact  
\( \beta \) = A factor shown relative position of a node on its contacting segment  
\( \varepsilon \) = Effective stain rate  
\( \dot{\varepsilon} \) = Volumetric strain rate  
\( \dot{\varepsilon}_{ij} \) = Stain rate  
\( \varphi \) = Variational functional for electrical contact  
\( \gamma \) = Film resistance per-unit-area  
\( \lambda_i, \lambda_k, \lambda \) = Lagrange multiplier  
\( \mu \) = Friction coefficient  
\( \rho, \rho_1, \rho_2 \) = Electrical resistivity  
\( \rho_c \) = Contact electrical resistivity  
\( \rho_{f} \) = Contact electrical resistivity film  
\( \rho_{\text{contaminant}} \) = Contact resistivity from surface film  
\( \theta \) = Angle  
\( \zeta \) = Coefficient  
\( \omega \) = Coefficient  
\( \omega_a \) = Coefficient for the penalty parameter  
\( \delta_{ij} \) = Kronecher delta  
\( \sigma_{nc} \) = Normal pressure at interface
Nomenclature

\( \sigma_{ij} \) = Cauchy stress
\( \sigma_{ij}' \) = Deviator stress
\( \bar{\sigma} \) = Effective stress
\( \sigma_{s, \text{soft}} \) = Yield stress of the softer material in contact members
\( \eta \) = Coefficient
\( \tau \) = Holm radius
\( \psi \) = Variational functional for thermal contact
\( \Phi \) = Generic functional
\( \Phi_L \) = Functional based on the Lagrange multiplier method
\( \Phi_p \) = Functional based on the penalty method
\( \Delta \) = Increment
\( \Omega^C, \Omega^D, \Omega \) = Domain occupied a body
\( \Gamma_s \) = Boundary surface with prescribed traction
\( \Gamma_v \) = Boundary surface with prescribed velocity
\( \Pi \) = Functional

Superscript

\( e \) = Element
\( C, D \) = Body C, D
\( T \) = Transpose
\( (i-1) \) = Time step (i-1) in iteration

Subscript

\( 0 \) = Beginning of the current time step
\( c \) = Contact
\( n \) = Normal direction
\( t \) = Tangential direction
\( A, B, S, P \) = Point A, B, S, P
\( J \) = Electrical contact
\( T \) = Temperature
\( \lambda \) = Lagrange multiplier method
# Testing and Modeling of Contact Problems in Resistance Welding

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Chapter 1 Introduction

Resistance welding is a widely applied joining technique. In this chapter a brief introduction is given to the resistance welding technology, followed by an overview to mathematical modeling of the process and the objectives of the project.

1.1 Fundamentals of Resistance Welding

Resistance welding embraces that branch of the welding art in which the welding heat in the parts to be welded is generated by the resistance offered by these parts to the passage of an electrical current[1], referring to Fig.1.1. It dates back to more than one century ago, initiated

![Fig.1.1 Schematic resistance welding](image-url)
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with Thomson’s work in 1880s [2] [3]. Nowadays resistance welding has developed into one of the most sophisticated automated welding processes, widely applied in industry.

1.1.1 Heat Generation

Joule heating is the dominant heat source in resistance welding. When an electrical current \( I \) is passing through a conductor, heat will be generated in the conductor. According to Joule’s law,

\[
Q = \int_0^t R(t) \cdot I^2(t) \cdot dt
\]  

(1.1)

where

- \( Q \) = heat developed [J],
- \( R \) = total ohmic resistance [\( \Omega \)],
- \( I \) = welding current [A],
- \( t \) = time [s].

Besides Joule heating, other thermo-electrical phenomena exist, such as the Peltier effect and the Thomson effect [4] [5] in the resistance welding process.

The Peltier effect, named after the French physicist Jean C. A. Peltier, is the phenomenon in which the direction of current influences the weld quality due to different Fermi levels of the metals to be joined.

Fermi level is the term used to describe the top of the collection of electron energy levels at absolute zero temperature. In an atom electrons cannot exist in identical energy states, they orbit its nucleus in defined paths. Energy is emitted when the electrons occupy a lower energy level than before. So at absolute zero they pack into the lowest available energy states and build up a Fermi sea of electron energy states. The Fermi level is the surface of that sea at absolute zero where no electrons will have enough energy to rise above the surface. The Fermi energies of metals are in the order of electron volts, but ordinary electrical and thermal processes involve energies of a small fraction of an electron volt. This implies that the vast majority of the electrons cannot receive energy from those processes.

Electrons change orbits when a current, \( I \), passing through the faying surface of two materials with different Fermi levels. The procedure is accompanied by release or absorption of energy for different current direction, thus the Peltier heating / cooling takes effect. In comparison with Joule heating, Peltier heating is proportional to \( I \) and dependent on the current direction.

\[
Q_p(t) = \int_0^t E_p(t) \cdot I(t) \cdot dt
\]  

(1.2)

where

- \( Q_p \) = heat developed from the Peltier effect, [J],
- \( E_p \) = Peltier coefficient,
And
\[ E_p = T \frac{dV_{th}}{dT} \]  \hspace{1cm} (1.3)

where
\[ T \] = temperature of the contact surface [K]
\[ V_{th} \] = thermal voltage.

Clearly the Peltier effect may have some influence only when difference in the Fermi levels of the mating materials is big. And even in that case, Joule heating still dominates because it is proportional to \( I^2 \) while Peltier effect to \( I \), referring to equation (1.1) and (1.2). But in some applications the Peltier effect could be important, for instance, in some microwelding applications using capacitor discharge machine.

During the resistance welding process, there is a steep temperature gradient in the workpieces and the electrodes, which will cause the mobile electrons to have different velocities, thus there will be a net flow of electrons from hot regions to cold ones. This is called Thomson effect, named after William Thomson. The effect is negligible in resistance welding [5].

### 1.1.2 Variants of Resistance Welding Processes

It was realized that the resistance welding process can be used in a wide range soon after it was invented. Today, the process has developed into a big family of variants, as illustrated in Fig.1.2.

![Fig.1.2 Resistance welding processes [1]](image)
Among all the processes, spot welding and projection welding are the most widely used examples, which are of interest in this work. In the foregoing, resistance welding refers specifically to spot welding and projection welding, though the discussion and methods may apply to other resistance welding processes as well.

A spot weld (refer to Fig.1.3) is made by pressing two or more pieces of overlapping metal sheets together while an electrical current is passed through a localized contact area to heat the metal to the welding temperature and form the weld nugget. Size and shape of electrodes are critical in spot welding because all the welding current is concentrated in the electrode tip.

Projection welding (Fig.1.4) is a resistance welding process in which the current and heating during welding are localized at a predetermined position by the design of the parts to be joined. Usually projection or embossment is made on one or both of the workpieces. Shape and the size of electrode tip are not of such significant importance as in spot welding.

### 1.1.3 Equipment

There are different kinds of resistance welding machines available in the market, ranging in size from small bench type welding machines to large installations automated with computer controls. A wide range of machinery and various tooling has made resistance welding very flexible.

A resistance welding machine consists of at least three major systems:

- The mechanical system
- The electrical system
- The control system

The mechanical system is used to load the workpieces and the tools. It controls the motion of the movable part and exerts mechanical force at the specified level usually utilising a hydraulic or pneumatic system.
The major part of the electrical system is a transformer for power generation. Electricity used in resistance welding is of lower voltage and high current which is converted from the high voltage and lower current electricity of the main power supply. According to characteristics of the current used for welding, resistance welding machines can be categorized into three groups, AC, DC and CD machines, which utilize alternating current, direct current and current from capacitor discharge, respectively.

The welding controller is used for the adjustment of welding parameters, such as welding time, current and force, etc.

In addition, large resistance welders are often facilitated with a cooling system to cool down the electrical components, the electrodes and the welding tools.

To meet the requirement for better quality, higher manufacturing speed, cost reductions and near 100% reliability, tremendous progress has been made in resistance welding machine [6] [7] [8] [9].

1.1.4 Features of the Process

In recent decades some novel techniques like laser welding and clinching, have replaced some traditional resistance welding applications. Yet resistance welding still stands out against many joining techniques because of its unique features.

- **No extraneous materials are required.** Resistance welding eliminates utilization of extraneous materials such as filler rods, fluxes, rivets and other added materials; this means not only material saving but also better quality in many cases because the metallography of the weld is not complicated by the addition of extraneous materials.

- **Mechanical force is applied.** Mechanical force is present before, during and after the application of electric current. Heat and pressure together determine the final results. That is why these processes are also referred to as *resistance pressure welding*. In fusion welding processes such as laser welding, TIG welding, etc., no mechanical force is applied. While in cold welding, only pressure is present, there is no application of heat. The application of force results in many advantages. On the one hand, the parts are pressed together and deformed before welding. So geometry variations are allowed in the workpieces. In contrast, shape error in parts may lead to serious quality problem in some other processes like laser welding. Further more, the force, which results in plastic deformation, in combination with welding heat refine the grain structure, thus producing a weld with physical properties, in most cases, equal to the parent metal, and sometimes even superior.

- **High manufacturing speed.** Resistance welding is characterized by short welding time — normally between 10 and 500 ms. High productivity is one of the benefits from resistance welding.

- **Good quality of products.** With resistance welding high quality of products can be achieved, both in appearance and physical quality.

- **Flexible and easy to automate.** On the one hand, resistance welding machines are available in a wide range of sizes, as is mentioned earlier; On the other hand, the standard types of welding machines can be used for several different types of welds by
utilizing different electrodes and tooling. And it is easy to put to mass production with automotive industry as a well known example.

The unique characteristics of resistance welding have kept the process in the forefront of the manufacturing industry, continuing refinements in equipment and improvements in procedures have paved the way for more efficient production and better quality of products. The prospects for resistance welding processes seem secure in the foreseeable future [7].

1.1.5 Applications

Resistance welding is a flexible method of joining metals, and it is applicable to a great range of sizes, shapes and materials.

As for size, in the upper ranges, sheets and plates for resistance welding can have thickness of 30 mm(for steel). While at the lower scale, there are such examples as the welding of tiny alloy tips to fountain pen points and joining wires of 0.02 mm in diameter. It is common to join copper wires to foils (tin, nickel, silver) of 12.5 $\mu$m in thickness in electronic circuit connections employing resistance welding [1] [10].

Resistance welding applies to a wide range of materials. Every metal product is a possible application, from mild steel, stainless steel, high strength steel, to aluminum, copper, nickel, etc. Plastic coated steels, pre-painted steels and titanium are suitable for joining by resistance welding. Besides metal, resistance welding is one of the most successful bonding techniques for thermoplastic composites [11] [12] [13].

As a productive and competitive joining technology, resistance welding has been widely applied in automotive, aerospace, electrical, electronic and mechanical industry as well as many other industrial sectors, being used extensively in the manufacture of composite products and the welding of small attachments. Resistance welds can be found everywhere, from automotive and aircraft parts, industrial equipment, office equipment, to domestic goods such as furniture, cooking utensils heating appliances, and great many miscellaneous hardware items.

The automotive industry has utilized resistance welding to the utmost for bodies, frames, levers, housings, braces, wheels, seats, and such smaller parts as spark plugs and electrical system components. Various resistance welding processes are employed in manufacture of vehicles. For instance, most of the sheet metal is resistance spot welded, usually in two layers, but many in three layers, and these sheets are often of galvanized metal; studs for the power brake booster are usually projection welded; metal gas tanks are seam welded, and some parts like the filler ring are added also with resistance welding; the steel wheels are often welded with flash butt welding. In a car body, there are usually 7000 to 12000 [14] spot welds depending on the size of a car.

Fig.1.5 illustrates some resistance spot welding equipment employed in the fabrication of the side panels of car bodies.
Fig. 1.5 Spot welding equipment guided by robots in the fabrication of car body side panels

Fig. 1.6 shows some typical applications in electrical and electronic industry.

Fig. 1.6 Electrical and electronic components assembled applying resistance projection welding and brazing of copper to silver alloys

In addition to the large scale resistance spot welding applications, small and micro-scale resistance spot welding, see Fig. 1.7, on the other hand is intensified due to the requirements of miniature in industry, from automotive electronics, to telecommunications components as well as medical products.
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Fig.1.7 Parallel gap welding nickel wire to nickel pad that is fused to a printed circuit board substrate, the wire is 0.38 mm in diameter and the pad is 0.38x3.8x3.8 mm³[10]

With the development in both equipment and process technology, the application of resistance welding is becoming even broader.

1.2 Resistance Welding Process: a Systematic Overview

1.2.1 Sequence of a Process

The typical resistance welding procedure can be broken down to three distinct periods, as illustrated in Fig.1.8.

- **Squeezing time.** The parts to be joined are brought into contact by a mechanical force but no current is passed through. The workpieces are deformed elastically or plastically. The time interval is to assure that the electrodes contact the workpieces and to build up a specified amount of force before welding current is applied.

Fig.1. 8 Process of resistance projection welding
- Testing and Modeling of Contact Problems in Resistance Welding -

- **Welding time.** Current is applied to the joint while the force is continuously applied. Heat is generated in the assembly. Local temperature is increased to the melting point, and finally a weld nugget is formed. Deformation in the workpieces continues.

- **Holding time.** The current is ceased but the welding force is maintained. As temperature decreases due to heat transfer, the weld nugget cools and is forged under the force of the electrodes. Finally the welding force is released when the nugget has adequate strength.

### 1.2.2 A Systematic Overview to the Process

Resistance welds can be made quickly and easily. This may lead to the belief that the process is simpler than true is. Unfortunately, the process is much more complex than it might appear at first sight. There are a large number of factors which have influence on the final results and some are very difficult to identify experimentally. It is necessary to appreciate how these parameters can affect weld quality.

A systematic overview of the process, taking into account all the main factors in the procedure, is shown in Fig.1.9 [15].

![Fig 1.9 System of parameters in resistance welding](image)

There are five groups of factors which determine the final quality of the weld, namely workpieces, machine, electrodes, interfaces and process.

Needless to say the shape, size and material properties of the **workpiece** are the basis to study a welding procedure. The same is true for the **electrodes**. The electrodes are used to conduct current to the workpieces, transmit the force and help to dissipate heat from the area being weld. In a welding process what is of interest is not only the quality of the weld, but also the
life time of the electrodes because, besides cost, the wear of electrodes affects the weld quality. **Interface** properties are critical in resistance welding. The contact surface between the electrodes and workpieces and at the faying surfaces offer electrical resistivity and thermal conductivity, these parameters are dynamic and dependent on many other factors. **Process** parameters including force, current and welding time should be determined as a compromise of weld quality, stability and productivity. The process parameters are not always readily between welding machines due to different **machine** characteristics. Each individual welding machine has its own electrical and mechanical characteristics including its dynamic response to a rapid variance in current, load or movement.

The large number of parameters involved shows how complicated a process resistance welding is. To evaluate a resistance welding process, one must consider all the factors as a whole.

### 1.3 Mathematical Modeling of Resistance Welding

Though resistance welding is accomplished in a very short period, it ignites a complicated process involving electrical, magnetic, thermal, mechanical and metallurgical phenomena. Experimental studies can be used to analyze different parameters involved in the procedure. With experiments alone, however, one cannot easily comprehend the whole process due to its complexity. Nor can experiments accurately predict the complex behavior of the coupled processes. Still worse, they are often expensive, time-consuming and subject to constriction from available hardware.

Numerical simulation has become an indispensable tool in almost every engineering filed today. Yet that is not the situation in resistance welding. Unlike in other manufacturing sectors such as metal forming and casting numerical simulation has not gained wide applications in process design of resistance welding. Most welding engineers work in a traditional way relying on standards and recommendations, personal experiences and trial-and-error tests. These are awkward, time-consuming and very difficult to optimize the process parameters.

The benefits of numerical simulation cannot be exaggerated. Researchers have for a long time endeavored to develop some mathematical models to provide insight into the resistance welding technique, aiming at an ultimate tool for process development. With the computing power of the present day, mathematical modeling has become more promising as a tool for welding engineers.

#### 1.3.1 Physical Phenomena Involved in Resistance Welding

Referring to Fig.1.10, for complete representation of the resistance welding process, the following mathematical models are necessary corresponding to the physical phenomena involved.
• **Mechanical model.** To simulate the loading and deformation in the whole procedure, during the squeezing, welding and holding stages.

• **Thermal model.** To calculate the heat transfer and the temperature distribution during the welding and the holding stage.

• **Electrical model.** To compute distributions of the voltage and the current and heat generation in materials and electrodes during welding.

• **Metallurgical model.** To take into account the phase transformation, latent heat and temperature dependent material properties during the welding and the holding stages.

• **Magnetic model.** The dynamic current will generate a magnetic field which in turn affects the current distribution.

Moreover, the five models are interrelated as illustrated in Fig.1.10. Usually the interaction between the electric and magnetic field is negligible.

### 1.3.2 Mathematical Modeling: a Literature Review

In contrast to its wide application in industry, there are not so many scientific publications in the open literature on mathematical modeling of resistance welding, probably owing to the complexity of the process, as stated earlier.

In the literature, mathematical modeling of resistance welding started with some analytical models based on some assumptions. With the process often oversimplified, these early works helped to understand the process. In the second stage, finite difference based models pushed
the understanding a step further with some analysis of the electric field, heat generation and heat transfer. But the limitation of the method has prevented it from making realistic simulation of the complete process. Since the 1980s FEM technique has become more popular in analysis of resistance welding because it is easier to take into account all the physical phenomenon in the process and their interaction to make realistic simulation.

Bowden and Williamson (1958) studied the effect of passing an electric current through the interface between two contacting pieces of gold [17]. Based on the potential difference across the constriction, the temperature in the contact region is calculated. Though the work was not targeted at resistance welding, the similar electrical and thermal process was touched. They found softening of the metal locally at the interface between two solids under constant load which increased the contact area. Greenwood and Williamson [18] formulated a model to calculate the spatial distribution of the current and temperature between two semi-infinite solids in contact. Their model predicted current density singularities at the periphery of contact which should lead to more heat generated near the periphery. This was proved in their experiments.

In 1960, Archer [19] studied the temperature distribution in spot welding based on several assumptions. Though the process was oversimplified, his study provided insights to the dynamic response of the material to spot welding.

In 1961 Greenwood [20] developed the first heat conduction model to simulate the resistance spot welding process. His model was a linear, axi-symmetric heat transfer model which included internal Joule heating and constant material properties. Finite difference method was used to solve the partial differential equations. This model marked an important step in spot welding process simulation, with some major features of electro-thermal characteristics of the process. Numerical results showed temperature regularity at the periphery, same as was found in [18].

Rice and Funk (1967) [21] developed a one-dimensional multilayer model of the resistance spot welding process which used the differential equation for analysis. Besides bulk Joule heating, a step further was made by taking into account the contact resistance at interfaces and temperature-dependent thermal and electrical material properties.

Two theoretical models were developed by Houcens et al [22] in 1977 to simulate the resistance spot welding process. Both models were solved using the finite difference method. The first was a one dimensional heat transfer model with latent heat considered. Moreover, the material properties of the electrodes as well as the workpieces were temperature-dependent. The second was an axi-symmetric electro-thermal model including the geometry of a flat-end electrode, which was able to predict the current and temperature distribution the electrodes and the workpieces as well.

Gould (1987) [23] investigated weld nugget development during spot welding both experimentally and analytically. He used a one-dimensional heat transfer model similar to the one in [22], taking into account electrode geometry, temperature-dependent material
properties, melting, Joule heating, melting, effective heat transfer in the liquid, and contact resistance. A finite difference technique was employed to solve the nonlinear differential equations. Comparison between the analysis and the metallographic results showed a discrepancy, though the shapes of nugget matched. The model did not account for non-uniform current density distribution.

The modeling of heat distribution in spot welding was presented by some authors as H.S. Cho and Y. J. Cho [24], Han et al [25], and Wei et al [26], [27], using two-dimensional axisymmetric heat transfer models which were solved with the finite difference method.

A program named SPOTSIM has been developed at Aachen University [28] to simulate spot welding. The program is based on finite difference method and includes a material database with information about the thermo-physical and thermo-mechanical properties of some steels, as well as a database of welding machine and electrodes.

In the above mentioned publications, resistance welding has been modeled as an electrothermal process. The governing equations for this model can be easily solved with the finite difference method, and geometry of workpieces and electrodes are constant. But none of the above models incorporate the mechanical model to calculate large deformation in workpieces, due to difficulty in dealing with large plastic deformation with the finite difference method.

The finite element method was introduced for the problems of structural analysis by Turner, Clough, et al. Later the method was extended to many other fields by numerous researchers. With finite element method, all models in resistance welding can be readily incorporated.

Simulation of resistance welding with finite element method started in 1984, when Nied [29] developed an axisymmetric model which included the geometry of the electrode and workpiece and accounted for temperature-dependent thermal properties, melting and Joule heating. Thermo-mechanical coupling was also included in the work but no details were provided in the paper. Predictions of electrode and workpiece deformations as well as stress distributions along the interfaces were obtained. The simulation was performed using the commercial FEM code ANSYS.

David Dickinson et al (1990) modeled spot welding with ANSYS, too [30]. Three types of elements were used to model the coupled mechanical and thermal behavior, thermoelectric solid element for thermal analysis, isoparametric solid element for stress analysis and surface element for coupling. The three cycles of the spot welding process, i.e. squeezing, welding and holding were simulated. Effects of unequal plate thickness as well as spot welding of dissimilar materials were studied. But some important factors such as contact resistance at the interfaces were not included in the model.

thermo-electrical FE model with an artificial interface element. Sun [34] studied the effect of projection design on the projection collapse process and heat generation in projection welding.

Na and Park [35] suggested a simple model to calculate the thermal and electrical response in spot welding to investigate the influence of contacting forces on the formation of weld nugget. The contact resistance was modeled as a one-dimensional element by assuming that the contamination film is crushed to a number of pieces in the contact interface at regular intervals.

In 1998, Gupta and Amitava De [36] developed a model with spherical tip electrode and incorporated the electro-thermal aspect as well as thermal-elasto-plastic behavior of material in spot welding. And Feng et al [37] presented a coupled thermal-electrical-mechanical model for the spot welding process. The electric contact resistance was modeled as an explicit function of temperature, pressure and bulk resistivity, while the thermal contact resistance between the workpiece and electrode was not mentioned. The work was based on ABAQUS.

In 1999, J.A. Khan [38] developed an axisymmetric finite element model employing coupled thermal-electrical-mechanical analysis based on ABAQUS platform, and the Al-alloy spot welding was simulated.

In 2000, Khan et al [39] presented another model to predict the nugget development during resistance spot welding of Al-alloys. The model employs a coupled thermal–electrical–mechanical analysis. The FEM commercial code, ABAQUS, was used for the temperature-dependent solid mechanics modeling; the finite difference code accounts for the convective terms in the molten weld pool, identical grids were used for both models. The model featured a combination of FE and finite difference method and convective transport in the weld pool. But the authors found that convection effect is not significant for the weld nugget formation.

Most of the aforementioned works were based on general purpose commercial FE software platforms. Commercial software based work eliminates dealing with some complicated numerical methods thus lead to easy and fast implementation, with sacrifice of flexibility to optimize according to the characteristics of the process. And they are not easy to use for welding engineers who have good knowledge about the welding process but less of numerical simulation. As a result some authors tried to develop stand-along program dedicated to simulate resistance welding as in [14] [35] [36].

On the other hand, in most of above-mentioned work, the analysis is focused on one or several applications, and the intention was to gain an insight into the process but not to provide a handy tool for welding engineers. With finite element method being a well-developed tool and available for engineers in almost every field, there are today several programs ready to use for resistance welding professionals [40] [41].

SYSWELD has been developed for the simulation of heat treatment, welding and welding assembly processes, with the mathematical models shown in Fig.1.11. In principle, it can be used to calculate electromagnetic fields, make thermal, metallurgical and mechanical analysis taking into account other effects like diffusion. So resistance spot welding can be simulated
with SYSWELD. But the program is not specialized for resistance welding. Some critical factors such as interface behavior, and machine dynamics, are not taken into account. To make the simulation results reliable, the users have to provide these data, which is not always an easy task.

![Fig.1.11 Interrelated physical phenomena within SYSWELD [34]](image)

SORPAS® (Simulation Of Resistance Projection And Sport welding) has been developed specifically for simulation of resistance welding processes. The program was derived from a series of research works at the Technical University of Denmark [42]-[47]. The concept has been to provide a reliable and professional tool directly for welding engineers.

SORPAS® is a professional tool dedicated to resistance welding engineers. It incorporates electrical, thermal, mechanical and metallurgical phenomena among which the electrical and thermal models are fully coupled while the mechanical model is coupled step-wise with the electro-thermal model, as illustrated in Fig.1.12. The program has a user-friendly interface with four built-in databases, i.e. material, interface, electrode and workpiece database. Thermal contact resistance is modeled as a function of bulk material and temperature, while the electrical resistance is a function of bulk material, temperature, load and surface contaminant. Different machines, including AC, DC and CD machines are included in the program. Also taken into account are temperature dependence of different material properties. Fig.1.13 illustrates a comparison of simulation with the experimental results.
Aside from numerical modeling work on the process of resistance welding, which focuses on the thermal history as well as the shape and size of the nugget, R. Pan and D.F. Watt [51] [52] presented a model to predict the microstructure development in high-carbon steel cross-wire welding. Their work focused on metallurgical reaction kinetics, and the thermal, electrical and mechanical phenomena were greatly simplified. The temperature distribution, on which the analysis is based, was estimated with an empirical grain growth equation.
It is worth noting that little work has been done on projection welding. Most of the above mentioned work treated only spot welding except for those with SORPAS® and [34] [51] [52].

In summary, great progress has been made during the past several decades on simulation of resistance welding with finite element method. The complicated resistance spot welding process can be simulated with interrelated mechanical, thermal, electrical and metallurgical models, and the coupling effect has been well understood, taking into account machine types, electrodes and process dynamics.

1.4 Challenges on Numerical Modeling

Though FE simulation of resistance welding has reached the state of industrial application, there is still much room for improvement in view of accuracy, efficiency and applicability. If we examine the state-of-the-art of numerical methods on resistance welding according to the influencing factors as illustrated in Fig.1.9, we can get some hints on the challenges for numerical modeling.

Regarding workpieces, with geometric information and material data as the basis for analysis, the shape and size of welding pool and thus the nugget geometry can be simulated with programs available. But this is not adequate to predict the weld quality. In addition to nugget size, the microstructure of the weld is of practical importance in evaluating the weld quality. And the resistance welding process is likely to introduce significant residual stresses in the parts, thus prediction of the residual stress in the workpieces will be also valuable. Of course a 3D model, which will give rise to a series of challenges, is critical to simulate resistance welding process of complicated geometry. Regarding machine, there is lots of work to do to take into consideration electrical and mechanical properties of different machines, especially the dynamic response. Considering the interface, there is a lot which still puzzle the researchers on the interfacial properties. In addition, the contact algorithm is a topic seldom touched yet is of extreme importance.

These tasks require, in addition to numerical techniques, a thorough understanding of the process of resistance welding.

Among these topics, the interfacial behavior is of significant importance. On one hand, the numerical model will be severely restricted in the range of application without a reliable contact algorithm, only applications with small surface expansion can be simulated; on the other hand, the contact resistance is critical to obtain precise simulation results.

1.4.1 Contact Modeling in Case of Significant Surface Expansion

In resistance welding, load is applied all through the process and temperature change from room temperature to the melting point. So the workpieces undergo large plastic deformation. The shape and size of the contact surface vary continuously. The situation becomes especially severe in projection welding in which the projection collapses to form a weld. This variation
of contact area plays an important role in adjusting the amount of heat generated and it also explains one of the reasons why the load is influential in the process; in numerical simulation, the shape and size of the contact area are indispensable to calculate heat generated. Reliable modeling of the contact surface is critical to the final results.

No publications have been found on the contact problem in resistance welding. In literature on numerical simulation of resistance welding, different bodies are usually linked to each other on the faying surface. In other words, the faying surfaces are treated as if they were welded from the beginning. This way eliminates employing complex contact algorithms with a loss of accuracy. In most resistance spot welding, this is acceptable. But in some other applications such as in most projection welding, this may lead to severe decrease of precision and in some cases it may prevent the simulation from continuing.

