Modelling of Sand Dunes in Steady and Tidal Flow

Niemann, Sanne Lina; Fredsøe, Jørgen

Publication date: 2004

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Ph.D. thesis:
Modelling of sand dunes in steady and tidal flow

Sanne L. Niemann
MEK-DTU, Coastal and River Engineering Section.
Technical University of Copenhagen, Denmark

October 24, 2003
Preface

The present thesis is submitted as one of the requirements for the Ph.D. degree according to the notice no. 1177 of December 1999, and of notice no. 483 of May 2000, from the Danish Ministry of Education.

The Ph.D. study was carried out at The Department of Mechanical Engineering at the Technical University of Denmark (MEK-DTU) within the Coastal and River Engineering Group under the supervision of Professor Dr. Techn. Jørgen Fredsøe.

I wish to thank my advisor for guidance throughout the study. I acknowledge his ability of always focusing on the important aspects - I hope, I have learned a bit from that to bring with me on. I also wish to thank all my colleagues in the Coastal and River Engineering Group for creating an inspiring and pleasant working environment. Finally my family and friends are thanked for their help and encouragement - especially during the last few months.

Acknowledgement: The study was partly supported by the European Commission under the project ”Human Interaction with Large Scale Coastal Morphological Evolution” (HUMOR), contract no. EVK3-CT-2000-00037, and by the Danish Technical Research Council (STVF) by the programm ”Coasts and Tidal Inlets”, contract no. 26-00-0144.

Kongens Lyngby, October, 2003
Sanne L. Niemann
Abstract

Sand dunes are caused by a complicated feedback between the water couloum, the sediment transport and the erodible bed. The mechanisms of two-dimensional steady as well as unsteady dunes are the topic of the present Ph.D.-thesis.

A numerical model previously developed at the Department of Hydrodynamics and Water Ressources, DTU, is used to calculate steady and unsteady dunes of a fixed length from an initially perturbed bed. The model consists of a two-dimensional (vertical and horizontal direction resolved) fully-nonlinear flow model solving the Navier-Stokes equations with a k-ω turbulence model closure, and is coupled with a sediment transport model and a morphological model.

The morphology of dunes in the case of only bed load transport are studied in five situations: Steady and unsteady dunes in a unidirectional flow, dunes exposed to a reversal of the flow direction, and dunes in a pure tidal flow as well as in a superimposed tide and current flow.

Steady dunes in equilibrium with the flow are calculated for a variation of the dune length between 3 and 8 water depths. For each choice of the dune length, the dune height is found to follow the same trend for a variation in the Shields’ parameter as the semi-analytical expression for the dune height by Fredsøe (1982). Furthermore, the dune heights predicted by Fredsøe are approached for increasing dune lengths. The smaller dune height for the shorter dunes can be explained by the inequilibrium in the flow at the top of the dunes. The mechanisms explaining the stability of the dunes for a given height and shape are the same as already identified by Fredsøe (1982), Tjerry (1995) and Tjerry and Fredsøe (to be publ.). Stability is obtained when equilibrium between the following is obtained: phase lag due to turbulence relaxation mechanisms, phase lag due to curvature acceleration of the flow (over curved bedform), gravity effects on sediment transport on a sloping bed, flow acceleration due to contraction and expansion of the flow area.

Preliminary results for the flow resistance for the computed steady dunes are presented. The calculations show that the prediction of the crest height above the trough level is important for the form drag. Within the uncertainty in the prediction of the crest height, the results for the flow resistance compare reasonably to the flow resistance model by Engelund (1966, 1967). The form drag is furthermore calculated from the Carnot-formula. Including a variation in the Carnot-coefficient found by Engelund (1977), a good agreement with the numerically predicted drag is found.

The time scale for a change in the geometrical properties after an abrupt change in the flow is calculated for unidirectional unsteady dunes. The change in the top level is found to compare well with the analytical findings by Fredsøe (1979). Changes in the trough level are found to be of the same order as the changes in the top level. The numerically predicted rate of change of the trough level is verified by estimating the changes in the trough from the calculated sediment transport field immediately after the change in the flow field in combination with physical considerations. The morphology during a simplified flood wave is calculated. The same main
Abstract

processes as found to govern the morphology after an abrupt change is identified in the bedform development during the flood wave. A hysteresis effect of the dune height also observed in nature caused by the lagged response of the morphology to the flow is seen.

By the use of the steady dunes mentioned above for a dune length of four water depths, the mechanisms and changes in the morphology after a reversal of the flow direction are studied. The morphological behaviour are found to compare well with observations of the bedform shape found during a tidal flow as described in the literature. The time scale for a full reversal of the dunes is found.

Tidal sand dunes in equilibrium with the periodically varying flow are calculated for a constant dune length of four water depths. The tidal flow is modelled by using a stepfunction approach, where only the flow direction changes every half tidal period. Only active tidal dunes are modelled - i.e. bedforms which change considerably during the tidal period. The dune height relative to the water depth, $H/D$, is found to increase with a decreasing ratio between the volume of sediment transported within each tidal cycle to the volume of the bedforms. Furthermore, the results indicates a similar variation of the dune height with the Shields’ parameter as in a steady flow. Further calculations are necessary to draw any conclusions.

For tidal flow it is demonstrated, that it is necessary to include the time variation of the bedform during the tidal cycle in order to obtain the correct equilibrium bedform. This phase-resolved feedback in the physical system was found to have a dampening effect on the bedforms.

Bedforms in a flow situation of a coexisting tide and current situation is exemplified in two morphological calculations. In the case of a strong current superimposed by a weak tide, the bedform morphology has strong similarities with the unsteady unidirectional dunes. The dunes in the situation of a strong tide and weak superimposed current develop during the tidal period similar to dunes in a pure tidal flow. However, even if one of the components (being either the tidal flow or the current) is weak, it is demonstrated that it is of high importance to include this component in the flow field for the evaluation of the migration rate.
Dansk resumé


Modellen består af en to-dimensionel, (vertikal og horisontal opløsning) fuldt ikke- lineær strømningsmodel, koblet til en sediment transport model og en morfologisk model. Strømningsmodellen løser Navier-Stokes ligheder med en $k-\omega$ turbulens model.

Den morfologiske udvikling af sandbanker, under tilstedevarsel af udelukkende bundtransport, undersøges i fem situationer: stationære og instationære sandbanker i ensrettet strømning, sandbanker i en situation hvor strømretningen vender, sandbanker i en tidevandsstrømning og sandbankeri kombineret tidevand og strøm.


Tidsskalaen for ændringen af de geometrisk beskrivende størrelser for instationære sandbanker beregnes for en pludselig ændring af stromstyrken i en ensrettet støm. Ændringen af toppens niveau findes at være i overensstemmelse med de analytiske resultater af Fredsøe (1979). Ændringer i trugets niveau findes til at være af samme størrelseorden som ændringerne i toppens niveau. De numerisk forudsagte ændringshastigheder af trug niveauet, verificeres ved en estimering baseret på sedimenttransport feltet umiddelbart efter ændringen af stromstyrken, kombineret med
fysiske betragtninger. Morfologien i løbet af en simplificeret flodbølge beregnes. For
denne findes, de vigtigste processer for morfologien at være sammenfaldende med
processerne for tilfældet med en pludselig ændring af strømstyrken. En hysterese
effekt findes for sandbankehøjden. Denne observeres også i naturen, og skyldes en
faseforskydning mellem morfologi og strømning.

Ved brug af de ovennævnte stationære sandbanker, med længde svarende til
fire vanddybder, undersøges mekaniserne for og ændringerne i morfologien, efter
e en vending af strømretningen. Den morfologiske udvikling findes at svare godt til
beskrivelser i litteraturen af observationer af bundens form ved tidevand. Tidsskalaen
for fuldstændig vending af sandbanken findes.

Tidevands sandbanker i ligevægt med den i tiden periodisk varierende strøm-
ning beregnes for en fastlagt sandbanke længde svarende til fire vanddybder. Tidevan-
det modelleres ved brug af en stepfunktion, i hvilken kun strømretningen ændres hver
halv-periode. Kun aktive sandbanker modelleres - dvs. kun bundformer hvis form
ændres væsentligt i løbet af tidevandsperioden. Forholdet mellem sandbankehøjden
og vanddybden, H/D, findes at forøge med Shields’ parameteren, og ydermere at
forøge med aftagende forhold mellem volumen af sediment transport og volumen af
bundformerne.

For tidevand demonstreres nødvendigheden af at medtage tidsvariationen af
bundformen i løbet af tidevandsperioden, for at opnå den korrekte ligevægts bund-
form. Denne faseoploste tilbagekobling til det fysiske system, findes at have en dæmp-
pende effekt på bundformerne.

Bundformer i tilfældet af kombineeret tidevand og strøm er eksemplificeret i
to morfologiske beregninger. I tilfældet af en stærk strøm, overlejer med en svag
tidevandsstrøm, ligner bundformerne de tilsvarende fra tilfældet med instationære
ensrettede sandbanker. I tilfældet af et stærk tidevand og en svag strøm, udvikles
sandbanker i tidevandsperioden som i tilfældet med rent tidevand. Selvom en af
komponenterne er svage (strøm eller tidevand), demonstreres det dog, at denne er
vigtig for bestemmelse af vandringshastigheden.
Contents

Preface 2
Abstract 3
Dansk resumé 5

1 Introduction 10
  1.1 Objectives ........................................... 10
  1.2 Instability mechanisms .................................. 11
  1.3 Review ................................................ 13
    1.3.1 Dunes in a unidirectional current .................. 13
    1.3.2 Tidal sand waves .................................. 15

2 Numerical model 18
  2.1 Flow model .......................................... 18
    2.1.1 Turbulence model .................................. 19
    2.1.2 Boundary conditions ................................ 20
    2.1.3 Flow solver ....................................... 20
    2.1.4 Grid-generation .................................... 20
  2.2 Sediment transport model .............................. 21
  2.3 Morphological model ................................... 23
    2.3.1 Morphological scheme .............................. 23
    2.3.2 Avalanche function ................................ 24
    2.3.3 Morphological filter ............................... 25
    2.3.4 Morphological time-stepping ......................... 28
  2.4 Nondimensional description and parameters ............ 29
  2.5 Flow over dunes ..................................... 34
    2.5.1 Comparison to data by Nelson et al. ................ 35

3 Steady dunes in unidirectional flow 40
  3.1 Methodology .......................................... 40
  3.2 Basic equations ...................................... 43
  3.3 Flow and sediment transport at equilibrium .......... 45
  3.4 Dune shape and properties ............................ 46
    3.4.1 The Fredsøe model for dune height, steepness and shape in a
          steady current .................................... 47
3.4.2 The approach by Tjerry and Fredsøe for dune steepness . . . . 50
3.4.3 Dune shape ........................................ 51
3.4.4 Dune height for the calculated dunes .................... 53
3.4.5 Dune steepness for the calculated dunes .................. 57
3.4.6 Discussion ......................................... 59
3.5 Flow resistance ....................................... 61
  3.5.1 The resistance model by Engelund ...................... 63
  3.5.2 The Carnot formula for expansion loss ................ 64
  3.5.3 Comparison with experimental results by McLean, Wolfe and
         Nelson ............................................. 65
  3.5.4 Comparison by numerical calculations by Yoon and Patel .. 70
  3.5.5 Flow resistance for the calculated dunes ............... 74
3.6 Numerical issues .................................... 78
3.7 Open ends ........................................... 83
  3.7.1 Dune length and steepness determination .............. 83
3.8 Summery ............................................. 86

4 Unsteady dunes in a unidirectional flow 89
  4.1 Methodology ......................................... 89
  4.2 Flow and sediment transport immediately after a change in flow
         properties ......................................... 90
  4.3 Morphology after an immediate change in the flow .......... 92
    4.3.1 Change in dune height and shape .................... 92
    4.3.2 The Fredsøe model for dune height change ............ 99
    4.3.3 Initial time scale: Comparison with the Fredsøe model . 103
    4.3.4 Initial time scale: A model for the trough .......... 104
    4.3.5 Time scale for the new equilibrium situation ......... 106
  4.4 Morphology during a flood wave ........................ 110
    4.4.1 Methodology .................................... 111
    4.4.2 Change in dune height and bedform ................ 111
  4.5 Numerical issues .................................... 115
    4.5.1 Variable morphological time step ................... 115
    4.6 Summery .......................................... 117

5 Unsteady dunes due to reversal of the flow direction 119
  5.1 Methodology ......................................... 119
  5.2 The flow and sediment transport field at flow reversal ....... 120
    5.2.1 Immediately before the flow reversal ............... 121
    5.2.2 Immediately after the flow reversal ................. 121
  5.3 Morphology after flow reversal ........................ 123
    5.3.1 Initial morphological development .................. 123
    5.3.2 Towards new equilibrium situation ................. 124
  5.4 Dune height amplification ............................ 124
  5.5 Time scale ......................................... 129
  5.6 Numerical issues .................................... 130
5.7 Summery ................................................................. 130

6 Dunes in a tidal flow ................................................. 134
  6.1 Methodology ......................................................... 134
  6.1.1 Flow description and impact on morphology .......... 134
  6.1.2 Morphological calculations - test-matrix .......... 138
  6.2 Tidal bedform shape ............................................. 141
  6.3 Geometrical properties ......................................... 143
  6.3.1 Bedform height ............................................. 149
  6.4 Active layer ...................................................... 153
  6.5 Discussion ....................................................... 156
  6.6 Stability of tidal bedforms .................................... 157
  6.6.1 Streaming .................................................... 158
  6.6.2 Time-averaged bed load: Phase-averaged and phase-resolved
        morphology ............................................... 158
  6.7 Numerical issues and stability ................................ 160
  6.8 Open ends ....................................................... 161
  6.9 Summery ......................................................... 163

7 Unsteady dunes in periodically reversing flow with a net current ........................................ 164
  7.1 Methodology ...................................................... 164
  7.2 Strong current and weak reversing flow ................. 165
    7.2.1 Dune height and shape during a tidal period .......... 165
    7.2.2 Migration velocity ..................................... 168
  7.3 Weak current and strong reversing flow ............... 170
    7.3.1 Dune height and shape during a tidal period .......... 170
    7.3.2 Migration velocity ..................................... 171
  7.4 Open ends ....................................................... 173
  7.5 Summery ......................................................... 174

8 Conclusion ........................................................... 175

9 References ........................................................... 179

10 LIST OF SYMBOLS ................................................... 183
Chapter 1
INTRODUCTION

This thesis is a study of two-dimensional sand dunes. Dunes are rhythmic bedforms found in rivers as well as at the sea bed - at the sea bed they are often denoted sand waves. A numerical model is applied to study the dunes in flow situations characterizing their environment. In rivers the flow is unidirectional, but the flow may change with time. At the sea bed, tidal currents cause a slowly varying flow with a periodic change of the flow direction.

The length of dunes in rivers is 4-8 times the water depth. At the sea bed the variation is larger, 2-18 times the water depth. The height of the dunes is less than 1/3 of the water depth (Stride, 1982).

Dunes and ripples are found in the subcritical flow regime and are as such the most common. From a practical point of view, dunes are the most important type of bedform. In rivers they carry a large part of the flow resistance and are hence important for determination of the discharge and water depth, for instance, during flood situations. In the sea, knowledge on bedforms such as dunes is relevant for engineers in relation to dredging as well as to burying of pipe lines in the sea bed. To protect the pipe lines from the hydrodynamic forces originating from currents and waves, they are buried in the bed, and the activity of the bed due to eventual migration of bedforms is important. Dredging of harbors and navigation channels is a costly and on-going activity and understanding of the sedimentation processes in relation to backfilling of dredged channels, as, for instance, the growth and migration of sand dunes, can optimize the planning of dredging.

Due to the complex interaction between the water, the sediment transport and the erodible bed, dunes also represent a challenge seen from an academic point of view. More research in the area is still needed to obtain a full understanding of the physics related to sand dunes.

1.1 Objectives
The purpose of this study is to calculate and study the mechanisms of two-dimensional dunes using a numerical non-linear flow model coupled with a morphological model. The aim is to obtain a better understanding of the physical mechanisms leading to growth and stability of the bedforms, and to understand the processes leading to changes in the dune shape and - mainly - the geometrical properties. Only bed load is included in the sediment transport description.

Steady and unsteady dunes are covered in this thesis. The development of dunes is calculated from an initial bed and until a bedform in equilibrium with the
given flow condition has developed. Five different flow situations are considered:

1. Steady dunes in a unidirectional current (river flow)
2. Unsteady dunes in a unidirectional current (e.g., during a flood situation in a river)
3. Dune morphology after an abrupt reversal of the flow direction
4. Dunes in a tidal flow
5. Dunes in a coexisting tide and current flow situation.

The situation, where the flow direction is changed is rare in nature and in this thesis mainly serves as an introduction to the tidal flow calculations. In each of the flow situations the morphological development of the dunes and the change in the geometrical properties are studied and the main physical processes are identified. Instability problems and uncertainty in the results in morphological modelling are common. Much effort has been put into obtaining stable and reliable results.

1.2 Instability mechanisms

An erodible bed is rarely plane. The fluid forces acting on the bed and the response of the sediments causes the bed to be unstable. The feedback and interaction between the water, the sediment and the bed causes bedforms to be generated; only the interaction between these three items causes the bed to be unstable. In this respect, the formation of sand dunes is a sensible process. The fluid forces respond to the bed itself, and not for instance to a structure like a bridge pier. The following is a short introduction to the instability mechanisms generating bedforms in an erodible bed. The subject is treated in Fredsøe (1995), in which further references can be found.

Fig. 1.1 illustrates the sediment transport over a perturbed bed. If the sediment transport on both slopes of the bedform are the same, i.e. \( q_1 = q_2 \), the bed will migrate downstream, but since the same volume of sediment is carried towards the top as away from the top it will not grow. It will be stable. However, if \( q_1 > q_2 \) more sediment will be carried toward the top than away from the top, and the bedform will grow. On the other hand if \( q_1 < q_2 \) the bedform decays. If we take \( q_1 \) and \( q_2 \) both very close to the top of the bedform (but still on each side of the top), we see that in the case of \( q_1 \neq q_2 \), a lag between the maximum bedform level and the maximum sediment transport rate is necessary to explain the instability. When \( q_1 > q_2 \) the maximum of the sediment transport is before the top, while if \( q_1 < q_2 \) the maximum sediment transport will be found after the top.

Such a phase lag can be explained by 1) a lag between the bed shear stress and the bed and 2) a lagged response of the sediment transport to the flow condition. The second point is mostly connected to suspended sediment transport, because it takes time for material already suspended in the water column to settle and adjust to a new flow situation. Parker (1975) also describes a lag in bed load due to inertia of sediment particles. This lag is however much smaller and is often neglected.
Instability mechanisms

The phase lag between the bed shear stress and the bedform is due to the interaction between the near-bed fluid, the pressure gradient and the bed. In this interaction, the role of the bed is two-folded: First, the flow area increases and decreases along the bedform, and causes the flow to accelerate and decelerate in the horizontal direction. Second, and equally important, the friction between the bed and the flow, reduces the velocity or the near bed flow, such that a boundary layer is generated. Thus, the flow close to the bed has less momentum and hence responds to the forcing differently than the high momentum outer flow.