Large deformation can be simulated in SORPAS® [49]. The contact properties are modeled with a contact layer, which is connected on both sides to the contacting pairs. When the interface expands during welding, a contact model is activated to calculate the contact pressure and area, etc. Yet the current contact model was not successful, which can be seen from discontinuity in pressure and temperature at interface. For a realistic simulation of such class of problems which involves drastic change of the interface area, a reliable contact algorithm is required.

1.4.2 Dynamic Contact Resistance

For spot welding of two sheets as illustrated in Fig.1.14, there are seven electric resistances involved, namely the bulk resistance of the workpieces and electrodes (R_3, R_5, R_1, R_7), the contact resistance of the interface between workpiece and electrode (R_2, R_6), and the contact resistance of the faying surface (R_4).

![Fig.1.14 Resistances during resistance welding process [16]](image)

All the resistances are dynamic (Fig.1.14), or they all change continuously during welding. And they together determine heat generation in the assembly and thus the final weld quality.
Among them, the bulk resistances are dependent on temperature and material. For a specific material, the variation of resistivity with temperature can be found experimentally. In contrast, the contact resistances are much more complicated.

Contact resistance is one of the most critical parameters in resistance welding. The ultimate goal of the welding process is to generate sufficient heat between the workpieces being welded so that the metal will melt, fuse together and form a weld. For this to happen, the contact resistance must be well controlled. Usually the contact resistance is bigger than the bulk resistance, thus more heat is generated in the vicinity of the interface and the welding pool lies in the interface. In numerical modeling, with all the models well developed and coupled in proper ways, the analysis could still completely fail if the model of the contact resistance were inappropriate. An insight into the dynamics of contact resistance is extremely important for industry, including not only numerical modeling, but also online control, machine design, et al.

Being the most important parameter in the process, the contact resistance has drawn attention of many researchers. Despite of lots of study, most of which are experimental, contact resistance is still not well addressed. The knowledge about contact resistance is mainly qualitative and not quantitative. It is known that the contact resistance is dependent on many factors, such as material, pressure, temperature, surface condition, etc. But no generally recognized model of contact resistance has been developed which can take into account all the main influencing parameters.

Needless to say, the contact resistance model is critical to the performance of a program for simulation of resistance welding. However, it is not described in most of the above mentioned work on numerical modeling. This perhaps results from complexity of contact resistance.

Wei and Wang [27] presented an empirical model of contact resistance. Feng et al [37] used another model derived from the experiments, yet both models are constricted to some particular applications and not easy to use as a general model.

Zwolsman [5] presented a dynamic model as,

$$R_c = \frac{F}{2} \sqrt{\frac{\pi H_b (T_v - T)}{3F(T_v - T_k)}} \left( \frac{T}{T_k} \right)$$  

(1.4)

where,

- $R_c$ – the contact resistance
- $\rho$ – the bulk resistivity of the material,
- $H_b$ – the Brinell hardness at room temperature,
- $F$ – the load,
- $T$ – temperature, K,
- $T_v$ – softening temperature, K,
- $T_k$ – room temperature, K.
The model is derived based on the assumption of linear change of material hardness, which is over simplified in many situations. Still worse, the resistance from surface film, which is the major part of the total contact resistance in some cases, is not considered.

The contact resistance model used in SORPAS® [49] is,

\[
\rho_c = 3\xi \left( \frac{\sigma_{s,soft}}{\sigma_{nc}} \right) \left( \frac{\rho_1 + \rho_2}{2} + \rho_{contaminant} \right)
\]  

(1.5)

where

\[ \sigma_{nc} = \frac{F}{A_c} \] is the contact normal pressure, F is load,

\[ \sigma_{s,soft} \] is the flow stress of the softer material in contact,

\[ \rho_1, \rho_2 \] is the resistivity of the metal in contact,

\[ \rho_{contaminant} \] is the resistivity of surface contaminant,

\[ \xi \] is a constant.

This model is quite convenient to use. The user should define the correct value of flow stress of the softer contact member and the resistivity of contaminant. But it is not well proven with experiments, especially as regards the resistance due to surface contamination.

1.5 Objectives of the Project

Numerical simulation not only offers an opportunity to explore the mechanism of the resistance welding process, but also provides some direct guidance to process design and optimization. As a part of the efforts towards a professional and reliable numerical tool for resistance welding engineers, this Ph.D. project is dedicated to refining the numerical modeling related to the interface behavior by developing necessary numerical techniques for resistance welding, and looking into the process for a better understanding of related phenomena.

As is mentioned earlier, in resistance welding especially in projection welding, the contact area undergoes severe change, reliable modeling of the variation of the contact surface as well as the contact pressure is extremely important for realistic simulation. Little work has been done on this topic.

On the other hand, electrical contact resistances at the interfaces between workpieces and electrodes and at the faying surfaces have great influence on the welding process and the weld quality. Though some research has been performed on the electrical contact property, there is no generally accepted model which is suitable to use in simulation, which should cover a wide range of temperature. Reliable testing and accurate modeling of the electrical contact properties could significantly improve the accuracy of numerical modeling of resistance welding.
So the objectives of this Ph.D. project are twofold:

- The first objective of the project is to develop a contact algorithm, dealing with the dynamic development of the interfaces between plastic deforming bodies during the welding process, and solving the thermal and electrical contact problems, too. The algorithms will be implemented into the FEM program for simulation of the mechanical, electrical and thermal contacts.

- The second objective of the project is to develop effective test methods and procedures for electrical contact resistance in a wide range of temperatures up to the melting point. Based on the results of the tests, the current model for contact resistance used in SORPAS® will be examined. Other mathematical models may be developed for contact resistances in resistance welding.

### 1.6 Overview of the Thesis

The proposed work will be presented as follows. In chapter 2, a contact algorithm is presented and implemented to the FE program. Some experiments are carried out to validate the algorithm in Chapter 3. Chapter 4 and Chapter 5 are dedicated to the electrical contact resistance. In the former, some general theory on contact resistance will be reviewed followed by an analysis of the SORPAS® model, while in the latter, some experiments are performed to study the contact resistance between some frequently used materials in resistance welding. The influencing parameters to the contact resistance are analyzed. In Chapter 6, the contact models are examined in realistic resistance welding processes followed by numerical analysis of several resistance welding applications. Chapter 7 lists the main conclusions achieved in the project. This dissertation is concluded in Chapter 8 with a proposal for future work.

### References

Chapter 2 Contact Modeling

A reliable, efficient and robust contact algorithm is crucial to the FE simulation of resistance welding processes. In this chapter, a contact algorithm based on the penalty method is developed, dealing with the planar contact problems involving coupled thermal, electrical and mechanical phenomena between deformable bodies. For the mechanical model, frictionless, sticking as well as frictional sliding contacts are solved. The algorithm is incorporated into the finite element program. Some numerical examples are shown for preliminary validation of the mechanical contact model.

2.1 Contact Problems in Resistance Welding

The analysis of contact problem is of common concern in engineering practice. In almost all mechanical and structural systems, there exists a situation in which one body comes into contact with another whereby loads and displacements are transmitted. Examples include manufacturing techniques like metal forming, industrial devices like gears and bearings, biological joints such as elbows and knees, and unintentional contact as in car accidents, etc. Contact problems range from frictionless contact in small-strain elastic analysis, to frictional contact in general large-strain plastic analysis, and are extremely important in many engineering fields. Therefore the development of numerical methods for solving contact problems has drawn the attention of many researchers.

In resistance welding, very complicated processes occur at the interface, involving plastic deformation, electrical conduction, heat generation and thermal conduction. In the procedure, interface temperature varies from room temperature to the melting point with sophisticated metallurgical transformations.

During squeezing, the workpieces are pressed together between the electrodes. The pressure is transmitted from the electrode-workpiece interface to the workpiece, further to the faying surface of the electrodes, and through all the parts involved. The pressure is increased till the prescribed level in a certain time. During this process the workpieces undergo elastic or plastic deformation in a confined region. The contact areas in the faying surfaces change in
size and shape under the load until equilibrium is finally reached. This procedure involves only mechanical contact. Referring to Fig.1.10, the parameters to be determined are the pressures at the interfacial surfaces and the contact areas.

During the welding time, thermal and electrical contacts arise when an electric current is passed through the interfaces and parts in question. Heat is generated in the interfaces as well as in the bodies involved. The temperature increases continuously and thus the material properties (mechanical, thermal, electrical) vary. As a result, the pressure and the contact area change continuously. Large-strain plastic deformation occurs especially in projection welding applications. During welding time, contact takes place involving not only mechanical phenomena but also electrical and thermal ones.

During holding, the parts cool down. Due to change of material properties with temperature, the pressure varies dynamically, and so does the contact area. During this period, thermal and mechanical contact occurs.

To summarize, varying contact conditions occur during the whole procedure of resistance welding involving interrelated mechanical, thermal and electrical phenomena. The contact problem plays a critical role.

In literature on the simulation of resistance welding, interfaces are treated as bulk material without a proper contact algorithm. In other words, nodes on the interface are handled as if they were interior ones. The contact problem is thus cast into a bulk deformation problem. But this solution is limited to those applications with small interface expansion. It does not introduce too much error for most spot welding process, where tangential sliding at interface is often negligible. While in spot welding of dissimilar materials, the solution may lead to significant errors because of sliding at interface. In other applications such as many projection welding processes where the electrical conduction area changes significantly during welding, this solution may lead to severe decrease of precision and in some other cases may prevent the simulation from proceeding.

In contrast to many other processes, surface expansion during resistance welding has an effect on heat conduction and electrical current flow, which in turn affect heat generation and thus surface deformation. In other words, the resistance welding process gives rise to coupled mechanical-electrical-thermal contact problems. In literature on contact modeling, most of the work dealt with mechanical contact, or thermo-mechanical contact in engineering fields such as metal forming. No published work has been found in modeling the contact problems in resistance welding.

### 2.2 Mechanical Contact

Contact problems have been studied extensively. Various numerical schemes are proposed. A majority number of published work dealt with mechanical contact, which is the subject of this section. Once the mechanical contact problem has been solved, the same algorithm can be applied to the thermal and electrical contact in resistance welding.
2.2.1 General Contact Problem

Mechanical contact often involves multiple bodies. For ease of presentation, consider only two bodies, both deformable, as illustrated in Fig.2.1, where $\Omega^C$, $\Omega^D$ denotes the domain occupied by body $C$ and body $D$, respectively. Load and boundary conditions are applied to the contact bodies. The contact bodies deform following a series of laws of deformation. The rigid body movement is not of interest in the context, neither is the dynamic effect. The problems discussed herein are static or quasi-static.

Within domain $\Omega^C$, $\Omega^D$, the deformation is governed by the differential equations of force equilibrium which can be expressed in tensor notation as

$$\sigma_{ij,j} + b_i = 0, \quad i, j = 1, 2$$  \hspace{1cm} (2.1)

where $\sigma_{ij}$ denotes the Cauchy stress tensor, and $b_i$ is the body force component. During plastic deformation the body force is usually set to zero because weight of the structure is much less than the load for deformation.

The Cauchy stress depends on material properties and deformation, which is stated in the constitutive equation. Following Levy-Mises equations,

$$\sigma_{ij} = \frac{2\sigma}{3\varepsilon} \Delta_{ij}, \quad i, j = 1, 2$$  \hspace{1cm} (2.2)

where $\sigma_{ij}^\prime$ represents the deviator stress component defined by

$$\sigma_{ij}^\prime = \sigma_{ij} - \delta_{ij} \sigma_{ii}, \quad i, j, l = 1, 2$$  \hspace{1cm} (2.3)
where \( \delta_{ij} \) is Kronecher delta.

\( \bar{\sigma} \) represents the effective stress which can be given as

\[
\bar{\sigma} = \sqrt{\frac{3}{2} \bar{\sigma}_{ij} \bar{\sigma}_{ij}}
\]

(2.4)

\( \dot{\varepsilon} \) is the effective strain rate defined as,

\[
\dot{\varepsilon} = \sqrt{\frac{2}{3} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}}
\]

(2.5)

where \( \dot{\varepsilon}_{ij} \) denotes strain rate component.

In absence of geometry non-linearity, the strain rate can be expressed as,

\[
\dot{\varepsilon}_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})
\]

(2.6)

where \( u \) is the velocity.

Notice that rate form is adopted here because a flow formulation is employed in SORPAS®.

The system is subjected to boundary traction, \( q_s \), on boundary surface \( \Gamma_s \), namely,

\[
\sigma_{ij} e_j = q_{si}, \quad \text{on} \quad \Gamma_s
\]

(2.7)

where \( e_j \) is the unit vector on \( \Gamma_s \) and \( q_{si} \) is the component of the prescribed boundary force \( q_s \).

In addition, velocity is prescribed along the boundary surface \( \Gamma_v \),

\[
u_i = \bar{u}_i, \quad \text{on} \quad \Gamma_v
\]

(2.8)

where \( \bar{u}_i \) is the prescribed velocity.

In SORPAS®, a rigid-plastic formulation [1] [2] is adopted; the material is regarded as incompressible, or the volumetric strain rate is equal to zero,

\[
\dot{\varepsilon}_v = \dot{\varepsilon}_{ii} = 0
\]

(2.9)

Equations (2.1), (2.2) and (2.6)-(2.9) are the governing equations for deformation of continuum, which are the basis to solve deformation of the bodies. For a contact problem as
shown in Fig.2.1, some conditions for boundary harmonization must also be satisfied in addition to the governing equations.

The fundamental contact condition is that no material particle of one body can penetrate into the other, or no material overlap can take place. The condition is often referred to as *impenetrability condition* in the context of contact problem.

Let $u_n^D$, $u_n^C$ denote the normal velocity of a pair of contacting particle on body $D$ and $C$, respectively, then the impenetrability condition holds,

$$u_n^D - u_n^C \geq 0$$  \hspace{1cm} (2.10)

Besides, for a releasable contact, it is assumed that the normal force on the contact boundary can only be compressive. In other words, no welding occurs for contacting particles on different bodies so the normal traction cannot be tensile. Then we have,

$$F_n^C \leq 0$$  \hspace{1cm} (2.11)

where $F_n^C$ is the normal force on the contact boundary of body $C$.

Another contact condition is the well know Newton’s 3rd law, which states that every action has an equal and opposite re-action. Or,

$$F_n^C = -F_n^D$$  \hspace{1cm} (2.12)

From equation (2.10) and (2.11) one can write,

$$F_n \cdot (u_n^D - u_n^C) = 0$$  \hspace{1cm} (2.13)

which states that either the first term equals to zero, implying no contact occurs, or the second term is zero, meaning the contact points move together in the normal direction.

For frictional contact, the tangential traction should satisfies a law of friction,

$$F_t \leq f_r$$  \hspace{1cm} (2.14)

where $F_t$ is the tangential force, and $f_r$ is the friction.

These conditions, equation (2.10) – (2.14), in addition to the governing equations of the particular phenomena in question, provide the basis to tackle contact problems.

A contact problem is by nature a boundary-value problem. The modeling of contact between solids poses mathematical and computational difficulties. Contact problems are inherently nonlinear because the contact boundaries are not known *a priori* but evolves continuously as a part of the solution. Contact problems are further complicated due to unsmooth response in
Chapter 2 Contact Modeling

both normal and tangential (in frictional case) directions. Therefore contact problems are highly nonlinear and remain one of the challenging problems in computational solid mechanics despite of extensive study over the years.

2.2.2 A Rigid-plastic Formulation in Absence of Contact

Considering a plastic forming problem governed by equations (2.1), (2.2) and (2.6)-(2.9). Mathematically, this can be regarded as a particularization of the more general family of constrained minimization problems which can be expressed as,

\[
\text{Minimize } \Phi(u_1, u_2, \ldots, u_n) \quad \text{with} \quad w_i(u_1, u_2, \ldots, u_n) \leq 0, \quad i=1,2,\ldots,M
\]

(2.15)

where both \( \Phi \) and \( w_i \) are generic functions of multiple variables, \( M \) and \( n \) are positive integers.

In rigid plastic formulation, the objective function is the energy rate,

\[
\Phi = \int_\Omega \bar{\sigma} \varepsilon d\Omega - \int_{\Gamma_p} q_n u_t d\Gamma
\]

(2.16)

According to the principle of minimum total potential energy, among the kinematically admissible velocity fields (which satisfy the velocity boundary conditions), the actual velocity field taken by the body is the one that makes the total potential energy stationary. Therefore the plastic deformation can be regarded as an optimization problem of the objective function (2.16) subjected to constraint (2.9).

As a standard approach, the optimization problem with constraints can be transformed to optimization problem of a new objective function without constraints. Most widely used methods are \textit{the penalty method} and \textit{the Lagrange multiplier method}.

2.2.2.1 The Penalty Method

The penalty method helps to reformulate an optimization problem with constraints as one without constraints. For optimization problem in (2.15), the objective function is reformulated as,

\[
\Phi_p = \Phi(u_1, u_2, \ldots, u_n) + \frac{1}{2} \alpha \sum_i^M w_i(u_1, u_2, \ldots, u_n)^2
\]

(2.17)

where \( \alpha \) is a pre-assigned weight parameter, called the \textit{penalty parameter}. The solution to the modified problem is given by the following equations:

\[
\frac{\partial \Phi_p}{\partial u_i} = 0, \quad i = 1,2,\ldots,n
\]

(2.18)
The solution \((u_1, u_2, \ldots, u_n)\) of these equations will be a function of the penalty parameter. The larger the value of \(\alpha\), the more exact are the constraints satisfied in a least-squares sense, and the solution approaches the actual value as \(\alpha\) approaches infinity.

Similarly, the incompressibility condition can be incorporated into the formulation by the penalty method, leading to a new functional,

\[
\Phi_p = \int_\Omega \sigma \varepsilon d\Omega - \int_{\Gamma_\tau} q_s u_i d\Gamma_s + \int_\Omega \alpha \dot{\varepsilon}_i^2 d\Omega
\]  

(2.19)

The velocity field can be obtained iteratively from the non-linear equations, following a standard procedure of discretization and linearization. When an incremental updated-Lagrangian formulation is adopted, the system equations can be expressed as,

\[
K\Delta U = \Delta F
\]  

(2.20)

where \(K\) is known as the tangent stiffness matrix, \(U\) is the vector of nodal incremental velocity, and \(\Delta F\) the vector of external nodal force.

### 2.2.2.2 The Lagrange Multiplier Method

With the Lagrange method, the problem (2.15) is reformulated as one of determining the stationary points of the modified functional

\[
\Phi_L = \Phi(u_1, u_2, \ldots, u_n) + \sum_{i=1}^{M} \lambda_i w_i (u_1, u_2, \ldots, u_n)
\]  

(2.21)

subjected to no constraints. Here \(\lambda_i\) denotes the Lagrange multiplier. The solution to the problem is obtained by setting partial derivatives of \(\Phi_L\) with respect to \(u_i\) and \(\lambda\) to zero,

\[
\frac{\partial \Phi_L}{\partial u_i} = 0, \quad i = 1, 2, \ldots, n
\]

\[
\frac{\partial \Phi_L}{\partial \lambda_k} = 0, \quad k = 1, 2, \ldots, M
\]  

(2.22)

which gives \((n + M)\) equations in \((n + M)\) unknowns. It’s obvious that the Lagrange multiplier method (also referred to as Lagrangian method) adds unknowns to the problem.

To impose the incompressibility condition, the objective functional is formulated as,

\[
\Phi_L = \int_\Omega \sigma \varepsilon d\Omega - \int_{\Gamma_\tau} q_s u_i d\Gamma_s + \int_\Omega \lambda_i \dot{\varepsilon}_i^2 d\Omega
\]  

(2.23)

Following a standard approach, the velocity field as well as the Lagrange multiplier can be solved from (2.23) using an iterative scheme.
2.2.3 A Review of Mechanical Contact Algorithms with FEM

Much effort has been devoted to the study of contact problems due to its importance and complexity. A great number of papers have been published on this topic, employing experimental, analytical and numerical methods.

Experimental study of the contact problem is subjected to limitations in many cases. It may be expensive and time-consuming, especially for those large-scale and complicated contact problems. Though experimental approaches are still widely employed today [3], they are mainly for investigation of the related mechanism, if they are not for validation of various mathematical models.

Analytical analysis of contact phenomenon goes well back to more than 100 years ago. The first successful analytical analysis of contact was attributed to Hertz, who in 1882 gave a solution to the dynamic contact of two elastic bodies, which has been considered a milestone in the field. Analytical analysis of contact problems continued before the advent of digital computers. Those solutions employed the theory of elasticity and were limited to simple linear cases of contact. Moreover, their application was further restricted because for most problems the exact contact surfaces are not known \textit{a priori}.

With the computing power of digital computers, it is possible to model many of the complications involved with the analysis of contact problems thanks to the flexibility and ability of the numerical techniques. Various methods, such as the finite difference method, the boundary element method and the finite element method, have been employed. Among these the finite element method is probably the most widely used [4] [5] [6].

Numerical contact algorithms were proposed in the early 1970s to handle the complex nature of the physical and numerical behavior of contact problems. Some pioneers are Conry and Seireg [7], Chan and Tuba [8], Hughes [9]. Great progress has been made during the past several decades. Nowadays, complicated three-dimensional contact problems between deformable bodies including friction, material non-linearity and large deformation can be solved numerically. Review papers on contact problems have been give by Zhong and Mackerle [10] and Mijar and Arora [11].

Mathematically, contact problems can be regarded as a constrained minimization problem. Denote the objective functional with $\Pi$, examples of which can take a form of (2.19) or (2.23). An additional set of contact conditions which is valid along the interface of the bodies in contact should also be satisfied. The basic constraint is equation (2.10), which is often written as,

$$g = \{g_i\} \geq 0 \quad (2.24)$$

where $g$ is the gap function at interface, $g_i$ denote the penetration at boundary point $i$.

In treating inequality constraints arising from mechanical contact with the finite element method, two main directions exist for tackling the contact problem; one uses special
optimization techniques for the resulting non-linear problem, while the other directly transforms the inequalities to equalities by enforcing the constraint conditions. Following the latter route, the most commonly used approaches for contact analysis are the penalty method, the Lagrange multiplier method, the perturbed Lagrangian method and the augmented Lagrangian method.

2.2.3.1 The Penalty Method [12-22]

In FE formulation using the penalty method, the contact problem is reformulated to an optimization problem as

\[
\Pi(U) + \frac{1}{2} \sum_i g_i^2(U) \rightarrow \text{min}
\]

This leads to the system equations for the penalty method,

\[
(K + K_p) \Delta U = \Delta F + R_p
\]

where \(K\) is the tangent stiffness matrix from standard FE formulation without contact, \(K_p\) is the additional contact stiffness matrix for the penalty method, \(R_p\) is the term arising from penetration.

Theoretically the constraints are only approximately satisfied using the penalty method. Another drawback of this method is that the accuracy of the approximate solution strongly depends on the penalty parameter employed. On the other side, the penalty method adds no unknowns to the system equations, thus it is easy to implement and is quite widely applied.

2.2.3.2 The Lagrange Multiplier Method [5] [9] [23-26]

With the Lagrange multiplier method, the contact problem is reformulated as

\[
\Pi(U) + \sum_i \lambda g_i(U) \rightarrow \text{min}
\]

The system equations for the Lagrange multiplier method is:

\[
\begin{bmatrix}
K & K_{\lambda} \\
K_{\lambda}^T & 0
\end{bmatrix}
\begin{bmatrix}
\Delta U \\
\Delta \lambda
\end{bmatrix}
=
\begin{bmatrix}
\Delta F \\
0
\end{bmatrix}
+
\begin{bmatrix}
R_c \\
P_{\lambda}
\end{bmatrix}
\]

(2.26)

where \(R_c\) are contributions from the updated forces after the previous iteration, \(\Delta \lambda\) represents the increments of Lagrange multipliers, \(P_{\lambda}\) is the vector of penetration, \(K_{\lambda}\) is the additional contact matrix, with \(K_{\lambda}^T\) as its transpose. The Lagrange multipliers associated with the
constraints on velocity have the physical meaning of contact forces, thus forces become primary unknowns, mixed with velocity.

The main merit of the Lagrange method is that the constraint conditions are accurately satisfied. However, it adds the equations associated with the constraints to the global equation set. This enlarges the total number of unknowns which is subject to continuous change in case of non-linear contact. Increased number of unknowns not only makes implementation more difficult but also increases CPU-time to solve the problem. Another disadvantage lies in the indefinite stiffness matrix associated with the Lagrange multiplier, which may lead to numerical difficulties in a direct solution process.

2.2.3.3 The Perturbed Lagrange Multiplier Method [25-28]

This method combines the merits of penalty function method and the Lagrange multiplier method. The contact problem is reformulated as

\[ \Pi(U) + \sum_i \lambda g_{ni}(U) - \frac{1}{2\alpha} \sum_i \dot{\lambda}^2 \rightarrow \text{min} \]

The system equations for the perturbed Lagrange multiplier method is:

\[
\begin{bmatrix}
K & K_{\dot{\lambda}} \\
K_{\dot{\lambda}}^T & -\frac{1}{\alpha}
\end{bmatrix}
\begin{bmatrix}
\Delta U \\
\Delta \dot{\lambda}
\end{bmatrix}
= \begin{bmatrix}
\Delta F \\
0
\end{bmatrix}
+ \begin{bmatrix}
R_c \\
P_c + \frac{1}{\alpha} \lambda^{(i-1)}
\end{bmatrix}
\]  

(2.27)

where \( \lambda^{(i-1)} \) represents the Lagrange multiplier for the previous iteration.

There is no zero on the main diagonal of the stiffness matrix, and an accurate result can be obtained without a very large penalty parameter. But it usually takes more solution time than the classical Lagrange multiplier method.

2.2.3.4 The Augmented Lagrange Multiplier Method [29-33]

This method is also a combination of the penalty function method and the Lagrange multiplier method. The contact problem is reformulated as,

\[
\Pi(U) + \frac{1}{2} \sum_i g_{\alpha n}(U)^2 + \sum_i \lambda g_{\alpha n}(U) \rightarrow \text{min}
\]

The system equations for the Lagrange multiplier method is:

\[(K + K_p)\Delta U = \Delta F + R_p + R_c\]  

(2.28)
There is no zero on the main diagonal of the stiffness matrix, and an accurate result can be obtained without a very large penalty parameter. It is reported that this method can save considerable CPU-time in solving large scale problems when the parameters are appropriately chosen [33].

2.2.4 Contact Algorithm

As is mentioned earlier, there are more than one solution to deal with the contact problems in resistance welding. Each method has its advantages as well as drawbacks. Among the four methods mentioned above, the penalty method is the simplest to incorporate into the current finite element code and when the penalty parameters are properly determined, satisfactory accuracy and efficiency can be achieved. The Lagrange multiplier method is theoretically more accurate compared with the penalty method, but is more difficult for implementation. Moreover, the resultant force along the interfaces could be unsmooth [13]. The hybrid methods are not only complicated for implementation but also require careful selection of the numerical parameters. As a compromise among accuracy, efficiency, robustness and complexity in implementation, the penalty method is employed in modeling the contact problems in resistance welding.

2.2.4.1 Criteria for Contact and Release

Within the framework of the penalty method, a new variational functional is formulated taking into account a penalty term arising from contact. So criterion of contact and release are necessary to find the contact elements.

In SORPAS®, the deformable bodies are discretized by 4-node quadrilateral isoparametric elements, as shown in Fig.2.2. Without losing generality, only two bodies are considered in the context.
In Fig 2.2, two bodies, the contactor and the target, are brought into contact, node $S$ is sufficiently close to the target segment $AB$, with $e_1$ and $e_2$ as its base vectors, or,

$$e_1 = (\cos \theta, \sin \theta)^T; \quad e_2 = (-\sin \theta, \cos \theta)^T$$  \hspace{1cm} (2.29)

where the superscript $T$ denotes transpose of a vector or matrix, $\theta$ is the angle of segment $AB$ relative to the horizontal basis.

A normal gap function, $g_d$, is defined to judge if a boundary node is in contact or not. From Fig.2.2, the gap between the contactor node $S$ and its nearest target segment is given by the scalar product

$$g_d = (X_S - X_A)^T e_2$$  \hspace{1cm} (2.30)

where $X_S$, $X_A$ are the vectors of coordinates of node $S$ and $A$, respectively, with, for example, $X_S = (x_s, y_s)^T$, where $x_s, y_s$ are the coordinates of node $S$. Then a boundary node is in contact if

$$g_d \leq g_{d0}$$  \hspace{1cm} (2.31)

where $g_{d0}$ is a small positive value.

Otherwise, the node is regarded as free and should be released if it was in contact in last time step.

### 2.2.4.2 Frictionless Contact

As a starting point, the contact problem is handled neglecting friction at interface. Though in practice, friction never vanishes, the assumption of frictionless contact provides acceptable approximation to many applications. For example, along well lubricated surface, friction is very small; in such a case, a frictionless assumption does not introduce too much error.
In the penalty method, the interfaces are treated like contact elements placed between surfaces wherever penetration is detected. Every boundary node is a potential contact node. In practice, all the potential nodes are checked using equation (2.30) and (2.31). As a result, a candidate set is found which contains all the nodes that satisfy (2.31). Among the candidate set, each node, together with its corresponding segment, form a contact element, as illustrated in Fig.2.3. Point $P$ is the projection of node $S$ on segment $AB$.

Based on the penalty method, a new variational functional is formulated incorporating the contribution of the contact elements, which take the form of point-to-segment. Some elemental parameters, including the vectors of nodal velocity, velocity increment and velocity at the beginning of this time step can be written as

\[
\begin{align*}
\mathbf{U}^T &= (u_{S1}, u_{S2}, u_{A1}, u_{A2}, u_{B1}, u_{B2}) \\
\mathbf{V}^T &= (v_{S1}, v_{S2}, v_{A1}, v_{A2}, v_{B1}, v_{B2}) \\
\mathbf{U}_0^T &= (u_{0S1}, u_{0S2}, u_{0A1}, u_{0A2}, u_{0B1}, u_{0B2})
\end{align*}
\]

Since the primary unknown in a flow formulation is the velocity, the contact constraints are also enforced on velocity. For the active set of contacting nodes, the following contact law should be satisfied to avoid penetration,

\[ g_n \leq 0 \quad (2.32) \]

where $g_n$ is the gap function of velocity.

Among the candidate set, those nodes that violate equation (2.32) forms an active set.

Thus the task for frictionless contact is to find a solution to the minimization problem of the variational functional in equation (2.19) with an additional constraint of (2.32).

Based on (2.19), the discrete penalty functional is reformulated by incorporating a penalty term $\Pi_{cn}$

\[ \Pi_p = \Phi_p + \Pi_{cn} = \Phi_p + \sum_{i} \frac{1}{2} \alpha_n g_{ni}^2 \quad (2.33) \]

where

- $lp$ – number of nodes in the active set,
- $\alpha_n$ – penalty parameter for contact in the normal direction,
- $g_{ni}$ – the velocity gap in the normal direction at node $i$.

An approximate solution can be obtained by determining the variation of equation (2.33) with respect to $V$. 

\[ \partial \Pi_p = \partial \Phi_p + \partial \Pi_{en} = \partial \Phi_p + \sum_{i}^{lp} \alpha_i g_n \partial g_{ni} = 0 \quad (2.34) \]

In the right hand side of equation (2.34), the first term leads to the standard finite element formulation, while the second term arises from the contact contribution, which is computed in the following.

For one contact element, referring to Fig.2.3, the velocity gap is given by scalar product

\[ g_n = (U_S - U_P)^T e_2 \quad (2.35) \]

where \( U_S \) and \( U_P \) represent the vectors of velocity of node \( S \) and point \( P \), respectively. With, for instance,

\[ U_S = (u_{S1}, u_{S2})^T \quad (2.36) \]

In the typical iterative scheme to solve the governing equations, an incremental scheme is employed. Denote the increment of velocity in one time step as \( V \) and the velocity of a node after last time step as \( U_0 \), then

\[ U = U_0 + V \quad (2.37) \]

Following a linear interpolation along segment \( AB \), velocity and its increment of point \( P \) can be obtained as

\[ U_P = \beta U_A + (1 - \beta)U_B \quad (2.38) \]
\[ V_P = \beta V_A + (1 - \beta)V_B \quad (2.39) \]

Inserting equations (2.37) – (2.39) into (2.35) yield

\[ g_n = \left[ U_{0S} + V_S - \beta(U_{0A} + V_A) - (1 - \beta)(U_{0B} + V_B) \right]^T e_2 \]
\[ = \left[ U_{0S} - \beta U_{0A} - (1 - \beta)U_{0B} + V_S - \beta V_A - (1 - \beta)V_B \right]^T e_2 \quad (2.40) \]

Stipulate

\[ N^T = \begin{bmatrix} e_2^T, -\beta e_2^T, -(1 - \beta)e_2^T \end{bmatrix} \quad (2.41) \]

Inserting (2.41) into (2.40) gives

\[ g_n = N^T \overline{U_0} + N^T \overline{V} = g_{n0} + N^T \overline{V} \quad (2.42) \]
where $g_{n0}$ represents the initial gap in the normal direction when the current time step starts, which can be written as

$$g_{n0} = N^T \overline{U}_0$$  \hspace{1cm} (2.43)

From (2.42)

$$\delta g_n = N^T \delta \overline{V}$$  \hspace{1cm} (2.44)

Thus from equation (2.34), the contribution of one contact element to the variation can be given by

$$\partial \Pi^e_{cn} = \alpha_n g_n \delta g_n$$  \hspace{1cm} (2.45)

Inserting equation (2.42) and (2.44) into (2.45) yield

$$\partial \Pi^e_{cn} = \alpha_n (g_{n0} + N^T \overline{V})(N^T \delta \overline{V})$$

$$= \alpha_n \delta \overline{V}^T N(g_{n0} + N^T \overline{V})$$

$$= \delta \overline{V}^T (\alpha_n N \cdot g_{n0} + \alpha_n NN^T \overline{V})$$  \hspace{1cm} (2.46)

Stipulate

$$K_{cn} = \alpha_n NN^T$$  \hspace{1cm} (2.47)

$$R_{cn} = \alpha_n N \cdot g_{n0}$$  \hspace{1cm} (2.48)

Then we have

$$\partial \Pi^e_{cn} = \delta \overline{V}^T (K_{cn} \overline{V} + R_{cn})$$  \hspace{1cm} (2.49)

It is clear that $K_{cn}$ is the tangent stiffness matrix from the contribution of the contact element $SAB$, and $R_{cn}$ is the contact force.

To enforce the normal contact constraints, the contact stiffness matrix and contact force, shown in equation (2.47) and (2.48), respectively, should be assembled to the system equations arising from the standard finite element procedure.

### 2.2.4.3 Sticking Contact

In many resistance welding processes, such as spot welding of sheets of same material, the tendency of tangential sliding at the interface is minimal. The assumption of sticking contact approximates the situation well.
In sticking contact, the normal constraint is still valid. Besides equation (2.32), the following constraint should be satisfied

\[ g_t = 0 \quad (2.50) \]

where \( g_t \) is the relative velocity of point \( S \) and \( P \) for contact element \( SAB \).

Notice that equation (2.50) is an equality instead of an inequality as in (2.32).

From Fig.2.2, \( g_t \) can be computed as

\[ g_t = (U_S - U_P)^T e_1 \quad (2.51) \]

The discrete penalty functional in equation (2.33) now should be reformulated as

\[ \Pi_p = \Phi_p + \Pi_{cn} + \Pi_{ct} = \Phi_p + \sum_{i}^{\mathbb{p}} \frac{1}{2} \alpha_n g_{ni}^2 + \sum_{i}^{\mathbb{p}} \frac{1}{2} \alpha_t g_{ti}^2 \quad (2.52) \]

The variation of equation (2.52) with respect to \( V \) is

\[ \delta \Pi_p = \delta \Phi_p + \delta \Pi_{cn} + \delta \Pi_{ct} \\
= \delta \Phi_p + \sum_{i}^{\mathbb{p}} \alpha_n g_{ni} \delta g_{ni} + \sum_{i}^{\mathbb{p}} \alpha_t g_{ti} \delta g_{ti} \quad (2.53) \]

In equation (2.53), the normal contact term, \( \delta \Pi_{cn} \), has been obtained in frictionless contact. Therefore, in the following only the contribution of tangential contact is considered.

Inserting equations (2.37) – (2.39) into (2.51) we obtain

\[ g_t = [U_{0S} - \beta U_{0A} - (1 - \beta)U_{0B} + V_S - \beta V_A - (1 - \beta)V_B]^T e_1 \quad (2.54) \]

Stipulate

\[ N_1^T = [e_1^T, -\beta e_1^T, -(1 - \beta)e_1^T] \quad (2.55) \]

and inserting (2.55) into (2.53) leads to

\[ g_t = N_1^T \overline{U}_0 + N_1^T \overline{V} = g_{t0} + N_1^T \overline{V} \quad (2.56) \]

where \( g_{t0} \) represents the initial gap of tangential velocity defined by

\[ g_{t0} = N_1^T \overline{U}_0 \quad (2.57) \]

The variation of equation (2.56) is
\[ \delta g_t = N_1^T \delta \bar{V} \]  

Hence the tangential contribution to the variation in equation (2.34) can be expressed as

\[ \delta \Pi_{ct}^e = \alpha_t g_t \delta g_t \]  

Inserting equation (2.56) and (2.58) into (2.59) yield

\[ \delta \Pi_{ct}^e = \alpha_t (g_{t,0} + N_1^T \bar{V}) (N_1^T \delta \bar{V}) \]
\[ = \alpha_t \delta \bar{V}^T N_1^T (g_{t,0} + N_1^T \bar{V}) \]
\[ = \delta \bar{V}^T (\alpha_t N_1 \cdot g_{t,0} + \alpha_t N_1^T \bar{V}) \]

The contributions of the tangential contact to the contact stiffness and the contact force are then obtained as

\[ K_{ct} = \alpha_t N_1 N_1^T \]  

\[ R_{ct} = \alpha_t N_1 \cdot g_{t,0} \]  

In implementing of sticking contact, the contact stiffness of the contact element, which consists of equation (2.47) and (2.61), should be assembled to the system equations together with the contact force expressed in equation (2.48) and (2.62).

### 2.2.4.4 Frictional Sliding Contact

In engineering practice, interface friction never vanish when contact occurs. From a numerical point of view, some additional constraints should be satisfied regarding the normal and tangential forces in the interface. Frictional phenomena occurring in the interface of colliding bodies are incorporated into the formulation of contact problems by friction laws. The most frequently applied friction laws include Coulomb’s law and the constant friction law.

- **Coulomb’s law.** Coulomb’s friction law is probably the most widely used in literature, although its validity at high normal pressures is questionable. The Coulomb’s law can be expressed as,

\[ |F_t| < \mu |F_n| \iff u_t = 0 \]
\[ |F_t| = \mu |F_n| \iff u_t \neq 0 \]  

where \( F_t \) and \( F_n \) are the tangential and normal traction due to friction, respectively; \( \mu \) is the friction coefficient, \( u_t \) is the tangential velocity.
Coulomb’s law originated from rigid body contact. It states a non-smooth and multi-valued relation between the frictionless force and the tangential force. In case of plastic deformation, the friction is often overestimated under considerably high normal pressures.

- **Constant friction.** In this model, friction is regarded as unchanged,

\[ F_i = mk \]  

(2.64)

where \( m \) is a coefficient and \( k \) is the shear yield stress. In contrast to Coulomb’s law, this model does not overestimate friction at high stresses, but under low normal pressure, the friction may be exaggerated.

There are some other models providing improvements to above two models. One example is the general friction model proposed by Wanheim and Bay [32]. Choice of model is not extensively discussed herein considering the fact that the procedure to impose the friction is similar for different friction models.

Despite of extensive studies, the frictional contact remains as one the most challenging problems. Variational equality approach is usually employed to deal with the frictional contact problem, but rely on careful numerical schemes and numerical difficulties are often encountered because of highly nonlinearity.

In frictional contact, the constraint in the normal direction is enforced in the same way as described previously, while another term due to friction is added to the total functional, which takes the form as

\[
\Pi_f = \Phi_p + \Pi_{cn} + \Pi_{cf} = \Phi_p + \sum_i \frac{1}{2} \alpha_n g_n + \sum_i F_{ti} \cdot g_{ni} 
\]  

(2.65)

where

- \( \Pi_{cf} \) – Variational functional due to friction,
- \( F_{ti} \) – Friction at point \( i \).

Frictional sliding forces are applied forces, they do not have a potential. Therefore, they need not be incorporated into a penalty functional. In this view the third term in equation (2.65) is a kind of pseudo-potential.

The variation of equation (2.65) with respect to \( \nu \) is,

\[
\delta \Pi_f = \delta \Phi_p + \delta \Pi_{cn} + \delta \Pi_{cf} 
\]

\[
= \delta \Phi_p + \sum_i \alpha_n g_n \delta g_n + \sum_i (\delta F_{ti} g_{ni} + F_{ti} \delta g_{ni}) = 0 
\]  

(2.66)
In the right hand side of equation (2.66), the first two terms are the same as in frictionless contact, so only the last term needs to be considered.

Based on the Coulomb’s law, the discrete analogue of the last term of (2.66) for one contact element can be expressed as

$$\delta \Pi_{cf} = \delta (\mu F_n) g_i + \mu F_n \delta g_i$$

(2.67)

With the penalty method, the normal force is modeled as

$$F_n = \varepsilon_n g_n$$

(2.68)

Substituting equations (2.40), (2.56) and (2.58) into (2.67), we obtain

$$\delta \Pi_{cf}^e = \delta \bar{V}^T (K_{cf} \bar{V} + R_{cf})$$

(2.69)

with

$$K_{cf} = \mu \alpha_n (NN_1^T + N_1^T)$$

(2.70)

$$R_{cf} = \mu \alpha_n (N \cdot g_0 + N_1 \cdot g_{n0})$$

(2.71)

The additional contact stiffness arising from friction is a symmetric matrix.

As mentioned previously, the classical Coulomb’s law of friction is prone to overestimating the friction at high interface stresses. To solve large plastic deformation using a rigid plastic formulation, the constant friction model is probably a better alternative.

The law of constant friction shown in equation (2.64) is not a continuous function. In the context of plastic forming modeling, it is often modified as

$$F_i = \frac{2}{\pi} mk \cdot \tan^{-1} \frac{\Delta u}{\omega}$$

(2.72)

where $\Delta u$ is the relative velocity at the contact point, $\omega$ is a small positive constant compared with $\Delta u$.

Insert equation (2.72), (2.56) and (2.57) into the last term of equation (2.66) and consider one contact element,

$$\delta \Pi_{cfs}^e = \delta \bar{V}^T (K_{cfs} \bar{V} + R_{cfs})$$

(2.73)

where
To summarize, the frictional contact problem can be enforced by incorporating the contact stiffness, which is given in equations (2.47) and (2.74), and the contact force shown in equations (2.48) and (2.75).

### 2.3 Thermal Contact

In resistance welding, when mechanical contact takes place, thermal contact also arises because the nodes usually have different temperatures when they were brought into contact. In SORPAS, heat generation and conductance within the material are handled in the thermal model, details of which can be found in [37].

In last section, the mechanical contact problem was solved. The same methodology can be applied to the thermal contact.

In mechanical contact the primary unknown, velocity, is a vector. In contrast, the primary unknown in the thermal model, temperature, $T$, is a scalar. The vector of nodal primary unknowns, temperature and temperature increment of the contact element shown in Fig.2.3 can be expressed as

$$\bar{T}^T = (T_S, T_A, T_B)$$

$$\Delta \bar{T}^T = (\Delta T_S, \Delta T_A, \Delta T_B)$$

The enforcement of the thermal contact condition corresponds to a constraint to the standard thermal problem,

$$g_T = 0$$

(2.76)

with temperature penetration

$$g_T = T_{0S} + \Delta T_S - T_{0p} - \Delta T_p$$

(2.77)

where the subscript $\theta$ represents the value at the beginning of the current time step.

With the same procedure as in the mechanical tangential contact, the constraint (2.76) can be enforced by adding to the variational functional a penalty term as
The contribution of contact elements to the conductance matrix and vector of heat source can be obtained from the variation of equation (2.78), for one contact element

$$
\delta \psi^{e}_{cT} = \alpha_{c} g_{n} \delta g_{n}
$$

Employing linear interpolation of temperature, the temperature gap can be obtained

$$
T_{p} = \beta T_{A} + (1 - \beta) T_{B}
$$

Substituting equation (2.80) into (2.77), we get

$$
g_{T} = T_{S} - \beta T_{A} - (1 - \beta) T_{B} = N_{T} T^{T}
$$

where

$$
N_{T} = [1, \beta, (1 - \beta)]
$$

So that from equation (2.81),

$$
\delta g_{T} = N_{T} \delta T^{T}
$$

Substituting equation (2.81) and (2.83) into (2.79), and consider one contact element following the same procedure as in the mechanical contact algorithm, we get

$$
\delta \psi^{e}_{cT} = \delta(\Delta T)^{T} (\alpha_{c} N_{T}^{T} \cdot g_{T0} + \alpha_{c} N_{T}^{T} N_{T}^{T} \cdot \Delta T)
$$

where

$$
g_{T0} = T_{0S} - \beta T_{0A} - (1 - \beta) T_{0B}
$$

Stipulate

$$
K_{cT} = \alpha_{c} N_{T}^{T} N_{T}^{T}
$$

$$
R_{cT} = \alpha_{c} N_{T}^{T} \cdot g_{T0}
$$

Then we have

$$
\delta \psi^{e}_{cT} = \delta \Delta T^{T} (K_{cT} \Delta T + R_{cT})
$$
where $K_{cT}$ is the conductance matrix from the contribution of the contact element, and $R_{cT}$ is the heat source arising from contact, which should be assembled to the system equations from standard finite element procedure.

### 2.4 Electrical Contact

The electrical contact in resistance welding can be tackled in the same way as the thermal contact. The details of the electrical model without contact in SORPAS® can be found in [38].

The primary unknown in the electrical model is the electrical potential, $J$, which is also a scalar. The elemental parameters including the vector of the nodal potentials, $\bar{J}$, and the increment, $\Delta \bar{J}$, can be expressed as

\[
\bar{J}^T = (J_s, J_A, J_B)
\]

\[
\Delta \bar{J}^T = (\Delta J_s, \Delta J_A, \Delta J_B)
\]

The constraint to the standard thermal problem is

\[
g_J = 0 \quad (2.89)
\]

with the electrical potential gap as

\[
g_J = J_{0s} + \Delta J_s - J_{0p} - \Delta J_p \quad (2.90)
\]

where the subscript $0$ represents the value at the beginning of this time step.

The constraint (2.89) can be imposed by adding to the variational functional a penalty term arising from the contact active set as

\[
\varphi_{cJ} = \sum_i^{I_p} \frac{1}{2} \alpha_j g_{ji}^2 \quad (2.91)
\]

The contact element analogy to equation (2.91) is

\[
\delta \varphi_{cJ} = \alpha_j g_{ji} \delta g_{ji} \quad (2.92)
\]

The electrical potential gap can be obtained employing linear interpolation along the segment, or

\[
J_p = \beta J_A + (1 - \beta) J_B \quad (2.93)
\]
Substituting equation (2.93) into (2.90) leads to
\[ g_J = J_S - \beta J_A - (1 - \beta)J_B = N_T^T \delta J \]
Therefore from (2.94)
\[ \delta g_J = N_T^T \delta J \]
Substituting equation (2.94) and (2.95) into (2.92) yield
\[ \delta \phi^c_J = \delta \Delta J^T (K_{cJ} \Delta J + R_{cJ}) \]
where \( K_{cJ} \) is the additional conductance matrix from the contribution of the contact element.
\[ K_{cJ} = \alpha_J N_T^T N_T \]
\[ R_{cJ} = \alpha_J N_T^T \cdot g_{J0} \]
with \( g_{J0} \) representing the initial temperature gap defined by
\[ g_{J0} = J_{0S} - \beta J_{0A} - (1 - \beta)J_{0B} \]
The electrical constraints can be enforced using (2.97) and (2.98).

### 2.5 Implementation of the Algorithms

The above-presented formulations for mechanical, thermal and electrical contact, are incorporated into the finite element code SORPAS® to solve the contact problems in resistance welding.

#### 2.5.1 Implementation

The interfaces are treated like another element class in the current model. The contribution of the contact element is assembled to the system equations. In applying the penalty method, each boundary node is checked through the contacting surface for geometrical contact to find the candidate set of boundary nodes. Further check is performed for the nodes in the candidate set for velocity penetration. When there is no penetration, nothing is done. If penetration occurs, the node is an active node. The system equations are then modified using the formulations shown above. Take the mechanical model as an example. The stiffness matrix should be reformed taking into account the changing connectivity along the interfaces, then the contact contribution to the tangent stiffness and force are added to the system equations. In other words, the contact conditions are applied to only the active set. The thermal and electrical contacts involve only equality constraints, and system equations should be modified according to all the nodes in the active set.
A flow chart of the algorithm is shown in Fig.2.3.

![Flowchart of contact algorithm](image)

Fig.2.4 Flowchart of contact algorithm

The resultant governing equations are then solved to obtain the mechanical, thermal and electrical variables under the contact conditions.

### 2.5.2 Symmetric Treatment of Interfaces

Some researchers divide the contact surfaces into the master surface and the slave one. In a node-to-segment contact element, the node is always lying on the slave surface while the segment is on the master surface. The interface forces are exerted between the slave node and its master surface. As a rule, the harder material of the contact pair is chosen as the master surface.

This asymmetric treatment of the interfaces may work well in some processes such as metal forming, where there is significant difference in stiffness of the contacting bodies (tools and
workpieces). But in resistance welding, the contact bodies in concern are of more complex combination. In the process, temperature varies drastically, so do material properties. And the load employed is usually high; the process is often very unstable. On the other hand, the material in contact may change properties in different patterns, the harder material of the contact pair in the beginning may become the softer one during the process. This highly nonlinearity makes it difficult to deal with using asymmetric treatment of interfaces.

The asymmetric method is not a suitable scheme for the present work. It is easier to run into numerical difficulties. Instead, a symmetric treatment of the contact surfaces is employed in the present work as in [13]. In this approach, each of the contact surfaces is treated as the master and the slave surface once.

![Fig.2.5 Interface elements](image)

Take the two bodies in Fig.2.5 for example. The contactor, where nodes 5, 6, 7 and 8 lie, is in contact with the target body, which has the nodes 1, 2, 3 and 4. With the symmetric method, there are 6 candidate contact elements, namely 287, 376, 465, 643, 732 and 821, depicted with the nodes of the element. Computation tests have shown that this symmetric scheme makes calculation more stable.

### 2.5.3 Coupling Scheme of the Three Models

Resistance welding is a coupled electrical-thermal-mechanical problem, as described in Chapter 1. The coupling scheme of the models in SORPAS® is depicted in Fig.1.12. When the contact formulations are incorporated into the three models, they follow a similar coupling scheme.

The coupling pattern of the mechanical, thermal and electrical contact models is shown in Fig.2.6.

### 2.5.4 Penalty Parameters

The fundamental problem associated with the penalty method lies in the choice of an appropriate penalty parameters. The accuracy and efficiency of the algorithm rely heavily on the value of the penalty parameter. The penalty parameter should, in principle, be an arbitrary large number, too small a penalty parameter leads to significant interpenetration at the interface; however, for a computer with a limited number of digits, too large a penalty
parameter may result in ill-conditioning of the governing equations. Thus the penalty parameter should be large enough to prevent an unacceptable penetration between objects; but not so large that the governing equations become ill-conditioned. The choice represents a compromise between significant loss of accuracy due to poor conditioning of the tangent matrix and unacceptable violation of the contact conditions.

![Flowchart](image)

**Fig.2.6 Couple scheme of models with contact**

For contact problems, a few guidelines are available to provide the optimum penalty parameter. Hallquist [14] chose penalty stiffness to be of approximately the same order of magnitude as the stiffness of the elements normal to the contact interface. Or,

\[
\alpha = \frac{\omega_\alpha K A_n^2}{V_{ol}}
\]

(2.100)

where

- \(\omega_\alpha\) – coefficient,
- \(K\) – stiffness of the element,
- \(A_n\) – area of the element on interface,
- \(V_{ol}\) – volume of the element.

In the present work, the penalty parameter is computed from the maximum element on the main diagonal of the contact stiffness matrix. Or,
where $c_\alpha$ is a constant and $k_{ci}$ is an element on the main diagonal of the tangential stiffness matrix.

### 2.6 Numerical Verification

In this section, some numerical tests are carried out to evaluate the effectiveness of the above-presented mechanical contact algorithms. Since the algorithms for the thermal and electrical contact are based on similar theory as those for the mechanical contact, validation of the mechanical contact algorithm will be a useful proof of the whole contact algorithm.

#### 2.6.1 Numerical Verification I: Upsetting of Cylindrical Parts

Due to its complexity, the contact problem can be solved analytically only for some simple cases. A well-known example is Hertz contact problem which involves contact between elastic bodies. However, the analytical solutions based on elasticity theory are not suitable for verification of the contact algorithms in this context because these are implemented in a rigid plastic formulation.

Consider two upsetting processes between flat dies illustrated in Fig.2.7. Fig.2.7 (a) shows a cylindrical billet. And Fig.2.7 (b) depicts two identical cylindrical parts; the two parts are piled up, each one is half the height of the part in Fig.2.7 (a). The die-workpiece interface is free of friction, and all the parts are of the same homogenous material. Because of symmetry the two cases should undertake identical deformation when subjected to upsetting, and the parts will keep being cylindrical during forming. As with simulation, the only difference between the two cases is that the contact algorithm is activated in the latter. Thus the contact algorithm can be evaluated with this example excluding interference of extraneous factors.
2.6.1.1 Frictionless Contact

The two processes in Fig.2.7 are first simulated with SORPAS® employing frictionless mechanical contact. The yield stress of the material used in simulation is \( \sigma = 360 \varepsilon^{0.15} \varepsilon^{0.02} \). Contact between the tools and the parts are assumed to be frictionless. The two cases in Fig.2.7 lead to identical shapes of workpieces after compression by 50% of the total height (see Fig.2.8).

![Fig.2.8 Profile of parts from numerical simulation](image)

The stress distribution in vertical direction is shown in Fig.2.9. It can be seen that stress distribution is uniform through the parts, and continuity is preserved in the normal direction along the interface. This demonstrates the effectiveness of the frictionless contact algorithm.

![Fig.2.9 Distribution of Stress in Y direction](image)

Fig.2.10 shows the external load. The loads for the two cases show no difference.
This example shows that the contact conditions in the normal direction are imposed properly with the frictionless contact model.

### 2.6.1.2 Sticking Contact

The two upsetting processes are then simulated employing sticking contact algorithm. The results are nearly identical to those from frictionless contact. Only the external load is shown here, referring to Fig.2.11. It can be observed that the difference between Fig.2.11 and Fig.2.10 is negligible.

There is no tangential sliding because of symmetry in Fig.2.7 (b). Therefore the frictionless contact algorithm leads to the same results as the sticking contact does. In this case, both models can predict the deformation accurately.
2.6.2 Numerical Verification II: Upsetting of Wedges

For upsetting of two cylindrical parts, the sticking and frictionless contact models differs little because of geometry symmetry and homogenous deformation.

Now consider a compression process of two parts as illustrated in Fig.2.12. The two parts are of the same material as in Fig.2.7, but of different shapes. If welded along the interface, the two parts will form a cylindrical billet as in Fig.2.7 (a). When the two parts are piled up and compressed as shown in Fig.2.12, deformation is not necessarily homogenous. The final shapes of the parts depend on the friction condition on the interface. At the extreme situation,
when sticking contact is assumed, the deformation will be uniform. This example can be used to examine the different effects of the sticking contact algorithm and the frictionless one.

### 2.6.2.1 Frictionless Contact

The compression process is simulated with SORPAS® using frictionless contact. The friction between the workpieces and dies are set to zero. The shapes of the parts are shown in Fig.2.23 after 24% reduction in height.

![Simulation result after 24% reduction in height (frictionless contact)](image)

It can be seen that deformation of the two parts is not homogeneous. The lower part, which is less stiff than the upper one, undergoes larger deformation. As a result, tangential sliding takes place. The simulation result appears to be qualitatively correct.

### 2.6.2.2 Sticking Contact

The process is then simulated employing sticking contact. After 24% of height reduction, the shapes of the parts are shown in Fig.2.14.