Take again the situation in Fig. 1.1. If the Froude number, is small the water surface is nearly plane. On the downstream side of the bed, the flow area expands and the flow decelerates. The pressure thus increases and creates an adverse pressure gradient. As mentioned above, the near-bed flow has less momentum due to friction at the bed, and is therefore more easily retarded due to this unfavorable gradient in the pressure. Flow separation can occur if the unfavorable pressure gradient is large enough. When the near bed flow is retarded, the bed shear stress decreases. Along the upstream slope, the flow again accelerates. The acceleration of the flow is helped by the contraction of the flow area and a favorable pressure gradient. Along the upstream slope, the bed shear stress therefore increases. Furthermore the curvature of the bed creates pressure gradients in the direction towards/away from the center of the curvature of the bed, i.e. vertically as well as horizontally and adds additional acceleration/deceleration to the flow field. Again the near bed flow more easily responds to the forcing, the favorable pressure gradient and the effect of the curvature, than the outer flow due to the difference in momentum in the two wasser masses.

This balance between the convection of momentum in the flow and the pressure gradient explains the inequilibrium in the flow along the bedform and hence the phase lag between the bed shear stress and the bedform. The sediment transport reacts to the bed shear stress and the bed will be unstable. For short dune lengths, the slopes of the bedform will be steeper and the inequilibrium in the flow is larger. The phase lag hence increases with decreasing dune lengths, and according to this growth of these short bedforms should be largest.

The stabilizing mechanism in the situation with bed load only is the effect of gravity on the slopes of the dunes. This mechanism is however also larger for the smallest or rather steepest sand dunes. The effect of gravity stabilizes the inequilibrium in the shear stress between the upstream and downstream side of the bed. When the bedform in Fig. 1.1 grows in height, the effect of gravity will reduce q1 while q2 will increase due to the gravity effect. When q1 = q2 the bedform will be stable.

The same main mechanisms as sketched above apply to bedforms in the case of a reversing flow. The situation is shown in Fig. 1.2. When the flow is directed from left to right, we have the exact same situation as above. If the transport on the upstream side is exceeding the transport on the downstream side, the net transport is directed towards the top and the bedform grows. When the flow is reversed the same happens, and as the net transport is again (in the figure) directed towards the top, this also adds to the growth of the bedform. A stable bedform is found, when the effect of gravity balances the bed shear stress such that the net sediment transport
is zero.

1.3 Review

1.3.1 Dunes in a unidirectional current

Dunes in a unidirectional current are triangular in shape, but have a gently curved upstream face. The length of the dunes in rivers and laboratory dunes is approximately 3-8 times the water depth and the height of the sand dunes is less than 1/3 of the water depth (Stride, 1982). Guy, Simons and Richardson (1966) performed thorough investigations of the various types of bedforms developing in a laboratory flume. The dunes migrate in the direction of the flow.

Pioneers in understanding the mechanisms of an erodible bed (as outlined above) were Kennedy (1963), Engelund (1970) and Smith (1970). Kennedy described a phase lag between the bedforms and the sediment transport and Engelund and Smith identified this lag as being connected to the fluid friction. Fredsøe (1974) included the effect of gravity on the sediment transport, which showed to change the stability of the dunes. The flow over dunes has been described in many details by several researchers in the more recent years, where the focus in measurements as well as modelling has been on the turbulence and shear stress distribution along the dunes. A review can be found in Tjerry (1995). Measurements over dunes have been presented by (among others) McLean et al (1994), Nelson et al (1995), Bennett and Best (1995), and McLean, Wolfe and Nelson (1999). McLean and Smith (1986), Nelson (1989), Yoon and Patel (1995, 1996), Tjerry (1995), Mendoza and Shen (1990) and Tjerry and Fredsøe (to be publ.) presented numerical calculation.

Engelund (1966 and 1967) developed a model for flow resistance over dunes in equilibrium with the flow. Mendoza and Shen (1990), and Yoon and Patel (1995) investigates the flow resistance over dune shapes of varying geometry using numerical models. Raudkivi (1967) and McLean, Wolfe and Nelson (1999) presents results for form drag and skin friction obtained from measurements of flow and pressure along dunes in the laboratory.

However, the literature is more sparse on the morphology of fully developed dunes. Fredsøe (1982) coupled measurement of the relaxation of the flow downstream a backwards facing step with the continuity equation for sediment and geometrical
Figure 1.2  Stability of a sandy bed in a tidal flow.
considerations to find the dune height in a steady flow as a function of the Shields’ parameter and the dune steepness. Applying the ideas by Fredsøe (1982), but now using a numerical model to determine the relaxation of the flow over the bedforms, Tjerry (1995) and Tjerry and Fredsoe (to be published) found that the curvature effect on the boundary layer over the bedforms is important for the determination of the steepness of the dunes, especially for the low dunes. Van Rijn (1984) presented results for the geometrical properties of dunes in steady flow based on semi-empirical relations.

Fredsøe (1979) studied the behavior of unsteady dunes in a unidirectional current by using physical considerations on the change of the skin friction distribution along the dunes after a change in the flow. Julien (2000) presented results of measurements of the flow as well as the bedforms (dunes) in two Dutch rivers during a flood situation, and analyzed the flow resistance.

Only three studies using numerical morphological modelling on the issue of dunes are known by the author - one of these are on tidal sand dunes and is mentioned in the next section. In the early work by John, Chesher and Soulsby (1990) the initial development of a measured dune in the Taw Estuary, England, was compared with calculations using a numerical model. The numerical model was a one-equation model, using a transport equation for the turbulent kinetic energy, combined with a mixing length approach. The horizontal pressure gradient was derived hydrostatically from the surface. Roulund (1996) was in his master thesis able to perform morphological calculations of steady dunes in equilibrium with the flow using a k-ε and a Reynolds–stress model. These early calculations are the starting point of this thesis. Morphological calculations on the development and backfilling of a trench (having some similarities with dunes) are performed by Jensen et al. (1999) using a three-dimensional k-ε model.

1.3.2 Tidal sand waves

Observations

Dunes opposed to tidal currents can be found offshore or in a tidal inlet, such as a river mouth. The shape of the bedforms (which are often called sand waves offshore) can be highly variable depending on the hydrodynamic environment. Langhorne (1982) performed a detailed study of sand waves in a tidal environment, which illustrates the dynamics of tidal dunes well: Close to the time where the flow reverses in the tidal cycle, the sand waves studied by Langhorne have a shape much like the triangular shape of the unidirectional sand dune. When the flow reverses, the top of the sand dune is eroded and sediment is carried over the crest where it accumulates. Langhorne noticed a difference in volume of sand eroded and deposited in each tidal cycle depending on whether it was spring or neap conditions. Furthermore, he concluded from the observations that the wind stress and surface waves occasionally interrupt the trends in the bedform shape, which occurs during tidal flow alone.

Stride (1982) presented a review of observations of offshore bedforms, including dunes but also other types of bedforms. The dunes were found to have a height to depth relation less than 1/3, and the length of the sand waves was between 2 and 18 times the water depth. The more asymmetrical dunes were found to have gentle
slopes of about 0.5-4° and steep slopes of 4-35°, where the small sand waves in general has the steepest slopes, 17-35°, commonly above 20°, while the larger sand waves have steep lee sides slopes that are less than 20°. The symmetric dunes have angles between 4-14° on both sides. For the asymmetric as well as the symmetric dunes, the steepest slopes were found for the most active dunes, i.e. the dunes where the minimum velocity for sediment transport is exceeded often. A correlation between the steepest slope and the net migration direction was found. For the tidal sand waves the migration direction is governed by the direction of the net sediment transport (which is in general that of the ebb or flood current that reaches the higher peak).

Regarding the plane shape of the dunes, the dune crests are perpendicular to the flow direction (Dalrymple and Rhodes, 1995). For very large dunes, however, the crest lines can be oblique to the flow direction. Dalrymple and Rhodes (1995) concluded on the basis of several field studies, that the small dunes are very straight-crested and two-dimensional in their appearance. For increasing flow velocity they noticed that the shape is more three-dimensional since scour pits are formed in the trough and causes the trough line to be sinusoidal. The crest line is still uniform.

Physical mechanisms

Most researchers have so far been concerned with the processes leading to the initial growth of the tidal bedforms, including sand waves, from a flat or slightly perturbed bed, among these Hulscher et al (1993.1996), Komarova and Hulscher (2000) and Blondeaux et al (to appear, 2003). The stability problem of sand bars and sand waves is treated in the three papers by Hulscher. In the 1993-paper the problem was studied using a depth averaged model solving the shallow water equations. The morphology was treated under the assumption that the average sediment balance over the tidal period is sufficient to describe the long-term bed evolution. By using a linear stability approach, the appearance of bedforms on a scale comparable to sand bars, but not to sand waves, are predicted. In the 1996 and 2000-papers the turbulent stresses are modelled by means of a constant eddy viscosity (1996) and a parameterized eddy viscosity (Komarova and Hulscher, 2000) which is (linearly) dependent on the local bed height and time-dependent, but still constant over the depth. Using this modified flow description, instabilities on a length scale comparable to sand waves were found by Hulscher et. al. in addition to the length scale of sand bars. Blondeaux et al (2003, to appear) applied basically the same approach, performing stability analysis, as Hulscher et al, but they include also the effect of wind waves and residual currents.

According to Stride (1982) the dunes opposed to a tidal current can be considered as dunes in a unidirectional current, which is modified by the tidally reversing current. The same line of thinking is found in Deigaard and Fredsøe (1986) who treat offshore sand waves by using physical analogies to dunes in a unidirectional current. Deigaard and Fredsøe (1986) study dunes in the case of a current and surface waves, however due to the highly non-linear relation found between the migration velocity of the sand waves and the depth averaged velocity, they noted that even a small asymmetry in the tidal current can produce sand waves that are dominated by the direction of the strongest tidal current. This supports the findings by Stride (1982).
Idier and Astruc (2003) compared measurements of the development (on a short time scale [hours]) of a large bedform superposed with smaller dunes with numerical results. The applied model is a 3D model based on the Navier-Stokes equations but under the assumption of hydrostatic pressure. The model predicts the dynamics of the large scale bedform, but the smaller overlying dunes disappear.

To conclude, the previous research on sand waves in a tidal environment has hence neglected the non-linear effects and there is a lack of research on the mechanisms leading to equilibrium.
Chapter 2
NUMERICAL MODEL

The computational modelling system, Dune2D, consists of three parts: a flow model, a sediment transport model and a morphological model. The flow model is a so-called 2DV-model solving the two-dimensional Reynolds-averaged Navier Stokes equations under the constraint of mass conservation. The flow is hence resolved fully resolved in the horizontal as well as the vertical direction, which furthermore allows for a correct description of the non-hydrostatic pressure field, and therefore the separation of the flow behind a dune. To calculate the sediment transport and morphology of dunes, a model which is capable of resolving the boundary layer and determine the correct phases between the bedform and the flow is crucial. Depth averaged models do not fulfill these requirements.

The flow and sediment transport modules in Dune2D was originally developed by Tjerry in 1995, but has in the period up to the present study been continuously further developed and applied to various problems of flow, sediment transport and morphology by other Ph.D. students at the department. Tjerry indeed developed the model to study dunes and thus verified it thoroughly with respect to the flow field over dunes. However, he used a different turbulence closure (k-ε, and Reynolds-stress model). Another application of the model was a study of morphological evolution of trenches, see Jensen and Fredsøe (2001), where the model was verified against experimental measurements of oscillatory flow over a trench. Andersen (1999) applied Dune2D with the turbulence closure also used in the present work (the k-ω model) to study ripples, and hence verified this part of the model. Bits and pieces have also been added to the modelling system in the present study, primarily to the morphological module.

The model is hence not the work of this study and is for the present application considered to be very well verified in advance. With respect to the physical aspects important for sediment transport and morphology, mainly the near-bed flow and the bottom friction, the previous applications of the model are similar. A rather brief introduction is therefore given in the following and for details reference is made to the following: Tjerry (1995), Tjerry and Fredsøe (to appear), Andersen (1999), Jensen and Fredsøe (2001).

2.1 Flow model

The governing equations of motion for turbulent flows in the model are the Reynolds-averaged Navier-Stokes equations and the continuity equation, which in a Cartesian coordinate system read:
\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = g_i - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu_T + \nu \right) \frac{\partial u_i}{\partial x_j}
\]  
(2.1)

\[
\frac{\partial u_i}{\partial x_i} = 0
\]  
(2.2)

- where \( u_i \) is the velocity, \( p \) is the pressure, \( g_i \) is the gravity component, \( \nu \) is the kinematic viscosity and \( \nu_T \) is the eddy viscosity. The latter is determined by a turbulence closure, in the present work a \( k-\omega \) model is applied.

A "rigid lid" is assumed. In relation to flow over dunes the effect describing the free water surface is negligible for Froude-numbers much less than unity, Fredsøe (1982).

2.1.1 Turbulence model

A turbulence model is a necessity to calculate the eddy viscosity as a function of space and time. The turbulence closure applied is a standard \( k-\omega \) turbulence model following the approach of Wilcox (Wilcox, 1993). Compared to the more widely used \( k-\varepsilon \) model, the \( k-\omega \) turbulence model performs better in areas of an adverse pressure gradient, and hence handles separation better (Wilcox, 1993). This ability was originally the motivation for choosing the \( k-\omega \) turbulence model for the present work, such that a better description of the separation zone behind the dune crest could be obtained. The separation zone in the morphological calculations, however seems to be underestimated for other reasons than the turbulent relaxation (Chapter 3).

The eddy viscosity in the \( k-\omega \) turbulence model is given as:

\[ \nu_T = \gamma^* \frac{k}{\omega}, \quad \gamma^* = 1 \]

- where \( k \) is the turbulent kinetic energy, \( \omega \) might be characterized as the specific dissipation rate (no specific physical relation is given for \( \omega \), as is the case with \( \varepsilon \) in the \( k-\varepsilon \) model being the dissipation of turbulent kinetic energy). \( \gamma^* \) is a model constant. The model equations for \( k \) and \( \omega \) are:

\[
\frac{\partial k}{\partial t} + u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu + \sigma^* \nu_T \right) \frac{\partial k}{\partial x_j} - u_j u_j \frac{\partial u_i}{\partial x_j} - \beta^* k \omega
\]  
(2.3)

\[
\frac{\partial \omega}{\partial t} + u_j \frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu + \sigma^* \nu_T \right) \frac{\partial \omega}{\partial x_j} - \frac{\omega}{k} \left( u_j u_j \frac{\partial u_i}{\partial x_j} \right) - \beta k \omega^2
\]  
(2.4)

-which is solved with the closure coefficients:

\[
\sigma^* = \frac{1}{2}, \beta^* = \frac{9}{100}, \gamma = \frac{5}{9}, \sigma = \frac{1}{2}, \beta = \frac{3}{40}
\]  
(2.5)
2.1.2 Boundary conditions

A periodic boundary condition is applied between the "inlet" and the "outlet". This means that if the domain contains one dune, the flow over this dune can be considered to be influenced by a "train" of similar dunes upstream. At the surface a rigid lid assumption is adopted. The boundary condition is hence a symmetry condition for all variables except the vertical velocity which is 0 at the surface. The bed boundary conditions is a no-slip condition implying \( u = v = 0 \), and the inclusion of the viscous sublayer in the model causes \( k \) to vanish at the wall, i.e. \( k=0 \). A specific value for \( \omega \) is set at the bed, based on the roughness of the wall:

\[
\omega = u_f \frac{S_R}{\nu}
\]  

(2.6)

-where \( u_f \) is the friction velocity at the wall, and \( S_R \) is found by Wilcox (Wilcox, 1993):

\[
S_R = \begin{cases} 
\left( \frac{50}{k_N^+} \right)^2, & k_N^+ < 25 \\
\frac{100}{k_N^+}, & k_N^+ \geq 25
\end{cases}
\]  

(2.7)

-\( k_N^+ \) is the roughness in wall values, \( k_N^+ = \frac{u_f k_N}{\nu} \), and \( k_N \) the Nikuradse grain roughness.

2.1.3 Flow solver

Details of the flow solver are found in Tjerry (1995). The flow equations are solved on a curvilinear, orthogonal grid, which allows for the grid to follow the bed. The finite volume solution method is used, applying a PISO algorithm - which is basically rewriting the continuity equation into an equation for the pressure - following the approach by Patankar (1980). The convective terms in the equations of motions are discretized using the standard QUICK-scheme (see ex. Abbott and Basco, 1989). The QUICK-scheme is second order accurate in space. Problems of numerical diffusion in the separation zone was improved when switching from the first order upwind scheme. The discretization of the model-equations is implicit, except for the discretization of the convective terms, which by the implementation of the QUICK scheme is semi-implicit.

The driving force is a volume force. This volume force is either specified explicitly by specifying a slope term or found by iteration by specifying a volume flux.

2.1.4 Grid-generation

An example of the grid used for the morphological calculations are shown in Fig. 2.1. To resolve the boundary layer, the number of grid points is concentrated near the bottom. In the \( k-\omega \) model, the lowest grid points must be within the viscous sub-layer in order to resolve this layer. According to findings of Wilcox (1993) the height of the cell closest to the bed should be less than \( \Delta y^+ = 2 \), where \( y^+ = \frac{u y}{\nu} \). The
lowest grid cell is in the calculations, unless otherwise mentioned, $\Delta y^+=1$. 40 points are used everywhere across the vertical.

In the morphological calculations, the resolution in the streamwise direction at the bed is $\Delta x = 0.05D$, where nothing else is mentioned. For the dune lengths tested, this is 60-160 grid points pr. dune length. It should be noted that each of the main result-chapters in this thesis includes a section denoted "Numerical issues", and - among other numerical findings - grid-dependency is also treated.

### 2.2 Sediment transport model

Sediment transport can be divided into bed load, wash load and suspended load. Wash load is the very fine material which is transported in the water column only and never is in contact with the bed. For morphology this part of the sediment transport is hence unimportant. Bedload and suspended material on the other hand are interacting with the bed. In general bedload is considered the part of the sediments which is in continuous contact with the bed by rolling, jumping or sliding along the bed due to the direct bed shear stress between the sediment grains on the bed and the flow. The suspended material is transported in the water column. The turbulent fluctuations acts to pick the grains up from the bed and keep them in suspension.

Only bed load is considered in the present work. This is reasonable for low values of the Shields’ parameter, say approximately $\theta' < 0.15 - 0.2$. As an example assume that $\frac{d}{D} = 0.0004$ (when $k_N$ is defined as 2.5d, this corresponds to $\frac{k_N}{D} = 0.001$ as used in the calculations) and $\theta' = 0.15$. An estimate for the suspended transport can be found by using Einsteins approach (1950). The bed concentration for the suspended material, $c_b$, is calculated to $c_b = 0.006$ by using the empirical formula found by Zyserman and Fredsøe (1994). The non-dimensional fall velocity, the Rouse-parameter - $Z = \frac{w_f}{\kappa U_f}$ is chosen to 1.5, which is a reasonable value if excluding fine
Sediment transport model

sediment (dependent on the actual grain size of the material in suspension and the temperature (or the viscosity)). By using Einstein’s approach (a simplified diagram of Einstein’s results can be found in Deigaard (1980)), the suspended transport is now found to $\Phi_s \approx 0.11$. Since $\Phi_b \approx 0.25$ for $\theta' = 0.15$, approximately $1/3$ of the total transport will be in suspension.

Calculations are performed for values of the Shields’ parameter lower than 0.40. For the highest of these Shields’ parameters a considerable part of the total transport will hence in nature be found in suspension as illustrated by the example above, and the calculated dunes will naturally be influenced by this. The calculations, though, serves to illustrate the effects of the bed load only case.

For determining the bed load along the dunes, the bed shear stress is the most important parameter. In numerical model the bed shear stress on the bed is calculated as:

$$\frac{\tau_b}{\rho} = (\nu_T + \nu) S_{ij}$$  \hspace{1cm} (2.8)

- where $S_{ij}$ is the strain rate perpendicular to the bed, $S_{ij} = \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$, and in the $k-\omega$ model the eddy viscosity is found as $\nu_T = \gamma^* \frac{k}{s}$.