With the sticking contact formulation, deformation is uniform through the two parts, round outer profile is reserved. This reveals that the sticking contact algorithm acts as if the contacting points are welded, as it is supposed to be in the situation. This is another proof of the sticking contact model.
2.6.3 Numerical Verification III: Frictional Contact

It is not easy to evaluate the frictional model numerically because of lack of known analytical solutions to frictional contact problems. Thus the effectiveness of the frictional contact model will be shown schematically.

Consider an upsetting problem shown in Fig.2.15. The two workpieces are of copper and mild steel, respectively. Because of different material properties, tangential sliding will occur when the parts are compressed. The final shapes of the parts are dependent on the interface condition, or friction coefficient.
The process in Fig.2.15 is simulated employing frictional contact and with different friction. Other conditions are,

- Flow stress of copper: $\sigma = 451 \varepsilon^{0.1} \dot{\varepsilon}^{0.1}$,
- Flow stress of steel: $\sigma = 360 \varepsilon^{0.15} \dot{\varepsilon}^{0.02}$,
- Friction coefficient between parts and punch: 0.9.

The simulation results after 10% of height reduction are shown in Fig.2.16. When the friction stress, $m_k$, is chose as zero, the result is the same as that of frictionless contact. When the fiction is increased to 10 MPa, tangential sliding in the interface is a little smaller, as shown in Fig.2.16 b); when friction is further increased to 50 MPa, the distance of tangential sliding is even smaller, as illustrated in Fig.2.16 c); while the friction stress is 500 MPa, no obvious interface sliding is found, in effect it is similar to that of the sticking contact algorithm. These examples demonstrated schematically the effectiveness of the frictional sliding contact model.

### 2.7 Influence of the Penalty Parameter

For the penalty method, the penalty parameters are critical to avoid physically inadmissible penetrations and numerical instability during the solution process. Because of lack of guidelines some numerical tests are carried out on the influence of the penalty parameter.

Firstly, different penalty parameters are employed to simulate the upsetting process shown in Fig.2.7 b). Besides the geometry and material data presented above, the conditions for simulation are as follows:

- No. of elements: 300, almost uniform,
- Constant punch velocity: 10 mm/s,
- Time step: 0.1 ms,
- Height reduction: 20%,
- Convergence accuracy: 0.0001.

All the calculations are carried out on a PC (Pentium® IV 2.2 GHz) and the frictionless contact algorithm is employed. The computing time is shown in Table 2.1.

<table>
<thead>
<tr>
<th>Penalty coefficient $c_\alpha$</th>
<th>Calculation time (hour:minute:second)</th>
<th>Penetration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0 : 3 : 25</td>
<td>yes</td>
</tr>
<tr>
<td>0.1</td>
<td>0 : 3 : 22</td>
<td>no</td>
</tr>
<tr>
<td>1.0</td>
<td>0 : 3 : 23</td>
<td>no</td>
</tr>
<tr>
<td>100</td>
<td>0 : 3 : 22</td>
<td>no</td>
</tr>
<tr>
<td>1000</td>
<td>0 : 3 : 22</td>
<td>no</td>
</tr>
<tr>
<td>10000</td>
<td>Not converge</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 Penalty parameter vs. computing time (steel-steel contact)
Fig. 2.16 Simulation results with different friction stress after 10% reduction in height (frictional sliding contact)
When the penalty coefficient \( c_\alpha \) varies from 0.1 to 1000, the simulation time is almost unchanged, and no penetration in interface is observed. As \( c_\alpha \) is increased to 10000, convergence cannot be achieved, while a value of 0.001 leads to obvious penetration, as is illustrated in Fig.2.17.

![Simulation result (c_\alpha=0.001, height reduction 20%)](image)

If the upper part is of copper, then the deformation is no longer uniform. Simulation is made under the same conditions except for the material of the upper part. The resultant calculation time for a series of penalty parameters is shown in Table 2.2.

<table>
<thead>
<tr>
<th>Penalty coefficient ( c_\alpha )</th>
<th>Calculation time (hour:minute:second)</th>
<th>Penetration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0:03:42</td>
<td>no</td>
</tr>
<tr>
<td>1.0</td>
<td>0:48:22</td>
<td>no</td>
</tr>
<tr>
<td>1000</td>
<td>1:48:25</td>
<td>no</td>
</tr>
</tbody>
</table>

Table 2.2 Penalty parameter vs. computing time (copper-steel contact)

It can be seen that when the penalty parameter is increased from 0.01 to 1000, computing time is enlarged by more than 30 times. This reveals the strong influence of the penalty parameter on efficiency of the algorithm.

From the two simple processes, it can be concluded that the optimal penalty parameter is process-dependent. In the first example in which the deformation is homogenous, the penalty parameter can vary in a quite wide range without significant loss of efficiency. In contrast, change of the penalty parameter results in marked variation of efficiency in the second...
process with obvious interface variation. In more complicated cases, proper choice of the penalty parameter is crucial for efficiency and accuracy of the algorithm.

Choice of the optimal penalty parameter remains a challenge. In the current work, the default penalty parameter coefficient in equation (2.101) is set to 1. Although this value may not be optimal in some cases, it can give satisfactory results at reasonable efficiency in many applications. All the simulation results presented throughout this thesis are obtained using this value, unless otherwise notified.

2.8 Conclusions

An FE algorithm for the contact problems in resistance welding is developed in this chapter, dealing with the coupled mechanical-electrical-thermal contact problem. The penalty method is used to impose the contact conditions for the electrical and thermal contact, as well as frictionless contact and sticking contact for the mechanical model. A node-segment contact element is the basis for the formulation, and the interfaces are treated in a symmetric pattern. The frictional sliding contact is also solved employing a constant friction law.

The algorithms developed are incorporated into the finite element code. Some preliminary verification is carried out using numerical methods. Since the same theory is applied to thermal and electrical contact problems, only the mechanical contact algorithm is tested in some forming processes. The frictionless and sticking contact algorithms are tested with upsetting processes. And frictional sliding contact is shown schematically the effectiveness in an upsetting test of different materials. The tests demonstrate the effectiveness of the resolution.

References

37. Wenqi Zhang, Finite element modeling of the thermal conditions in resistance welding – fundamentals, Department of Manufacturing Engineering and Management, Technical University of Denmark, May 1997.
38. Wenqi Zhang, Finite element modeling of the electrical conditions in resistance welding – fundamentals, Department of Manufacturing Engineering and Management, Technical University of Denmark, May 1997.
Chapter 3  Experimental Validation of the Mechanical Contact Model

In this chapter, the mechanical contact algorithm presented in Chapter 2 is validated experimentally. Two types of experiments, namely upsetting of cylindrical specimens and compression tests of disc-ring combinations, are carried out both at room temperature. The experimental results are compared with simulations using SORPAS® to examine the validity of the contact algorithm.

3.1 Introduction

In the last chapter, contact algorithms were presented handling the mechanical, electrical and thermal contact problems in resistance welding. Subsequently some numerical tests were performed to verify the mechanical algorithm.

Numerical testing is advantageous in some aspects. With numerical tests, we can focus on the influence of a particular factor of interest while keeping all other parameters unchanged, thus the factor can be examined without disturbance of extraneous factors. Therefore it is possible to make a fair comparison in numerical tests.

Numerical tests are usually carried out in two ways, one is to compare the numerical simulation to the known analytical solution, so that the effectiveness of the model can be directly evaluated; the other is to compare the numerical solution to another simulation based on the same conditions except for the model to examine. The first way entails an analytical solution of the problem, and is thus confined to some simple cases; while the second way is an indirect method of validation.

Experimental testing is a direct way to validate the effectiveness of a numerical model. However, the precision of the experiment is often subjected to the influence of uncontrolled conditions. Sometimes it is not easy to isolate the factor of interest from others.
In this chapter the mechanical contact algorithms are verified in two experiments, i.e., cold upsetting tests of cylindrical parts and disc-ring combinations.

3.2 Upsetting Tests of Cylindrical Parts

In the upsetting tests illustrated in Fig.3.1, the parts undergo large deformation in the tangential as well as in the normal direction, resulting in varying curvatures along the interface and contour depending on materials combination. Contact takes place between the tools and the parts, as well as between the two parts. Therefore upsetting tests of cylindrical parts can be employed to validate the contact algorithm. This kind of contact is similar to the one found in resistance spot welding.

![Fig.3.1 Cylindrical parts for upsetting tests](image)

3.2.1 Specimens and Tools

3.2.1.1 Shapes and Sizes of the Specimens, Punch and Anvil

All the specimens used in the contact experiments are cylindrical and of the same size (Φ30 x 10 mm), referring to Fig.3.1.

The flow stresses of the materials are determined using upsetting tests. The specimens used in the tests are also cylindrical, with dimension of Φ30 x 30 mm.

Both the punch and the anvil are forging tools with flat working surface, as illustrated in Fig.3.1.

3.2.1.2 Materials of the Specimens

The materials used in the experiments are brass (W.Nr. 2.1090) and aluminum alloy (AA 2014). The difference in flow stress between brass and aluminum is large enough to ensure
inhomogeneous deformation in the parts but not so large that deformation concentrates in one material only.

The brass workpieces are annealed before the experiments (kept at 510°C for 1 hour in argon), while the aluminum workpieces remain as after machining.

Flow stresses of the materials employed are determined using upsetting tests.

### 3.2.1.3 Machine

The experiments are carried out with a 60-ton hydraulic press, see Fig.3.2.

![The hydraulic press for experiment](image)

### 3.2.2 Procedures and Other Conditions

#### 3.2.2.1 Flow stress tests

The upsetting tests for flow stress are performed in the press. The contact surfaces between the specimen and tools are lubricated with Molycote DX. The final height reduction of the specimen is about 50%, which is reached in a series of (no less than six) steps of upsetting processes. The height of the specimen and the corresponding load are recorded after each step of deformation to determine the flow stress. For each material, three tests are performed.
3.2.2.2 Contact tests

Experiments are carried out with various material combinations. The combination and the corresponding height reduction in the experiments are listed in Table 3.1. The combinations marked with a ‘-’ are not adopted in the experiments.

<table>
<thead>
<tr>
<th>Materials</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass - Al - Brass</td>
<td>22%</td>
<td>32%</td>
<td>35%</td>
</tr>
<tr>
<td>Al – Brass - Al</td>
<td>20%</td>
<td>34%</td>
<td>-</td>
</tr>
<tr>
<td>Al - Brass</td>
<td>24%</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.1 material and deformation in the experiments

One of the main difficulties in the experiments is to keep the workpieces from asymmetric, tangential sliding occurring due to small inaccuracies in workpiece shape or imperfect alignment, see Fig.3.3.

In the experiments, all faying surfaces of the parts are grinded with sandpaper (200#). The rough surfaces help to diminish the tendency to asymmetric deformation.

Before the tests, both the anvil and the punch are cleaned with alcohol. No lubricant is applied to the interface between the tools and specimens.

The upsetting speed in the tests is about 1 mm/s.
3.2.3 Results and Discussions

Numerical simulation leads to numerous output parameters for comparison. The parameters that are available from the experiments are the total load and workpiece profile. In the present work, the contour of the parts including the interface after deformation are the main items for comparison, and the loads are also used to verify the contact model.

As presented in Chapter 2, the mechanical contact algorithm is implemented in three versions, namely with zero friction, sticking friction and sliding friction. All the three models are employed in the simulations. The results are compared with the corresponding experiments, as shown in the following.

3.2.3.1 Yield Stresses Used in Simulation

The experimental data for the flow stresses of aluminum and brass are shown in Fig. 3.4. The flow stresses are determined by interpolation using a power function, as adopted in SORPAS®.

![Graph showing flow stresses of brass and aluminum](image)

Fig. 3.4 Flow stresses of brass and aluminum used in the tests: experimental results and interpolated power functions

The resultant flow stresses of brass and aluminum are listed in (3.1) and (3.2), respectively.

\[
\sigma_{s_{-brass}} = 503.6\varepsilon^{0.506} \varepsilon^{0.01} \quad (3.1)
\]

\[
\sigma_{s_{-alu\text{min}}} = 186.4\varepsilon^{0.230} \varepsilon^{0.01} \quad (3.2)
\]
The punch and anvil are regarded as rigid in simulation.

### 3.2.3.2 Brass – Aluminum Upsetting

The upsetting process of brass-aluminum combination is simulated with SORPAS® employing frictionless contact, sticking contact as well as sliding frictional contact. The material data are shown in (3.1) and (3.2), the total number of elements in simulation is 300, which are uniformly distributed. Other settings in simulations are as follows:

- Time step increment: 0.1 ms,
- Punch velocity: 10 mm/s,
- Convergence accuracy: $10^{-4}$,
- Friction between specimens and tools: constant friction model, with friction coefficient of 0.2.

These settings are employed in the simulation of sandwich upsetting as well.

In Fig.3.5, the contour of the specimens is compared with the simulation result from the sticking contact model. It can be seen from Fig.3.5 a) that the specimens, which had the same shape and starting dimension, are of different shapes and sizes in the end. Neither of the parts is cylindrical after upsetting. The aluminum part undertook more deformation as a result of smaller yield stress than brass. The brass part has a larger height and a smaller diameter than that of the aluminum specimen. In the cross section of the brass part, the free surface contour is nearly a straight line, with the diameter increasing from the tool-specimen interface to the faying surface. In contrast, the outside profile the aluminum part is a convex curve. The final faying surface is not flat. Instead, the surface is slightly concave in the aluminum side, and is convex in the brass side. Tangential sliding is observed, leading to a bulge at the corner of the aluminum specimen.

The sticking contact model predicts most of these features well. For instance, the shape of the brass part, curvature of the faying surface and the outside profile of the aluminum are of
reasonable accuracy. Naturally, no tangential sliding is observed. The two specimens deform as if they were welded together. The result shows, on the one hand, the sticking contact model works well because it imposes the sticking condition successfully; on the other hand, the assumption of sticking contact does not fit well to the actual situation, the interface friction is not so large as to enforce a full sticking contact. Compared with the experimental results, the sticking contact predicts more deformation in the brass specimen and less deformation in aluminum part than reality. In effect, the aluminum part appears to be more rigid in the calculation than in the real case. And naturally, sliding of the faying surfaces is not predicted.

Fig.3.6 illustrates a comparison between the experimental result and the numerical prediction assuming the frictionless contact. The results agree well in many aspects such as the contour of the brass part, curvature of the faying surface as well as the free profile of the aluminum. Unlike sticking contact, the frictionless contact algorithm predicts tangential sliding in the interface and a larger one than observed in the experiment. The result reveals that the assumption of frictionless contact does not agree well to the real situation, either, i.e. sliding friction does exist in the real situation. Contrary to the sticking contact, the frictionless contact predicts more tangential sliding of the aluminum part, less deformation in the brass specimen and more deformation in aluminum part in comparison with the experimental results. The aluminum part appears to be less rigid in the calculation than in the real case.

In practice, friction appears in the interface. The friction is not so small as to be neglected, as is presumed in frictionless contact; nor is it so large as to prevent tangential sliding which is the basic assumption in the sticking contact model. Consequently neither of the models gives a perfect prediction of the deformation.

In Fig.3.7 the simulation result from the frictional sliding contact model is shown as well as the experimental result. The simulation was made with a friction factor $m=0.3$. Obviously the result agrees better with the experimental results than those of the sticking contact and the frictionless contact model. With better knowledge of the actual friction in the interface, the choice of the friction factor can be refined and an even better match could be achieved to the experiment.
3.2.3.3 Sandwich Upsetting of Aluminum – Brass – Aluminum

In Fig.3.8, the final contour of the specimens in the experiment is compared with simulation results based on the sticking contact model. The final shape is symmetric with respect to a horizontal line. This is observed in both the experiment and the simulation. The curvature of the faying surfaces between the brass and the aluminum are also predicted well in the simulation. On the brass side, the surface is concave near the horizontal center, gradually changing to be convex in the middle, and again becomes concave near the side. The concave side of the brass is predicted in simulation, as well as the convex side the aluminum specimens. In a word, the simulation result agrees qualitatively to that of the experiment. Nevertheless, tangential siding in the faying surfaces is observed in the experiment, leading to small bulges at the corners of the aluminum specimens. Differences are found between the experimental results and the simulation indicating that the assumption of full sticking contact is not a perfect match to the real situation.
The process is then simulated with the frictionless contact model, and the result is compared with the experimental one, see Fig.3.9. The frictionless contact model predicted tangential sliding on the faying surfaces to a greater extent than observed in the experiment. The curvature of the faying surface is not simulated with the frictionless contact model, giving rise to a nearly flat final interface. And deformation of the brass part in simulation is much smaller than that in the experiment, as if the brass specimen were more rigid than it is. In other words, the frictionless model, although can predict tangential sliding, does not apply to this situation because of missing influence of friction.

In Fig.3.10, a comparison is shown on the experiment and the simulation result based on the frictional contact model. Again a constant friction model is adopted in simulation with a friction factor of 0.3. The simulation result is between that of the sticking contact model and the frictionless one, as expected. Compared with the experiment, the deformation in brass is less than that in the experiment, indicating that a larger friction factor should be used in the simulation.
3.2.3.4 Sandwich Upsetting of Brass – Aluminum – Brass

The shapes of the specimens in the experiment are shown together with the simulation result based on the sticking contact model in Fig. 3.11 when the total height reduction is 35%. Generally speaking, the simulation matches the experiment quite well. The contour of the brass part agrees with those in the experiment, the simulated curvatures of the faying surfaces agree with the experiment. On the aluminum side, the interface is convex in the middle, and smoothly transits to be concave near the side. Because of limitation of the model, tangential sliding is not observed in the simulation. Consequently, the aluminum apart undertook less deformation in the simulation, resulting in a little larger thickness and smaller diameter in the end.

![Fig. 3.11 Brass - aluminum – brass upsetting -I (total height reduction 35%)](image1)

The same sandwich upsetting process is computed with the frictionless contact model and the results are shown in Fig. 3.12. The simulation result differs significantly from the experimental one. Deformation is concentrated in the aluminum part, and more tangential sliding is observed than in the experiment. In this process, the frictionless assumption introduces too much error.

![Fig. 3.12 Brass - aluminum – brass upsetting –II (total height reduction 35%)](image2)
The frictionless contact model is then applied to analyze the process with a friction factor of 0.3. The result is illustrated in Fig.3.12. The simulation is in good accordance with the experiment. Tangential sliding along the interfaces is predicted, and the profile of the interfaces as well as the shapes of the periphery agrees well with what is observed in the experiment.

![Image](image.png)

**Fig.3.13 Brass - aluminum – brass upsetting –III (total height reduction 35%)**

### 3.2.3.5 Comparison of Loads

Comparison of loads in experiments and simulations is shown in Table 3.2.

<table>
<thead>
<tr>
<th>Specimens Combination</th>
<th>Height Reduction Ratio</th>
<th>Load (kN)</th>
<th>Experiment</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sticking</td>
<td>Frictionless</td>
</tr>
<tr>
<td>Al- Brass</td>
<td>4.72</td>
<td>181.5</td>
<td>167.1</td>
<td>156.6</td>
</tr>
<tr>
<td>Al- Brass - Al</td>
<td>10.10</td>
<td>220.8</td>
<td>211.8</td>
<td>186.5</td>
</tr>
<tr>
<td>Brass - Al- Brass</td>
<td>9.50</td>
<td>261.0</td>
<td>297.7</td>
<td>237.3</td>
</tr>
</tbody>
</table>

*Table 3.2 Comparison of loads in the experiments and simulation*

### 3.2.3.6 Final Comments to the Tests

The deformation patterns in the above presented upsetting processes are similar to that in many spot welding examples. In the present experiments the results are affected by some factors such as the surface conditions. In the experiments, the interface is grinded to avoid asymmetric tangential sliding, thus friction is not so small as to be negligible, nor is it large enough to result in full sticking. Consequently when the experimental results are compared with those from the sticking contact models or the frictionless one, there are some errors.
These are not due to the models themselves, but due to the fact that the real frictional conditions differ from what the models are based on.

The sticking contact model results in errors of different type with those of the frictionless contact model. In the former model, the bodies deform as if they were welded along the interface. In effect this makes the softer material in the contact pair harder. On the contrary, the frictionless contact model makes the softer material even softer than it is. The frictional sliding contact leads to results in between. The model can predict the results accurately if the friction is correctly modeled.

Comparisons between simulations and experiments have demonstrated the effectiveness of the contact models. The accuracy is satisfactory considering factors like errors in material data, lack of accurate information on friction, etc.

3.3 Experimental Verification in Disc-Ring Tests

In the sandwich upsetting tests, it is difficult to avoid tangential sliding, resulting in asymmetric deformation. The part surfaces are ground before the experiments to prevent tangential sliding. This leads to different surface conditions in experiments which affects the final results. Thus the choice of contact algorithm, i.e., sticking, frictionless or frictional contact, makes much difference on the simulation results. As a consequence, it is difficult to make fair comparison due to lack of knowledge on friction.

By choosing a more stable, self-aligning geometry in form of a ring projection contacting a plane plate, see Fig.3.14, the problem of asymmetric deformation is avoided. This geometry fortunately resembles a common class of resistance welding process, namely those of the solid projection type. With this upsetting test of disc-ring combination, the contact algorithm is additionally validated in this section.

3.3.1 Specimens

Various combinations of disc and ring are employed in the tests. The former is a round plate with an annular ring projection machined at the periphery on one side, while the latter is a flat round plate with a central hole, as illustrated in Fig.3.14.

The materials chosen for the disc are mild steel (W.Nr.1.0037), stainless steel (W.Nr.1.4301) and brass (W.Nr.2.0401); the rings used in the experiment are of mild steel (W.Nr.1.0338) and stainless steel (W.Nr.1.4301). These materials are widely applied in resistance welding, and their flow stresses cover a quite wide range.

In order to ensure proper alignment, the disc is provided with a central stud on the opposite side of the projection, referring to Fig.3.14. The height of projection is 1 mm for all the discs. For the stainless steel discs two projection angles are adopted, namely $\zeta = 75^\circ$ and $90^\circ$, while for brass and mild steel discs, the projection are of the same angle, namely $90^\circ$. The stainless
steel disc with a projection angle of 75º has a thickness of 3 mm, and all other discs are 2 mm thick, referring to Table 3.3. The ring is 2 mm thick in all the experiments.

No heat treatment is performed after machining of the specimens.

Table 3.3 lists the geometry and material combinations as well as the approximate deformation in the experiments. For each combination, two tests are performed. The deformation ratio in the table was calculated as the punch travel (or the total height reduction) divided by the initial height of the projection.

<table>
<thead>
<tr>
<th>Deformation</th>
<th>Disc</th>
<th>Steel 1.0037</th>
<th>St. steel 1.4301</th>
<th>Brass 2.0401</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>90º(t=2)</td>
<td>75º(t=3)</td>
<td>90º(t=2)</td>
<td>90º(t=2)</td>
</tr>
<tr>
<td>Steel 1.0338 1 (~40%)</td>
<td>i</td>
<td>iii</td>
<td>v</td>
<td>vii</td>
</tr>
<tr>
<td>2 (~80%)</td>
<td>ii</td>
<td>iv</td>
<td>vi</td>
<td>viii</td>
</tr>
<tr>
<td>St. steel 1.4301 1 (~40%)</td>
<td>ix</td>
<td>xi</td>
<td>xiii</td>
<td>xv</td>
</tr>
<tr>
<td>2 (~80%)</td>
<td>x</td>
<td>xii</td>
<td>xiv</td>
<td>xvi</td>
</tr>
</tbody>
</table>

Table 3.3 Combinations of geometry & materials with the series no.
3.3.2 Machine, Tools and Experimental Setup

Electrodes, which are normally used in projection welding, are employed in the experiments to hold and align the specimens. The setup is shown schematically in Fig.3.15. The disc is centered by the central stud fitting into the central hole in the upper electrode, whereas the ring is centered in a recess in the lower electrode.

![Fig.3.15 Schematic setup in upsetting test of disc – ring](image)

Both electrodes are made of ISO 5182 A2-2. There is no lubricant on the surfaces of the disc, ring and the electrodes.

The experiments are carried out with the same 60-ton press and an upsetting speed of 1mm/s as in the previous experiments. After experiments, the specimen pair is cut near the central point, then molded in epoxy, grinded and polished until the central point is reached. Comparison is made on the contour of the deformed parts from experiments and simulation.

3.3.3 Conditions for Simulation

The upsetting processes of various disc-ring combinations are simulated with SORPAS®. The flow stresses of the materials were tested in [1], the results are listed in (3.3) – (3.6),

\[
\sigma_{s_{1.0037}} = 354.3 \varepsilon^{0.13} \varepsilon^{0.22} \quad \text{(3.3)}
\]

\[
\sigma_{s_{1.0038}} = 360 \varepsilon^{0.150} \varepsilon^{0.02} \quad \text{(3.4)}
\]
The number of elements in the simulations is 400 for series iii, iv, xi and xii. For all other series, the total number of elements is 300, with the initial mesh illustrated in Fig. 3.16.

Other settings in simulations are as follows:

- Time step increment: 0.1 ms,
- Punch velocity: 10 mm/s,
- Convergence accuracy: $10^{-4}$,
- Friction between specimens and tools: constant friction model, with friction factor of 0.8 (the default value in SORPAS®).

![Initial mesh](image1)

**Fig. 3.16 Initial mesh (for all the series excluding series iii, iv, xi and xii)**

![Simulation results](image2)

**a) Frictionless contact**
**b) Sticking contact**

**Fig. 3.17 Simulation results using different algorithms**
**Brass disc – stainless steel ring (series xvi, height reduction 85%)**
Deformation of the electrodes is not considered in simulation.

Unlike in the sandwich upsetting processes in the previous experiments, choice of different friction models does not lead to significant difference in simulation results due to confined global deformation dictated by the parts geometry. Fig.3.17 illustrates the simulation results employing the frictionless contact model and the sticking model. The difference is minimal. The materials involved are brass (disc) and stainless steel (ring) with a reduction ratio of 85%. The series no. is shown in Table 3.3.

Since choice of the contact algorithms is not critical, only the sticking contact model is employed in simulation of all the cases.

3.3.4 Results and Discussions

Fig.3.17 illustrates the results from experiments and the corresponding simulation for series ii. Only the part near the projection is shown.

From Fig.3.18, it can be seen that simulated deformation of the disc-ring pair agrees well with that in the experiment. For the disc, deformation is confined to the vicinity of the projection. The angle of the projection is enlarged, and the side of the disc is no longer vertical because of material flow outward. The ring is compressed by the disc, resulting in a groove under the projection. At a reduction ratio of 96%, the simulated projection angle is slightly larger than observed in the experiment, and the depth of the groove is a little smaller in the simulation. This discrepancy is probably due to error in the flow stress used in the simulation. Since no heat treatment was performed after machining, the real flow stress of the disc may well be larger than what was used in the simulation owing to the work hardening effect in machining the projection.

The final contours of the stainless steel disc - steel ring pair with height reduction of 80% are shown in Fig.3.19. The pair corresponds to series iv. The projection angle is 75° and the disc thickness is 3 mm. Because of difference in hardness between the disc and the ring, the
projection is only slightly deformed penetrating into the ring like a punch. This is seen in the experiment and predicted in the simulation too, see Fig.3.19.

![Fig.3.19 Stainless steel disc (75°) – steel ring (series iv, height reduction 80%)](image)

In series v and vi, the projection angle is 90°. Similar results are obtained from both simulation and experiment, referring to Fig.3.20 and Fig.3.21, which correspond to total height reduction of 49% and 59%, respectively. The projection undergoes slightly larger deformation than in the experiment probably because of work hardening effect of the disc.

![Fig.3.20 Stainless steel disc (90°) – steel ring (series v, height reduction 49%)](image)
In Fig.3.22 and Fig.3.23 the simulation results are compared with experiments for combinations of brass disc and steel ring. The figures correspond to series vii and viii, with height reduction of 59% and 96%, respectively.

It is seen that in the experiment the disc deforms mainly in the projection. Height of the projection is decreased and the angle is enlarged. The ring is pressed resulting in a groove under the projection. In the simulation results for both cases, the deformation is minimal in the disc and is confined to the ring.

The agreement between simulation and experiment is thus not satisfactory. This is probably due to incorrect material data as a result of work hardening of the disc.

The experimental result for the steel disc – stainless steel ring combination is depicted in Fig.3.24 together with the simulation result, corresponding to a reduction ratio of 57%. In the simulation, the deformation is concentrated in the projection of the disc, while the ring
deforms little. The interface at the projection is nearly flat. In contrast, in the experiment the ring is slightly indented by the disc projection and a shallow groove is formed.

Thus the simulation is not in perfect match with the experiment. The difference probably originates from work hardening of the disc during machining. Consequently, the material data of the disc material used in simulation, which were determined from specimens without work hardening, are smaller than the real values.

Shown in Fig.3.25 is a comparison of the experiment and the simulation when both the disc and the ring are of stainless steel, corresponding to a height reduction of 63%. The thickness of the disc is 3 mm and the projection angle is 75°.

In the experiment, the disc projection is not deformed very much, only the tip of the projection is slightly enlarged in angle. A deep groove is formed in the ring. In the simulation result the projection is nearly flattened. And the groove on the ring is less deep than in the experiment. The difference is probably due to the disc used in the experiment is harder than that employed in simulation because of work hardening.
In Fig.3.26 the result from simulation as well as experiment for series xiii is shown. Again it is found that the simulation predicts smaller deformation in the disc and larger deformation in the ring. This can also be originated from work hardening of the disc.

Fig.3.26 Stainless steel (90°) – stainless steel (series xiii, height reduction 36%)

Fig.3.27 and Fig.3.28 are results for the brass disc and the stainless steel ring combinations, the corresponding height reduction is 37% and 85%, respectively. In both cases, the simulation results match the experiments quite well, e.g., the profiles of the discs, and the shapes of the grooves formed on the ring.

Fig.3.27 Brass disc – stainless steel ring (series xv, height reduction 37%)

It can be observed in Fig.3.27 that the calculated deformation of the disc is a little less than in the experiment, in accordance with series vii and viii, see Fig.3.21 and Fig.3.22. This also suggests that the flow stress of the disc used in the simulation might be larger than reality. Because the rings in series xv and xvi are of stainless steel, which has a larger flow stress than mild steel, the influence of inaccurate flow stress of the disc is not so big as in series vii and viii where the rings are of steel.
3.3.5 Summary

In this section the upsetting tests of disc-ring combinations are carried out. The deformation of the parts resembles that of a class of projection welding operations. By comparison of the experimental results with the simulation results, the mechanical contact algorithm is validated. Generally speaking, the simulation results match the experiments well. In some cases, however, discrepancies are observed. These may be attributed to inaccurate material data used in the simulations.

The aforementioned verification involves only the mechanical model. The thermal and electrical model can be regarded as proven because they are based on the same theory. The experimental validation covering all the mechanical, thermal and electrical phenomena will be presented later, see Chapter 6.

3.4 Conclusions

The mechanical contact model developed in Chapter 2 is verified in this chapter in two groups of experiments.

The first test is on upsetting of cylindrical parts. Because of marked interface sliding, the deformation pattern depends heavily on interface conditions of the parts. Accordingly, different models in simulation lead to results of obvious difference. The frictionless, sticking as well as the frictional sliding contact models are examined by the experiments. It is noticed that the assumption of sticking contact leads to less deformation of the softer material and more deformation of the harder material than experimentally observed; while the frictionless
contact model results is in the opposite, indicating the algorithm works satisfactorily. In all the tests, the actual deformation of the parts is between the prediction of the frictionless contact model and the sticking one; and the frictional sliding contact model gives the best simulation result.

The second test is on compression of disc-ring pairs of different materials and sizes. In this experiment surface condition does not play such a critical role as in the first test. In most cases, the simulation results from the contact model are in good agreement with the experiments. In some tests, good conformity is not reached because of work hardening of the disc.

The validity of the mechanical contact model is confirmed by the experiments.

Reference

Chapter 4  Electrical Contact Resistance in Resistance Welding —
A Theoretical Investigation

The implementation of the contact algorithm has expanded the application of SORPAS®. To ensure accuracy of simulation the model of contact resistance should be examined. In this chapter, the general theory on electrical contact resistance is reviewed; the influence of some related parameters are discussed. The current model used in SORPAS® is analyzed followed by a proposal for improvement.

4.1 Introduction

Electrical resistance is the opposition that a substance offers to the flow of electric current in a circuit. For a metal rod as illustrated in Fig.4.1 a), the resistance $R$ is known as,
where
\[ R = \rho \frac{l}{A_d} \]  

Consider cutting the rod in Fig. 4.1 a) into two parts, bringing them together and pressing one against the other under a load \( F \), as illustrated in Fig. 4.1 b). The surfaces are not in perfect electrical contact at the interface because, on one hand, the surfaces are not perfectly smooth but possess some micro roughness, so the actual direct contact between the surfaces takes place at only a limited number of spots. Furthermore, there are usually surface films that are less conductive; as a result, the current flow across such an interface takes place by conduction through both the films and the spots in direct metal-to-metal contact, hence a resistance to the current flow exists. Since this resistance is confined to a very thin layer between the surfaces, it is called \textit{electrical contact resistance}. In addition, the contact interface also offers a thermal contact resistance. In this context, only the electrical contact resistance is considered.

Thus the total resistance in the two contacting rods in Fig. 4.1 b), \( R_2 \), is,
\[ R_2 = R + R_c \]  

where \( R_c \) is the contact resistance.

In general, the term \textit{electrical contact resistance} means the electrical resistance of a releasable junction between two conductors. These conductors may be called \textit{contact members}, or simply \textit{contact}, when no misinterpretation is likely [1].

Being employed to carry current, electrical contacts are found in almost every electric and electronic application. The reliability of electronic systems is largely influenced by the performance of the contacts and connectors within the assembly. For instance, electrical contacts are often the weak point in integrated circuits, so the electrical characteristics of contacts must be studied in order to determine under what circumstances they impact circuit performance or reliability.

Contact resistance (sometimes expressed as contact resistivity instead) is the basic parameter to evaluate the electric contacts, and thus one of the basic parameters by which different metal systems or metal preparation procedures are typically examined, compared and selected. Moreover it can provide useful insights into the stability and reliability of the contact.

In most cases, a low contact resistance is favorable. In electronic systems, the contact resistance should be kept below a prescribed threshold value. In electromechanical switching devices such as relays, the contact resistance should be minimized to decrease waste of
energy. However, there are some applications which make use of contact resistance, such as resistance welding. Contact resistance is indispensable in creating a resistance weld and it determines the shape, size and location of the weld nugget.

### 4.2 General Theory of Contact Resistance

Due to its critical role in electronic and electrical applications, the behavior of electrical contacts has drawn the attention of many researchers [1-8], leading to knowledge about many important aspects as regards contact resistance.

#### 4.2.1 Contact Surfaces

The existence of contact resistance is due to the properties of the surfaces of solid, mainly surface roughness and surface film.

##### 4.2.1.1 Surface Topology

In the real world, all solid surfaces are rough on the micro scale. No matter how carefully they are prepared the actual surfaces always have irregularities, or hills and valleys which are many atomic dimensions high, as illustrated in Fig.4.2. With the cutting-edge technology of the present day, the average surface roughness can be obtained as small as Ra=0.79 nm for SiC using a particular technique referred to as nanogrinding [9]. Actual electrical contact occurs where the atoms of one metal approach those of the other within several angstroms, corresponding to the normal atomic spacing of the metals. Thus for a practical surface, the irregularities are large compared with the dimension of atomic, to say nothing of normal surfaces prepared with conventional technology.

![Fig.4.2 Surface topography measured over a 80 x 80 µm area by Atomic Force Microscopy [10]](image)

Aside from the local peaks and valleys, a real surface is usually featured with curvature and undulations with spacing in the range of millimeters.

When two solids, both with nominal flat surfaces, are pressed one on top of the other (Fig.4.3), the whole covered area is often called the contact surface. It is more correct to call it the apparent contact surface because the surfaces are not ideally flat in reality.
When the two solids come into contact, there are only a few peaks in contact. The real contact area is much smaller than the apparent area. The hills are deformed either elastically or plastically under the load, thus the contact points are enlarged and simultaneously additional subareas are brought into contact. The process continues until equilibrium is reached. The real contact area, or the load bearing area, is the sum of all these subareas.

When a current runs through the interface, the current lines bundle together to pass through the separate conducting spots, as illustrated in Fig.4.4. The spot is often modeled as a circle of radius $a$ thus frequently called an "$a$-spot". Constriction of the electric current by contact spots reduces the volume of material used for electrical conduction hence giving rise to a resistance at the interface, named constriction resistance, or spreading resistance.

### 4.2.1.2 Surface Films

In general, clean mating metallic surfaces are not common in the real world. Most contacts, whether intentionally or not, have surface films of less conductive species, as illustrated in Fig.4.5. These films are of various composition, thickness and property. Depending on conductivity, the films can be categorized into three groups:
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Fig.4.5 Surface film

- **Conductive films.** An example is surface coating with some other metal such as tin or zinc.
- **Semi-conductive film.** Thin film (up to average 20 angstrom in thickness) is semi-conductive because it can conduct electricity by tunnel effect. And it can be easily fractured mechanically. An example is chemisorbed oxygen atoms to tungsten.
- **Insulating films.** Thick films (usually greater than 100 angstrom in thickness) of oxides, sulphides, grease, dirt, etc, are practically insulating.

The effective conduction area is further decreased when the load bearing area is partly or fully covered by semi-conductive or insulating films, resulting in additional resistance.

To sum up, the contact resistance consists of two components: first, the constriction resistance, $R_x$, resulting from the constriction effect of the contact subareas which produces non-uniform current flow; and second, $R_f$, a resistance due to less conductive surface films.

Films covering surfaces need to be electrically or mechanically fractured before metal-to-metal contact is formed. On rough surfaces the films are easier to fracture mechanically since films usually cannot follow the metal expansion. Unevenly distributed film may drastically change the current distribution in bulk material and the constriction resistance; in this case the film resistance is interrelated to the constriction resistance. For simplicity, the total contact resistance can be estimated as,

$$R_c = R_x + R_f$$

(4.3)

**4.2.2 Constriction Resistance**

Suppose the surfaces of contact members are clean, i.e. film-free and the constriction resistance is the only resistance at the interface. The spots where actual contact is made will be of various shapes and sizes located randomly over the apparent contact area, leading to non-uniform current flow and thus the constriction resistance.

**4.2.2.1 Constriction Resistance of a Single Spot**

Consider a single a-spot on the mating surface, as illustrated in Fig.4.6. For simplicity, the contact spot is assumed to be circular. This assumption provides an acceptable geometrical description of electrical contact spots on the average.
Assuming a circular contact surface, same material in both contact members and perfect symmetry, the contact resistance $R_s$ is,

$$R_s = \frac{\rho}{2a}$$

(4.4)

Formula (4.4) was first presented by J. C. Maxwell [12] for two infinite electrodes touching at a single circular spot of radius $a$. A detailed derivation can be found in [1].

Formula (4.4) has been experimentally verified by R. Holm [1] et al, and is widely used in the electrical contact literature and in problems such as the design of electrical contact. If the contact members are of different materials, then the constriction resistance is,

$$R_s = \frac{\rho_1 + \rho_2}{4a}$$

(4.5)

where $\rho_1$ and $\rho_2$ are the resistivity of the contact members, respectively.

Formula (4.4) applies to the situation where the contact area is much smaller than the apparent area. For more general cases, Kouwenhoven and Sackett [4] presented a model of the constriction resistance, in ohms, for a single circular contact area centrally located at the base of a solid, cylindrical conductor,

$$R_s = \rho \left( \frac{1}{d_2} - \frac{1.409}{d_a} + 0.296 \frac{d_2^2}{d_a^3} + 0.052 \frac{d_2^4}{d_a^5} + \ldots \right)$$

(4.6)

where $d_2$ is the diameter of the circular contact area and $d_a$ the diameter of the apparent contact area.
Timsit et al [2] [3] gave another generalization as,

\[
R_s = \frac{\rho}{2a} \left[ 1 - 1.41581 \frac{a}{r} + 0.06322 \left( \frac{a}{r} \right)^2 + 0.15261 \left( \frac{a}{r} \right)^3 + 0.19998 \left( \frac{a}{r} \right)^4 \right]
\]  

(4.7)

where

- \( r \) – the radius of the cylinder
- \( a \) – the constriction radius.

The difference between expression (4.6) and (4.7) is not significant, and they both reduce to (4.4) when the actual contact area is much smaller than the apparent contact area.

Constriction resistance for the contact spots of some other shapes, such as elliptic, can be found in [1]. According to the experimental study [4], the location of the contact spot does not have much effect on the constriction resistance until the eccentricity exceeds 60% for round specimens with circular spots and 50% for strip specimens.

In practice, the length of the constricted path is usually quite small. The effect of the length of the constricted path was experimentally studied in [4]. It turned out that the constriction resistance is independent of the constriction length. This is reasonable because the constriction resistance results from non-uniform current flow which is not interfered with by the length of the constricted path.

### 4.2.2.2 Constriction Resistance of Multiple Spots

Formula (4.4) – (4.7) applies only to single spot contact. In practice, the electrical contact members comprise a cluster of a-spots formed from the contacting asperities on the faying surface. Thus the constriction resistance is determined by the shape, size, number and distribution of the microcontacts. Assuming the contact spots are circular and far apart compared with the radii; the interaction between the spots is then negligible. In this case, the constriction resistance is,

\[
R_s = \frac{\rho}{2} \sum_{i} a_i
\]  

(4.8)

where \( a_i \) is the radius of the \( i \)th spot.

If the a-spots lie close to each other, the constriction resistance is more complicated.

Supposing all the a-spots have the same radius \( a \) and are distributed uniformly over the apparent contact surface, \( A_a \), with a distance of \( 2l \) to the neighboring spot, Holm [1] gave an approximate solution as,
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\[ R_s(n,a,l) = \frac{\rho}{2\pi n a} \arctg \frac{\sqrt{l^2 - a^2}}{a} - 0.6 \rho \frac{\sqrt{l^2 - a^2}}{A_a} + \frac{\rho}{4r} \]  \hspace{1cm} (4.9)

where \( r \) is the radius of the apparent area.

Details of the real contact areas are required in equation (4.9).

Kouwenhoven and Sackett [4] found in their experiments that dividing the actual contact area into \( n \) equal subareas uniformly and symmetrically distributed over the apparent contact area reduced the constriction resistance by the factor of \( n^{-\frac{1}{2}} \). But in their work only a small \( n \) was studied.

Consider a large number \( n \) of equal, circular spots distributed uniformly over a circular area of radius \( \tau \). The resistance was estimated by Holm as [7],

\[ R_c = \rho \left( \frac{1}{2na} + \frac{1}{2\tau} \right) \]  \hspace{1cm} (4.10)

where \( a \) is the radius of the spots.

There are two terms in (4.10); the first term originates from the constriction resistance of all a-spot in parallel, while the second is due to the interaction of a-spots.

Formula (4.10) was rarely used until Greenwood [8] demonstrated its validity. Also proved was that (4.10) can be generalized, to a good approximation, to unequal spots situated within a single cluster. And \( a \) is the mean a-spot radius defined as \( \sum a_i / n \). \( \tau \) is also referred to as the Holm radius. This implies that the number and spatial distribution of a-spots are not very important to evaluate the constriction resistance in many practical engineering applications where electrical contact occurs reasonably uniformly over the apparent area. This conclusion is supported by some authors. Kouwenhoven and Sachett [4] experimentally examined the effect of a-spot distribution on contact resistance. They found that the interface resistance of the same contact area was not affected significantly by the locations of the a-spots provided the a-spots are not limited to the periphery of the apparent contact interface. Under this circumstance, the resistance is increased by a factor of approximately 2. Nakamura et al [13] examined the contact resistance dependence on the location of the a-spots using the finite element method. Their results showed that the distribution of a-spots affects the constriction resistance but not significantly.

For many engineering applications where there are a large number of a-spots distributed within a Holm radius \( \tau \), formula (4.10) can be approximated as

\[ R_s = \frac{\rho}{2\tau} \]  \hspace{1cm} (4.11)
In this case the Holm radius is sufficient to evaluate the contact resistance. This suggests that details of surface roughness are relatively unimportant to estimate the constriction resistance in many engineering applications.

The Holm radius can be calculated from the real contact area as [11],

\[ A_h = \eta \pi \epsilon^2 \]  \hspace{1cm} (4.12)

where \( \eta \) is an empirical coefficient of order unity.

The load bearing area is related to the load \( F \) applied to the interface and to the plastic flow stress \( H \) of the softer material as

\[ F = A_h H \]  \hspace{1cm} (4.13)

Combining (4.11) – (4.13), the contact resistance can be expressed as,

\[ R_s = \rho \sqrt{\frac{\eta \pi H}{4F}} \]  \hspace{1cm} (4.14)

A similar express is shown in [14] as,

\[ R_s = \frac{\rho}{2} \sqrt{\frac{\pi H_b}{3F}} \]  \hspace{1cm} (4.15)

where \( H_b \) is the Brinell hardness.

In the derivation of (4.14) and (4.15), some factors such as work hardening, increase of the number of a-spots, are not taken into account. But according to Timsit [11], Equation (4.14) is not overly simplified.

Baycura [15] presented a model which takes into account the effect of work hardening. The equation is,

\[ R_s = c F^{-1/(2+h)} \]  \hspace{1cm} (4.16)

where \( h \) is the strain hardening coefficient and \( c \) is a constant which can be found experimentally.

The above discussions deal with some simplified cases. For the complicated interaction of load, geometry and material properties, it is impossible to analyze the constriction resistance analytically. Numerical methods, such as the finite element method (FEM) and the boundary element method (BEM), have been employed in analyzing the contact resistance during the last decade [13], [16-20]. With numerical tools, it is possible to study constriction from spots
of complicated shapes, as well as the evolution of the surface under mechanical deformation, etc. However, most of the work were based on the analytical models as discussed earlier.

### 4.2.3 Resistance from Surface Films

In the above discussions, the surfaces are regarded as completely clean, which is not the case in most industrial applications. Films give rise to an additional electrical resistance which is very complicated to model. A description of different films regarding electrical contact can be found in [1].

The film resistance differs with the property of the film. For thin film of adsorbed gases, or oil of approximately single molecular thickness, 30 angstrom or less, the surface resistance can be calculated as [1],

\[ R_f = \frac{\gamma}{A_a} \]  

(4.16)

where \( A_a \) is the contact area, and \( \gamma \) is the film resistance per-unit-area, which depends on the film thickness and the electron work function of the material.

For thick films which will be ruptured by the plastic deformation under the load exerted to the contact members, the film resistance is dependent on the total subareas, instead on the total contact area [1].

Films of water and liquid lubricants influence contact resistance only slightly because their upper layers are squeezed away at the contacts and the remaining mono-film is penetrable for tunneling electrons [1].

Analysis of contact resistance is mainly qualitative instead of quantitative because the behavior of the films differs widely. It is very difficult, if not impossible, to find a general model of contact resistance from films. Experimental investigations have been the main method to study the influence of different films.

### 4.2.4 Some Influential Parameters

Generally, contact resistance is determined by material properties and the surface conditions of the contact members. Thus any parameter that could influence these factors must have a role in the contact resistance.

One parameter is the contact force. The shape, size and distribution of the actual conducting spots are determined by the mechanical load. As a general rule, an increased load brings about a smaller contact resistance. Transverse sliding or wiping can be an important factor, especially in rupturing insulating surface films.
Another important factor is the surface topography, including macroscopic as well as microscopic features. Macroscopic features include surface curvature and undulations with spacing in the range of millimeters. Microscopic features including small-scale roughness, fine asperity, and pits may have dimensions less than 0.1 mm (<100 microns).

Many material properties, such as electrical resistivity and yield stress, are temperature-dependent, and so are the surface films. As a result, the contact resistance is affected by temperature. Usually a higher temperature leads to increased resistivity and decreased flow stress. The latter decreases the contact resistance by enlarging the real contact area under same load, while the former increases the contact resistance. On the other hand, a higher temperature facilitates driving off of surface films such as adsorbed vapor and accelerates the formation of oxide. In all, the effect of temperature depends on the joint interaction of many factors.

As for the surface film, the chemical composition, the crystal structure, the physical and mechanical properties have much influence on the contact resistance.

Another mechanism of deformation is creep, which is related to time. The contact resistance decreases with time due to continued thermal diffusion of atoms, as shown in Table 4.1.

<table>
<thead>
<tr>
<th>Contact</th>
<th>Load (N)</th>
<th>Temperature (°C)</th>
<th>Time (hour)</th>
<th>Contact Resistance(µΩ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag-Ag</td>
<td>350</td>
<td>18</td>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.5</td>
<td>16.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>15.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>144</td>
<td>15.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>288</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>960</td>
<td>34.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2640</td>
<td>32.6</td>
</tr>
<tr>
<td>Cu-Cu</td>
<td>350</td>
<td>150</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1 Change of contact resistance with time [1]

From Table 4.1, the time span taken to affect the contact resistance is of the order of hours. For resistance welding process which is usually accomplished within less than half a second, it is safe assuming time-independent contact resistances.

The electrical contact involves complicated interaction between contact members of complex geometry and it is very difficult to model. Analytical models are available only for the constriction resistance of some simplified cases. The behavior of surface film is very complex to model. Knowledge of more general cases is not available yet.

Finally it is worth noting that only static contact is discussed up to this point, omitting the dynamics involving friction, fretting, mechanical wear and other dynamic factors which make the contact resistance even more complicated. Little is available in literature on the dynamic contact resistance.
4.3 Contact Resistance in Resistance Welding

Contact resistance is complicated itself, yet resistance welding adds more complexity to it. Being the most critical parameters in resistance welding, the contact resistance still puzzles the welding engineers, in spite of extensive research.

On analyzing the contact resistance in resistance welding, one must consider at least the following factors.

- **Wide range of pressure.** In most electrical contacts such as electric switches and electronic devices, the pressure at the interface is low and the plastic deformation is constrained to the surface asperities. In contrast, the interface pressure in resistance welding is so large that not only the interface asperities but also the base material undergoes severe plastic deformation. Load increases from zero to a prescribed level during squeezing time, fluctuates during welding and holding time because of thermal expansion and variation in material properties and decreases to zero when unloading. The wide range of pressure affects the contact resistances heavily.

- **Wide range of temperature.** Unlike in most other circumstances, where over-heating is regarded as unacceptable, the temperature at the interfaces change drastically in resistance welding, from room temperature to the melting point, and there is a steep temperature gradient in the contact members.

- **A variety of materials are involved.** In resistance welding, contact may occur between similar as well as dissimilar materials, and the material properties are changing because of temperature variation.

- **Diverse surface conditions.** The surfaces may vary extensively because of different ways of surface treatment, previous processing, storage and base metal properties.

- **Significant change of contact area.** Severe plastic deformation occurs at the contact area, accompanied with fretting, fritting, etc., and the geometry of the interfaces undergo significant change within a very short time, especially in projection welding.

- **Dynamic effects.** All the influencing parameters are strongly dynamic. The above-mentioned factors are interrelated and changed in a short period; they combine to determine the final contact resistance.

To sum up, the electrical contact conditions in resistance welding are of extreme nature.

4.4 Contact Resistance Model in SORPAS®

A reliable model for the contact resistance is crucial for realistic simulation of resistance welding. The contact resistance is dependent on the interface normal pressure and temperature
which are continuously changing in welding. A contact resistance model that can be applied in numerical simulation should, without oversimplification, take into account the influence of the main influential factors. A model for the contact resistance across the interface between the electrode-to-workpiece and the faying surfaces was presented by Zhang [21] and has been implemented in SORPAS®.

### 4.4.1 The Model of the Contact Resistance in SORPAS®

In SORPAS®, the contact interface is modeled with an interface layer which has its own mechanical, thermal and electrical properties. Details of the model can be found in [21]. For convenience of analysis, the model is described in brief as follows.

Assuming the thickness of the contact layer is \( l_c \) and the contact area is \( A_c \) (apparent area), the contact resistance \( R_s \) can be calculated by formula (4.1),

\[
R_s = \rho_c \frac{l_c}{A_c} \quad (4.17)
\]

According to Wanheim and Bay’s friction model [22], the real contact area, or load bearing area \( A_b \) between rough surfaces is dependent on the load \( F \) and the flow stress of the softer metal \( \sigma_{s,\text{soft}} \) in the contact members. At low normal pressures, where no interaction in the plastic deformation between neighboring asperity contacts occurs, the following relationship exists:

\[
A_b = \frac{F}{3\sigma_{s,\text{soft}}} \quad (4.18)
\]

Stipulating \( R_{s1}, \rho_1, l_1, R_{s2}, \rho_2, l_2 \) as the contact resistance, resistivity and contact layer thickness of surface 1 and 2, respectively, the contact resistance of metal 1 is then calculated,

\[
R_{s1} = \rho_1 \frac{l_1}{A_b} \quad (4.19)
\]

Inserting equation (4.18) into (4.19) leads to

\[
R_{s1} = \frac{3\sigma_{s,\text{soft}}}{F} \rho_1 l_1 \quad (4.20)
\]

The total resistance can be obtained by adding the contact resistance of the metals in contact,

\[
R_s = R_{s1} + R_{s2} = \frac{3\sigma_{s,\text{soft}}}{F} \left( \rho_1 l_1 + \rho_2 l_2 \right) \quad (4.21)
\]
Combining (4.17) – (4.21) and assuming \( l_1 = l_2 = \frac{1}{2} l_c \), the contact resistivity \( \rho_c \) is then obtained,

\[
\rho_c = \frac{3}{2} \left( \frac{\sigma_{s,\text{soft}}}{\sigma_{nc}} \right) \left( \rho_1 + \rho_2 \right)
\]

(4.22)

where \( \sigma_{nc} = \frac{F}{A_c} \) is the average contact normal pressure.

In order to include the influence of contaminants on the contact resistivity, an extra term is added to equation (4.22),

\[
\rho_c = \frac{3}{2} \left( \frac{\sigma_{s,\text{soft}}}{\sigma_{nc}} \right) \left( \rho_1 + \rho_2 \right) + \rho_{\text{contaminant}}
\]

(4.23)

where \( \rho_{\text{contaminant}} \) is the film resistivity which is dependent on temperature and the base material.

For a better fit to the real welding processes, the model was modified [23] to introduce the influence of pressure to film resistivity as,

\[
\rho_c = 3 \xi \left( \frac{\sigma_{s,\text{soft}}}{\sigma_{nc}} \right) \left( \rho_1 + \rho_2 \right) + \rho_{\text{contaminant}}
\]

(4.24)

where \( \xi \) is a constant ranging from 0.1 to 10.

In SORPAS® the contact resistance is calculated with equations (4.17) and (4.24).

4.4.2 Evaluation of the Model

Heat generation is calculated in SORPAS® based on the contact resistance model shown in equations (4.17) and (4.24). This model makes it convenient to prepare a simulation with SORPAS®. Parameters to be determined by the users include \( \xi \), \( \sigma_{s,\text{soft}} \), \( \rho_1 \), \( \rho_2 \) and \( \rho_{\text{contaminant}} \). These parameters, most of which are temperature dependent, need to be found experimentally. But the values for a large number of frequently applied materials have been built into the program. When all these contact parameters are defined, the contact resistance can be calculated automatically.

In the model, the complicated contact resistance is simplified in an elegant form and related to the process parameters and material properties of the contact members. Using this model a number of resistance welding applications have been successfully analyzed with reasonable
accuracy. With the requirement for precision of simulation higher and higher, the model needs to be refined. After all, contact resistance is a fundamental parameter which affects heavily on the reliability of the simulation results.

There are two components in the model, corresponding to the constriction resistance and the resistance from the surface film (grease, water, oxide, etc), respectively. The following analysis deals with the two parts separately.

### 4.4.2.1 Constriction Resistance in the Model

The constriction resistance is calculated using equation (4.17) originating from expression (4.1) which is usually applied to calculation of bulk resistance. In other words, the model shown in equation (4.24) is based on the assumption that the real contact area is continuous rather than formed by discrete spots.

In using the model shown in equation (4.22), the constriction resistance is simulated in SORPAS® with an artificial contact layer, which has an area of the apparent contact area $A_c$, and a thickness of $l_c$.

Inserting (4.22) into (4.17) leads to,

$$ R_s = \frac{3}{2} (\rho_1 + \rho_2) \frac{\sigma_{s\text{soft}}}{F} l_c $$

(4.25)

This implies that the contact resistance relies on material properties, process parameters and the choice of the thickness, $l_c$, of the contact layer. The constriction resistance given in equation (4.25) is qualitatively correct, decreasing with increase of temperature and load, and decrease of resistivity of the base metal.