The nondimensional bed shear stress, the Shields’ parameter, is the most important parameter for bed load transport and hence for the morphological calculations in this thesis. The Shields’ parameter is formed by relating the stabilizing forces (friction, immersed weight) on a grain to the agitating forces (drag and lift). The definition of the Shields’ parameter is:

$$\theta' = \frac{\tau_b}{\rho(s-1)gd}$$  \hspace{1cm} (2.9)

- where $s$ is the specific density of the sediment ($\frac{\text{mass}}{\text{volume}}$), which for sand is approximately $2.65$, and $d$ is the grain size.

The nondimensional bed load is calculated by the well-known Meyer-Peter and Müller bed load formula, Eq. 2.11. The relation between the nondimensional and dimensional bed load is given by Eq. 2.12, where $q_b$ is volumetric $[\text{m}^3/(\text{s} \cdot \text{m})]$ rate of sediment transport. The formula is known to overestimate the bed load for high Shields’ parameters, but for Shields’ parameters below approximately 0.4, this formula will suffice. The critical Shields’ parameter, $\theta'_c$, is defining the situation when the agitating forces on a sand grain equals the stabilizing forces. The nondimensional bed shear stress must hence exceed $\theta'_c$, for the sand to be mobile. $\theta'_c$ is dependent on the grain Reynolds number ($\text{Re}_g = \frac{U_d d}{\nu}$) according to the Shields’ diagram (see ex. Deigaard and Fredsøe, 1992), but takes a constant for fully turbulent flow; usually this constant is chosen in the range 0.045-0.06. In the present work $\theta'_c = 0.05$ is used. On a sloping bed, gravity is an additional agitating force. The influence on the transport is included by including a slope term, $\mu \frac{\partial h}{\partial x}$, in the effective Shields’ parameter as derived by Fredsøe (1974) - see Eq. 2.10. Fredsøe showed that $\mu$ takes a value of approximately 0.1.
\[
\theta'_{\text{eff}} = \theta' - \mu \frac{\partial h}{\partial x} \tag{2.10}
\]

\[
\Phi_b = 8 \cdot (\theta'_e - \theta'_c)^{3/2} \tag{2.11}
\]

\[
\Phi_b = \frac{q_b}{\sqrt{g(s - 1)d^3}} \tag{2.12}
\]

2.3 Morphological model

Besides the numerical solution scheme for updating the bed according to the sediment transport, the modelling of the avalanche process, a morphological smoothing process to treat numerical instabilities and morphological timestepping is treated in this subchapter. Of the three parts in the modelling system (flow, sediment transport and morphological model) the morphological model has been payed most attention in this study.

Numerical instabilities is a general problem in morphological modelling. Using a 2DV model in morphological modelling (or another model which resolves the boundary layer), the instabilities are believed to be more likely to "blow up" the solution than in depth-averaged models, due to the strong interaction and feed back between near bed flow (and hence the bottom shear stress) and these instabilities. A filter to avoid/dampen these are required (at least using the present model), but this is not without problems. Thus, much effort has been put into investigating the effects of the filter on the morphology. This has resulted in a "trough-correction" to the filter, which will be presented below.

The processes in the morphological module is therefore following the sequence below:

1. Update of bed according to the sediment transport field
2. Avalanche/sliding procedure if the bed slope exceeds angle of repose at any point
3. Morphological filter
4. Trough correction to filter

2.3.1 Morphological scheme

The continuity equation for sediment is simply stating that the divergence of the bed load field equals the rate of deposition:

\[
\frac{\partial h}{\partial t} + \frac{1}{1 - n} \frac{\partial q_b}{\partial x} = 0 \tag{2.13}
\]
The equation is solved using an Adam-Bashforth method to update the bed with time (see ex. Ferziger and Peric, 1999). The method is of the predictor-corrector type and the discretized version of Eq. 2.13 read:

$$h_{i}^{n+1} = h_{i}^{n} - \frac{\Delta t}{12} \left( 23 \frac{\partial q_{b}}{\partial x} \bigg|_{i}^{n} - 16 \frac{\partial q_{b}}{\partial x} \bigg|_{i}^{n-1} - 5 \frac{\partial q_{b}}{\partial x} \bigg|_{i}^{n-2} \right)$$  \hspace{1cm} (2.14)

where index $n$ is the time step and index $i$ the spatial grid point. The spatial gradient in the sediment transport, $\frac{\partial q_{b}}{\partial x}$, is discretized using a simple central difference. Other schemes has been tested in the course of the study, trying to minimize numerical instabilities. Among these a Leap-Frog type of scheme for the updating with time and a spacial difference including more neighboring points than the simple central difference for the spatial gradient in the sediment transport. The differences on the morphology were minimal, and none of these methods were found to be better at dampening the numerical instabilities than the Adam-Bashforth scheme.

2.3.2 Avalanche function

When the bed slope exceeds the angle of repose, approx. 30 degrees, avalanching will take place. This has been attempted modelled in two different ways:

1. Increasing the Shields’ parameter and hence the sediment transport exponentially when the bed slope exceeds angle of repose (as also applied by Andersen, 1999).

2. Moving/sliding the sediment at the location where the bed exceeds the angle of repose down the slope until the bed at all locations are less than the angle of repose, see Fig. 2.2.

A model of type (1) was found hard to control. With increasing sediment transport along the dune, the stronger an avalanche process is needed and hence the bed slope ended up being larger for dune calculations with a large transport. Whether the angle of repose is 28 or 32 degrees, was not really seen as a serious problem. However, this type also created numerical problems. Since the bed load is greatly increased at grid points where the angle of repose is exceeded, large gradients in the bed load and therefore also in the deposition field and final bedform is the result. By keeping the morphological time step low, however, only small corrections are necessary at each time step, and the model works acceptable.

(2) is much easier to use. A sliding function that works in the following steps was implemented:

- Locate the point, closest to the top where the bed slope exceeds angle of repose.
- Lower the uppermost grid point to fullfil the bed slope to be angle of repose. Calculate the volume of sediment subtracted from the bed in that process
- Raise the next point down slope according to the mass balance for sediment.
Morphological model

This procedure is repeated until the bed slope at all locations are less than angle of repose. On a dune front where the bed slope exceeds angle of repose close to the top, the sediment will "slide" down the slope, point by point, until it deposits at a location, where the bed slope is less than angle of repose. The bed slope is by the avalanche routine reduced to 29.5° and a bed slope up to 30.5° is accepted before avalanching takes place in the model. This procedure has been found to work well and is used in the final morphological calculations.

2.3.3 Morphological filter

In the present work it has not been found possible to avoid the use of a filter. Several approaches was tested in the course of the study among these filtering of the skin friction and the bed load. In the end a filter on the bedform was applied using the following scheme:

\[
\begin{align*}
\hat{h}_i &= \frac{1}{2} h_{\text{raw}, i} + \frac{1}{4} (h_{\text{raw}, i-1} + h_{\text{raw}, i+1}) \\
{h}_i &= \frac{1}{2} h_i - \frac{1}{4} (\hat{h}_{i-1} + \hat{h}_{i+1})
\end{align*}
\]  

(2.15)

The unfiltered bedform is denoted \( h_{\text{raw}} \), and the filtered bed is \( h_i \). The same smoothing routine was applied by Tjerry(1995) also on dunes. Jensen et al (1999) used a slightly different approach with morphological calculations of a trench. They applied a two step approach for the filtering routine as follows:

- Filtering of \( h_{\text{raw}} \) with first step in Eq. 2.15 \( \rightarrow \hat{h}_i \).

- Do 1..N

\[-\Delta h_{\text{filt}} = h_{\text{raw}} - \hat{h} \]
- Filtering of $\Delta h_{filt}$ with the first step in Eq. 2.15 $\rightarrow \Delta \hat{h}_{filt}$
- $h = \hat{h} + \Delta \hat{h}_{filt}$

- Enddo

The filter is therefore smoothing the bedform, but since the difference is between the "raw" and the filtered bed is added - after having been filtered - the effect of the filter is to smooth only on length scales of the grid size, if choosing $N$ high enough. Increasing the value of $N$, furthermore reduces the effect of the filter.

It turns out that the filter by Jensen et al (1999) and the filter in Eq. 2.15 are exactly the same when choosing $N = 1$. It was not found possible to use a value higher than $N=1$ in the morphological calculations of the dunes (and thereby reduce the effect of the filter), and the filter applied therefore corresponds to Eq. 2.15.

A filter must be applied with most care in morphological calculations. No matter which type of filter that is applied, the effect on the bedform can be seen as "artificial" erosion or deposition. The criterion used to evaluate the "filter-effects" in this thesis, is that the effects should be negligible in comparison with the physical erosion/deposition. Any effects of the filter will cause a feed back on the flow field, which means that the influence is not only important on the time step where the filter is applied, but causes the bedform to develop in an erroneous way in the following time steps. Furthermore the full effect of the filter is extremely hard to judge.

The effect of the filter applied on a dune migrating downstream is shown below. This is actually stepping a bit quickly forward in the thesis, but since a correction to the filter has been developed and is included in the morphological model, this is found necessary. In a single time step the effect of the filter can be evaluated by comparing the erosion/deposition pattern as calculated directly from the sediment transport field and the actual erosion/deposition pattern after the bedform is filtered. This is shown in Fig. 2.3. Studying the filter effects, the following can be concluded:

1. Along the upstream slope of the bedform, from just downstream the trough until just after the top, the filter effects are negligible in comparison with the rate of change of the bedform.

2. Just downstream the trough, the filter causes more erosion than calculated from the sediment transport field.

3. On the downstream slope, a large deposition is calculated from the sediment transport field to be located on the upper part of the slope. The filter causes this deposition to be more evenly distributed along the downstream slope.

4. In the area between the top and the crest a more complex situation is found. This will be commented on in the Section 2.5.1.

Effects 2-4 are the problematic ones. Number 3 is seen as the least serious of those, but further comments will be added to this bullet-point in Section ?? in Chapter 3.
Figure 2.3  Effects of morphological filter on erosion/deposition. Effect of the trough-correction.

**Trough correction to filter**

The artificial erosion caused by the filter removes sediment from the trough (bullet-point 2 above) and hence deepens the trough more than the calculated sediment transport field predicts. The calculated trough level might therefore be too low, and hence the total dune height is too large.

A correction to the filter is added to remove the effect of the filter in the trough. The sediment artificially eroded by the filter is simply added back on in the trough area. The sediment should be added ”smoothly” to avoid numerical problems, hence, the most natural way of handling the problem that comes into ones mind - to just add the artificially eroded material in the trough area back on where eroded - will not work. Instead a thin and ”smooth” layer is added. The shape of the added layer is here calculated by using a normal distribution, Eq. 2.16, but of course any other smooth shape will probably work.

In the normal distribution there are two free parameters: one is the amplitude, a, and the other is the extention of the layer, \( L_{cor} \). \( x' \) in Eq. 2.16 is the distance from the center of the correction layer and \( L_{1\%} \) is defining, where the normal distribution has a value of 0.01 of the amplitude, which is chosen as the cut-off point for correction to the bedform.

\[
\Delta h_{\text{troughcor}} = a \cdot e^{\frac{1}{2} \left( \frac{x'}{L_{\text{cor}}} \cdot \frac{1}{L_{1\%}} \right)^2}
\]  (2.16)
Morphological model

The local maximum of artificial erosion caused by the filter located in the trough area, determines the amplitude of the layer, a, and the location of the maximum artificial erosion determines the center of the normal distributed correction-layer. The extension of the layer \( L_{cor} \) is found by requiring that the same volume of sediment should be added with this trough-correction as is eroded artificially in the trough area.

The correction layer in the trough is thus done by the following steps:

1. Find the trough
2. Find the local maximum in artificial erosion in the trough area
3. Calculate the amplitude of the artificial erosion as \( h_{raw} - h \)
4. Find the length of the trough correction, such that the amount of the sediment added by the correction layer equals the artificially eroded material.
5. Calculate a correction by Eq. 2.16
6. Update the bed by \( h = h + \Delta h_{trough, cor} \)

Bullet point no.1-6 is repeated three times

An example of the effect of the trough-correction is shown in Fig. 2.3.

2.3.4 Morphological timestepping

Ideally we should be able to find the morphological time step from a morphological Courant-number criterion:

\[
Cr_{morp} = a \frac{\Delta t}{\Delta x} < 1 \tag{2.17}
\]

- where \( a \) is the migration velocity of the bedform, \( \Delta t \) and \( \Delta x \) respectively the morphological time step and the spatial step. For a steady migrating dune the migration velocity is constant along the bedform, and the bedform change is given by (see also Chapter 3):

\[
\frac{dh}{dt} = \frac{\partial h}{\partial t} + a \frac{\partial h}{\partial x} = 0 \tag{2.18}
\]

By combining the above equation with the continuity equation (Eq. 2.13), the migration velocity is found as

\[
a = \frac{\partial q_b}{\partial h} \tag{2.19}
\]

-which can also be rewritten as:
\[ a = \frac{\partial q_b}{\partial h} = \frac{\partial q_b}{\partial x} / \frac{\partial h}{\partial x} \]  \hspace{1cm} (2.20)

-where the porosity is included in \( q_b \) for simplicity: The same expression for the migration velocity can be found from a non-steady bedform from the Exner equation alone however in this situation the migration velocity is a \textit{local} migration velocity of a disturbance. The Courant-criterium can then be expressed as:

\[ \Delta t < \frac{\partial h}{\partial x} \frac{\partial q_b}{\partial x} \cdot \Delta x \]  \hspace{1cm} (2.21)

From the principle of similarity of the flow, we can find that the gradient in the bed load is proportional to the rate of the bed load, i.e. \( \frac{\partial q_b}{\partial x} \sim q_b \). This will be further explained in Chapter 6 and is just stated as a fact here. Given that the bed load is found from the Meyer-Peter and Müller formula (Eq. 2.11), the morphological time step is thus inverse proportional to \( (\theta'_{eff})^{3/2} \).

\[ \Delta t \sim \frac{1}{(\theta'_{eff})^{3/2}} \]  \hspace{1cm} (2.22)

In the morphological modeling of the dunes it has been found that a much smaller time step is needed for stability reasons. This is partly due to the calculated flow field around the crest of the dunes (where smoothing is needed) and partly due to numerical instabilities in the flow field. In a morphological calculation with an unsteady flow situation the time step is scaled by a relation like 2.22.

### 2.4 Nondimensional description and parameters

The numerical model is formulated in nondimensional parameters. The nondimensional description and the nondimensional parameters governing the morphology of dunes are given in the following. The formulation of the governing equations in nondimensional parameters has the advantage that the number of parameters can be reduced by the number of "fundamental dimensions". The kind of physical systems, studied here, has three fundamental dimensions: length, time and mass. Mass is left out in the description of the equations of motion, since all terms are divided by the density, \( \rho \), and this leaves only two fundamental dimensions; length and time.

The system of equations in the present work can hence be described by two parameters less, when they are made dimensionless, than in the dimensional versions. The number of governing equations are the same, but the number of independent parameters are reduced by two. When recalculating the solutions described by a combination of dimensionless parameters, there are hence two "free" parameters in the dimensional description. However, as will be shown in the following, there are physical bindings on some parameters, which limit the choice of these free parameters, like for instance the gravity parameter, \( g = 9.82 \text{ m/s}^2 \), and the molecular viscosity, \( \nu \sim 10^{-6} \text{ m}^2/\text{s} \).
Flow calculations

The momentum equations can be non-dimensionalized by a characteristic velocity and a length scale, \( V \) and \( D \), which is respectively the mean depth averaged velocity and the mean water depth (i.e. corresponding to the water depth measured from the mean bed level).

\[
\frac{\partial \left( \frac{u_i}{V} \right)}{\partial t} + u_j \frac{\partial \left( \frac{u_i}{V} \right)}{\partial x_j} = \frac{D}{V^2} g I_i - \frac{1}{\rho} \frac{\partial \left( \frac{p}{V^2} \right)}{\partial (z_i/V)} + \frac{\partial}{\partial (x_i/V)} \left( \left( \frac{\nu_T}{VD} + \frac{\nu}{VD} \right) \frac{\partial u_i}{\partial (x_i/V)} \right)
\] (2.23)

Two non-dimensional parameters scales the momentum equation; the Froude number and the Reynolds number, here defined as:

\[
Fr^2 = \frac{V^2}{g D}
\]

\[
Re = \frac{VD}{\nu}
\]

Furthermore the dimensionless roughness, \( \frac{k_N}{D} \), enters the equations trough the boundary conditions at the wall. The driving force in a steady flow, the mean energy slope, \( I \), is normalized with the Froude number, \( I/Fr^2 \). The nondimensional parameters governing the system is hence \( \left( \frac{k_N}{D}, Re, Fr^2 \right) \) and in dimensional parameters \( (k_N, D, V, g, \nu) \). In the present calculations a ”rigid lid” is assumed. The driving force is therefore not necessarily the gravity term, \( g \cdot I \) (or \( I/\text{Fr}^2 \) in nondimensional parameters), but can instead be anticipated as a volume force, \( S \), necessary to drive the discharge, \( Q=V \cdot D \), in the system. \( S \) is here rather an acceleration term, than a force, since the momentum equation is divided by the ’mass’, \( \rho \). When \( V \) and \( D \) (and hence the discharge) are specified, the volume force in the steady flow case is found by iteration. Alternatively the volume force could be specified, and the discharge found by iteration.

Since gravity is now left out of the system, the remaining nondimensional parameters are \( \left( \frac{k_N}{D}, Re \right) \) and the dimensional parameters are \( (k_N, D, V, \nu) \). When recalculating the solution, given in the nondimensional set of parameters into the dimensional parameters, two of those can be chosen independently according to the introductory remarks to this section. From these and the nondimensional parameters specified the other dimensional parameters will be given. The natural value of the molecular viscosity of water restricts the choice of one of the parameters, \( \nu \). This means, that if (for instance) the mean velocity is chosen as the second parameter, \( D \) and \( k_N \) are given.

For a fully turbulent flow situation (if the bed is fully rough, \( k_N^* \gtrsim 70 \)) the flow is independent on the molecular viscosity since the momentum exchange by the turbulence is much larger than the momentum exchange by the viscous forces. The flow is thus not dependent on the Re-number. For practical applications, the molecular
viscosity can be excluded in the problem-description, and one non-dimensional parameter (Re) as well as one dimensional parameter ($\nu$) is removed from the problem. There are hence two ”free” parameters in the system, since none of the remaining dimensional values have physical bindings (except that the Froude-number is required to have a small value due to the ”rigid lid” assumption mentioned in Section 2.1). The Reynolds number must, however, still be specified in the k-\omega turbulence model, since the viscous layer is included.

In the case of an instationary flow, a time scale, $T$, is added to the flow problem. One more dimensionless ($T_D$) as well as a dimensional parameter ($T$) are hence introduced. The time is in Dune2D made dimensionless with the flow parameters, $V$ and $D$:

$$t_D = \frac{tV}{D} \quad (2.24)$$

Furthermore a variation in the forcing is necessary. This can be done in two different ways. A variation, $f(t)$, in the volume force term, S, or a variation in the mean velocity $V$ can be introduced, such that these are given by $S \cdot f(t)$ or $V \cdot f(t)$. $f$ is a nondimensional function and $S$ and $V$ are now respectively a reference volume force or a reference velocity. $f$ will therefore be given by $S'(t)/S$ or $V'(t)/V$, where $S'$ and $V'$ are respectively the time-varying volume force or time-varying depth averaged velocity. It is preferred to specify a variation in the velocity $V$, since (as will be seen in Chapter 3) the energy lost behind the dunes is underpredicted, and then specifying a volume force is a less ”stable” method of obtaining a given flow than specifying the velocity. The depth averaged velocity in the instationary case is hence $V'(t)=V \cdot f(t)$. The non-dimensional parameters are now ($k_N, D, V, \nu, V'(t), T$). If the time scale is given in the flow problem, for instance if a tidal flow is calculated, the values of $T$ and $\nu$ specifies the two free parameters by their natural values.

**Sediment transport**

When calculating sediment transport, information about the grains in motion and the scaling of the forces driving the particles must be specified. The grain size, $d_{50}$, and the Nikuradse roughness height are in this work coupled by:

$$d_{50} = \frac{k_N}{2.5} \quad (2.25)$$

and any new nondimensional parameters are hence not added to the problem. The sediment is assumed uniform.

The nondimensional parameter, that scales the driving forces to the stabilizing forces with regard to the sediment transport, is the Shields’ parameter, $\theta'$:

$$\theta' = \frac{U_f^2}{(s - 1)gd_{50}} \quad (2.26)$$
Rewriting the Shields’ parameter, using the nondimensional parameters for the flow, V and D, we obtain:

\[ \theta' = \frac{2.5 \frac{U'^2}{V^2}}{(s - 1) \frac{k_N}{D}} Fr^2 \]  

(2.27)

When the Shields’ parameter is specified, the Froude number is hence given, when the roughness/grain size, \( \frac{k_N}{D} \), is given. The nondimensional values specifying the problem are now \((\frac{k_N}{D}, Fr, Re)\) or \((\frac{k_N}{D}, \theta', Re)\). The dimensional set of parameters are \((k_N, D, V, g, \nu)\). The natural choices of the gravity parameter, g, and the molecular viscosity, \( \nu \), means that there are no free parameters left in the dimensional set of parameters in the system, when the non-dimensional values have been chosen. However, taking the Re-independence into account for the fully rough flow case leaves for practical applications one dimensional parameter free.

If a time scale is added in the flow problem, \((T_D, f(t_D))\) and \((T, V'(t))\) are respectively added to the non-dimensional and dimensional set of parameters. There are now three parameters in the dimensional set which have ”natural” values: \( g, \nu, \) and \( T \). The number of non-dimensional parameters can hence be reduced by expressing the time scale, \( t_D \), in terms of the Froude number and Reynolds number together with the (natural) dimensional values for \( g, \nu \):

\[ t_D = t \cdot \frac{(Fr \cdot \sqrt{g})^{4/3}}{(Re \cdot \nu)^{1/3}} \]  

(2.28)

Discarding the Reynolds-dependence in the flow, \((T_D, f(t_D))\) and \((T, V'(t))\) can be added to the non-dimensional and dimensional set of parameters without this paraphrasing of the non-dimensional time.

In the morphological calculations of the dune, the Shields’ parameter, \( \theta'_u \), on a undisturbed bed, i.e. a flat channel bed, is often given in the text. This Shields’ parameter is obtained by combining Eq. 2.27 and Eq. 2.25 with Eq. 2.29 below, which is the resistance law for a rough channel bed:

\[ \frac{V}{U'_{f,u}} = 6 + 2.5 \ln \left( \frac{D}{k_N} \right) \]  

(2.29)

\[ \left( \frac{V}{U'_{f,u}} \cdot \frac{k_N}{D} \cdot Fr \right) \Rightarrow \theta'_u \]

As will be shown in the following, \( \theta'_u \) is however also an approximate value for the nondimensional skin friction in the morphological calculations performed in this thesis. Einstein and Barbarossa (1952) expressed the skin friction for a dune covered bed with Eq. 2.30:
\[ \frac{V}{U_f'} = 6 + 2.5 \ln \left( \frac{D'}{k_N} \right) = 6 + 2.5 \ln \left( \frac{D'/D}{k_N/D} \right) \]  \hspace{1cm} (2.30)

D’ can be anticipated as an average boundary layer thickness, see Fig. 2.4. For a fully developed channel flow, the boundary layer extends across the entire water column. In this case D’/D=1, and Eq. 2.30 is similar to Eq. 2.29. For a dune covered bed, the D’/D turns out to be approximately in the range [0.3;1]. In the morphological calculations performed in this thesis, \( \frac{k_N}{D} \) is kept constant. The variation in \( \frac{V}{U_f'} \) for the above-mentioned change in D’/D can hence be found to 20.3-23.3, or a maximum difference of approximately 15 %. \( \theta'_e \) is therefore an approximate value of the nondimensional skin friction, also in the dune covered cases studied in this thesis.

**Morphology**

Non-dimensionalizing the continuity equation for sediment with the same scaling parameters as the flow equations, V and D, we obtain:

\[ \frac{\partial \left( \frac{h}{D} \right)}{\partial \left( \frac{x}{D} \right)} + \frac{1}{1 - n} \frac{\partial \left( \frac{q_s}{VD} \right)}{\partial \left( \frac{x}{D} \right)} = 0 \]  \hspace{1cm} (2.31)

The bed load is hence scaled with V·D, i.e. the water discharge in the case V and D are chosen as the mean velocity and the mean water depth.

The nondimensional description of the morphological equations above is based on the mean variables for the flow. This is how the numerical model works. In the analysis of the dunes in the chapters later in this thesis, non-dimensionalizing is based on parameters relevant for the sediment transport and morphology of dunes, i.e. the nondimensional version of the sediment transport, \( \Phi \), and a time scale related to the sediment properties. The continuity equation for sediment (Eq. 2.13) in these nondimensional parameters can be rewritten as:

\[ \frac{\partial \left( \frac{h}{D} \right)}{\partial \left( \frac{t}{D^2} \right)} = \frac{1}{1 - n} \frac{\partial \Phi}{\partial \left( \frac{t}{D} \right)} \]  \hspace{1cm} (2.32)
This reveals a time scale for the morphological development, \( t^* \), defined as:

\[
t^* = t \sqrt{\frac{(s - 1) g d^3}{D^2}}
\]  
(2.33)

The relation between the flow time scale and the morphological time scale is then:

\[
t^* = t_D \cdot \frac{1}{F_r} \cdot \sqrt{(s - 1) \cdot \left(\frac{k_N}{D} \cdot \frac{1}{2.5}\right)^3}
\]  
(2.34)

This time scale, Eq. 2.33, can also be expressed as:

\[
t^* = \frac{\sqrt{(s - 1) g d^3}}{q_b} \cdot \frac{q_b \cdot t}{D^2} = \frac{1}{\Phi} \cdot \frac{V_{sed}}{D^2}
\]  
(2.35)

- where \( q_b \) is the sediment transport when only bed load is considered. \( V_{sed} = q_b \cdot t \) is the volume of sediment transported across a given vertical section of the sand bed within the time interval \( t \). For the calculations with unsteady morphology of dunes in this work, this paraphrasing of the time scale turns out be helpful in the understanding of the morphodynamics. The length scales (height and length) of the bedforms, studied in this work, scale with the water depth. The water depth squared, \( D^2 \), in the denominator in Eq. 2.35 is therefore a scale/measure for the height and length of the bedforms and hence also for the volume of those bedforms. The relation \( \frac{V_{sed}}{D^2} \), the second term in Eq. 2.35, can hence be put into the physical context of a measure for the part of the volume of the bedform, which is being transported within a time interval \( t \).

This time scale is preferred here instead of using a typical time scale related to the flow variables. Since we operate only on slowly varying flow conditions (tide, flood wave), all the flow situations with regard to the sediment transport are comparable to current conditions more than to unsteady flows like waves.

### 2.5 Flow over dunes

In the present section the main characteristics of flow over dunes is described. A more detailed description can be found in (among others) Tjerry (1995). Dune2D’s ability to model flow over dunes was also thoroughly verified by Tjerry (1995) using respectively a k-\( \varepsilon \) and a Reynolds Stress turbulence model. Anderson (1999) verified the k-\( \omega \) turbulence model. The purpose of the following is hence not to verify the model, but a comparison of the flow field to data by Nelson, McLean and Wolfe (1993) is shown and shortcomings relevant to the morphological calculations are discussed. In general the model performs well. Especially problems in handling the separation at the crest, however causes the separation zone to be predicted too short when the crest is included in the dune formulation (as in the present approach) - more about this later.
The flow over dunes in a unidirectional flow is characterized by a separation zone after the crest followed by a wake and a boundary layer flow on the upstream part of the dune, see Fig. 2.5. The flow has in this way similarities with the flow after a backward facing step. Whereas the turbulence models in general have problems predicting the flow and especially the decay of turbulence after a backward-facing step, the flow over dunes seem to be easier to model. Due to the contraction of the flow area in the dune situation after the separation zone, a favorable pressure gradient will be induced, which is not found after a backward facing step. The increased decay of turbulence in the flow with a contracting flow area seems to be the reason for this.

Besides from the discussion of the model performance in the present section, a discussion and comparison of pressure and skin friction measurements with the model results is found in Chapter 3.

2.5.1 Comparison to data by Nelson et al.

Comparison between a measured and modelled flow over dunes are shown in Fig. 2.7-2.9, where profiles of the horizontal and vertical velocities as well as profiles of turbulent kinetic energy is shown. The experimental data are measured by Nelson, McLean and Wolfe (1993) over fixed concrete dunes using a LDA-technique. The dunes are cosine shaped with a linear stoss side. The dune length and height are respectively 80 cm and 4 cm and the water depth 21.5 cm. The roughness in the numerical calculations are set to \( k_N/D = 0.002 \) (according to personal communication between Tjerry (1995) and J. Nelson).

The numerical calculations are performed by using two different approaches, in the first approach the front of the dune is included and in the second the front is left out and replaced by a vertical wall. The two different grids are shown in Fig. 2.6. In the first approach a grid with the front is generated (upper subfigure in Fig. 2.6 - a similar grid is used in the morphological calculations) and boundary conditions as explained in Section 2.1.2 are applied. In the second approach the front is excluded (lower subfigure in Fig. 2.6). The periodic boundary conditions between the inlet and outlet section is handled by interpolation in the grid cells above the crest. In grid cells below the crest in the "inlet" section, a wall-boundary condition is used. The advantage of using the second approach is that the bend in the grid lines at the crest is avoided.

The velocities computed in the model compare well with the data. In the wake the velocities at the bed is underestimated - the same tendency is found by Tjerry
Flow over dunes

Figure 2.6 Two types of grid is used in calculations for comparison with data by Nelson et al. (1995) using the k-ε model as well as the Reynold stress model. Also the profiles of turbulent kinetic energy fit the data well as seen in Fig. 2.9. The shear layer between the separated region and the upper flow region are less marked in the computed profiles, the reason might be numerical diffusion. No significant difference in the profiles is seen in the two different numerical approaches. The approach without the front is applied later in this thesis to calculate the bottom pressure over dunes.

At the crest Including the crest in the calculations as in the upper subfigure in Fig. 2.6 a peak is seen in the near-bed velocity (see example in the upper subfigure in Fig. 2.10). The same behavior was found by van der Knaap et al (1991) when calculating flow over dunes using a k-ε model and by Tjerry (1995). The peak is caused by the inability for the flow model to separate the flow at the correct location. When the grid lines follow the bed curvature and the cells in the grid is ”bended”, this effect is enhanced according to Jess Michelsen, MEK-DTU (personal communication).

When the flow does not separate right at the crest, the streamlines at the bed is ”forced” around the high curvature at the crest. This modifies the pressure (since a pressure gradient is needed to force the flow around a ”corner”) to have a local minimum just at the crest, this seen in the lower subfigure in Fig. 2.10. Furthermore the length of the separation zone is reduced. The flow after the separation reattaches approximately 6 step heights (dune heights) downstream a step (crest of a dune), but in the example in Fig. 2.10 the reattachment point is approximately 3 step heights from the crest, since the point of separation is not on the top of the stoss slope.
This reduced separation zone is reflected in the morphological calculations and will be commented on in the result chapters.

The peak is naturally reflected in the calculated skin friction and hence the calculated deposition/erosion in this area. It is rather difficult to predict the exact flow field in this area. An attempt to "correct" the near-bed flow parameters was tested just to see the effect on the morphology of a steady migrating dune. The "peak" and the slight depression close to the crest was removed in the flow parameters used in the evaluation of the bed shear stress, i.e. $u$, $v$, $k$ and $\omega$, and then the bed shear stress was evaluated. The correction was a guess of a more correct velocity-distribution, the curvature in the flow variation was found to be nearly constant close to the top, so this curvature was used to "correct" the flow variables. The first tests seem promising, however problems arise when the flow is strong and the peak larger. The correction method is not applied in any of the morphological calculations presented in this thesis.
Figure 2.8  Vertical velocity calculated with Dune2D compared to experimental data by Nelson et al 1993.

Figure 2.9  Vertical velocity calculated with Dune2D compared to experimental data by Nelson et al 1993.
Figure 2.10  Curvature acceleration around the crest are due to the peak in the near bed flow velocity.
Chapter 3

STEADY DUNES IN UNIDIRECTIONAL FLOW

In this chapter, steady dunes in a unidirectional flow are studied using the numerical model, Dune2D, described in Chapter 2. Steady dunes in equilibrium with the flow field are calculated from an initial bed by continuously updating the bed according to the sediment transport field. The equilibrium dune is reached, when the interaction between the water, the sediment transport and the bedform is balanced. Only bed load is taking into consideration in the sediment transport. The computations are hence only valid for small values of the nondimensional bed shear stress, $\theta'$. The period boundary conditions applied in the numerical calculations mean that the calculation can be interpreted as the development of a large number of similar dunes in a row. The forcing in the system (in excess of the forcing from the mean bed slope) is purely coming from the dune bed itself. The morphological development of the erodible bed into dunes is hence caused by the instability of the bed due to the interaction with the flow.

The methodology of using a morphological model to solve the continuity equation for sediment is a 'next step' in research aiming at understanding the morphology of fully developed dunes at The Technical University of Denmark. Previous work was undertaken by Fredsøe (1982) and by Tjerry (1995) and Tjerry and Fredsøe (to be publ.). Fredsøe found the dune height, shape and steepness by combining continuity of sediment with physical considerations and a conceptual model for the friction over dunes. Tjerry (1995) and Tjerry and Fredsøe (to be publ.) applied a similar approach to calculate the shape and steepness of dunes behind a backward facing step, but used the flow and sediment transport modules in Dune2D.

In the following, the calculated dunes are studied with respect to the dune properties (combination of height, shape, length) and the flow resistance. The dune length is held constant during each morphological computation. The dune heights and shape are hence found under the constraint of a given dune length. The results for the steady dunes in equilibrium with the flow are found to compare well with the previous findings by Fredsøe and Tjerry. The morphological model are hence found to give consistent results with respect to the physical mechanisms.

3.1 Methodology

Dunes in equilibrium with the given flow field are calculated from the morphological development of an initially perturbed bed. The steady dunes are calculated for different values of the nondimensional bed shear stress (the Shields’ parameter) and the dune length. The calculation method and the test matrix for the steady dune
computations is given in this section.

The general calculation method in the morphological computations (in this
and the following chapters) is following the procedure below:

1. An initial bedform is chosen
2. The flow field is calculated with the flow model for the given bed at the time
   \[ t = t_n \].
3. The bed shear stress is calculated from the flow field, according to Eq. 2.8
4. The bed load field is calculated from the distribution of the bed shear stress
   along the bed from the Meyer-Peter and Müller bed load formula (Eq. 2.11)
5. The bedform is updated according to the divergence of the sediment transport
   field (Eq. 2.13), and a new bed to the time \( t_{n+1} = t_n + \Delta t \) is hence found.
6. 2)-5) is repeated, in this chapter until a steady migrating dune is found

In the beginning of this study, the initial bed in the morphological calculations
was always a small sinusoidal perturbation of 2.5% of the water depth. However, a
negligible influence of the initial bed on the final steady bedform was found. The
initial bedform for the calculations presented here, is therefore often a "guessed"
bedform. This bedform can either be a bedform from a previous calculation or occasion-
ally a bedform obtained with a courser calculation grid. Both methods have the
great advantage of reducing the calculation time to obtain a steady state condition
for the morphology.

Periodic boundary conditions are applied in the calculations between the inlet
and outlet section. This means that within the calculation domain, the bedform can
develop freely - under the restriction that the changes taking place at the left and
right boundaries are always the same. The bedform is therefore "allowed" to split (or
merge - if more than one dune is present) within the domain, but due to the constant
domain length and the periodic boundary conditions, the sum of the dune lengths is
constant, and a whole number of dunes is hence always found within the domain.

For the calculations performed, the domain length is of such a length that this
"splitting" and "merging" of dunes does not happen, since only one dune is found
in the domain. In reality the length of the dunes are therefore set by the length of
the domain. Dune lengths between 3 and 8 water depths are tested, which is roughly
the range of dune lengths observed in nature (Stride, 1982). In some calculations
with a long domain, the bed was not found stable. This is further commented on in
Section 3.7. Moreover, two test calculations with the purpose of studying the length
determination processes are presented in this section.

For each dune length tested, steady dunes is calculated for different rates of
the nondimensional bed load, \( \Phi_b \). The parameter governing the bed load rate is the
Shields’ parameter, and Shields’ parameters in the range \( \theta'_u = 0.062 - 0.40 \) are applied.
\( \theta'_u \) is the nondimensional skin friction on an undisturbed channel bed for the given set
of flow parameters and with the same roughness of the bed. The undisturbed bed is


understood as a bed without bedforms (ripples, dunes), i.e. a flat and non-erodible bed. $\theta_u$ can be found from the resistance law for a rough channel bed (see Section 2.4).

The variation of the Shields’ parameter in the numerical model is applied by choosing the nondimensional parameters ($\frac{k_N}{D}$, $Fr$, $Re$). In the numerical model a choice of the input parameters ($\frac{k_N}{D}$, $Fr$, $Re$) defines the flow and sediment transport as outlined in Section 2.4. Together with the Shields’ parameter for an undisturbed bed, $\theta_u$, in the given flow situation, the input parameters are given in Table 3.1 for all the morphological calculations performed with steady dunes.

The discharge in each calculation is by this methodology constant, and the forcing necessary to drive the discharge is found by iteration in the model. The link between the nondimensional and the dimensional parameters in the system is described in Section 2.4, and is exemplified in the following: Run s3L4 is used as an example, this means the length of the domain/bedform is four water depths and the Shield parameter on a undisturbed bed is $\theta_u = 0.11$. From Section 2.4 it is known, that there is one so-called ’free’ dimensional parameter in the system, when sediment transport is calculated (and no time scale is applied), and the dependence on the molecular viscosity (the Reynolds-number) is discarded. The grain size is hence chosen to $d = 1$ mm. Since the nondimensional roughness is $\frac{k_N}{D} = 0.001$, and the relation between the roughness height and the grain diameter is $k_N = 2.5d$ (Eq. 2.25), the mean water depth can be found to $D=2.5$ m and the length of the domain to $L=10$ m. From the Froude number, $Fr = \frac{v}{\sqrt{gD}} = 0.20$ in run s3L4, the depth averaged velocity corresponding to the mean water depth is found to $V=1$ m/s.