Compare equation (4.4) and (4.19), the contact resistance of a spot of radius $a$ is equal to that of a bar of the same radius and $\frac{\pi}{2}a$ in length. In SORPAS®, the thickness $l_c$ is a small number (about 0.05 mm for spot welding). This implies that the constriction resistance may be underestimated in the model.

### 4.4.2.2 Film Resistance in the Model

In (4.23), the film resistivity, $\rho_{contaminant}$, is independent of the normal pressure. This is not true in many cases. On the contrary, the mechanical force affects the film resistance heavily on surface film resistance because it helps rupturing the film and establishing metal-to-metal contact.

Compared with (4.23), formula (4.24) introduces influence of load to the film resistivity. However, in formula (4.24), the influence of load on the film resistivity is the same as on the constriction resistivity, which is without sound theoretical or experimental basis. Moreover,
the flow stress of the softer contact member plays an important role in the formula. One possible argument would be that the flow stress of the base metal can be related to temperature. In practice, the flow stress decreases with the increase of temperature, so does the film resistivity. Thus in the model the relation between film resistivity and temperature is qualitatively correct. As mentioned earlier, it is very difficult to find the influence of surface film. Model (3.24) is acceptable in many cases though it is reasonable to assume different influence of temperature on constriction resistance and film resistance.

To summarize, the model used in SORPAS® provides a simple approximation of the contact resistance, which still needs to be refined.

4.4.2.3 Magnitudes of the Two Components in the Model

With the model shown in equation (4.24), the contact resistance in resistance welding can be represented in reasonable accuracy, provided the factors are properly chosen.

In the model the contact resistance consists of two parts and the difference of the two is determined by the resistivity of the base material and the film. In using SORPAS®, the latter, $\rho_{\text{contaminant}}$, is usually greater than the resistivity of base material, hence the constriction resistance is smaller than the film resistance. Thus the constriction resistance is only a small part of the total contact resistance.

Taking mild steel (AISI 1008) as an example, the bulk resistivity and the resistivity of steel-to-steel interface in SORPAS® is illustrated in Fig. 4.7. The contact resistivity shown in the figure has been verified in some spot welding applications and is built in the program. It is obvious from Fig. 4.7 that bulk resistivity is only a small fraction of the interface resistivity. And the interface resistivity can be enlarged or decreased both by a factor of 10 in SORPAS® based on the shown default value to take into account diverse surface conditions. A
comparatively much larger film resistance can diminish the influence of an imprecise constriction resistance.

It is the total resistivity instead of one specific value that matters to achieve accurate simulation. Consequently, in the model $\rho_{\text{contaminant}}$ is not necessarily the contaminant resistivity only; instead, it may compensate for part of constriction resistivity, too. The values can be determined by comparison between the simulation and experimental results.

On the other hand, it is usually the final result that is of interest quantitatively. More than one contact resistivity function may exist which can result in the same nugget. In other words, even though the contact resistance employed in simulation is not identical to that of the real process, it is still possible to get nearly the same or even identical prediction of the weld nugget; of course the dynamic process of nugget development will not be identical. As to the film resistance, the function of film resistivity relies on at least three factors, namely the effect of pressure, the influence of temperature and the film resistance. Inaccuracy in one or two factors may be compensated for by the other factor/factors.

In order to refine the contact resistance model, the relation between contact resistance and load as well as yield strength need experimental verification.

### 4.5 A New Model of Contact Resistance

As mentioned earlier, model (4.24) is based on the assumption of continuous contact area, neglecting the resistance originating from the constriction of separated a-spots to the current flow. This may lead to underestimation of the contact resistance. The model can be improved by taking this effect into account.

From equation (4.12) and (4.18), the Holm radius can be estimated as,

$$
\tau = \sqrt{\frac{A_h}{\eta \pi}} = \sqrt{\frac{F}{3 \eta \pi \sigma_{s\_soft}}} \quad (4.26)
$$

Inserting (4.26) into (4.11), the constriction resistance is,

$$
R_s = \frac{\rho}{2} \sqrt{\frac{3 \pi \eta \sigma_{s\_soft}}{F}} \quad (4.27)
$$

If the contact members have different bulk resistivity, then (4.27) should be modified as,

$$
R_s = \frac{\rho_1 + \rho_2}{4} \sqrt{\frac{3 \pi \eta \sigma_{s\_soft}}{F}} \quad (4.28)
$$
Expression (4.28) represents only the constriction resistance. For a real surface, another term is necessary for the resistance from surface film.

The overall contact resistance is the sum of the two parts,

\[ R_c = R_s + R_f \]  \hspace{1cm} (4.29)

In practice, the surface conditions vary over a wide range and the effect of pressure on film resistance depends on many factors. For simplicity, assume pressure affects the film resistivity in the same way as it affects the constriction, or

\[ R_f = \rho_{fv} \sqrt{\frac{3\pi\eta\sigma_{s,soft}}{F}} \]  \hspace{1cm} (4.30)

where \( \rho_{fv} \) is a coefficient that can be determined experimentally.

This is the overall resistance with no influence of the thickness of the contact layer. In SORPAS\textsuperscript{®}, the contact layer has a thickness and an area for convenience of calculation of heat generation, etc. Thus the model would be more convenient to implement into the program if (4.30) is rewritten to a form like (4.17). Then we obtain the total resistivity, \( \rho_c \), of the contact layer as,

\[ \rho_c = R_c \frac{A_c}{l_c} \]  \hspace{1cm} (4.31)

Inserting (4.28) – (4.30) into (4.31), the equivalent resistivity of the contact layer is,

\[ \rho_c = \frac{1}{l_c} \left( \rho_1 + \rho_2 + \rho_{fv} \right) \left( \frac{3\pi\eta A_c \sigma_{s,soft}}{\sigma_{nc}} \right) \]  \hspace{1cm} (4.32)

And the constriction part is,

\[ \rho_s = \rho_1 + \rho_2 \frac{A_c}{4l_c} \sqrt{\frac{3\pi\eta\sigma_{s,soft}}{F}} = \rho_1 + \rho_2 \frac{3\pi\eta\sigma_{s,soft}A_c}{4l_c \sigma_{nc}} \]  \hspace{1cm} (4.33)

The resistivity in (4.32) and (4.33) is for an artificial layer used to simulate the real constriction resistance, thus it depends on the thickness. The thickness \( l_c \) is not necessarily equal to that of the real contact layer, even if it is possible to find the true value. In the model, influence of load (pressure) is not as heavy as in (4.24).
4.6 Conclusions

Electrical contact resistance is studied in this chapter, aiming at a more reliable model for numerical simulation of resistance welding.

The general theory on contact resistance is reviewed. The contact resistance can be categorized in two parts, namely the constriction resistance and the resistance originating from surface films. The former has been extensively studied and can be estimated with available models; while the latter is more complex and is not well understood. In resistance welding the contact resistance is more complicated because of some factors like strong dynamics of the process.

The model currently employed in SORPAS® is evaluated. It was found that the model may underestimate the real constriction resistance because it is based on the assumption of continual contact area. A new model is proposed for the constriction resistance in resistance welding process based on discrete distribution of the real contact areas.

References

Chapter 5  An Experimental Study of the Electrical Contact Resistance

In this chapter, an experimental investigation is carried out on the electrical contact resistance, studying the influence of interface normal pressure, temperature and material properties.

5.1 Introduction

As earlier mentioned, the contact resistance consists of two components, i.e., the constriction resistance and the contact resistance resulting from surface films. The former is known from earlier theoretical and experimental studies, while the latter is not well understood.

In resistance welding, the process parameters such as interface pressure and temperature are continuously changing, affecting the contact resistance in a complex manner which is difficult to predict quantitatively by theoretical modeling. Instead experimental analysis is applied to establish data for the contact resistance. This is especially necessary as regards the film resistance which varies heavily.

In Chapter 4, the contact resistance in resistance welding is reviewed together with the model employed in SORPAS®. A new model was proposed to improve the model. These models, however, are not based on experimental investigation.

Both models, i.e. the one being used in SORPAS® and the one proposed by the present author, comprise two parts, corresponding to the two components of contact resistance. From the earlier analysis the constriction resistance in the former model may well be underestimated. However, this factor can be compensated for in the second part of the model. In other words, it is the total contact resistance that determines the validity of a model, rather than a particular
component of the model. So the effectiveness of both models needs to be experimentally verified.

On the other hand, in a model of contact resistance aiming at practical applications in numerical simulation, it is desirable to minimize the number of user-input parameters if the precision can be kept at a reasonable level. That is one of the reasons why in both models influence of the process parameters such as normal pressure is the same on the two components. This treatment, however, needs experimental proof.

To summarize, an experimental investigation of the contact resistance is essential to validate the model of contact resistance.

5.2 Experimental Study: A Literature Review

In resistance welding, knowledge of the surface conditions, such as size, number and distribution of contacting asperities, is very difficult to obtain, and it is also difficult to describe the surface films accurately. Experimental studies have been dominating to obtain deeper insight into contact resistance in resistance welding [1-17].

It is evident that the objective is to find the influence of basic parameters like pressure, temperature, surface conditions, etc., on the contact resistance, and each of these parameters covers a wide range. To fulfill this aim, two methodologies have been applied in the experimental work in literature, which may be called the static and the dynamic method, respectively. In the static method, the contact resistance across the interface is measured under some well-controlled conditions, keeping parameters such as load, temperature, etc. stationary during measurement. The parameters can be changed over a range to obtain their influence on the contact resistance; in the dynamic method, the contact resistance is examined in situ during resistance welding. A history of contact resistance is recorded together with the simultaneous variation of many factors in the process.

Examples of the static method are [1-5]. As early as in 1939, Frank J. Studer [1] performed some experiments to investigate the contact resistance during resistance welding. In the experiments, load was exerted with a press, and the specimens could be heated up to 750°C within a furnace, thereby recording the contact resistance in a wide range of temperature and load. Stainless steel and low-carbon steel sheets of various shapes were studied. The influence of force and temperature on the contact resistance was analyzed. The author finally gave the diagram of schematic evolution of contact resistance in welding which is widely accepted today. However, the bulk resistances were mixed with the contact resistance in the experiments, and as the author reported, the results were subjected to significant scatter. Vogler and Sheppard [2] carried out extensive testing on the influence of load and temperature on the contact resistance involved in spot welding of thin sheet steels. A special fixture was applied which allowed the contact resistance at the electrode-to-sheet interface and the faying surface to be measured over a range. In the work of other researchers [3-5], different factors, such as load, surface condition, electrode shapes, etc., were examined studying their effect on the contact resistance for a series of materials, but these studies were
limited to static resistance at room temperature. Experiments in [3] showed that the effect of electrode force on the initial contact resistance can be ignored, which is in contradiction to other work as in [1] [2]. Another surprising result was that the resistance at the electrode-to-sheet interface appeared to be higher than for the steel-to-steel samples, which is contrary to what is normally observed.

The dynamic method has been employed by more authors [6-17]. Tylecote [6] made in 1941 some tests on the contact resistance across the sheet-sheet interface during welding. Variation of some parameters such as load was shown, as well as the dynamic resistance in the procedure. Kohei Ando and Mituo Hasegawa published their work [7] in 1943 on experimental study of electrical resistance. The resistance measured in the experiments is the total resistance including the bulk resistances and the contact resistance. Roberts [8] explored resistance changes during resistance welding. In the experiments, the static as well as the dynamic resistance were measured during welding of stainless steel, low-carbon steel and aluminum sheets with electrodes of different shapes. Shunt effect was also studied. Similar results of resistance variation as showed schematically in [1] were observed in the experiments. The author also reported inconsistency in the experimental results. Savage et al [10] reported experimental studies of the dependence of dynamic resistance on factors like electrode force, current, material surface condition etc., for both uncoated and galvanized auto-body steel.

In the static method, measurement is made under equilibrium state; each parameter of interest is known. It is thus convenient to investigate the effect of a particular variable. The drawback is that the influence of dynamic effects on the contact resistance is not considered.

With the dynamic method, some researchers present the contact resistance as a function of time, others as functions of load or temperature. However, all the concerned parameters change simultaneously during the dynamic welding process. This makes it impossible to extract the influence of a particular variable. In other words, all the parameters are interrelated and in combination they determine the resistance. In this view, the analysis on the influence of individual factors on contact resistance was not meaningful but misleading in those empirical works based on the dynamic method. These work were, however, still of significance because they helped gaining better insight into the dynamics of the process. Actually all of the above-mentioned experimental works were intended to provide an insight into the welding technology via the critical parameter in the process. In this sense they were valuable, but none of them has led to a mathematical model that can be applied in numerical simulation.

It is worth mentioning that high degree of scatter in the empirical research of contact resistance is reported by many authors [1] [2] [6] [8]. According to Studer’s [1] comment on his experiments, in spite of all care to maintain uniform conditions over a surface, the contact resistance is usually considerably different when measured at different positions, and indeed, when measured a second time in the same region. Robert [8] found that the measured resistance of apparently identical specimens may differ widely, by as much as 10:1, especially at low electrode force.
The striking inconsistency in experiment results reveals how variable an interface can be. A seemingly slight alteration of the interface may result in a large deviation of contact resistance. Though prepared in apparently the same way, the surface conditions of the specimens may change at different circumstances, especially when exposed to the air. If the measurement is carried out in vacuum or in protective gases, the divergence could be smaller.

To sum up, the above-mentioned experimental work, though providing some knowledge of contact resistance in resistance welding, is not adequate to leading to a mathematical model that can be used in numerical simulation.

5.3 Experimental Study of Contact Resistance

In this section experiments are carried out to investigate the effects of temperature and interface normal pressure on the contact resistance, aiming at a mathematical model of contact resistance for numerical simulation.

5.3.1 Objectives and Methodology

The fundamental task of the experiments is to investigate the influence of interface normal pressure and temperature on contact resistance and to obtain a better understanding of the dynamic contact resistance in resistance welding dependent on the process parameters; another objective is to examine the earlier contact resistance models (4.23), (4.24) and validate the new model (4.28).

Clearly only the static method is suitable for these experiments since it is necessary to isolate the influence of one parameter from that of the other. The dynamic method would not enable such a parametric study.

To fulfill the aims, the contact resistance should be measured at different normal pressures and temperature. Besides, in both formulas (4.24) and (4.28), the contact resistance is calculated using the flow stress and electrical resistivity of the contact members. These two parameters are temperature dependent and should also be determined experimentally.

5.3.2 Equipment for the Tests

In the experiments the temperature at the interface should change over a wide range, from room temperature to near the melting point, and the normal pressure at the interface should also vary significantly, covering the pressure range in real resistance welding.

To fulfill these purposes, dedicated equipment is required to heat the specimens and load them together. Presses equipped with furnaces were used by some researchers [1] [2]. Yet the Gleeble® system [18] [19] is a better choice.
The Gleeble system is a dynamic testing machine that can simulate a wide variety of thermal/mechanical and metallurgical situations. A sample can be heated and mechanically worked following a prescribed program while various performance parameters of interest are measured and recorded for later analysis. And the process can be controlled accurately and efficiently.

The Department of Manufacturing Engineering and Management (IPL) at the Technical University of Denmark (DTU) has a Gleeble 1500 system from 1982, see Fig.5.1. The machine is equipped with a high speed heating system, a servo hydraulic unit, a computer control and data acquisition system.

![Gleeble 1500 used in the experiments](image)

Heating of the specimens in the machine, which is done by resistance heating, is fast and of satisfactory accuracy. The thermal system of the machine can heat at high rate: up to 10,000 °C per second for 6 mm diameter bar with 15 mm free span plain carbon steel using a 480 volt power line. The accuracy of the thermal servo is ±1°C at equilibrium and ±10°C at heating rate of 1500°C per second.

With the hydraulic unit, the mechanical system can generate a maximal force of 80 kN (tension or compression). The stroke of the piston is maximal 102 mm with a resolution of ±0.001 mm for the stroke transducer. In addition, the machine is equipped with a pneumatic AIRRAM system. The pneumatic controlled punch can hold the jaws, while the hydraulic punch is drawn back for acceleration.

With the Gleeble system complex physical simulations can be performed. The process can be partially or fully controlled by a computer program, which is written in GPL (the Gleeble Programming Language) by the user [20].

Obviously with the Gleeble the planned experiments on contact resistance can be carried out. Besides, there are some advantages using the Gleeble. One of these benefits is that the Gleeble heats the specimen with resistance heating, exactly the same as in resistance welding. Identical mechanism of heating may provide the experiments with better analogy to the real
resistance welding process. For instance, high current employed during heating can break some thin film thus affecting the contact resistance, which cannot be considered in those static experiments where a small current is passed through the specimens. Furthermore, semi-dynamic tests can be made with the Gleeble, which will be discussed later.

5.3.3 Specimens

Specimens of a wide range of shapes can be handled with the Gleeble. Since the aim of the test is a fundamental study of some parameters, it may be desirable to employ samples of simple shapes that are easy to handle.

The specimens in the experiments are round rods of 7.5 mm in diameter; see Fig.5.2. The specimens are of three different lengths L, namely 25 mm, 11.5 mm, 6.0 mm, being employed in the experiments to determine resistivity, flow stress and contact resistance, respectively.

![Fig 5.2 Specimen for the tests](image)

The specimen materials are low carbon steel (W.Nr.1.0037), stainless steel (W.Nr.1.4301) and aluminum (Al 99.5, semi hard). These materials are widely applied in industry, and their mechanical and electrical properties cover a wide range. The surface roughness is unimportant to contact resistance; all the specimens are of the same surface roughness. All the specimens of one particular material were made from the same batch of bars and stored in air.

5.3.4 Physical Setup

The experimental setup is illustrated schematically in Fig.5.3 and the photograph is shown in Fig.5.4.

Referring to Fig.5.3, the specimens are held between the anvils of the Gleeble machine. One of the anvils is mounted on the punch. During the tests, the Gleeble heats the samples to the specified temperature and adjusts the load according to the program. The voltage drop across the faying surface between the specimens and the current are recorded, thus enabling calculation of the contact resistance. This whole procedure of loading and heating is
controlled by a GPL program, and measurement of temperature, load and deformation is done with the GPL program as well.

![Fig.5.3 Schematic setup of the experiments](image)

**Fig.5.3 Schematic setup of the experiments**

![Fig.5.4 Photograph of the setup for contact resistance tests](image)

**Fig.5.4 Photograph of the setup for contact resistance tests**

Accuracy of temperature measurement is crucial in the experiments. The temperature is measured using thermal couple wires (Chrome-Alumel, or type K) which are percussion welded to the samples. Since it is difficult to connect the wires exactly at the interface, the temperature gradient neat interface should be sufficiently small to ensure reliable temperature measurement. Some measures are taken to minimize the gradient in the interface. Firstly, tantalum-foils are placed between the anvils and specimens for thermal insulation; secondly, the anvil is made of tungsten carbide, which is electrically conductive but of poor thermal conductivity, and the jaw to hold the anvil is of stainless steel (so called hot jaw); thirdly, measurements are started after a certain period of heating to allow for sufficient heat transfer.
within the specimen. In addition, graphite foil between the anvils and the tantalum foil, and molybdenum sulphide between the tantalum foil and the specimen reduce the friction.

Similar setup is used in tests for material flow stress, as shown in Fig.5.5. The details for determining the flow stress can be found in [21].

![Fig.5.5 Flow stress test](image)

The resistivity is determined with a similar setup as illustrated in Fig.5.3. In the experiment, two wires, which are around 15 mm apart, are welded to the specimen. Referring to Fig.5.6, the voltage drop \( V \), the current \( I \) and the distance between the wires \( d_i \) are measured. The resistivity can then be derived using equation (4.1).

![Fig.5.6 Test for the resistivity](image)

5.3.5 Measurement

5.3.5.1 Parameters to Be Measured

During each experiment on contact resistance measurement, the following parameters are measured:

- Force
- Deformation of the specimens
- Interface temperature
- Heating current
- Voltage drop across the interface

Among these the first three parameters are recorded with the control unit of the Gleeble, while the others should be measured with extra data acquisition devices.

### 5.3.5.2 Devices for Measurement

The heating current is measured using a Rogowski coil (TECNA – 1430, current range 0-200 kA) encircling the specimens. When current passes through the specimens a potential is induced in the coil due to the large magnetic field, from which the current can be calculated.

The voltage drop is measured using two wires connected to each side of the faying surface. The wires should be as close as possible to the interface. They are twisted around each other to minimize the influence of the magnetic field.

Both signals are collected with a data acquisition board (DAQPad-6020E from National Instrument, input resolution: 12 bit, maximum sampling rate 100 ks/s, input range ±0.05 to ±10 Volt) connected to a PC. A program is written based on LabView® to configure the device, define the settings of measurement and record the parameters. The current for heating is 50 Hz in the Gleeble. To ensure that enough data samples are recorded, the sampling rate is 10,000 per second in the experiments.

### 5.3.5.3 Ensuring Satisfactory Precision in Measurement

The Gleeble is a hostile environment for instrumentation because it applies alternating current of high amplitude. During the experiments care must be taken to ensure accurate measurement of the information of interest.

At the high current used in the Gleeble, the primary noise is caused by induction. One technique that is very effective to minimize induced voltages is the twisted pair of wires. The induced voltage is, according to Faraday’s law, determined by the rate of current change, area of the loop and its orientation. By twisting the two signal-carrying wires tightly, the area of the loop is minimized and its orientation is continuously altered thus reducing the induced noise to zero. So, in the experiments shielded twisted wires are used wherever possible. The Rogowski coil is placed perpendicular to the specimens (the direction of current) to ensure that the correct current is measured.

The load, temperature and deformation are measured by the Gleeble transducers instead of additional devices. The precision is guaranteed by the Gleeble. Yet for temperature measurements, the problem lies in how to ensure that the temperature at the position of the thermal couple wire tip is identical to that of the interface. Since the thermal couples are not located exactly in the interface (the position error of the wire is within 0.5 mm in the tests), the temperature gradient near the interface should be small enough.
In order to guarantee adequately small gradient near interface, tungsten carbide anvils and hot dies are employed in the experiments, and tantalum foils are place between the tools and specimens. In addition, the samples are kept at the prescribed temperature for a short period before measurement. The period should be long enough to allow for homogenous temperature distribution in the vicinity of the interface, but not so long that the interface is spoiled too much by extra oxidation, etc.

Some preparatory tests were made to determine the minimum time needed for each material. Temperatures at the interface and 3 mm from the interface are measured. Fig.5.7 shows the results for a pair of mild steel specimens.

![Fig.5.7 Temperatures at interface and 3 mm from interface for steel specimens](image)

It can be seen that the temperature at the interface is a little higher than that measured at 3 mm from the interface. The time delay is around 13 seconds. Similar results were found for stainless steel and aluminum. It turned out that 20 seconds holding time is sufficient for specimens of all materials to achieve uniform temperature near the interface. Based on these observations a pre-heating time of 30 seconds was chosen in the tests. The thermal couple wire (type K) is 0.25 mm in diameter, and its position in the test is within 0.5 mm from the faying surface, thus ensuring a reliable temperature measurement.

In the tests for the resistivity, the distance between the two wires measuring voltage drop is about 15 mm. To ensure homogenous temperature between the wires, measurement is carried out after 1 minute holding time.

### 5.3.6 Plan of the Experiments

In order to investigate the influence of pressure and temperature on the contact resistance, one way is to measure the resistance under a series of discrete loads and temperatures, also referred to as the \textit{static} method. Another method is to alter one parameter continuously under
Testing and Modeling of Contact Problems in Resistance Welding

a series of constant values of the other parameter. Analysis is easier by the former method, while the latter test method resembles the situation in real resistance welding better. Since only one parameter is changed during each test, thus the method may be called semi-dynamic.

In the practical tests, only load can be dynamically changed because the temperature must be kept for a short period to ensure precise measurement. Both static and semi-dynamic methods are used in the experiments.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Steel-steel</th>
<th>Stainless Steel-Stainless Steel</th>
<th>Aluminum-Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>100</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>200</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
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<td>Y</td>
</tr>
<tr>
<td>600</td>
<td>Y</td>
<td>Y</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.1 Experimental plan for tests with static method

The material and temperature combinations for static and semi-dynamic tests are listed in Table 5.1 and Table 5.2, respectively. The chosen combinations are marked with ‘Y’, while the others are marked with ‘-’.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Steel-steel</th>
<th>Stainless Steel-Stainless Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>100</td>
<td>Y</td>
<td>-</td>
</tr>
<tr>
<td>200</td>
<td>Y</td>
<td>Y</td>
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<tr>
<td>300</td>
<td>Y</td>
<td>Y</td>
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<td>400</td>
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<td>1000</td>
<td>-</td>
<td>Y</td>
</tr>
<tr>
<td>1200</td>
<td>-</td>
<td>Y</td>
</tr>
</tbody>
</table>

Table 5.2 Experimental plan for tests with semi-dynamic method

For steel and stainless steel specimens, the loads range is 0 - 10 kN at room temperature, and for aluminum the load range is 0 - 6 kN. Because the Gleeble can control the movement of the anvil very accurately, the stroke of the piston is used as the control parameter in the program to achieve the planned load. SORPAS® is used to estimate the stroke to achieve the loads. The largest stroke in the tests is 0.6 mm, and the speed of the punch is 0.1 mm/s.

The settings of the tests, including temperature, heating rate, stroke, speed, etc, are written into GPL programs to control the Gleeble machine.
Similar to the plan for testing contact resistance, the planned tests for measuring the stress–strain curves are listed in Table 5.3. The flow stress is determined at different temperatures for each material with a strain up to 0.5. The strain rate is 7.6 in all the tests.

<table>
<thead>
<tr>
<th>Temperature(°C)</th>
<th>Steel</th>
<th>Stainless Steel</th>
<th>Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>200</td>
<td>-</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>400</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>500</td>
<td>-</td>
<td>-</td>
<td>Y</td>
</tr>
<tr>
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<td>Y</td>
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<td>-</td>
</tr>
<tr>
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<td>Y</td>
<td>Y</td>
<td>-</td>
</tr>
<tr>
<td>1200</td>
<td>Y</td>
<td>Y</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.3 Flow stress tests

The planned tests for measuring resistivity are listed in Table 5.4.

<table>
<thead>
<tr>
<th>Temperature(°C)</th>
<th>Steel</th>
<th>Stainless Steel</th>
<th>Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>100</td>
<td>Y</td>
<td>-</td>
<td>Y</td>
</tr>
<tr>
<td>200</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>400</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>600</td>
<td>Y</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>800</td>
<td>Y</td>
<td>Y</td>
<td>-</td>
</tr>
<tr>
<td>1000</td>
<td>Y</td>
<td>Y</td>
<td>-</td>
</tr>
<tr>
<td>1200</td>
<td>-</td>
<td>Y</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.4 Resistivity tests

5.3.7 Experimental Procedure

The procedure of the contact resistance test is as follows:

1) Start the Gleeble machine, load the GPL program. The air pressure for the AIRRAM load is set to a value of 70 N.
2) Percussion weld the thermal couples and wires for measuring voltage to the specimens.
3) Set AIRRAM mode to *Tension*.
4) Setup as shown in Fig.5.3. Care must be taken to ensure good aliment of the specimens.
5) Start the pump, move the hydraulic ram until the force reaches 0.1 kg.
6) The AIRRAM mode is set to *Compression*.
7) Shut down the pump using panic stop.
8) Start the LabView program for measurement.
9) Start the Gleeble using the GPL program. The specimens are heated to the specified temperature and held for half a minute, then compressed by a specified amount. All these procedures run automatically controlled by the GPL program.
10) The test on contact resistance under specific conditions should be repeated at least once (in case of consistent results are obtained). A scatter of 30% in the final contact resistance is regarded as acceptable.

5.4 Results and Discussions

Data collected from the experiments are analyzed in this section.

5.4.1 Data Analysis

Some preparatory treatment of the experimental data is necessary before they can reveal any valuable information about the contact resistance.

5.4.1.1 Calculation of Contact Resistance

When studying the contact resistance with the dynamic method, some authors calculate the resistance only at the peak current values to avoid the induction noise. In Fig.5.8, the recorded current and voltage drop across the interface are shown. The data are extracted from the experiment with stainless steel at 800 °C.