An example of the morphological development from an initial bedform (here a sinusoidal bedform with the initial height 0.05D and $L/D=4$) to an equilibrium dune is shown in Fig. 3.1. The bedform starts out from the initial bed in the lowermost part of the figure. Only one dune has been calculated (i.e. the domain length is $L/D=4$), but three is shown for visual reasons. The flow direction is from left to right in the figure and the dune is hence seen to migrated downstream in the direction of the current. The bedforms grows up and deforms with time as shown in the figure. A steady state dune is reached in the uppermost part of Fig. 3.1, where the dune simply migrates downstream without changing its shape. The development in the dune height corresponding the morphological change in Fig. 3.1 is shown in Fig. 3.2. The dune height approaches a constant value, which in the calculations for the steady

<table>
<thead>
<tr>
<th>Run</th>
<th>L/D</th>
<th>$\frac{k_N}{D}$</th>
<th>$\theta_u$</th>
<th>Fr</th>
</tr>
</thead>
<tbody>
<tr>
<td>s1L*</td>
<td>3.4,5,6.7,8</td>
<td>0.001</td>
<td>0.062</td>
<td>0.15</td>
</tr>
<tr>
<td>s2L*</td>
<td>3.4,5,6.7,8</td>
<td>0.001</td>
<td>0.08</td>
<td>0.17</td>
</tr>
<tr>
<td>s3L*</td>
<td>3.4,5,6.7,8</td>
<td>0.001</td>
<td>0.11</td>
<td>0.20</td>
</tr>
<tr>
<td>s4L*</td>
<td>3.4,5,6.7</td>
<td>0.001</td>
<td>0.20</td>
<td>0.27</td>
</tr>
<tr>
<td>s5L*</td>
<td>4.6</td>
<td>0.001</td>
<td>0.30</td>
<td>0.33</td>
</tr>
<tr>
<td>s6L*</td>
<td>3.4,5,6</td>
<td>0.001</td>
<td>0.40</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Table 3.1  Morphological calculations with steady flow - steady dunes. $Re=2.5e6$ in all calculations. * = L/D in the namegiving of the runs.
dunes is chosen as the criterium for equilibrium.

### 3.2 Basic equations

Before the results for the steady dunes in remaining of the flow situations are presented, the basic equations for the sand dune problem are given. In a coordinate system following the steady dunes, the shape is given by:

\[ h = h(x - at) \]  

(3.1)

where \( h \) is the local dune height and \( a \) is the migration velocity. Differentiation with respect to time yields:

\[ \frac{dh}{dt} = \frac{\partial h}{\partial t} + a \frac{\partial h}{\partial x} \]  

(3.2)

The first term on the right hand side is the local rate of change in the bed. From the continuity of sediment the rate of change of the local dune height, equals the divergence of the sediment field, Eq. 3.3. When the sediment is deposited, the pore volume, \( n \), must be taken into consideration, which accounts for the factor \( \frac{1}{1-n} \). The porosity is chosen to \( n=0.4 \) in the present work.

\[ \frac{\partial h}{\partial t} = -\frac{1}{1-n} \frac{\partial q}{\partial x} \]  

(3.3)

where \( q \) is the rate of sediment transport. For steady dunes, the left hand side of the equation 3.2 is 0, and the spatial change in the sediment transport field hence balances the change in the bed due to pure migration:

\[ -\frac{1}{1-n} \frac{\partial q}{\partial x} + a \frac{\partial h}{\partial x} = 0 \]  

(3.4)

This equation can be integrated to read:

\[ q = q_0 + (1-n)ah \]  

(3.5)

Furthermore, this gives the important result, that the sediment transport is proportional to the local dune height for a steady migrating dune:

\[ q \sim h \]  

(3.6)

In the case of pure bed load, the sediment transport within the separation zone in the trough area is 0 (the velocities in the separation bubble might be strong enough to move sediment, but this is negligible), in which case we arrive at:
Figure 3.1  Development of the bed from an initial sinusoidal bed (in the lower part of the figure) to a fully developed dune (in the upper part of the figure). The flow is from left to right. $\theta'_u = 0.20$. $L/D = 4$. $k_N/D = 0.001$. Note: The domain length is $4D$ and hence only one bedform is calculated.
Figure 3.2 The change in the dune height corresponding to the bedform development in the figure above.

\[ a = \frac{1}{1 - n h} q \]  

(3.7)

- where \( h \) is measured from the trough. Taking both quantities, \( q \) and \( h \), at the top we find that the migration velocity of the dunes can be found as the relation between the sediment transport at the top and the total dune height.

\[ a = \frac{1}{1 - n \bar{H}} q_T \]  

(3.8)

### 3.3 Flow and sediment transport at equilibrium

Next the flow, sediment transport, and erosion/deposition field for dunes in equilibrium with the flow are described. Two flow situations - one with a low sediment transport rate (\( \theta_u = 0.08 \)) and one with a higher sediment transport rate (\( \theta_u = 0.20 \)) are shown in Fig. 3.3 and Fig. 3.4 together with the respective bedforms found in the morphological calculations. The following flow and sediment transport mechanisms can be observed:

**At the steep downstream slope**

Downstream the dune front there is a rapid increase in the local water depth, which causes the streamlines to diverge and thus the flow resistance decreases. The flow separates (if the unfavorable pressure gradient is large enough), and the flow velocity decreases and the skin friction drops rapidly as seen in the upper figures. The capacity for moving the sediment hence decreases (middle figures). The flow is then unable
to carry the sediment further downstream, and a zone of deposition is found at the upper part of the front. This is seen in the lower figures, where the deposition/erosion pattern is shown by the local rate of change of the bed, 
\[ \frac{\partial (h/D)}{\partial t} = - \frac{\partial \Phi_e}{\partial h (x/D)}. \]
From the upper part of the front the sediment will slide down due to avalanching. This process is not included in \[ \frac{\partial (h/D)}{\partial t} \], but the influence of the avalanche process on the equilibrium bedform is described in Section 3.6.

At the gentle upstream slope

After the reattachment point, the flow accelerates in the downstream direction. Sediment transport begins, where the effective skin friction exceeds the critical level \( (\theta'_{\text{eff}} > \theta'_c) \). The flow accelerates along the upstream slope of the dune due to the convergence of the streamlines caused by the reduced flow depth. This increases the bed load and causes erosion from the bed to take place along the gentle upstream slope. The maximum effective skin friction and the maximum bed load rate is found at the top of the dune.

A similar erosion and deposition pattern is found along the bedforms for the two different Shields’ parameters in Fig. 3.3 and Fig. 3.4; the same general features are found - only the bed load is less and the rate of change of the bedform is smaller for the lower Shields’ parameter. Since the bedforms are in equilibrium with the flow, they will not change the shape as they migrate downstream (due to the deposition at the front and erosion along the downstream front) after the equilibrium situation is found.

The bed load and the local dune height are proportional along the major part of the dune, all along the gentle upstream slope and until just before the crest. In this area the steadiness equation for a steady migrating dune (Eq. 3.6) is therefore fulfilled. At the front area, the deviation from equilibrium is caused by two different aspects in the numerical modelling procedure: 1) the avalanche function 2) the numerical filter. Both of these modify the erosion/deposition pattern as it is calculated directly from the divergence in the sediment transport field. Whereas the avalanche process is physical, the numerical filter is not. A further description of the equilibrium situation and the influence of the avalanche process as well as the numerical filter is given in Section 3.6 below.

The ”peak” in \( \theta' \) and \( \Phi_b \) at the dune crest was discussed briefly in Section 2.5.1. It is, however, seen to also influence the erosion/deposition pattern in the crest area. If no filter is applied on the bed, a bedform much closer to the shape of the bed load distribution would be found, i.e. the dune would have a ”nose” at the front.

3.4 Dune shape and properties

The objective of this section is to find the properties of the dunes in a steady current and understand the mechanisms that determine the height, shape and length of the dunes. The equilibrium bedforms calculated for different Shields’ parameters for a fixed (given) dune length are analyzed in the following. First the approach taken by Fredsøe (1982), Tjerry (1995) and Tjerry and Fredsøe (to be publ.) are described
Dune shape and properties

Figure 3.3 Flow, sediment transport and erosion/deposition for run s3L4. $\theta'_u = 0.08$.

Since the basic physical mechanisms and results in their work are an important basis for the analysis of and comparison with results from the present work.

3.4.1 The Fredsøe model for dune height, steepness and shape in a steady current

Fredsøe (1982) used geometrical considerations and a simple expression for the variation of the skin friction over the dunes to find the dune height, shape and steepness. The basic equation is the continuity equation for a steady migrating dune, Eq. 3.6, used in the form:

$$\frac{h}{H} = \frac{\Phi_b}{\Phi_{b,\text{top}}}$$

(3.9)

- or:

$$\frac{\partial \Phi_b}{\partial x} = \frac{\Phi_{b,\text{top}}}{H} \frac{\partial h}{\partial x}$$

(3.10)

Fredsøe (1982) used a perturbation method to arrive at the dune height as a function of the Shields’ parameter. The same expression for the dune height was found by Fredsøe and Deigaard (1992) by a different method outlined below. Since the non-dimensional bed load rate, $\Phi_b$, is a function of the Shields’ parameter, $\theta'_\text{eff} - \theta'_c$, alone, Eq. 3.10 can also be written as:
Figure 3.4 Flow, sediment transport and erosion/deposition for run s5L4. $\theta'_u = 0.20$. 
\[
\frac{\partial \Phi_b}{\partial \theta'_{\text{eff}}} \cdot \frac{\partial \theta'_{\text{eff}}}{\partial x} = \frac{\Phi_{b,\text{top}} \partial h}{H \partial x} 
\]

where:

\[
\theta'_{\text{eff}} = \theta' - \mu \frac{\partial h}{\partial x} 
\]

Only bed load is considered and thus, the sediment transport can be found using the simple Meyer-Peter and Müller bed load formula, Eq. 3.13.

\[
\Phi_B = 8 \left( \theta'_{\text{eff}} - \theta'_c \right)^{\frac{3}{2}} 
\]

\[
\frac{\partial \Phi}{\partial \theta'_{\text{eff}}} \text{ is hence found as:}
\]

\[
\frac{\partial \Phi}{\partial \theta'_{\text{eff}}} = 12 \cdot \sqrt{\theta'_{\text{eff}} - \theta'_c} 
\]

The second part on the left hand side of Eq. 3.11, \( \frac{\partial \theta'_{\text{eff}}}{\partial x} \), is the critical term. The variation of the skin friction along the dune can be written as consisting of two terms (Eq. 3.15), one is the cross-sectional effect and the other, \( f(x) \) is the memory effect. The memory effect hence includes "anything" else than the effect of the local mean velocity, which increases/decreases when the local cross sectional water depth decreases/increases.

\[
\theta' = \theta'_{\text{top}} \cdot f(x) \cdot \frac{V(x)^2}{V_{\text{top}}^2} = \theta'_{\text{top}} \cdot f(x) \cdot \frac{(1 - \frac{h}{H})^2}{(1 - \frac{h}{H})^2} 
\]

- where \( \theta'_{\text{top}} \) is the Shields’ parameter on the top of the sand dune, \( V(x) \) is the local mean velocity and \( V_{\text{top}} \) is mean velocity at the top. \( f(x) \) is the normalized skin friction, i.e. the skin friction coefficient, \( c_f \), downstream the former crest, divided by its value on the top. \( h \) is in the following the local bedform height measured from the trough. The gradient in the Shields’ parameter is hence:

\[
\frac{\partial \theta'_{\text{top}}}{\partial x} = \theta'_{\text{top}} \cdot \left( \frac{\partial f(x)}{\partial x} \right)_{\text{top}} + \frac{2}{(1 - \frac{h}{H})} \cdot \frac{\partial h}{\partial x_{\text{top}}} 
\]

Fredsøe (1982) found from experimental data strong similarity between the skin friction variation behind a backward facing step (see Fig. 3.5) and along triangular dunes, and argued that the skin friction distribution over dunes are similar. He assumed, that the variation in \( f(x) \) was negligible at the top, in which case the term
\[ \frac{\partial f(x)}{\partial x} \bigg|_{\text{top}} \] can be ignored in 3.16. In physical terms this corresponds to assuming that the shear stress at the crest of the dune is in equilibrium with the bedform. By inserting Eq. 3.14 and Eq. 3.16 into Eq. 3.11 an expression for the dune height can be found (Fredsoe and Deigaard, 1992):

\[ \frac{H}{D} = \frac{2(\theta'_{\text{top}} - \theta'_{c})}{7\theta'_{\text{top}} - \theta'_{c}} \] (3.17)

The dune height variation with the Shields’ parameter at the top using this expression is later in the present chapter compared to dune heights for the computed dunes (Fig. 3.10).

The shape of the dunes can be found by a first order differential equation in \( h \) obtained by combining Eq. 3.6 with 3.13, 3.15 and using \( \theta'_{\text{eff}} = \theta' - \mu \frac{\partial h}{\partial x} \) for the effective skin friction and thereby including the effect of gravity on a sloping bed in the sediment transport:

\[ \frac{\partial h}{\partial x} + \left( \frac{\theta'_{\text{top}} - \theta'_{c}}{\mu} \right) \left( \frac{h}{H} \right)^{2/3} = \frac{\theta'_{\text{eff}} - \theta'_{c}}{\mu} \] (3.18)

Fredsoe used measurements of the skin friction behind a reverse step to determine the variation in \( f \). The iteration of the shape of the dune is carried out such that \( \frac{h}{H} = 1 \) for \( \theta' = \theta'_{\text{top}} \). \( \theta'_{\text{top}} \) is given. As the dune is supposed to end at the top of the dune, this calculation will also give the steepness of the dune for a given choice of the dune height.

Fredsoe was able to predict the dune shape, height and steepness by these relatively simple physical principles. With regard to the flow description, it is noted that the only effects included in the shear stress distribution are 1) the effect of the cross section on the local mean velocity and 2) the turbulence relaxation downstream the crest of the dunes, as given by the skin friction coefficient downstream a backward facing step.

### 3.4.2 The approach by Tjerry and Fredsoe for dune steepness

Tjerry (1995) and Tjerry and Frederse (to be publ.) applied a turbulence model (k-\( \varepsilon \) and Reynolds-stress model) to determine the steepness and shape of the dunes, by using an approach similar to the approach by Frederse (1982): the turbulence model was used to determine the skin friction along the dunes and the shape equation (Eq. 3.18) was applied to determine the dune shape by iteration. The effect of the upstream dune was modelled by a reverse step in front of the dune, but now the shape of the dune itself influences the flow field by other means than the effect of the reduced cross section on the mean velocity. Tjerry and Frederse was not able to determine the dune height by this method and used Frederse’s relation for the dune height (Eq. 3.17).

An important difference from the method by Frederse (1982) is that no assumption regarding the skin friction along the dunes was made in the later work by Tjerry and Frederse. The main finding in the work with regard to dunes at low Shields’ parameters (excluding suspended material) was that the effect of curvature
acceleration along the dunes on the skin friction is important for the steepness of the lower dunes.

The process of curvature acceleration acts by locally modifying the pressure and hence also modifies the pressure gradient along the bed: The streamlines at the bed follows the bedform. To balance the centrifugal force acting on the water particles in the direction away from the "center" of the "curve of the bed", a pressure gradient towards the center of the curve is induced. This increases the pressure, where the bed has a concave shape and reduces the pressure where the bedform is convex. The effect of this is to accelerate the flow, when the bedform in the streamwise direction is going from concave to convex (or for a weakly convex shape to a stronger convex shape) and the other way round. The curvature effect were found to reduce the steepness of the dunes - especially the low dunes, by moving the location of the maximum shear stress further downstream. The turbulent relaxation downstream of the crest for these dunes was found to take place long before the top of the dune, and was hence not able to explain the considerable long dune lengths found for the low dunes. A more constant dune length than found in the work by Fredsøe (1982) was hence predicted by Tjerry and Fredsøe. The results by Tjerry/Fredsøe for the dune steepness were compared to data and to the empirical relation found by van Rijn (1984) and can be seen in Fig. 3.6: The expression for the steepness by van Rijn (1984) is:

$$\frac{H}{L} = 0.015 \left( \frac{D}{d_{50}} \right)^{0.3} \left( 1 - e^{-T} \right) (25 - T)$$

(3.19)

-where $T$ is a so-called transport stage parameter given by $T = \frac{u^* y_{50}}{\gamma_c}$.

3.4.3 Dune shape

In the following the calculated dunes are presented. First the dune shape and the general features are evaluated and in the proceeding sections a closer look is taken
at the dune height and dune length (or rather steepness), followed by a discussion of the results.

The calculated dunes are shown in Fig. 3.7 ($\frac{L}{H}$, $\frac{b}{H}$), Fig. 3.8 ($\frac{L}{H}$, $\frac{h}{H}$) and Fig. 3.9 ($\frac{L}{H}$, $\frac{h}{H}$). The dunes have the well known triangular shape with a gentle curved upstream slope and a steeper front. The dune shape is more rounded at the top as the Shields’ parameter decreases, while the dunes for the high Shields’ parameters are more triangular.

The dunes for the large Shields’ parameters ($\theta_u' > 0.2$) have a front slope corresponding to the angle of repose (in the model set to 30°). This angle is not reached for the dunes for the lower Shields’ parameters. Furthermore, the height of the crest above trough level for these dune is very low. Also the distance between the top and the crest of the dunes decreases with an increasing Shields’ parameter.

The described difference in the dune shape between the dunes for the small and the larger Shields’ parameters can be caused by the relative influence of gravity. Gravity acts as a diffusion term in the erosion/diffusion field (combine Eq.3.3 with Eq. 3.10-3.12) in a similar way as the diffusion term in the Burger equation for a migrating and steepening wave (see eg. Ames (1977)). Since the gravity term is relatively more important for the small Shields’ parameters, the effect of the diffusion term will be largest for the small dunes.

However, numerical issues might also influence the dune shape in the crest area. Three different numerical aspects should be considered: 1) Prediction of the separation point in the model 2) Grid spacing 3) The numerical filter. The tendency for the model to separate further downstream from the top than expected (described in Section 2.5.1), might also influence the crest location (to be further downstream), and the crest height above the trough level. Grid dependency and the effect of the numerical filter is treated in Section 3.6. Here it is just noted that refining the grid and reducing the effect of the filter increases the front slope angle.
No firm evidence of the difference in the bedform shape has been found in the literature, however Guy, Simons and Richardson (1966) describe dunes with a sharp crest as well as a rounded crest. Due to the scaling of the bedforms (the length of the dunes are many times larger, 16 to more than 100, than the dune height, especially for the lower dunes) this shape is also hard to investigate.

Normalizing the dunes with their respective length and height, similarity in the dune shape for a similar Shields’ parameter, $\theta'_{u}$, is found, see Fig. 3.9. The similarity between dunes of a different length is greatest for the dunes in the flow situations with a high Shields’ parameter. When the Shields’ parameter is high the effect of the gravity on the sloping bed is negligible. The short dunes are steeper than the longer dunes for the same Shields’ parameter (as will be shown later in the chapter). However, when the Shields’ parameter is high, the difference in steepness is less important for similarity between dunes of different lengths than for dunes with a low Shields’ parameter.

3.4.4 Dune height for the calculated dunes

The height of the dunes is the subject in this section. The results found in the present work are compared to the findings by Fredsøe (1982) outlined in Section 3.4.1. The dune heights for the calculated dunes are shown in Fig. 3.10, as a function of the variable $\theta'_{top} \left( 1 - \frac{H}{D} \right)^2$, which corresponds to the mean Shields’ parameter. The dune heights are compared to the expression by Fredsøe, Eq. 3.17. The dune heights increases with the Shields’ parameter and approaches an upper limit for an increasing Shields’ parameter as well as increasing dune length.

The calculated dune heights are in general lower than the dune height predicted by Fredsøe, but as the length of the calculation domain (and hence dune length) increases, the dune height approaches the dune height predicted by Fredsøe. To understand this, the assumption made by Fredsøe regarding the skin friction variation (in excess of the cross section effect) downstream the crest being similar to the skin friction variation downstream a rearward facing step, must be considered. For a flow behind a rearward facing step the maximum skin friction is found approximately 16 step-heights downstream the step due to relaxation in the flow (Fig 3.5).