![Fig.5.8 Instantaneous current and voltage drop, recorded in the test of stainless steel contact at 800 °C](image)

Little phase shift is observed between voltage and current implying that the contact resistance can be calculated directly by dividing the voltage drop by the current point by point. In practice RMS values of both parameters are found every half cycle, and the contact resistance is derived by the RMS values of the voltage and current.
5.4.1.2 Synchronization of Data

There are two groups of data from the tests; one includes those measured with the Gleeble, i.e., load, temperature and displacement of the anvil (deformation of the specimens). The other group includes current and voltage drop, which were collected with an external board. All the data are recorded in a time sequence. The two groups of data must be synchronized because data acquisition in the two devices was not triggered at the same time.

When a test finishes, the GPL program shuts down the machine, data acquisition in the Gleeble is ceased, and so is the current. So at the same time, the current measured in the second group should return to zero. In other words, the point in time where measured current has decreased to zero corresponds to the end of the data acquired with the Gleeble. Thus synchronization can be implemented referring to the common ending point in current.

5.4.1.3 Balance of the Number of Samples

After synchronization, the data should be examined to ensure the same number of data samples for each parameter. Because of the restriction of the Gleeble 1500, the data acquisition rate for the Gleeble is smaller than that of the external board. As a result, different numbers of samples were collected for parameters in different groups. The number of data samples of current and voltage are altered when using their RMS values. In case of unequal number of samples for different parameters, linear interpolation is employed for the parameters with less samples to balance the numbers for all the parameters.

5.4.1.4 Apparent Contact Area

The apparent contact area is used in the models to calculate the average contact pressure. The apparent contact area is regarded constant in the experiment.

In reality, the real apparent contact area is a little smaller than the nominal apparent contact area because the alignment of the specimen pair is not perfect. On the other hand, the contact area is enlarged during the test because of compression and thermal expansion at high temperatures. In the tests, the largest reduction in length is 0.6 mm, or 5% of total length of a pair of specimens. So the surface expansion is not large. Assumption of constant apparent contact area will not introduce unacceptable errors.

5.4.2 Flow Stresses of Specimen Materials

5.4.2.1 Experimental Results

The measured flow stress-strain curves of mild steel at different temperatures are shown in Fig.5.9. As a general trend, the flow stress decreases with increasing of temperature. Another fact is that the strain-hardening effect changes significantly during the course of deformation. Take 50 °C as an example, the work-hardening effect is large at strains up to around 0.15, while at strains above 0.2, the work-hardening effect is much smaller. In other words, there is
a threshold value, \( \varepsilon_{th} \), in the stress-strain curve; the work hardening rate is large before the threshold value of strain and is much smaller beyond this value. This is observed at all temperatures tested.

![Graph showing flow stress of mild steel at different temperatures](image)

**Fig.5.9 Flow stress of mild steel at different temperatures**

At small strains, the stress-strain curve is nearly a straight line. Linear interpolation in the form of \( \sigma = (c_1 \varepsilon + c_2) \), with \( \sigma \), \( \varepsilon \) the stress and strain, \( c_1 \) and \( c_2 \) the constants, can approximate the curves with satisfactory precision. The coefficients of the curves are listed in Table 5.5.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>( \varepsilon_{th} )</th>
<th>( \varepsilon &lt; \varepsilon_{th} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( c_1 )</td>
<td>( c_2 )</td>
</tr>
<tr>
<td>50</td>
<td>0.088</td>
<td>4388</td>
</tr>
<tr>
<td>200</td>
<td>0.091</td>
<td>5328</td>
</tr>
<tr>
<td>400</td>
<td>0.051</td>
<td>8713</td>
</tr>
<tr>
<td>800</td>
<td>0.073</td>
<td>1774</td>
</tr>
</tbody>
</table>

**Table 5.5 Coefficients of stress-strain curves for mild steel at different temperatures**

\( \varepsilon_{th} \) is a threshold strain value, \( \varepsilon^2 \) is the R-squared value.

![Graph showing flow stress of stainless steel at different temperatures](image)

**Fig.5.10 Flow stress of stainless steel at different temperatures**
Fig. 5.10 shows the stress-strain curves for stainless steel. The existence of threshold strain values is clear. The curve is steep before the threshold while nearly flat afterwards at all temperatures. At temperatures higher than 200 °C, the flow stress approximates a linear hardening-ideal plastic pattern.

Interpolation is performed using a linear function; the coefficients are shown in Table 5.6.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>$\varepsilon_{th}$</th>
<th>$\varepsilon&lt;\varepsilon_{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_{th}$</td>
<td>$c_1$</td>
</tr>
<tr>
<td>50</td>
<td>0.116</td>
<td>6051</td>
</tr>
<tr>
<td>200</td>
<td>0.092</td>
<td>6315</td>
</tr>
<tr>
<td>400</td>
<td>0.12</td>
<td>4940</td>
</tr>
<tr>
<td>800</td>
<td>0.07</td>
<td>5512</td>
</tr>
<tr>
<td>1200</td>
<td>0.06</td>
<td>811</td>
</tr>
</tbody>
</table>

Table 5.6 Coefficients of stress - strain curves for stainless steel at different temperatures, $\varepsilon_{th}$ is a threshold strain value, $e^2$ is the R-squared value

Table 5.6 Coefficients of stress - strain curves for stainless steel at different temperatures, $\varepsilon_{th}$ is a threshold strain value, $e^2$ is the R-squared value

Fig. 5.11 shows the stress-strain curve of aluminum. The tests were performed at four temperatures. Aluminum is not studied as extensively as regards contact resistance as the other two materials, only the static tests were employed.

Table 5.7 lists the coefficients for the stress-strain curves from linear interpolation.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>$\varepsilon_{th}$</th>
<th>$\varepsilon&lt;\varepsilon_{th}$</th>
</tr>
</thead>
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<td></td>
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<tr>
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<tr>
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<td>3059</td>
</tr>
<tr>
<td>500</td>
<td>0.051</td>
<td>39.15</td>
</tr>
</tbody>
</table>

Table 5.7 Coefficients of stress - strain curves for aluminum
5.4.2.2 Flow Stress for the Contact Resistance Models

In the models of contact resistance shown in equation (4.24) and (4.28), yield stress is employed to compute the real contact area, which in turn determines the constriction resistance.

In the models the flow stress corresponds to that at the contact surface rather than the average flow stress of the whole part. In other words, the flow stress should be the local value at the interface and be calculated using the local strain.

N. Bay and T. Wanheim et al have studied the average deformation of the surface asperities in metal working processes [22]. The average effective strain in the surface layer is a function of the nominal normal pressure and the initial slope of the surface asperities. For the specimen surfaces prepared by machining, the initial slope of the surface asperities is larger than 10°. With this slope the average effective strain of the asperities is about 0.15 [22]. Referring to Fig.5.9 – Fig.5.11, the materials employed in the experiment show different work hardening patterns in different strain ranges separated by the threshold strain values. At strains above the threshold, work hardening is very small. The threshold values for the materials at most of the temperatures are less than 0.1. So the average effective strain at the interface is larger than the threshold strain value, implying that the flow stress can be regarded as constant when the temperature is constant, see Fig.5.9 – Fig.5.11.

Among the three materials employed, the assumption of ideal plasticity at large strains holds good for stainless steel and aluminum, referring to Fig.5.10 and Fig.5.11, while for mild steel it is acceptable as well, see Fig5.9. With this assumption the yield stresses can be obtained without knowing the local strain.

The stresses used in calculation are listed in Table 5.8.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Flow stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stainless steel</td>
</tr>
<tr>
<td>50</td>
<td>710</td>
</tr>
<tr>
<td>200</td>
<td>611</td>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>800</td>
<td>376</td>
</tr>
<tr>
<td>1200</td>
<td>104</td>
</tr>
</tbody>
</table>

Table 5.8 Flow stresses used in calculation of contact resistance

5.4.3 Resistivity of the Materials

The resistivity measured in the experiments is illustrated in Fig.5.12 for the three materials employed in the tests.
5.4.4 Results & Discussion of the Static Method

In a test based on the static method, interface temperature is kept constant while the load changes stepwise and the contact resistance changes accordingly. The variation of contact resistance with load for steel-to-steel contact at 200 ºC is illustrated in Fig.5.13. In this way, the influence of pressure on the contact resistance can be investigated without intervention of temperature. In these tests, the SORPAS® models of contact resistance, as well as the one proposed in the present work are examined experimentally.
5.4.4.1 Experimental Results

For steel-to-steel contact, the experiments are carried out at five different temperatures, referring to Table 5.1. Fig.5.14 shows the measured contact resistance versus the normal pressure and temperature.

![Fig.5.14 Contact resistance of mild steel contacts](image)

It can be seen that the interface normal pressure has great influence on the contact resistance. As a general rule, the contact resistance decreases with increased normal pressure. This applies to all tested temperatures. For instance, at 50 °C, the contact resistance decreases from 300 µΩ to about 75 µΩ when normal pressure increases from 70 MPa to 295 MPa. The rate of decrease in contact resistance with pressure is less steep at high pressures than at low ones.

Pressure has an effect in at least two aspects: enlarging the real contact area and facilitating rupture of the surface film. The former effect helps to decrease the constriction resistance and the latter decreases the film resistance, thus the total contact resistance is lowered if the pressure is increased. At high pressures, when the real contact area approaches the apparent one and the surface film has been ruptured to a large extent, both effects become less influential.

Temperature also plays an important role. The test results show that the contact resistance is highest at 50 °C, and decreases at 100 °C, increasing at 200 °C and then drops consistently after 200 °C. This may be due to easier film rupture at high temperature as a result of smaller flow stress. As mentioned earlier, thick surface film is usually less deformable.

Another observation is that the variation in resistance due to varying temperature is less pronounced as pressure increases. This is probably because at high pressure the interface has been subjected to considerable deformation leading to a large contact area and rupture of the films.
Temperature affects contact resistance in several aspects. The mechanical properties change with temperature and so does the electrical properties such as the electrical resistivity. Under the same load, the real contact area is larger at higher temperature since the material is softer implying smaller constriction resistance. On the other hand, the resistivity increases with temperature for many materials, and this increases the constriction resistance. The surface films are also influenced by temperature. At high temperatures, some surface contaminants like oil and water vapor will be burned off; other relatively thick contaminant layers such as oxides can be ruptured more easily because of softer base metal, leading to smaller contact resistance; at the same time, the oxidation layer may grow at a higher rate at higher temperatures, resulting in higher contact resistance. The overall influence of temperature on the contact resistance is a joint effect among these factors. The increase of resistivity between 100°C and 200°C prevails the decrease of flow stress thus leading to a local peak of contact resistance.

Fig.5.15 illustrates the contact resistance between stainless steel specimens. The contact resistances of stainless steel contacts are much higher than those of the steel-to-steel counterparts under the same conditions, referring to Fig.5.14. This must attribute to the higher electrical resistivity and flow stress of stainless steel. Pressure reveals similar influence as is on steel contacts, and so does temperature. There is a local peak of contact resistance at around 200 °C.

![Fig.5.15 Contact resistance of stainless steel contacts](image)

The contact resistance of aluminum samples is shown in Fig.5.16. Again, similar influence of pressure and temperature are observed. The magnitude of contact resistance between aluminum specimens is smaller than that of steel and stainless steel because of smaller electrical resistivity and flow stress of aluminum. And contact resistances at all temperatures approach the same value with increasing pressure.
5.4.4.2 Verification of the Models

Employing the data measured in the tests, it is possible to evaluate the contact resistance model used in SORPAS® expressed in (4.17) and (4.24), as well as the newly proposed model (4.28)-(4.30).

These models employ the flow stress, the resistivity of the base materials and the contact resistivity. In addition, (4.17) requires the thickness of the contact layer. The yield stresses and resistivity have been determined experimentally as shown in Table 5.8 and Fig.5.12, respectively. The resistivity of surface films used in calculation is shown in Fig.5.17. These are the default values in SORPAS® for contaminant resistivity of materials involved and have been proved to be applicable in some spot welding processes. It is worth noting that this is not a fair comparison because the contaminant resistivity is application dependent and it may vary.
in a wide range. The thickness of contact layer influences only the magnitude of the contact resistance, the value is chosen as 0.05 mm. And the constant $\eta$ in (4.28) – (4.30) is set to 1.0.

Shown in Fig.5.18 is a comparison of the measured contact resistance of stainless steel specimens at 50 ºC with those calculated using different models. The constriction resistance labeled with $Constriction\_new$ is computed by equation (4.28), while the value labeled with $Constriction\_SORPAS^®$ is calculated by equation (4.17) and (4.22), while the total contact resistance is calculated using (4.17) and (4.24).

![Fig.5.18 Contact resistances from test and different models (stainless steel, 50 ºC)](image)

It is clear that the constriction resistance calculated with the SORPAS® model takes up only a very small fraction of the total contact resistance. Under high pressures, the difference becomes smaller but is still very large. The total contact resistance calculated with SORPAS® model is larger than the experimentally measured, but since the constant $\zeta$ in equation (4.24) varies in the range of 0.1 to 10, the experimental results lie in the range which may be covered by the model. The model approximates the experiment pretty well at high pressures while at low pressures the model predicts a steeper decrease of contact resistance than observed in the experiment. Compared with the total resistance, the calculated constriction part is negligibly small.

![Fig.5.19 Contact resistance from test and different models (stainless steel, 50 ºC)](image)
The constriction resistance computed with the new model is shown in comparison with the SORPAS® model in Fig.5.19. The new model predicts much larger values than the old one does, still the constriction resistance is small compared with the experimental results, indicating that the film resistance is quite large in this case.

![Graph showing contact resistance comparison](image)

**Fig.5.20 Contact resistance from test and different models (stainless steel, 100 °C)**

Contact resistance of stainless steel specimens at 100 °C are shown in Fig.5.20. The same conclusions can be drawn as at 50 °C. In the new model the influence of the interface pressure is in better agreement with that observed in the experiment, and the SORPAS® model reveals a faster drop of contact resistance with increasing pressure at low pressures.

![Graph showing contact resistance comparison](image)

**Fig.5.21 Contact resistance from test and different models (stainless steel, 200 °C)**

The experimental as well as the theoretical results for stainless steel at 200 °C are shown in Fig.5.21. Similar results are obtained except that the calculated total contact resistance is smaller than the experimental values. This may originate from a small default contaminant resistivity chosen in SORPAS® for stainless steel-to-stainless steel contact at 200 °C.
Fig. 5.22 Contact resistance from test and different models (stainless steel, 400 °C)

Fig. 5.22 and Fig. 5.23 illustrate the results at 400 °C and 600 °C, respectively. The conclusion is largely the same except for some deviation in magnitude. Again the influence of the interface normal pressure in the new model is in better agreement with the experiment.

Fig. 5.23 Contact resistance from test and different models (stainless steel, 600 °C)

Similar results were obtained in the experiments on steel specimens. The results at 50 °C and 100 °C are shown in Fig. 5.24 and Fig. 5.25, respectively. It is worth noting that because the flow stress of the steel does not rigorously follow a linear hardening-ideal plastic pattern, the choice of the flow stress affects the magnitude of the calculated contact resistances, in addition to some other factors that affect the size of the calculated values.
The experimental results for aluminum at different temperatures are shown in Fig.5.26 – Fig.5.28. In all cases, the experimental curves are steeper than those obtained by the SORPAS® model and the new one. This is perhaps due to the fact that oxidation layers on the surfaces of aluminum specimens are thick and easy to rupture. So the interface normal pressure affects the film resistance greatly. Another fact is that the film resistivity at high temperature is underestimated in SORPAS® compared with experiments.
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Fig. 5.26 Contact resistance from test and different models (aluminum, 50 °C)

Fig. 5.27 Contact resistance from test and different models (aluminum, 100 °C)

Fig. 5.28 Contact resistance from test and different models (aluminum, 200 °C)
To summarize, the results of the experiments comply with the theoretical analysis in Chapter 4. The contact resistance resulting from constriction to the current flow is not adequately taken into account in the SORPAS® model, or equation (4.19). Another phenomenon observed in the comparison is that the relationship between the experimental results and the calculated total contact resistance is not consistent; sometimes the former is larger, while in other cases the latter. This is understandable, since the contaminant resistivity used in calculation may not fit the real situation. Although a coefficient appears in equation (4.24) to adjust the overall values, the dependence of the contaminant resistivity on temperature is critical to the validity of the model.

The contact resistance measured in the tests lie in the coverage of the SORPAS® model, implying that the SORPAS® model is applicable in a wide range, especially for those industrial sheets with broad range of surface contaminants. In effect, the contaminant resistivity in the model is not that of the real film, but is combined with some part of the constriction resistance. The model takes a simple form with only a few parameters for the user to determine according to the case of concern.

The new model, in contrast, predicts a much larger constriction resistance than that of the SORPAS® model. Since none of the specimen surfaces are perfectly film free, the new constriction model is not strictly validated in the experiments. Yet the model shows similar influence of pressure as the experiments on steel and stainless contacts, indicating a more precise representation of the relation between contact resistance and interface normal pressure in the new model. And once the film resistance, corresponding to the discrepancy between the experimental results and the constriction resistance, is determined, a more reliable model can be obtained.

As for the influence of normal pressure at the interface, the SORPAS® model predicts a steeper contact resistance-pressure curve than the new model does. In the tests, the gradient of the experimental curves usually lie in between, except for the aluminum contacts. This could be owing to the fact that the surface film on the aluminum specimens, e.g., oxides, is sensitive to pressure.

This implies that the influence of pressure on the overall contact resistance may vary with different surface films. Assumption of the same influence of pressure on the constriction resistance and the film resistance could be overly simplified. But for steel and stainless steel contacts, assumption of the same influence of interface normal pressure on the constriction resistance and film resistance is acceptable, as shown in the experiments.

5.4.5 Results & Discussions in the Semi-dynamic Method

With the static method presented in the previous section valuable information can be obtained, which was used in the examination of the theoretical contact resistance models. The method is straightforward and is suitable for parametric studies. But it only leads to some discrete results instead of providing the effects of a continuously varying parameter. In this section, more results are presented of the experiments in each of which the load is changed
continuously while the temperature is kept constant. In this *semi-dynamic* method, more data are collected for analysis and the dynamic effect of load can be included, if there is any. Only steel and stainless steel are studied in these tests.

**5.4.5.1 Stainless Steel Contacts**

A series of tests were carried out at different temperatures. The results for stainless steel samples are illustrated in Fig.5.29 and Fig.5.30. As a general trend, the contact resistance decreases when the normal pressure increases. The influence of pressure is more severe at low temperature. At 50 °C, contact resistance decrease rapidly with increase of pressure. At high...
temperatures, the gradients on the curves are smaller. Beyond 1000 °C, the variation of contact resistance is minimal, with the normal pressure varying in a much smaller range.

There is a local roof of contact resistance at 400 °C, as seen in Fig.5.30.

Some curves in Fig.5.29 intersect each other when the difference in contact resistance is limited, for example, the curves for 50 °C and 400 °C. This may be due to different behavior of the surface film at different temperatures. The seemingly identical specimens may have different surface conditions. Different work-hardening rates at different temperatures can have some influence, too.

Comparing the contact resistances measured by the semi-dynamic method with those of the static method, usually the former leads to smaller values, as is shown in Fig.5.31 for stainless steel at 200 °C. Dynamic effects of pressure may play a role, yet the discrepancy is more likely due to surface films. The semi-dynamic tests were performed about 3 weeks earlier than the static tests. During that period, the surface films of the specimens could develop thus resulting in larger contact resistance.

Needless to say, it would be useful if a mathematical function could be extracted from the experimental data on the relationship between contact resistance, pressure and temperature.

By performing a least square fit, the experimental data in Fig.5.30 are regressed using a power function for both temperature and pressure. The function turned out to be,

$$R_c = 5193 T^{-4.118} P^{-5.1169}$$ (5.1)

The interpolated function and the experimental data are shown together in Fig.5.32. The interpolated surface function is drawn as a mesh. The error from interpolation is very large.
Using exponential functions for pressure and temperature, the interpolated function is,

$$ R_c = 1176 e^{-0.001243T} e^{-0.002485P} $$

(5.2)

The original data and the interpolation surface are shown in Fig.5.33. Again (5.2) is not a good approximation to the experimental data.
Equation (5.1) or (5.2) can be applied for rough estimation of contact resistance in a wide range of pressures and temperatures. But the regression error is large, and neither equation can predict the local roof of contact resistance. They are not accurate enough for detailed numerical simulations.

The new model on constriction resistance is further examined with the experimental data. The comparison is shown in Fig.5.34. Notice that the experimental data represent the total contact resistance.
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![Graphs showing contact resistance at different temperatures](image)

**Fig. 5.34** Comparison of contact resistance at different temperatures measured in experiment and calculated constriction resistance using equation (4.28) (stainless steel)

It can be seen that generally the model approximate the experimental results well, especially at higher pressures. At low pressures, the calculated curves are steeper than the experimental ones. This could be due to the assumption that the flow stress is constant. In other words, at low pressure higher yield stress than real is applied in calculation thus larger contact resistance is deduced, consequently pressure appears more influential. Fortunately contact resistance at low pressure is not of interest. In resistance welding, contact resistance acts through Joule heating which takes place only during the welding time. Before welding starts, high normal pressures at the interface have developed. Low pressure appears only during the squeezing time, when no current is passing through, so contact resistance at low pressures does not influence the final result.

Both equation (4.24) and (4.28) describe contact resistance as power functions of pressure, but of different orders. When the experimental data are interpolated with power function in
form of $R_c = y\sigma^z$, the constants, $y$ and $z$, are listed in Table 5.9. The regressions are of reasonable precision.

<table>
<thead>
<tr>
<th>Temperature(°C)</th>
<th>$y$</th>
<th>$z$</th>
<th>$e^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>4289</td>
<td>-0.3987</td>
<td>0.8996</td>
</tr>
<tr>
<td>200</td>
<td>1426</td>
<td>-0.2114</td>
<td>0.8955</td>
</tr>
<tr>
<td>300</td>
<td>1008</td>
<td>-0.1578</td>
<td>0.9429</td>
</tr>
<tr>
<td>400</td>
<td>2378</td>
<td>-0.2499</td>
<td>0.7084</td>
</tr>
<tr>
<td>500</td>
<td>4701</td>
<td>-0.4522</td>
<td>0.8882</td>
</tr>
<tr>
<td>600</td>
<td>2831</td>
<td>-0.4197</td>
<td>0.8756</td>
</tr>
<tr>
<td>800</td>
<td>579</td>
<td>-0.1839</td>
<td>0.7789</td>
</tr>
</tbody>
</table>

Table 5.9 Coefficients at different temperatures and errors, $e^2$ is R-squared value (stainless steel)

The interpolation is of even higher precision than the R-squared values reveal because the error is bigger at low pressure, as shown in Fig.5.35, while the data at low pressure is not of concern in resistance welding.

From Table 5.9, it can be seen that the order of the power function is not identical at different temperatures, but all are greater than -0.5, or the value in equation (4.28). Since the SORPAS® model uses a value of -1, equation (4.29) gives a better approximation in these situations.

5.4.5.2 Mild Steel Contacts

The experimental results from mild steel contacts are shown in Fig.5.36 and Fig.5.37.
The results look very similar to that for stainless steel (referring to Fig.5.29). At low temperatures the resistance-pressure curves are much steeper than at high temperatures. There is a local roof at around 300 °C.
The experimental data in Fig.5.37 are interpolated using power function for both temperature and pressure, and the function turned out to be,

\[ R_c = 36108T^{-1.3362}P^{-3.004} \]  \hspace{1cm} (5.3)

The interpolated function and the experimental data are shown in Fig.5.38. The interpolated surface is drawn in mesh.

When the data is interpolated using exponential function for temperature and pressure, the function is,

\[ R_c = 596e^{-0.002799T}e^{-0.005736P} \]  \hspace{1cm} (5.4)

The function is drawn in Fig.5.39. It is clear that equation (5.4) gives an even poorer approximation than equation(5.3).
Interpolating all the experimental data using a power function, \( R_c = y \sigma^z \), the constants obtained are listed in Table 5.10. Constant \( y \) decreases with temperature, while \( z \) varies around -0.5. In the SORPAS® model, the corresponding value is -1.0, while in the new model, the number is -0.5. This also implies that equation (4.28) models the influence of interface normal pressure better in this situation.

<table>
<thead>
<tr>
<th>Temperature(ºC)</th>
<th>( y )</th>
<th>( z )</th>
<th>( e^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>3349</td>
<td>-0.4624</td>
<td>0.8749</td>
</tr>
<tr>
<td>100</td>
<td>3241</td>
<td>-0.6703</td>
<td>0.8998</td>
</tr>
<tr>
<td>200</td>
<td>2384</td>
<td>-0.6199</td>
<td>0.9355</td>
</tr>
<tr>
<td>300</td>
<td>1058</td>
<td>-0.3858</td>
<td>0.8227</td>
</tr>
<tr>
<td>400</td>
<td>749</td>
<td>-0.4115</td>
<td>0.8197</td>
</tr>
<tr>
<td>600</td>
<td>336</td>
<td>-0.3856</td>
<td>0.8880</td>
</tr>
<tr>
<td>800</td>
<td>272</td>
<td>-0.4375</td>
<td>0.7746</td>
</tr>
</tbody>
</table>

Table 5.10 Coefficients at different temperatures and errors (mild steel)

5.5 Conclusions

The SORPAS® model and the new model proposed in the present work were presented in Chapter 4. These models are examined in a series of tests in this chapter. The influence of
some basic parameters, such as interface normal pressure, temperature and material properties, is also studied.

The experiments show that the calculated constriction resistance with the SORPAS® model is very small, which is in agreement with the earlier analysis in Chapter 4. On the other hand, the measured contact resistances lie in a range which may be covered by the SORPAS® model, implying that the model is applicable if the constants are wisely chosen.

The new model is not directly validated by the experiments because the influence of surface film cannot be eliminated. If the tests are carried out in protective gas, it may be possible to verify the model. However, the experimental results are in favor of the new model in some aspects compared to the SORPAS® model. On one hand, the magnitude of the predicted contact resistances is compatible with those experimentally found; on the other hand, the new model presents a better approximation to the influence of pressure than the SORPAS® model does for the tests performed with stainless steel and mild steel.

The influence of pressure is quite consistent in the experiments: contact resistance decreases when pressure increases, while the influence of temperature is more complex. The contact resistance is not always dropping with increasing temperature as a result of the joint effects of the electrical and mechanical properties of the base materials and those of the films. So a simple function between the contact resistance and temperature, for example, a power function or an exponential function is not quantitatively accurate. It may, however, provide a rough estimation, which may be useful in analysis of the process.

References

19. Søren Rasmussen, Information about Gleeble 1500, Department of Manufacturing Engineering and Management, Technical University of Denmark, Publication Nr. TM 90.16.
20. Gleeble Programming Language, Version 4.5, Dynamic Systems, Inc., P.O. Box 1234, Poestenkill NY, 12140, USA.
22. N. Bay, T. Wanheim and A. S. Pernersen, \( R_a \) and the average effective strain of surface asperities deformed in metal-working processes, p. 77-84, Wear, 34, 1975.
Chapter 6  Experimental Verification with Resistance Welding Tests

Tests with the resistance welding process are carried out in this chapter in order to verify the whole package of the contact model including mechanical, thermal and electrical contact algorithms. The experiments include projection welding of disc-ring specimens of different materials under various conditions of loads and electrical current. The experimental results are compared with the corresponding numerical simulations. Finally several industrial welding applications are simulated to show the ability of the program.

6.1 Introduction

As described in Chapter 2 and Chapter 3, a contact model has been developed and implemented into the finite element program to deal with the coupled mechanical, electrical and thermal phenomena in resistance welding. The mechanical algorithm has been validated by some numerical tests as well as experiments.

The electrical and the thermal contact models are based on the same theory, thus validation of the mechanical contact algorithm can also be regarded as an indirect proof of the thermal and electrical algorithms. However this verification concerns the discrete algorithms only. The contact model is developed for resistance welding, which involves coupling effects of electrical, thermal and mechanical phenomena. Real resistance welding involves much more complex contact conditions than what appeared in the verification experiments, which were confined to mechanical deformation. The deformation is more severe under a situation with drastic change in temperature; more over, thermal as well as electrical contact appears which need to be properly dealt with. So a direct verification in resistance welding is necessary to obtain direct confidence in using the contact model.