Fredsøe assumed that at the top, the variation in $\frac{\partial f(x)}{\partial z}_{top}$ is weak and can be neglected. This is the same as assuming that the boundary layer at the top is fully developed and if the non-uniformity in the flow areas in horizontal direction is also weak (the top of the dune is rather flat), the shear stress is in (or close to) equilibrium at the top.

If this equilibrium is not fully reached, the effect of a gradient in $f$ can be studied by including the variation in $f$ in the shear stress when deriving the formulation for the dune height (Eq. 3.17). It appears that the effect of the term is to decrease the dune height (this was also shown by Tjerry (1995)). This is consistent with the observations in the present calculations. When the domain length decreases, the flow field downstream from the separation zone has a shorter distance to adapt to an equilibrium situation at the dune top. The dune height should hence be expected to deviate more from the dune height predicted by Fredsøe, and furthermore be smaller according to the discussion above. The variation in $f$ at the top of the dune is shown
Figure 3.7  Equilibrium dunes calculated in a steady current. Vertical and horizontal scale is normalized by the mean water depth, D. L/D=[3,8]. The x/D-value, where the dune ends indicates the dune length in the graph. Mean bed level is at h/D=0. $\theta'_u = [0.062, 0.40]$. 
Figure 3.8 Equilibrium dunes calculated for a steady current. Vertical and horizontal scale normalized by the dune height. L/D = [3, 8]. $\theta' = [0.062, 0.40]$.
Figure 3.9  Equilibrium dunes calculated in a steady current. The vertical scale is normalized with the dune height and the horizontal scale with the dune length. L/D=[3,8]. $\theta'_u = [0.062, 0.40]$. 
for $\theta'_{u} = 0.11$ for $L/D = 3.6$ in Fig. 3.11 and shows a weaker gradient in $f$ for a longer dune length. This is consistent with an increasing dune height found for a longer dune length.

The question is, what is the origin in the flow field is for the observed variation in the normalized skin friction at the top. The findings by Tjerry and Fredsøe explain this. The turbulence relaxation downstream the crest can according to the observation by Tjerry and Fredsøe only account for the non-equilibrium at the dune top for the higher dunes. For the lower dunes the turbulent relaxation takes place much before the top. If the curvature of the top, however, is not constant (which it is not for the present dunes), curvature acceleration will add to the non-equilibrium in the skin friction (according the discussion in Section 3.4.2) and hence set a smaller dune height. A larger gradient in the curvature, $\frac{\partial^2 h}{\partial x^2}$, is actually observed for the shorter dune lengths, compared with the larger dune lengths.

In the expression for the dune height by Fredsøe, the slope effect (gravity effect) on the Shields’ parameter has no influence on the dune height (only on the length and the shape). In the present calculations, where the calculation domain is limited and the gradient in $f$ is not zero, the role of the gravity term in the effective shear stress is to balance the variation in the friction coefficient at the top. This can be seen from a differentiation of $\theta'_{eff} = \theta' - \mu \frac{\partial h}{\partial x}$ combined with Eq. 3.15, in which case the following equation for $\frac{\partial \theta'_{eff}}{\partial x}$ at the top can be found:

$$\frac{\partial \theta'_{eff}}{\partial x}\bigg|_{top} = \theta'_{top} \cdot \left( \frac{\partial f(x)}{\partial x} \bigg|_{top} + \frac{2}{(1 - \frac{H}{D})} \cdot \frac{\partial h}{\partial x} \bigg|_{top} \right) - \mu \frac{\partial^2 h}{\partial x^2} \tag{3.20}$$

### 3.4.5 Dune steepness for the calculated dunes

For the large Shields’ parameters, where the effect of gravity is less important in the "only bed load" situation, the approach by Fredsøe predicts a dune length of 16H, corresponding to the distance to the maximum in the bed shear stress downstream a rearward facing step. At the smaller Shields’ parameters the influence of gravity must be taken into account. The maximum skin friction is hence found further downstream and Fredsøe predicted a decrease in the steepness. The dunes in the present calculations agree well with these observations as seen in Fig. 3.8. For the large Shields’ parameters, i.e. large dunes, the dune length approaches 16 dune heights and increases for the smaller Shields’ parameters where the effect of gravity is of relatively higher importance.

In Fig. 3.12 the dune steepness, $\frac{H}{D}$, is shown for the calculated dunes. Since the dune crest in the work by Fredsøe (1982) and Tjerry and Fredsøe is assumed to appear just downstream from the top of the dune, a different steepness of the dunes, the steepness of the upstream slope defined as $H/L_1$, is given in Fig. 3.13, where $L_1$ is the distance between the dune trough and top:

$$\frac{L_1}{D} = \frac{x}{D} \bigg|_{dune \ top} - \frac{x}{D} \bigg|_{dune \ trough}$$
Figure 3.10  Dune height variation with the Shields’ parameter.

Figure 3.11  Variation of the dimensionless skin friction for L/D=3-6 for $\theta'_u = 0.11$. The dune top is located at $x'/D=0$. 
The dune steepness varies with the height of the dunes. The higher dunes (or the dunes with a larger dune length) have a smaller steepness than the smaller dunes for the same Shields’ parameter. This was also found by Tjerry and Fredsøe - when setting a lower dune height than the dune height predicted by the Fredsøe-expression in the calculation of the dune length, the steepness increased.

The calculated steepness, $\theta_{\text{top}}$, is found to agree very well with the results by Tjerry/Fredsøe, which are calculated with a k-ε turbulence closure as well as a Reynold Stress model. For the small Shields’ parameters, the calculated steepness are within the range of the predicted steepness by the two turbulence models, but the best fit is seen for a dune length of $L/D=4.6$. For the larger Shields’ parameters/dunes, a short dune length of 3-4 water depths is seen to fit the results by Tjerry/Fredsøe better than the longer dune.

3.4.6 Discussion

The dune height and steepness for the calculated dunes have been compared to the results by Fredsøe and Tjerry above. The physical mechanisms explaining the dune height and length are the relaxation of the flow downstream the crest, the effect of curvature acceleration on the upstream curved slope of the dune, and gravity-effects on the sediment transport on a sloping bed. The effect of the gravity parameter ($\mu$) on the dune length has not been investigated here, but Tjerry shows that the steepness of the dunes increases for a decreasing gravity parameter. A variation of $\mu = 0.05 - 0.20$ gives a change in the dune length within 20% of $\mu = 0.10$.

The variation in the dune height follows the physical mechanisms revealed by Fredsøe: the dune height for the calculated dunes approaches the Fredsøe-expression
for the longer dunes, where the variation in the friction coefficient at the top is weaker, and the assumption of equilibrium in the flow at the dune top is fulfilled. For shorter dunes, the dune height is lower, and the effect of the variation in the skin friction coefficient reduces the dune height to approximately half of the dune height predicted by Fredsøe for a dune length of $L_1 = 3$. Tjerry (1995) compared the height predicted by Fredsøe to some field data, which show a good agreement, but also a large scatter in the data.

No conclusions with regard to the natural dune height and length can be drawn by combining the results from the present work (predicting the dune height) with the work by Tjerry/Fredsøe (predicting the dune length). Both methods have, though, the limitation that only one of the parameters can be predicted under the assumption that the other parameter is known, but using either method the response of the bed can be explained by the same mechanisms.

The three turbulence models (k-ε and Reynolds Stress models as applied by Tjerry/Fredsøe and the k-ω model as applied in the present work) predict roughly the same steepness. Tjerry and Fredsøe used the dune height by Fredsøe, which at least for the short dunes in the present work is somewhat larger than the dunes predicted in the present work. For a given Shields’ parameter and the same steepness as predicted by Tjerry/Fredsøe, the dune length (or $L_1$) predicted by the present method is hence also smaller than the dune length predicted by Tjerry/Fredsøe. A test of the steepness dependence on the choice of the dune height in the Tjerry and Fredsøe paper showed that a smaller dune height for a given Shields’ parameter causes a larger steepness (even larger than the maximum steepness of 0.06 for $\theta_{top} > 0.15$), and can hence not
explain the difference.

The steepness for the calculated dunes is in agreement with the findings by Tjerry. For the lower Shields’ parameter, the steepness for the all the dunes in the present work is within the range of the uncertainty in the steepness by the choice of turbulence model in the work by Tjerry. No selective dune length can hence be chosen as the ”best fit” with regard to steepness between the two methods. For larger Shields’ parameters ( $\theta'_\text{top} > 0.1$) the predicted steepness by Tjerry fit the steepness of the dunes in the present calculation for a domain length, $L$, of $3-5$.

The effect of the separation zone, which is too short in the present calculations (as seen in Fig. 3.3 and Fig. 3.4), should be commented on. In the present calculations a larger separation zone would cause the dune height to be lower for a given dune length. The distance for the flow to obtain equilibrium after the reattachment zone would be reduced, and this causes, as shown above, the dune height to decrease (the gradient in the skin friction coefficient at the top is increased), when the dune length is fixed. This is mainly important for the higher dunes. For the low dunes, where the separation zone is low compared to the dune length, the too short modelling of the separation zone is not likely to have any effect on the dune height. The effect of the separation zone is actually found in the difference between the Upwind and Quick scheme for the convective terms in the momentum equations. Using the Upwind scheme, more numerical diffusion is added, which reduces the separation length (even more than in the present calculation). By switching from the Upwind scheme to the Quick scheme, the dune height was found to decrease. In a natural environment, where the dune length is set by natural mechanisms the situation would most likely be different. The effect of the separation zone would be to increase the dune length (rather than reduce the dune height), since the reattachment point occurs further downstream. In any case the steepness would be smaller for higher Shields’ parameters.

The tests performed by Tjerry, show that the choice of turbulence model is also quite important for the dune length. It could be interesting to perform morphological calculations with the $k-\varepsilon$ or the Reynolds Stress model, and compare with the present work. Furthermore comparison with experimental and field data should actually be performed. Focus in the work in the present chapter has hence been to test if the results from the morphological method were consistent with the results by Tjerry/Fredsøe.

Finally it should be mentioned, that several tests have been performed in order to determine the dune length using the morphological model - the work has however not been finished within the present study but some preliminary results are presented in Section 3.7.

### 3.5 Flow resistance

One of the most important issues with regard to bedforms is the resistance they oppose on the flow. Once the bed is not flat but covered by bedforms of some kind (ripples, dunes) the flow resistance exceeds the resistance caused purely by the skin friction between the grains and the water on a flat bed. This is due to the pressure distribution on the bedforms, and Einstein and Barbarossa (1952) hence argued that
the total flow resistance can be divided into two contributions, skin friction and form drag:

\[ \tau = \tau' + \tau'' \]  
(3.21)

or using the nondimensional Shields’ parameter:

\[ \theta = \theta' + \theta'' \]  
(3.22)

where \( \theta \) denotes the total resistance, \( \theta' \) the skin friction and \( \theta'' \) the form drag caused by pressure. The form drag arises from the uneven pressure distribution on the bedforms. Especially the pressure drag will be commented on in the following. Averaging the bed shear stress and the bottom pressure along one sand dune, these contributions are found to be:

\[ \theta' = \frac{1}{L} \left( \frac{\tau'_b}{(s-1)gd_{50}} \right) dx \quad \theta'' = \frac{1}{L} \left( \frac{P_b}{(s-1)gd_{50}} \right) dy \]  
(3.23)

The skin friction and form drag over the bedform can also be represented by resistance coefficients, i.e. the skin friction coefficient, \( C_f \), the form drag coefficient, \( C_p \), and total drag coefficient, \( C_d \):

\[ C_f = \frac{1}{L} \left( \frac{\tau'}{\frac{1}{2} \rho V^2} \right) dx \quad C_p = \frac{1}{L} \left( \frac{p}{\frac{1}{2} \rho V^2} \right) dy \]  
(3.24)

\[ C_d = C_f + C_p \]  
(3.25)

The total drag coefficient can also be found from the slope of the water surface, \( I \):

\[ C_d = \frac{\rho g DI}{\frac{1}{2} \rho V^2} \]  
(3.26)

As will be shown, especially the pressure drag is very sensitive and not easily calculated by the numerical model. Therefore results from the model are first compared to the experimental work of McLean, Wolfe and Nelson (1999) and the numerical work by Yoon and Patel (1996). The skin friction, form drag and the total flow resistance are evaluated for the calculated dunes for a dune length of \( \frac{L}{T} = 4 \) and \( \frac{L}{T} = 6 \). The results should be anticipated as preliminary - further calculations are required to ensure that the results are indeed independent of the choice of the grid. The results are compared to results by Engelund (1966, 1967). Furthermore the form drag is calculated by the Carnot formula.
3.5.1 The resistance model by Engelund

Engelund (1966 and 1967) proposed a model for the flow resistance over steady dunes in equilibrium with the flow. The model is semi-empirical and based on 1) a logarithmic resistance formula for the skin friction over a rough bed (Eq. 3.27) and 2) experimental data for dune beds in flumes. Eq. 3.27 was already briefly described in Chapter 2 and is recognized as being similar to the well-known resistance formula for a steady uniform flow over a rough channel bed. The only difference is that the water depth is exchanged with $D'$, which is interpreted as the boundary layer thickness.

$$\frac{V}{U'_f} = 6 + 2.5 \ln \frac{D'}{k_N} \quad U'_f = \sqrt{gD'I}$$

(3.27)

For a dune covered bed with a slope, $I$, and a mean velocity, $V$, the skin friction can be determined from Eq. 3.27 by evaluating the boundary layer thickness, $D'$ (or the friction velocity $U'_f$). The Shields’ parameter for the skin friction can then be found from:

$$\theta' = \frac{D'I}{(s-1)d}$$

(3.28)

Using the principles of geometric and dynamic similarity, Engelund (1966) found that the total resistance of a dune covered bed was a function of the non-dimensional skin friction:

$$\theta = \theta(\theta')$$

(3.29)

He tested this by analysis of experimental flume data for sand dunes (Fort Collins experiments by Guy, Simons and Richardson, 1966) - see Fig. 3.14 - and established a relation between the total flow resistance and the skin friction for steady dunes in equilibrium with the flow. Engelund calculated the dimensionless skin friction from Eq. 3.27, and found from the measurement over dunes in the lower flow regime the relation between $\theta'$ and $\theta$ to:

$$\theta' = 0.06 + 0.4\theta^2$$

(3.30)

The relation is shown in Fig. 3.14 together with experimental data. The total resistance for a dune covered bed can hence be found from Eq. 3.30, and the form drag then as the difference between the skin friction and the total resistance.

$$\theta'' = \theta - \theta'$$

(3.31)
3.5.2 The Carnot formula for expansion loss

The form drag can be estimated by the expansion loss (energy loss in the separation zone), when the flow passes the top of the dune. The Carnot formula for expansion loss is:

\[ \Delta H'' = \alpha \frac{(V_1 - V_2)^2}{2g} \]  

(3.32)

where \( V_1 \) and \( V_2 \) are respectively the mean velocity upstream and downstream the dune crest, and \( \alpha \) is the Carnot coefficient and is a factor of the order 1 depending on the velocity profiles. The energy loss per length of the dune is hence estimated by \( \frac{\Delta H''}{L} \), and the form drag of the bed can be predicted by:

\[ C_p = \frac{\rho g D \left( \frac{\Delta H''}{L} \right)}{\frac{1}{2} \rho V^2} \]  

(3.33)

Expressing \( V_1 \) and \( V_2 \) by the discharge and the local flow depth, \( V_1 = \frac{Q}{D - \frac{H}{2}} \) and \( V_2 = \frac{Q}{D^2 + \frac{H}{2}} \), an approximate expression for the form drag, \( \theta'' \), using the nondimensional values from the numerical model is:

\[ \theta'' = \alpha \left( \frac{H}{D} \right)^2 \frac{F_d^2}{2} \frac{1}{(s - 1) \frac{d}{D} \frac{L}{D}} \]  

(3.34)
Engelund (1977) found an empirical variation of $\alpha$ by comparison with the Fort Collins experiments:

$$\alpha = 2.5 \exp(-2.5 \frac{H}{D}) \tag{3.35}$$

3.5.3 Comparison with experimental results by McLean, Wolfe and Nelson

The form drag has been found to be sensitive to the calculation of the pressure just downstream the crest. The sensitivity in the numerical calculations of the separation zone and hence of the bottom pressure along the dune does mainly arise from:

- the problems of separation in the model (see Section 2.5.1)
- the grid
- the choice of the convective scheme (Upwind 1st order scheme, Quick scheme)

The inability in the model to separate right at the crest and instead ”force” the near-bed stream line around the corner of the crest, has significant impact on the bottom pressure as mentionend in section 2.5.1. The pressure has hence been found to be most precisely calculated excluding the front of the dune in the pressure calculations. The QUICK scheme were found to improve the calculation of the separation zone.

The pressure calculation is hence verified by comparison with experimental data. McLean, Wolfe and Nelson (1999) performed bottom pressure measurements along fixed bedforms using small pressure ports in the bed and measured the pressure difference between these ports and a reference port. They also measured the slope of the water surface. They used two types of bedforms (see Fig. 3.15): Type A are composed of a linear front slope of 30 degrees and a cosine shaped upstream slope, Type B are composed of linear pieces of different slope - this type of bedform was also used by Mierlo and de Ruiter (1988) for experimental work.

Two different setups of the numerical model is tested for calculation of the pressure along the dune. These two different setups were previously introduced in Chapter 2 (Section 2.5.1, see Fig. 2.6) One (with front) is the regular setup used in the morphological calculation, where the front is included. In the second method (without front), the sloping front is excluded and instead a vertical front is added at the location of the crest. This enables to make a grid, without a strong curvature of the near-bed grid-lines at the crest. The boundary conditions is handled by interpolation to get the values in the ghost cells above the crest at the boundary, and a wall condition is applied on the vertical wall, which is exchanged for the front. For the pressure just after the crest this set-up has little or no influence, since the pressure is governed by the separation zone and not the shape of the bed at this location. For the numerical calculations the grid without the front slope give better results.
<table>
<thead>
<tr>
<th>Run</th>
<th>Type</th>
<th>H [mm]</th>
<th>L [mm]</th>
<th>D [mm]</th>
<th>V [m/s]</th>
<th>Fr</th>
<th>Re</th>
<th>$k_N/D$</th>
<th>$H/\rho$</th>
<th>$L/\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>A</td>
<td>40</td>
<td>807</td>
<td>158</td>
<td>0.39</td>
<td>0.31</td>
<td>61620</td>
<td>0.001</td>
<td>0.253</td>
<td>5.01</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>40</td>
<td>807</td>
<td>546</td>
<td>0.28</td>
<td>0.55</td>
<td>152880</td>
<td>0.001</td>
<td>0.073</td>
<td>1.47</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>40</td>
<td>408</td>
<td>300</td>
<td>0.54</td>
<td>0.31</td>
<td>162000</td>
<td>0.001</td>
<td>0.129</td>
<td>1.32</td>
</tr>
<tr>
<td>T5</td>
<td>B</td>
<td>80</td>
<td>1600</td>
<td>252</td>
<td>0.44</td>
<td>0.28</td>
<td>110880</td>
<td>0.001</td>
<td>0.317</td>
<td>6.34</td>
</tr>
<tr>
<td>T6</td>
<td>B</td>
<td>80</td>
<td>1600</td>
<td>334</td>
<td>0.55</td>
<td>0.30</td>
<td>183700</td>
<td>0.001</td>
<td>0.24</td>
<td>4.79</td>
</tr>
</tbody>
</table>

Table 3.2 Experimental parameters for experiments by McLean, Wolfe, Nelson, 1999, and nondimensional parameters for corresponding numerical calculations

Pressure variation along the bedforms

Five of the experiments performed by McLean, Wolfe and Nelson are chosen for verification of the pressure calculations. The flow-parameters for these experiments and the corresponding nondimensional parameters for the numerical calculations are given in Table 3.2. The roughness $k_N/D$ is set by the author. A higher roughness has also been tested and but has little or no influence of the calculated pressure.