In this chapter, some experiments on real resistance welding are performed to evaluate the performance of the contact algorithms including all the factors appearing in real resistance welding processes.
6.2 Welding Experiments of Disc-ring Combinations

Projection welding is suitable for validation of the contact algorithm. On one hand, large deformation occurs in many projection welding processes, leading to significant change of the shape and size of the interface; on the other hand, it is of practical importance in industry.

6.2.1 Specimens

The geometry combination for the welding experiment is the so-called disc-ring, the same as the specimens used in previous experiments shown in Fig.3.14. The combination includes a ring in form of a flat plate with a central hole, and a disc with a triangular-shaped ring projection machined at the periphery on one side. Two different thicknesses of the disc, 2 mm and 3 mm, and two different angles of the projection, namely 75° and 90°, are employed in the experiments. This kind of solid projections are widely seen in industry because, on one hand, it is stable and mechanically strong, implying few follow-up problems; on the other hand, ring projections make it possible to weld leak-tight joints. In the welding process, mechanical contact as well as thermal and electrical contact take place, thus the contact algorithms can be examined in this experiment.

In choosing specimen materials, not only mechanical properties but also electrical and thermal properties are considered. Two materials are used for the disc, namely mild steel (W.nr.1.0037) and stainless steel (W.nr.1.4301). The materials used for the ring are mild steel (W.nr.1.0338) and stainless steel (W.nr.1.4301). These materials are commonly applied in resistance welding, and they vary in mechanical, thermal and electrical properties over a quite wide range.

6.2.2 Equipment and Tools

An inverter machine from Expert provided with a Harms & Wende control unit is used in the
welding experiments, see Fig.6.1. Specifications of the welding machine can be found in [1].

The electrodes applied for mounting the disc and ring are the same as used in [2]. The upper electrode has a central hole to align the disc centrally; while the lower electrode has a recess to locate the ring centrally, see Fig.6.2.

![Fig.6.2 Experimental set up for resistance welding [2]](image)

### 6.2.3 Schedules of the Experiments

For a resistance welding process, the basic parameters include the electrical current, weld time, and the load. The parameters used in the experiments are as follows:

- **Load:** 2.8 kN and 5 kN,
- **Weld time:** 15 ms,
- **Squeezing time:** 2 s,
- **Holding time:** 0 s.

The welding current for each combination is determined via preliminary tests.

In the tests, welding is made for various disc-ring combinations of the same materials as well as dissimilar materials, as shown in Table 6.1.
6.2.4 Procedures of the Experiments

The procedure of the experiment is as follows:

- **Welding of disc-ring**
  For each combination listed in Table 6.1, a proper welding current is found by preliminary tests. The current should be chosen near the splash limit to ensure good welds. For each combination, two welds are made and all specimens are numbered for analysis.

- **Measuring the profile**
  All the specimens are molded in epoxy, sectional near the median plane and then polished to the center to obtain a macrograph.

6.3 Simulation Conditions

6.3.1 Material Properties

Simulation of the welding process requires comprehensive material data including mechanical, electrical as well as thermal properties such as thermal conductivity, heat capacity, electrical resistivity, thermal expansion coefficient, stress–strain curve, solidus, liquidus, etc. The material data used in the simulations, many of which were experimentally determined in [2], are listed in Appendix.

6.3.2 Numerical Settings in Simulation

Numerical settings in simulations are as follows:

- Time step increment: 0.1 ms,
- Convergence accuracy for the mechanical model: $10^{-4}$,
- Convergence accuracy for the thermal model: $10^{-4}$,
• Convergence accuracy for the electrical model: $10^{-4}$,
• Friction between specimens and tools: constant friction model, with a friction factor of 0.8.

The total number of elements for the setup, including electrodes, interfaces and workpieces is 400 for all the cases except for series v, vi, vii and viii, in which 500 elements are used in simulation. The initial mesh for the former series is shown in Fig.6.3. Notice that the electrode shapes are simplified, which will not influence the final results too much.

![Initial mesh with 400 elements for the welding setup](image)

Only the sticking contact model is employed to simulate the welding processes.

### 6.3.2 Model of Contact Resistance

It was experimentally shown in Chapter 5 that the new model of contact resistance, shown in equation (4.28), approximate the influence of normal pressure better. For the first step of improvement, the old model (4.24) is accordingly modified as (6.1) and used in the simulation.

$$
\rho_c = 3\xi \left( \frac{\sigma_{s\_soft}}{\sqrt{\sigma_{nc}}} \right) \left( \frac{\rho_1 + \rho_2}{2} + \rho_{conta\_min\_ant} \right)
$$

(6.1)

### 6.4 Results and Discussion

Simulations are made using SORPAS® for each combination listed in Table 6.1. The results are compared with the experiments to investigate the validity of the contact models. The
shape and size of the nugget as well as the profile of the weldment are the main items for comparison.

6.4.1 Final Electrical Currents Employed

In the preliminary tests, the electrical current for each experiment shown in Table 6.1 is determined as listed in Table 6.2.

<table>
<thead>
<tr>
<th>Series no.</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
<th>v</th>
<th>vi</th>
<th>vii</th>
<th>viii</th>
<th>ix</th>
<th>x</th>
<th>xi</th>
<th>xii</th>
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<td>5</td>
<td>2.8</td>
<td>5</td>
<td>2.8</td>
<td>5</td>
<td>2.8</td>
<td>5</td>
<td>2.8</td>
<td>5</td>
<td>2.8</td>
<td>5</td>
</tr>
<tr>
<td>Current (kA)</td>
<td>23</td>
<td>25</td>
<td>23</td>
<td>25</td>
<td>19</td>
<td>20</td>
<td>18</td>
<td>19</td>
<td>22</td>
<td>23</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 6.2 Current applied in the experiments

The current was selected near the splash limit of the corresponding experiment. It is noticed that for the same material combination, the current for expulsion increases slightly with load. For instance, in welding a steel disc to a steel ring (series i and ii), a DC current of 25 kA can result in a good weld under 5 kN while under 2.8 kN splash was observed with the same current. Higher load has decreased the contact resistance thus requiring higher current to make the same weld.

6.4.2 Welding Steel Disc to Steel Ring (Series i, ii)

In Fig.6.4, the contour of the weldment from the experiment is compared with the simulated one, showing the calculated temperature distribution after a weld time of 15 ms, a DC current 23 kA and a load of 2.8 KN (series i). Both the disc and ring are of mild steel.

![Fig.6.4 Macrograph from the experiment and temperature distribution from simulation after welding for series i](image)

It is seen that in the experiment, the deformation of the disc-ring combination is confined to the projection. The projection tip flattens and forms the weld. The weldment profile and temperature distribution from simulation are in good accordance with the macrograph.
A magnified picture of the projection, see Fig.6.5, reveals clearly the agreement between the experiment and simulation. Deformation is concentrated in the projection tip, whereby the nugget is formed. The nugget is located on the side of the disc, i.e. more material melts in the disc than in the ring, which is observed from both experiment and simulation. The dark dashed curves in Fig.6.5 b) are the contours of the heat affected zone. Another observation from Fig.6.5 is that there is obvious difference in the outer profile (right hand side in the figure) of the disc between the experiment and simulation. This is probably due to the error in the input geometry for simulation. The contour of the projection is curved instead of a straight line used in the simulation. When the figure is magnified the difference is obvious.

In Fig.6.5 b), the appearance of some melted material indicated by purple color is seen. It is worth noting that the melted material shown here does not always correspond to the nugget. During welding, the interface temperature undertakes drastic changes and is not necessarily increasing even when a DC current is applied because of significant expansion of contact area. An enlarged contact area leads to smaller contact resistance, and as a consequence, less heat will be generated and the temperature may drop in some period, thus melted material may solidify during the process.
Illustrated in Fig. 6.6 is a comparison between the experimental result and numerical solution for series ii. The load applied in this case is 5 kN and the welding current is 25 kA. All other conditions are the same as in series i, see Fig. 6.5. Similarly, the deformation and heat generation is confined to the projection tip. The simulation is in good accordance with the experiment.

6.4.3 Steel Disc — Stainless Steel Ring (Series iii, iv)

Shown in Fig. 6.7 and Fig. 6.8 are the results for welding steel disc to stainless steel ring employing different loads and currents, corresponding to series iii and iv. Again, deformation is located in the disc projection. Because stainless steel has a larger electrical resistivity than mild steel, melted material is located not only in the disc but also in the ring after termination of welding current. The contour shape of the projection from simulation agrees well with that in experiment.

The parameters are identical in series iii and series i, the only difference is the disc material. Comparing the temperature distributions in Fig. 6.5 and Fig. 6.7, a larger volume of melted material is seen in the latter. This is because series iii concerns a disc of stainless steel with larger electrical resistivity than mild steel in series i.
6.4.4 Stainless Steel Disc — Steel Ring (Series v, vi)

The experimental results and the numerical solution for the combination of stainless disc and steel ring are shown in Fig.6.9 and Fig.6.10, corresponding to series v and vi, respectively. The projection angle is 75°. In this combination, the ring has a smaller electrical resistivity than the disc. Consequently more material is melted in the disc after welding, as seen from the temperature distribution in Fig.6.9. The final shape of the weld shows good agreement between experiment and simulation.

Fig.6.9 Contour of the disc-ring weldment in the experiment and temperature distribution after welding for series v

Fig.6.10 Contour of the disc-ring weldment in the experiment and temperature distribution after welding for series vi
6.4.5 Stainless Steel Disc — Stainless Steel Ring (Series vii, viii)

The weld from experiment and the simulated temperature distribution illustrated in Fig.6.11 are for series vii. Both the disc and ring are of stainless steel, and the angle of the disc projection is 75°.

In the experiment, the projection tip is melted and material is squeezed out from the interface, indicating that expulsion has happened. The computed temperature distribution reveals similar conclusion. No melting material is observed after welding in the figure. This is because melting of a large volume of material results in marked enlargement of contact area which in turn decreases heat generation. When the current is terminated, the interface temperature has decreased so much that all melted material has solidified. The simulation result is in good agreement with the experiment.
Fig. 6.12 shows the results for series viii. It can be seen that expulsion has taken place in the experiment, a fact that is also observed from the contour of the projection in the simulation.

6.4.6 Stainless Steel Disc — Steel Ring (Series ix, x)

The experimental results as well as the numerical simulation illustrated in Fig. 6.13 and Fig. 6.14 show the combination of stainless steel disc and steel ring, with an angle of the disc projection of 90°, and a welding current of 22 kA. In comparison, the welding current is 19 kA for series v where the projection angle is 75°. The difference observed originated from stronger constriction to current flow or larger current density in the projection for series v.

In both series most part of the nuggets locates in the disc because of current concentration in the disc in addition to smaller resistivity of the ring material. In both cases the match between simulation and experiment is good.
6.4.7 Stainless Steel Disc — Stainless Steel Ring (Series xi, xii)

In Fig. 6.15 comparison between experiment and simulation is illustrated for the combination wherein both the disc and the ring are of stainless steel and the projection angle is 90°, corresponding to series xi. Compared with series vii, everything is the same except for the projection angle. From Fig. 6.11, it is seen that expulsion occurs while in Fig. 6.15, expulsion is observed neither in the experiment nor in the simulation. This is because less heat is concentrated in the projection tip in series xi as a result of a larger projection angle.

The contour of the disc determined in the simulation is in good agreement with that observed in the experiment.

Fig. 6.15 Contour of the disc-ring weldment in the experiment and temperature distribution after welding for series xi

Fig. 6.16 depicts a comparison for series xii, wherein load is 5 kN. The agreement between simulation and experiment is again satisfactory.

Fig. 6.16 Contour of the disc-ring weldment in the experiment and temperature distribution after welding for series xii
6.4.8 Summary

Projection welding of disc-ring combinations of various materials and geometries are carried out. The process entails a mechanical contact algorithm as well as a thermal contact model and an electrical contact model. With these projection welding experiments, the contact models are examined. In all the cases, the simulation result matches the experiment with good accuracy. Perfect match between experiment and simulation is not easy to obtain due to difficulty to make fair comparison. For example, the mechanical, electrical and thermal properties of the materials are often temperature dependent. The values employed for simulation may not be identical to the real data. Taken these factors into account, the experiments have verified the contact algorithms developed in this work to be applicable.

6.5 Solving Resistance Welding Problems Using SORPAS®

After being validated experimentally, the contact model is applied in this section to analyze some industrial welding processes involving contact problems.

6.5.1 Cross Wire Welding

As an example of a natural projection welding, cross wire welding finds a huge volume of applications in the manufacture of wire fence, grills, gratings, baskets, guards, racks, reinforcements, shelves and the like. In practice, it usually consists of welding a number of parallel wires at right angles to one or more wires or rods, as illustrated in Fig.6.17.

In many crosswire welding applications, appearance of the weld are of primary consideration instead of weld strength, thus optimization of process parameters are of practical importance, wherein the numerical simulation may help.
Fig. 6.18 depicts the simulated temperature distribution of a cross wire welding application after welding, the result was obtained based on assumption of plane strain. The wires are of mild steel (W.Nr.1.0338) and 1 mm in diameter, the parameters for welding are as follows:

- Squeezing time: 60 ms,
- Weld time: 40 ms,
- Current: 1 kA, DC,
- Load: 0.3 kN.

The temperature distribution in the weld is shown in Fig. 6.18. In simulation of cross wire welding, reliable modeling of contact is critical. The expansion of the contact area can be observed in the figure.

6.5.2 Welding of Series of Projections

It is commonly seen that multiple projections are welded simultaneously. In this situation, the variation in height between projections should not be too big. Excessive variation will cause expulsion or inadequate weld in some of the joints, implying the formation of unequal weld diameters.

Consider a welding process illustrated in Fig. 6.19, where a plane plate is to be joint to two parts (Part 1 and Part 2) with embossed projections. Due to errors in manufacture or setup, there is an error of \( dh \) in the height of the projections. This error may cause problem to the weldment.

Both plates are of stainless steel (W.Nr.1.4301) and 2 mm thick, and the process parameters are as follows:
- Squeezing time: 20 ms,
- Weld time: 20 ms,
- Current: AC current of 15 kA, heat setting 15%, corresponding to 1.443 kA in RMS,
- Load: 1.5 kN.

The process shown in Fig.6.19 is simulated with SORPAS® for different variation between projection heights employing the sticking contact model and assuming plane strain.

![Fig.6.19 Welding a plate two projections simultaneously](image)

Fig.6.19 Welding a plate two projections simultaneously

Fig.6.20 Temperature distribution in the setup at 39.5 ms, the projections are of the same height, simulation is made with a time step of 0.05 ms and 800 elements

![Fig.6.20 Temperature distribution](image)

Fig.6.20 Temperature distribution after welding

Fig.6.20 depicts the temperature distribution at 39.5 ms, when the maximum volume of melted material is obtained. The two projections were perfectly prepared and setup, with no
height variation between them. From the figure it can be seen that satisfactory welds are achieved in both projections.

Fig.6.21 shows the temperature distribution at 39.5 ms when the offset in projection height is 0.1 mm. It can be seen that a weld is obtained in both projections, but the nugget size in the left projection, which is originally lower, is a little larger than in the right one. This is because the contact area in the left projection is smaller and the contact resistance is larger. In the end more heat is produced in the left projection.

Fig.6.22 shows the temperature distribution at 39.5 ms when the offset in projection height is 0.3 mm. It can be seen that a weld is obtained in both projections, but the nugget size in the left projection, which is originally lower, is a little larger than in the right one. This is because the contact area in the left projection is smaller and the contact resistance is larger. In the end more heat is produced in the left projection.
When the offset in projection height is 0.3 mm, no weld, i.e. no melting, is obtained in the projections, as is observed from the temperature distribution shown in Fig. 6.22. In this case, the higher projection collapse to a large extent; hence the contact area in the right projection is large enough to let most of the current flow through. The contact resistance in the right projection is so small that no adequate heat is generated to form a nugget even though a higher current flow through it. In the left projection, though the contact resistivity is high, no weld is formed because of less current flow.

The current distribution at 29.5 ms is shown in Fig. 6.23 where the original height offset is 0.6 mm. In this case, only the right projection is in contact with the plate, and so much heat is produced that expulsion has taken place.

From the previous simulations, it is clear that the precision in geometry in case of multi-point projection welding is of critical importance to the weld quality. If a number of projections are to be welded simultaneously, the height variation of the projections must be confined within a small range.

**6.5.3 Influence of Shearing Burrs**

In preparing the weldment, it is imperative that shearing burrs be eliminated. Otherwise the welding process may run into problems.

Shown in Fig. 6.24 is an application of welding a plane plate to a plate with an embossed projection, thickness of the projection is 2 mm. In Fig. 6.24 a), the plates are well prepared without any shear burrs, while in Fig. 24 b) a shearing burr is not eliminated from the part with projection, which may influence the weld quality significantly.
a) No shearing burrs on the plates

b) A shearing burr remains on one side of the plate with projection

Fig.6.24 Projection welding between a plane plate and a plate with embossed projection

The welding parameters are:

- Squeezing time: 20 ms,
- Weld time: 20 ms,
- Welding current: AC current of 13 kA, with heat setting of 15% (corresponding to 1.340 kA in RMS),
- Load: 1.5 kN

The two cases are simulated with SORPAS® and the temperature distributions are shown in Fig.6.25 and Fig.6.26, respectively. Both figures correspond to 29 ms when the volume of melted material is the largest.

In Fig.6.25, the two plates are joined at the projection. The nugget is confined to the periphery of the projection. This is probably due to the variation of contact resistance along the interface. In the periphery of the projection, contact resistance is relatively larger than that in the middle of the projection as a result of smaller normal pressure, thus more heat is generated in the periphery.
The influence of the shearing burrs is obvious. In Fig.6.26, an expulsion has taken place at the shearing burr, and the temperature distribution in the projection is not symmetric, showing more material melted on the side of the burr. The shearing burr forms a shunting path when it is brought into contact with the upper plate. The shunting path affects not only current flow but also pressure distribution, leading to less current through the projection and lower pressure at the projection. Decrease of normal pressure results in higher contact resistance,
which in turn results in more heat generation; while less current means less heat generation near the projection. The shearing burr is not so strong as to undertake much load, but can significantly affect the current. As a result, in the projection more current flows through the side near the shearing burr, thereby leading to an asymmetric nugget.

As a general rule, shearing burrs must be eliminated ensuring consistent welds.

6.6 Conclusions

In this chapter, the contact models, including the coupled mechanical, thermal and electrical models, are further validated in selected resistance projection welding operations. Projection welding experiments of various combinations of materials and geometries are carried out; the results are compared with the corresponding simulations. In general, satisfactory agreement between experiment and simulation is obtained, indicating the validity of the contact models developed in this work.

The program is then applied to analyze a few typical industrial resistance welding involving contact problems to show the ability of the contact models. Three processes namely cross wire welding, series projection welding and projection welding with a shearing burr on the workpiece, are simulated and analyzed, showing that numerical simulation can facilitate a better understanding of the resistance welding process.

References

1. Operating instructions, Medium – frequency welding control unit HWS2000 PROFILE (IQ), Harms & Wende GmbH & CO. KG, Grossmoorkehre 9, D-21079 Hamburg, Germany.
Conclusions

Contact problems are extremely important in the simulation of resistance welding processes. This involves not only the numerical schemes for the mechanical, electrical and thermal contact phenomena arising during resistance welding, but also material properties of contacting interfaces which have not been well documented.

As a part of the efforts towards a professional and reliable numerical tool for resistance welding engineers, this Ph.D. project is dedicated to refining modeling of the contact problems in resistance welding.

The first part of the thesis is on the FE modeling of the contact problems in resistance welding, in which a contact algorithm is developed and validated, dealing with the coupled mechanical-electrical-thermal contact problem. The penalty method is used to impose the contact conditions for the electrical and thermal contact, as well as for the frictionless contact and sticking contact in the mechanical model. The frictional sliding contact is also solved employing a constant friction law. The thermal contact model is strongly coupled with the electrical one, while the mechanical contact model is coupled stepwise with the electrical and thermal models. A node-segment contact element is the basis for the formulation, and the interfaces are treated in a symmetric pattern.

The algorithms are incorporated into the finite element code. Verification is carried out using both the numerical method and experiments. The frictionless and sticking contact algorithms are tested numerically by analyzing the upsetting processes. The model for frictional sliding contact is schematically shown applicable in an upsetting test of different materials.

Experiments are then carried out to verify the contact model. The first test is on joint upsetting of cylindrical parts of dissimilar materials. Because of marked interface sliding, the deformation pattern depends heavily on interface conditions of the parts. The frictionless, sticking as well as the frictional sliding contact model are examined comparing with the experiments. It is noticed that the assumption of sticking contact leads to less deformation of the softer material and more deformation of the harder material than experimentally observed; while the frictionless contact model results is in the opposite, indicating that the algorithm works satisfactorily. In all the tests, the actual deformation of the parts is between the prediction of the frictionless contact model and the sticking one; and the frictional sliding contact model gives the best simulation results.
The second test is on compression of a disc provided with ring projection of triangular cross section towards a flat ring. The discs as well as the rings were of different materials and sizes. In this experiment surface conditions do not play such a critical role as in the first test. In most cases, the simulation results based on the contact model are in good agreement with the experiments. In a few tests, good conformity is not reached because of work hardening of the disc. The validity of the mechanical contact model is demonstrated in these experiments.

The second part of the thesis is on electrical contact resistance, aiming at a more reliable model for numerical simulation of resistance welding.

The general theory on contact resistance is reviewed. The contact resistance can be categorized into two parts, namely the constriction resistance and the resistance originating from surface films. The former has been extensively studied and can be calculated with some models; while the latter is more complex and not well understood. In resistance welding the contact resistance is even more complex due to factors like strong dynamics of the process.

The model currently employed in SORPAS® is evaluated. It was found that the model may underestimate the real constriction resistance because it is based on the assumption of continual contact area rather than discrete contact areas. A new model is proposed for the constriction resistance in resistance welding based on discrete distribution of the real contact areas, taking into account the constriction resistance.

A series of tests are performed on the contact resistance. The tests are carried out with a Gleeble machine which heats the specimens in a similar way as a resistance welding machine. A parametric study is carried out by using a static as well as a semi-dynamic method. The influence of basic parameters in resistance welding, such as the interface normal pressure, temperature and material properties of the base metal, are studied. The SORPAS® model and the new model in the present work for contact resistance are also examined comparing with the experiments.

It was found that the influence of the interface normal pressure is quite consistent in the experiments: contact resistance decreases when pressure increases, while the influence of temperature is more complicated. The contact resistance is not always dropping with increasing temperature as a result of the joint effects of the electrical and mechanical properties of the base materials and those of the films. So a simple function between the contact resistance and temperature, for example, a power function or an exponential function is not quantitatively accurate. It may, however, provide a rough estimation, which may be useful in analysis of the process.

The influence of interface normal pressure is dependent on the base metal. For instance, the contact resistivity of aluminum contacts decrease much faster than those of stainless steel and mild steel contacts. This could be due to different properties of surface films which rely on the base metal.

It was found that the calculated constriction resistance with the SORPAS® model is very small compared with the experimental results, which is in agreement with the theoretical
analysis. On the other hand, the measured contact resistances lie within the range covered by the SORPAS model, implying that the model is applicable if the constants are wisely chosen.

The new model is not directly validated by the experiments because the influence of surface film cannot be eliminated. However, the experimental results are in favor of the new model in some aspects compared to the SORPAS® model. The magnitude of the predicted contact resistances is compatible with those experimentally found; furthermore, the new model presents a better approximation to the influence of pressure than the SORPAS® model does in the tests performed with stainless steel and mild steel.

The work on contact modeling and electrical contact resistance are validated with resistance welding experiments in the last part of the thesis. Projection welding experiments with disc-ring specimens of various material and geometry combinations are carried out. In most cases, numerical results based on the contact model are in satisfactory agreement with the experiments, indicating the validity of the contact models developed in this work.

Finally the program is applied to analyze some typical industrial resistance welding operations involving contact problems to show the ability of the contact models. Three applications namely cross wire welding, series projection welding and projection welding with a shearing burr on the workpiece, are simulated and analyzed, showing that numerical simulation can facilitate a better understanding to the resistance welding process.
Future Work

Based on the work accomplished in the present Ph.D. project, the following topics are proposed to be studied in future:

On contact modeling:

- Implementation of other friction models. In the current work only the constant friction model has been implemented and tested, while Coulomb’s friction is only provided with the formulation. Since friction plays an important role in some processes like cross wire welding, it is beneficial to find the most suitable friction model for the contact algorithm.

- Numerical schemes for better efficiency of the contact algorithm. For frictional sliding model involving stick and slip, the calculation speed is not satisfactory. It is desirable to seek better numerical schemes to improve efficiency of the algorithm.

On the electrical contact resistance:

- Further experimental investigations of the constriction resistance. One possible solution is to perform the experiments in protective gas using specimens with clean surface. Experiments have shown that the influence of the process variables on the constriction and film resistance are different for some material. So a reliable contact resistance model should describe different influence of the process parameters between the components of contact resistance.

- Experimental investigation of the film resistance. The influence of surface films of different contamination and oxidation should be studied, aiming at an applicable mathematical model covering a wide range of base materials and surface conditions.

- Investigation on the influence of process parameters on the contact resistance in spot welding and in projection welding. The influence of the process variables on the contact resistance is different in the spot welding and the projection welding. Knowledge of this difference is of practical value for numerical simulation.
Appendix: Material Properties

Table A.1 – A.6 list some material data employed in the simulation.

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Table A.1 Thermal conductivity of the materials (W/m·K)
## Appendix: Material Properties

### Table A.2 Heat capacity of the materials (J/Kg·K)

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Table A.3 Resistivity of the material (µΩ·m)
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</tr>
<tr>
<td>800</td>
<td>12.1</td>
<td>14.7</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>13.8</td>
<td>13.8</td>
<td>18.4</td>
</tr>
</tbody>
</table>

Table A.4 Thermal expansion coefficients of the materials (10⁻⁶/ºC)
<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>W.Nr.1.0037</th>
<th>W.Nr.1.0338</th>
<th>W.Nr.1.4301</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c</td>
<td>n</td>
<td>m</td>
</tr>
<tr>
<td>20</td>
<td>354.3</td>
<td>0.13</td>
<td>0.22</td>
</tr>
<tr>
<td>100</td>
<td>395.4</td>
<td>0.18</td>
<td>0.16</td>
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<tr>
<td>200</td>
<td>407.5</td>
<td>0.19</td>
<td>0.13</td>
</tr>
<tr>
<td>300</td>
<td>492.7</td>
<td>0.14</td>
<td>0.10</td>
</tr>
<tr>
<td>400</td>
<td>483.0</td>
<td>0.16</td>
<td>0.11</td>
</tr>
<tr>
<td>500</td>
<td>354.2</td>
<td>0.11</td>
<td>0.16</td>
</tr>
<tr>
<td>600</td>
<td>252.1</td>
<td>0.06</td>
<td>0.16</td>
</tr>
<tr>
<td>700</td>
<td>148.4</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>800</td>
<td>135.6</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>900</td>
<td>105.6</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>1000</td>
<td>64.1</td>
<td>0.19</td>
<td>0.19</td>
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<tr>
<td>1100</td>
<td>45.6</td>
<td>0.17</td>
<td>0.17</td>
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<tr>
<td>1200</td>
<td>51.4</td>
<td>0.18</td>
<td>0.18</td>
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<tr>
<td>1300</td>
<td>39.3</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>1400</td>
<td>10.0</td>
<td>0.30</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table A.5 Constants for the flow stresses (MPa) in form of \( \sigma_f = c \varepsilon^n \dot{\varepsilon}^m \),

with \( \varepsilon \) and \( \dot{\varepsilon} \) as the strain and strain rate, respectively.
### Appendix: Material Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>W.Nr.1.0037</th>
<th>W.Nr.1.0338</th>
<th>W.Nr.1.4301</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solidus (°C)</td>
<td>1500</td>
<td>1560</td>
<td>1430</td>
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<tr>
<td>Liquidus (°C)</td>
<td>1510</td>
<td>1580</td>
<td>1440</td>
</tr>
<tr>
<td>Latent heat (kJ/kg)</td>
<td>277</td>
<td>277</td>
<td>114</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>200</td>
<td>200</td>
<td>193</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
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<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Mass density (kg/m³)</td>
<td>7900</td>
<td>7871</td>
<td>8020</td>
</tr>
</tbody>
</table>

Table A.6 Other material properties