The bottom pressure for run 2 along the dunes are shown in Fig. 3.16 along with the results by McLean, Wolfe and Nelson. The expected variation of the pressure along the bottom is found in the measurements as well as the numerical calculations. The pressure at the top of the dune is low due to the high streamwise velocities. This low pressure is also found just downstream of the crest and along the front slope, because of the flow structure in this area: right after the crest the streamlines above the crest are still horizontal due to the flow separation and hence the flow above the separation zone is not subject to diverging flow conditions until further downstream. The divergence of the streamlines when the height of the separation zone decreases causes the pressure to increase. The maximum pressure is found around the reattachment point - located around 5-6 dune heights downstream of the crest - after which the pressure starts to decrease again towards the top due to the contraction of the streamlines.

The QUICK scheme and a first order Upwind scheme for the convective terms in the flow equations are tested. This variation of the pressure is predicted most precisely with the QUICK scheme in the situation with the front as well as without the front. Using the upwind scheme, more numerical diffusion is introduced than when using the QUICK scheme, and this decreases the height and the length of the separation zone. The effect of the upwind scheme is mostly pronounced in the situation with the front slope and less so when in the situation when the front is presented by a vertical wall. This has probably to do with the grid: in the situation with the front the cells are bent around the crest, and the flow tends to be forced around the crest instead of separating exactly at the corner. In the case of cells with a very uneven relation between the height and length as is the case at the bottom where the boundary layer must be resolved, this effect is more pronounced (personal communication with Jess Michelsen, Fluid Mechanics Section, MEK-DTU).

Of course this effect is present in the case of using the Quick scheme as well as the Upwind scheme.

For the pressure calculations where the front is included, the above presented
problems means that 1) the separation point is located slightly downstream of the crest when the front is included. To generate curvature acceleration to force the streamlines around the crest, a low pressure at the crest is needed - this is the reason for the extreme low pressure seen in the calculated pressure right at the crest. 2) the height and the length of the separation zone is in general underpredicted by the model. The streamlines are found to diverge right downstream the front (especially using the upwind scheme) and hence the pressure will start to increase as seen in Fig. 3.16 instead of keeping at the low pressure level at the top. The pressure on the front slope hence tends to be overestimated. 3) The short separation length means that along the upstream slope the pressure is too high and the location of the maximum pressure is too close to the front. The total pressure along the upstream slope of the bedform is therefore too high, but due to the nearly horizontal bedform in the area where the error is largest - between the trough and the reattachment point - the influence is not very large in the integrated pressure as will be seen in the following.

When these numerical effects in the predictions of the pressure has been evaluated the conclusion is still that the pressure distribution is fairly well predicted when the Quick scheme is used - for the situation with and without the front - and also with the upwind scheme, when the front is excluded. The numerically predicted bottom pressure is compared to the measured pressure for run 2, 3, 6, T5 and T6 is shown in Fig. 3.17.

**Resistance coefficients**

The total skin friction and form drag on the bedform are calculated by integrating the shear stress and the pressure along the bedforms (using Eq. 3.23) and compared to the results evaluated by McLean, Wolfe and Nelson (1999). The front slope is excluded as described above. The results are shown in Table 3.3. In Table 3.4 the numerically predicted slope is listed along with the experimentally found slope of the water surface. In case of the numerically predicted slope, two values are given: the output of the model, $S_{Dune}$, and the slope found using the momentum principle, $S_{mom,Dune}$, by integration of the skin friction and the pressure on the bottom, i.e. calculating the slope from Eq. 3.26 using Eq. 3.24 and Eq. 3.25.

The following comments are related to the results in Table 3.3 and 3.4:

- The real bedform, i.e. with the front, is used for the integration of $C_f$ and
Figure 3.16  Comparison of bottom pressure measurements (McLean, Wolfe, Nelson(1999)) with numerical calculations with/without the dune front (see text) and with respectively Upwind and Quick Scheme for convective terms in momentum equations.

Figure 3.17  Comparison of bottom pressure over dunes measured by McLean, Wolfe and Nelson, 1999 with Dune2D calculations (excluding the front).
Flow resistance

<table>
<thead>
<tr>
<th>Run</th>
<th>( C_f, \text{Dune} )</th>
<th>( C_f, \text{MWN} )</th>
<th>( C_p, \text{Dune} )</th>
<th>( C_p, \text{MWN} )</th>
<th>( C_d, \text{Dune} )</th>
<th>( C_d, \text{MWN} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.0029</td>
<td>0.0084</td>
<td>0.0123</td>
<td>0.0109</td>
<td>0.0152</td>
<td>0.0193</td>
</tr>
<tr>
<td>2 (with front)</td>
<td>0.0030</td>
<td>0.0084</td>
<td>0.0095</td>
<td>0.0109</td>
<td>0.00126</td>
<td>0.0193</td>
</tr>
<tr>
<td>3</td>
<td>0.0022</td>
<td>0.0056</td>
<td>0.0071</td>
<td>0.0056</td>
<td>0.0093</td>
<td>0.0112</td>
</tr>
<tr>
<td>6</td>
<td>0.0017</td>
<td>0.0066</td>
<td>0.0173</td>
<td>0.0140</td>
<td>0.0190</td>
<td>0.0206</td>
</tr>
<tr>
<td>T5</td>
<td>0.0030</td>
<td>0.0134</td>
<td>0.0206</td>
<td>0.0169</td>
<td>0.0236</td>
<td>0.0243</td>
</tr>
<tr>
<td>T6</td>
<td>0.0027</td>
<td>0.0099</td>
<td>0.0146</td>
<td>0.0134</td>
<td>0.0173</td>
<td>0.0206</td>
</tr>
</tbody>
</table>

Table 3.3  Cf, Cp and Cd coefficients calculated by Dune and compared to values from experimental work by McLean, Wolfe, Nelson, 1999

<table>
<thead>
<tr>
<th>Run</th>
<th>( S_{\text{Dune}}(\times 10^4) )</th>
<th>( S_{\text{mom,Dune}}(\times 10^4) )</th>
<th>( S_{\text{MWN}}(\times 10^4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6.9</td>
<td>7.3</td>
<td>9.5</td>
</tr>
<tr>
<td>3</td>
<td>0.64</td>
<td>0.67</td>
<td>0.82</td>
</tr>
<tr>
<td>6</td>
<td>8.9</td>
<td>9.1</td>
<td>10.2</td>
</tr>
<tr>
<td>T5</td>
<td>9.3</td>
<td>9.3</td>
<td>9.6</td>
</tr>
<tr>
<td>T6</td>
<td>7.5</td>
<td>7.8</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Table 3.4  Slope found numerically compared to the experimentally found slope by McLean, Wolfe, Nelson, 1999

\( C_p \) from the skin friction and pressure distribution, even though the front is excluded in the numerical calculations. It is assumed that the pressure and the skin friction is the same at the wall on the sloping front, as the calculated on the flat bed. Due to the separation, the error is small.

- The total resistance coefficient, \( C_d \), for the numerical calculations are found by Eq. 3.25

- \( C_p \) by McLean, Wolfe and Nelson are found from their Table 2, where the integrated pressure along the bedform is given.

- The total resistance coefficient, \( C_d \), for the results by McLean, Wolfe and Nelson are found from \( pgSD \) (in their Table 2), however, as also mentioned in their paper, this drag coefficient also includes the friction on the sidewalls of the flume used in the experiments and should be expected to be lower.

- The skin friction coefficient for McLean, Wolfe and Nelson, has here been estimated by the difference between \( C_d \) and \( C_p \). In reality this means that the friction from the sidewalls will be included in \( C_f \) - a smaller \( C_f \) value would be expected.

The numerically calculated resistance coefficients are seen to be in reasonable agreement with the experimentally found resistance coefficients. The slope found in the numerical calculations are on average 15 % lower than the experimentally found slope by McLean, Wolfe and Nelson. The disagreement can partly be due to the friction on the sidewalls in the flume used in the experiments. The form drag coefficients found numerically are on average 20 % larger than found in the
experiments, and the skin friction coefficients are smaller than the experimentally found, but since this coefficient includes the sidewall friction this is expected.

3.5.4 Comparison by numerical calculations by Yoon and Patel

The reason for comparing the results for resistance coefficients for flow over dunes from the present model with results found by Yoon and Patel (1996) are two-folded. One reason is to verify the pressure calculations and the resistance coefficients with results to another computational model. Yoon and Patel uses a numerical model much like the present (the only difference seems to be the pressure scheme) to calculate the flow over two dimensional sand dunes and presents results for the resistance coefficients for dunes of varying steepness. They compare these coefficients to the results calculated by the resistance formula of Engelund (1966, 1967) (see Section 3.5.1). In their paper, they find that the results compare very poorly and conclude that the total resistance is grossly underestimated by the Engelund model, and that the formula splits the contribution of skin friction and form drag incorrectly. The second reason is hence to investigate these conclusions.

The findings in this sections show that the numerically calculated resistance coefficients match those of Yoon and Patel well. The comparison with the method of Engelund, however, is not appropriate for the given dunes - at least the formula must be used in a different way.

Yoon and Patel (1996) calculates the flow over dune shaped bedforms like Type B in Fig. 3.15. This type of bedform is also used by Mierlo and de Ruiter (1988) with whose experimental results Yoon and Patel are verifying their model. They perform a parametric study of the resistance coefficients of dune-bed channels in which they vary the steepness of the dunes, by changing the height as well as the length of the dunes. Four dunes, which cover the dune height, length, and steepness range tested by Yoon and Patel, have been chosen for the present verification, see Table 3.5. The mean water depth, the grain size and the mean bulk velocity above the crest are kept constant in all the calculations by Yoon and Patel:

- Mean bulk velocity above crest: \( U_0 = 0.633 \text{ m/s} \)
- Mean velocity: \( V = 0.557 \text{ m/s (run 1 and 3)} \) and \( V = 0.519 \text{ m/s (run 2 and 4)} \)
- Mean Water depth: \( D = 0.332 \text{ m} \)
- Median grain diameter: \( d = 1.6 \text{ mm} \)
- Kinematic viscosity \((18^\circ)\): \( \nu = 1.06\times10^{-6} \text{ m}^2/\text{s} \)

The mean velocity (for the mean water depth) has been found from the bulk velocity. For the non-dimensional values for the present model, this means:

- \( k_N/D = 0.00482 \) ( \( k_N = d \), in this case, since this is used by Yoon and Patel)
- \( Fr = 0.31 \) (run 1 and 3) and \( Fr = 0.287 \) (run 2 and 4)
- \( Re = 174456 \) (run 1 and 3) and \( Re = 156290 \) (run 2 and 4)
Flow resistance

<table>
<thead>
<tr>
<th>Run</th>
<th>Dune height [m]</th>
<th>Dune Length [m]</th>
<th>H/D</th>
<th>L/D</th>
<th>H/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>1.6</td>
<td>0.24</td>
<td>4.8</td>
<td>0.050</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
<td>1.6</td>
<td>0.36</td>
<td>4.8</td>
<td>0.075</td>
</tr>
<tr>
<td>3</td>
<td>0.08</td>
<td>1.0</td>
<td>0.24</td>
<td>3.0</td>
<td>0.080</td>
</tr>
<tr>
<td>4</td>
<td>0.12</td>
<td>1.0</td>
<td>0.36</td>
<td>3.0</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Table 3.5 Dune properties for numerical calculation by Yoon and Patel, 1996.

Figure 3.18 $U_f/V$ as measured by Mierlo and Ruiter (1988) (from Yoon and Patel, 1996) along bed form, compared with Dune2D calculations.

Figure 3.19 Comparison of $c_f$ and $c_p$ values as calculated numerically by Yoon and Patel, 1996, and Dune2D. Data from their Fig. 5.
Flow resistance

<table>
<thead>
<tr>
<th>Run</th>
<th>$C_f$, Dune2D (excl. front)</th>
<th>$C_f$, Dune2D (incl. front)</th>
<th>$C_f$, YP (incl. front)</th>
<th>$C_f$, Engelund*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0040</td>
<td>0.0041</td>
<td>0.004</td>
<td>0.0084</td>
</tr>
<tr>
<td>2</td>
<td>0.0046</td>
<td>0.0041</td>
<td>0.004</td>
<td>0.0097</td>
</tr>
<tr>
<td>3</td>
<td>0.0033</td>
<td>0.0033</td>
<td>0.003</td>
<td>0.0087</td>
</tr>
<tr>
<td>4</td>
<td>0.0038</td>
<td>-</td>
<td>0.003</td>
<td>0.00102</td>
</tr>
</tbody>
</table>

Table 3.6 Friction coefficients. *See method in Section 3.5.4.

<table>
<thead>
<tr>
<th>Run</th>
<th>$C_p$, Dune2D (excl. front)</th>
<th>$C_p$, Dune2D (incl. front)</th>
<th>$C_p$, YP (incl. front)</th>
<th>$C_p$, Engelund*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0144</td>
<td>0.0087</td>
<td>0.011</td>
<td>0.0103</td>
</tr>
<tr>
<td>2</td>
<td>0.0303</td>
<td>0.0246</td>
<td>0.019</td>
<td>0.0251</td>
</tr>
<tr>
<td>3</td>
<td>0.0188</td>
<td>0.0150</td>
<td>0.017</td>
<td>0.0133</td>
</tr>
<tr>
<td>4</td>
<td>0.0397</td>
<td>-</td>
<td>0.028</td>
<td>0.0331</td>
</tr>
</tbody>
</table>

Table 3.7 Form drag coefficients. *See method in Section 3.5.4.

The flow field is calculated using the same approach as in the comparison with the experimental results by McLean, Wolfe and Nelson: the flow has been calculated using the same two types of grids, respectively including and excluding the front slope. Yoon and Patel calculates their results including the front and uses a very similar grid.

Verification of friction and pressure coefficients The friction velocity is compared to the measurements by Mierlo and de Ruiter (1988) in Fig. 3.18 and the computed distribution of the friction and pressure coefficient calculated along the dune (run 1) are shown in Fig. 3.19 compared to the distribution calculated by Yoon and Patel. Agreement between the results are found using both types of grid for the friction coefficient. Excluding the front, the flow obviously is different in the area close to the bed and this shows up in the friction coefficient. The length of the separation zone is over-predicted compared to the measurements by Mierlo and de Ruiter, when the front is excluded. Including the front, the pressure increases too close to the crest. This was also seen to be the case in the pressure calculation by Yoon and Patel. Comparison with the experimental data by McLean, Wolfe and Nelson (1999) in the previous section showed that the increase in the pressure related to the divergence of the streamlines are predicted too close to the crest using this calculation method. For the calculations excluding the front, the pressure distribution along the bed were found to have a more reliable shape.

The resistance coefficients are shown in Table 3.6-Table 3.8. Convergence could not be reached for the flow over the very high and steep dune in run 43, when the front is included. The resistance coefficients obtained with Dune2D are very similar to the ones computed by Yoon and Patel. The form drag coefficients, and hence the total drag coefficients, calculated without the front, show that the calculations including the front predict the form drag coefficients 17-30% lower than the drag coefficient, when the front is excluded.
### Comparison with the Engelund resistance model

Yoon and Patel (1996) compares their results to the Engelund model. They conclude in their paper that the Engelund method underestimates the total resistance grossly and splits the contributions from the skin friction and form drag wrong.

Since the Engelund method used as described in Section 3.5.1 (and in the paper by Yoon and Patel) is only applicable for dunes, which have the dimensions and the shapes which are in equilibrium with the flow, the method is not appropriate for calculating the flow resistance over random dunes. Even though the dunes, used by Yoon and Patel, are within the dune regime with regard to the steepness, they are not in equilibrium with the flow field.

The methodology by Engelund can however be used in a slightly different way to split up the contributions of skin friction and form drag for ”random” dunes, when the slope and the water depth are given:

1. The skin friction is evaluated by Eq. 3.27 and 3.28.

2. The total resistance is found from: \( \theta = \frac{Dl}{(s-1)d} \)

3. The form drag is calculated as the difference between the total resistance and the skin friction: \( \theta'' = \theta - \theta' \)

Using the slope found numerically, the \( C_f \) and \( C_p \) coefficients are calculated - these are included in Table 3.6-3.8. The skin friction is seen to be overestimated by the Engelund method and hence the form drag is underestimated compared to the values found by the present model. For very steep dunes like those used by Yoon and Patel, Eq. 3.27 is over-estimating the skin friction since the separation extent over a relatively large part of the dune length. The analogy to a channel flow is then less applicable. Since the numerically found slope from Dune is used, the total drag coefficient is naturally the same.

It should be noted here that personal communication with Yoon revealed that some data in the paper (including the Engelund resistance coefficients) were misplaced. In an internal (earlier) report from the department (Report No. 362, Iowa Inst. of Hydraulic Research, Univ. of Iowa, 1993, June), different numerically calculated slopes are given, and different resistance coefficients were found using the Engelund method, and the conclusion hence not the same as in the paper. This does however not change the key point in this section that the Engelund method using the semi-empirical relation between the total resistance and skin friction as found by Engelund, is not applicable to these dunes.

<table>
<thead>
<tr>
<th>Run</th>
<th>( C_d \text{Dune2D} ) (excl. front)</th>
<th>( C_d \text{Dune2D} ) (incl. front)</th>
<th>( C_d \text{YP} ) (incl. front)</th>
<th>( C_d \text{Engelund}* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0435</td>
<td>0.0387</td>
<td>0.031</td>
<td>0.0433</td>
</tr>
<tr>
<td>2</td>
<td>0.0221</td>
<td>0.0183</td>
<td>0.021</td>
<td>0.0220</td>
</tr>
<tr>
<td>3</td>
<td>0.015</td>
<td>0.0129</td>
<td>0.015</td>
<td>0.0186</td>
</tr>
<tr>
<td>4</td>
<td>0.0394</td>
<td>0.031</td>
<td>0.031</td>
<td>0.0348</td>
</tr>
</tbody>
</table>

Table 3.8  Total drag coefficients. *See method in Section 3.5.4.
3.5.5 Flow resistance for the calculated dunes

The investigations with respect to the flow resistance include several interesting questions to be answered: Most important is of course how the flow resistance compares with the flow resistance for the experimentally found dunes (the Fort Collins data analyzed by Engelund, 1966,1967). But it is also interesting to see how the drag compares with the expansion loss that can be calculated by the Carnot formula. Another question is how the flow resistance changes with the length of the calculated dunes. Since the ”natural” length of the dunes has not been found, the influence of the dune length on the flow resistance should be investigated.

It turned out to be slightly problematic to calculate the flow resistance for the computed dunes. The results presented here should hence be anticipated as preliminary. The pressure distribution along the dune was found to be grid dependent to some degree. It is not exactly clear what the problem is. A higher Reynolds number in the present calculations than in the comparison with Yoon and Patel and Mclean, Wolfe and Nelson, requires a more refined grid at the bed in the k-ω model. This might be a part of the problem and should be tested. The problem arises around the crest area. The method sketched in the two previous sections seems overall to work well as shown. Improvements in the (special) period boundary condition applied only in these flow calculations and verification of the grid dependency, however, seems necessary and has not been finished due to a lack of time within the present study.

Methodology

The flow resistance has been found roughly by the same method as outlined above in the comparison with McLean, Wolfe and Nelson, 1999, and Yoon and Patel, 1996, which means calculating the flow along the dunes excluding the front. A few additional comments should be added.

Location of the front

The location of the front is important for the pressure calculation as will be revealed. As mentioned in the discussion about the dune shape there is uncertainty about whether the location of the crest is as found in the morphological calculations or closer to the top. The two extremes positions of the crest are therefore tested, see Fig. 3.20:

- A: At the crest as found in the morphological calculations, i.e. where the curvature of the bedform is largest, and

- B: At the top of the dune.

Grid generation

The dunes used in the experiments by McLean, Wolfe and Nelson and also in the numerical calculations by Yoon and Patel have a horizontal tangent at the top just before the crest. This is not the case for the calculated dunes, especially for the low transport rates. They have a slightly curved shape between the top and the crest. A special method is therefore needed for generating the grid. The grid is therefore created by the method sketched in Fig. 3.21. First the bedform is mirrored at the
Figure 3.20  Bedforms used for flow resistance calculations.  A: the entire dune from morph. calculations is used but front is exchanged with vertical wall.  B: the dune stops at the top and front is a vertical wall.

crest. Then a grid is created with the grid generator for this double bedform. Half of the grid is used in the flow calculations. This creates a grid, which has vertical boundaries at the inlet and outlet - as necessary for the special periodic boundary condition as applied for the pressure calculation here. It is not able to get a fully orthogonal grid close to the right boundary using this method. Since the angle at the front is small this only creates a minor error in the flow field according to personal communication with Jess Michelsen, MEK-DTU.

Results for the flow resistance - comparison with Engelunds resistance model

The total flow resistance, $\theta = \theta' + \theta''$, for the dunes in Fig. 3.7 for dune lengths of $L/D = 4$ and $6$ are presented in Fig. 3.22. The nondimensional skin friction, pressure drag and total resistance are calculated according to Eq. ?? and Eq. 3.23 by methods A and B as sketched in Fig. 3.20. The difference in the location of the crest reduces the skin friction slightly when the crest is located on the top of the dune - the flow resistance curves are shifted to the left, when using method B. The reason is that the length of the separation zone, where the skin friction is low, increases when the front increases in height. The location of the crest, however, has a larger effect on the pressure drag on the dunes. The energy loss behind the dune increases, when the height of the separation zone (starting from the crest) increases. Method B hence predicts a larger flow resistance than method A. Furthermore the length of the dune is reduced, when using method B, which means that the 'energy loss per length of the river/sea bed' expressed in $\theta''$, increases.

The flow resistance is nearly the same for the dunes of different length if the
Figure 3.21  Grid generating method for pressure calculations where the front is excluded. Upper: the bed is mirrored at the crest and a "double" bed is generated. Lower: half of the grid is used in the calculation (A finer grid than shown is used).

nondimension skin friction is the same. The dune height increases with an increasing dune length. Even though the drag is larger on the highest dunes, the average drag per length is nearly the same since the larger dunes are also longer.

In comparison with the Engelund model outlined in Section 3.5.1, the results for method A have a lower total flow resistance for the smaller values of $\theta'$, while the results by using method B have a better agreement. The difference is hence the location of the front. For the higher values of $\theta'$, methods A and B both predict a larger flow resistance than the Engelund method, with method B giving a larger flow resistance than method B.

**Comparison with the Carnot loss**

The pressure drag for the computed dunes is calculated by the Carnot loss formula, Eq. 3.34. The dune height $H'$ in the equation is taken as the height at the crest in both methods A and B. By using the nondimensional skin friction found from the flow calculations (methods A and B), the total resistance is found from Eq. ???. In Fig. 3.23 a comparison with the total flow resistance found by using Eq. 3.23 is shown, where $\theta'$ is found from the bed shear stress in dune and $\theta''$ is found from the Carnot loss Eq. 3.34. The Carnot-coefficient, $\alpha$, is set to 1.0 (left hand side subfigure) and the empirically expression found by Engelund in Eq. 3.35 (right hand side subfigure).

The choice of the $\alpha$—coefficient is seen to be important for the prediction of the form drag by the Carnot loss method. When using the $\alpha$—coefficient found by
Engelund (1966, 1967)

L/D=4, A
L/D=4, B
L/D=6, A
L/D=6, B

Figure 3.22 Total flow resistance for calculated dunes compared to Engelund’s resistance formula. L/D=4 and L/D=6. A: crest located as found in morphological calculation. B: crest located at the top of the dune.

Engelund, the fit to the total flow resistance is very good.

Discussion

The form drag calculations are found to be sensitive to the height of the crest above the trough level. Effects of the numerical filter applied in the morphological calculation and of the horizontal grid-spacing are most pronounced in the area of the crest. The two extreme locations of the crest were hence tested in the calculations of the flow resistance: 1) as computed with the morphological model and 2) at the top of the dune. This dependence on the crest height is found when using the numerical model as well as the Carnot loss formula for the prediction of the form drag.

The flow resistance is compared to the flow resistance presented by Engelund (1966, 1967). For dunes for the large Shields’ parameters, the numerically calculated flow resistance exceeds the prediction by the Engelund resistance model. For the small dunes, the flow resistance computations with the two crest locations were found to be respectively higher (crest at the top of the dune) and lower (crest as computed in the morphological model) than predicted by the Engelund model. The best agreement with the Engelund model for the smaller dunes, however, is found with the crest located at the top.

When the form drag were estimated by the Carnot formula, and the Carnot-coefficient found empirically by Engelund (1977) was applied, nearly identical results with the numerically calculated flow resistance are obtained for the total flow resistance.

The two dune lengths tested show nearly the same total flow resistance for
the entire variation of the Shields’ parameter. Although the flow resistance for the computed dunes do not compare well the Engelund formula, the dunes seem to follow the relation of $\theta(\theta')$ found by Engelund using similarity principles.

3.6 Numerical issues

Some attention should be payed to numerical issues. First of all since numerical problems are one of the major problems in morphological modelling in general, so experience gained should be shared. Second, because it serves to verify the presented results.

Equilibrium - influence of the avalanche process and the numerical filter

The equilibrium condition (Eq. 3.6) is not fulfilled by the bed load field caused by the skin friction alone as found in Section 3.3. The avalanche process and the numerical filter are needed to ‘fulfill’ the equilibrium condition for the calculated dunes. The proportionality between the bed load and the local dune height, can also be described by Eq. 3.2 for steady migrating dunes ($\frac{dh}{dt} = 0$), and yield:

$$\frac{\partial h}{\partial t} = -\frac{\partial h}{\partial x}$$

(3.36)

which simply states that the local erosion/deposition (given by $\frac{\partial h}{\partial t}$) must equal the local change of the bed due to the downstream migration ($\frac{\partial h}{\partial x}$). The filter and the avalanche function in the morphological calculation procedure are both applied to the bedform after the bed is updated according to the divergence in the bed load field (as calculated from the skin friction). The change in the bed within one time step, $\frac{\partial h}{\partial t}$, also includes an effect of these two steps in the morphological model. The equilibrium
of the bed is studied from the point of equation 3.36 in the following (rather than 
Eq. 3.6 even though the sense of the two equations are the same).

Including the effect of avalanching and the numerical filter, the rate of change 
of the bed can be given as:

$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial t}_s + \frac{\partial h}{\partial t}_a + \frac{\partial h}{\partial t}_n$$

(3.37)

- where $\frac{\partial h}{\partial t}_s$ is the rate of change of the bed due to the divergence in the bed load field 
as calculated from the skin friction, $\frac{\partial h}{\partial t}_a$ is the rate of change of the bed caused by the 
avanche function and $\frac{\partial h}{\partial t}_n$ is the effect on the bed change cause by the numerical 
filter. The calculated bedforms are naturally the equilibrium bedforms for the sum 
of these terms, $\frac{\partial h}{\partial t}$. The relative influence on each of these terms is therefore 
important, since to accept the bedforms as ”physical” rather than ”numerical”, the 
umerical term ($\frac{\partial h}{\partial t}_n$ ) should be small in comparison with the physical terms ($\frac{\partial h}{\partial t}_s$ 
and $\frac{\partial h}{\partial t}_a$). In Fig. 3.24 the relative influence of these terms is compared to the right 
hand side of Eq. 3.36 for the front area and the upstream gentle slope respectively 
in a given time step for a steady dune. The migration velocity, a, is calculated as the 
migration of the center of gravity.

In the upper-left figure in Fig. 3.24, the effect of the avalanche at the front 
is seen. Erosion from a point on the upper part of the front slope ($\frac{7}{H} \approx 23.4$) takes 
place and deposition is seen one point further down ($\frac{7}{H} \approx 23.7$). The rate of change 
of the bed caused by the avalanche function is comparable to the deposition further 
up the front slope as calculated from the skin friction. Since the avalanche process 
is the only physical process to move the sediment down the slope this is a good sign. 
Eq. 3.36 is however not fulfilled by including only $\frac{\partial h}{\partial t}_s + \frac{\partial h}{\partial t}_a$ in $\frac{\partial h}{\partial t}$. This can 
partly be explained by the implementation of the avalanche process. The avalanche 
function works, as described in Section 2.3.2, by adjusting the bed slope, when he 
slope exceeds 30.5°, such that the bed slope is less than the angle of repose (29.5°). 
This approach causes a stepwise correction of the bed slope, which does not take 
place in every time step. In the next step the correction might take place a grid 
point further down, or not take place at all.

In the upper-right figure in Fig. 3.24, the effect of the filter is included in $\frac{\partial h}{\partial t}$. 
The smoothing process is necessary partly due to numerical wiggles, which in the 
present work are unavoidable without the filter, but also due to the local behavior 
(the 'peak') in the flow (and thus the sediment transport and erosion/deposition) 
at the crest. After the bed is filtered, the actual rate of change of the bed within 
the given time step as calculated by the skin friction distribution and the avalanche 
model, is smoothed out at the front. The filter causes an 'artificial deposition' on the 
lower part of the front which cannot be explained by the skin friction and avalanche. 
Furthermore, the filter smooths out the effect of the peak mentioned above. The rate 
of change of the bed is now much closer to $a \frac{\partial h}{\partial x}$, although some deviance is still seen 
on the upper part of the front. The variations from time step to time step in the 
avanche model and the filter account for this.
Figure 3.24  Influence of the filter and the avalanche process on the equilibrium situation for a steady dune (s5L4).

In the lower figure equilibrium is shown along the upstream gentle slope of the dune and until after the top of the dune (approx. four dune heights downstream the top in the present calculation) is shown. \( \frac{\partial h}{\partial t} \) is seen do be in equilibrium with \( a \frac{\partial h}{\partial x} \) and the filter effects are negligible in that area. In the front area, the morphological filter smooths out the erosion/deposition pattern and influences the bedform shape at the lower part of the front slope.

An important question is, whether the migration velocity is influenced by the filter. Uncertainty in the migration velocity might cause an uncertainty in the total dune height, since equilibrium requires Eq. 3.36 to be fulfilled. Since the filter has no influence between the trough and just upstream of the crest any influence of the filter is restricted to the area of the steep slope. Furthermore, for continuity reasons the bed load transported through the cross section just upstream from the crest, where the influence of the filter is first noticed, must be deposited at the front, since the bed load in the separation area is zero. The filter might therefore influence the shape of the front area, but not the migration velocity, since it is the volume of the deposition in that area that governs the migration velocity.
Influence of morphological time step

For small bedforms/low sediment transport rates the morphological Courant criterion (2.17) is further from being fulfilled, than for the larger bedforms/higher sediment transport rates. Furthermore, when starting out with a sinusoidal bedform a rather small time step must be used until the bedform has grown. Then the time step can be increased. Intuitively this is understood as being due to the morphological growth of the bedform - see Fig. 3.1. Small superimposed perturbations migrate along the bedform much faster than the bedform actually migrates and thus a lower morphological time step is needed according to 2.17.

The filter is applied at each time step, which means that when the morphological time step is increased the relative influence of the filter is reduced. The effect of the filter on equilibrium (and within one time step) was shown above. In Fig. 3.25 the filter on the morphology is shown when the time step is small (a factor of 20) and the effect of the filter on the equilibrium dune is larger. Especially the front and trough are smoothed by the filter and the bedform height is 13 % higher for the low time step.

Influence of horizontal grid spacing

Reducing the grid spacing in the horizontal direction has a similar effect as increasing the morphological time step as seen in Fig. 3.26. The front area and trough are more "rounded" when the grid spacing is larger and with a refined grid, the bed becomes slightly more asymmetrical and the bed slope approaches angle of repose. The difference in the dune height in Fig. 3.26 is 6 %.

Influence of trough correction to filter

The influence of the trough correction routine (outlined in Section 2.3.3) on the shape of a steady dune is shown in Fig. 3.27. Since "artificial erosion" takes place in the trough without the trough correction, the dune does not have the horizontal trough.
Figure 3.26  Effect of reducing the grid spacing in the horizontal direction. $\theta'_{u} = 0.11$. 

Figure 3.27  Influence of the trough correction routine on the steady bedform. $\theta'_{u} = 0.11$. $\frac{L}{D} = 4$. 
Figure 3.28 Numerical 'wiggles' in the morphology occur when a too large time step is used due to instabilities in the flow.

**Appearance of instabilities**

When the morphological time step is too large, wiggles appear. When calculating the flow with Dune2D along a sinusoidal bed instabilities in the flow just before the trough and just before the top can be found. Instabilities in the flow are very small, but can be identified in the forth derivatives of the velocity-components. The reason for these instabilities to occur in the flow is not clear, although several possibilities have been tested (among these instabilities in the grid, the relaxation coefficients in the flow solver, and the convective scheme). Instabilities in the morphology occur due to these instabilities. A typical example is shown in Fig. 3.28. The wiggles have a length of several grid points, and are hence not on ”grid-scale”, which makes them hard to remove by a numerical filter.

### 3.7 Open ends

#### 3.7.1 Dune length and steepness determination

The dune length in the analysis of the dune height is set by the domain length, and hence it is not a natural dune length. As mentioned previously no clear conclusions regarding the combination of the dune length and the dune height can be made from the work already presented. The results and considerations presented in the following shortly summarize the work that has been performed within the present study on the subject of dune length determination. The work has not been finished within this study.
First it should be mentioned that when calculating the steady dunes for L/D=8 and \( \theta_u = 0.20 \) and \( \theta'_u = 0.40 \), the bedform was not found to be stable. The bedform obtains a strange shape at the top/crest of the dune. The results have not yet been further investigated, but they might indicate a too long domain for the dunes to be steady.

The easiest way of finding the dune length, using numerical calculations, would be by using a very long domain (ex. 10 times the expected dune length) and let the morphological changes of the bedform choose its own length. This, however, is time-consuming due to a very long calculation time (months). Instead a number of other test-cases are studied. The main objectives of these are:

1. Study the interaction among sand dunes of different height and length
2. Study the splitting mechanism of dunes, observed when a long domain is tested

(1) aims at understanding the physical mechanisms responsible for the dynamic changes in the height and the length of the dunes when several dunes are present. Test cases with two or three dunes within the domain have been tested. The calculation with three dunes is presented below. The initial dunes have different heights or different lengths and the physical processes of changing these properties are studied.

**Interaction among dunes**

If a field of dunes change their length, an interaction among the individual dunes is required. In Fig. 3.29 the initial bedform of a test case is presented, where three dunes of the same length (L/D=4) are present in the calculation domain. Two of the dunes have an initial height of \( \frac{H}{D} = 0.077 \), while the third is twice as high, i.e. \( \frac{H}{D} = 0.155 \). The dunes are seen to change their length and height with time. In Fig. 3.30 change of the dune height respectively dune length with time is shown for the three dunes. A periodicity in the change of the dune height as well as the dune length is noted. The following observations is noted:

- When the dunes grow, their length in general also increases
- A phase lag between the maximum length and height is observed. The length has a minimum before the dune height has a minimum, or a respectively maximum before the dune height reaches a maximum
- When the *upstream* dune reaches a certain height, the dune height for the dune studied starts to decrease. The upstream dune continues to increase its height.
- The dune studied continues to decrease, while the upstream dune reaches the maximum dune height, and starts to decrease

No conclusions on the physical mechanism responsible for the behavior described above has been reached. However, the following mechanisms are in general
Figure 3.29  Initial bedform for calculation with 3 dunes within the domain. The dunes have initially the same length \((L/D=4)\) but one dune is twice as high as the two others. \(k_D/D=0.001, \theta'_u = 0.11\).

believed to be the governing processes. The growth of the dune of a constant length, requires due to conservation of sediment either:

1. a net supply of sediment or

2. a “self-modification” of the dune, in the sense that the dune height increases by relocating the sediment from the trough area to the top

If the dune length is not constant, sediment supply for increasing the dune height, can furthermore be added to the dune by:

3. a change of the length

The first bullet point requires that sediment is transported from one dune to the other in the trough. This does not seem to be the case, since the transport in this area \(q_0\) was found to be negligible. The second bullet point can be explained by the change in the boundary layer along the dune, when the properties (mainly dune height) of the upstream dune change. The considerations with respect to the third bullet point are as follows: If a dune reduces in the height with time and at the same time migrates downstream, the level of the trough increases. A volume of sediment is hence “left” behind, and is as such a net supply to the upstream dune.

It should be noted that the trough correction to the filter (see Section 2.3.3) is not included in the calculations. The upwind scheme is used for the convective terms in the momentum equations (i.e. more numerical diffusion is included than in the remaining morphological calculations) and the morphological time step is relatively low and can probably be increased (see the effect of a small time step in Section 3.6).

Splitting of dunes

Dunes in a unidirectional current are observed to have a length less than approximately 8 water depths. A calculation has been performed with long initial bedform, \(L/D=10\) water depths. The purpose was to see if the dune would split up. The original dune was a steady dune characterized by: \(L/D=4, \frac{k_\gamma}{D} = 0.001\), and \(\theta'_u = 0.11\).
Figure 3.30  Development in time of dune height (upper figure) and length (lower figure) of the three dunes.

This dune was then ”stretched” in the horizontal direction to have a length of 10 water depths. The flow parameters were unchanged. The grid-spacing was maintained at $\Delta x = 0.05$, but increased to $\Delta x = 0.10$ during the calculation to reduce the calculation time.

The dune was found to split up - the morphological development in the calculation is shown in Fig. 3.31. The dune splits ”slowly” up into two dunes in the beginning. As time passes the number of dunes within the calculation domain increases to four small dunes before the calculation was stopped. This dune length is most likely too small. It is uncertain what causes the dune to split up.

The calculation was performed with the same numerical characteristics as the above described calculation: without the trough correction to the filter, a first order upwind scheme for the convective terms in the momentum equations and a relatively low morphological time step.

3.8 Summery

Two-dimensional steady dunes in a unidirectional flow were calculated with a numerical morphological model. Only bed load was taken into considerations in the sediment transport.

Dunes in equilibrium with the flow are calculated for fixed dune lengths between 3 and 8 water depths for Shields’ parameters in the range $\theta^* = 0.06 - 0.40$. The dunes reach a steady state, when the local sediment transport rate is proportional to the local height of the dune.

Non-equilibrium in the flow at the top of the dunes was found to reduce the dune height. When the dune length is increased (for a constant Shields’ parameter), the flow is closer to equilibrium at the top, and the dune height approaches the
Figure 3.31  The morphological development of an initially very long (L/D=10) dune. The calculation domain is only 10 water depths, but twice the domain is shown in the figure for visual reasons.
dune height predicted by Fredsøe (1982), who assumed equilibrium at the top. The maximum dune height, however, seems to be lower than the dune height predicted by Fredsøe.

The steepness of the computed dunes was comparable to the findings by Tjerry (1995) and Tjerry and Fredsøe (to be publ.). The best agreement was found for dune lengths L/D=4-6 for the smallest dunes ($\theta' < 0.15$) and for dune lengths of L/D=4-5 for the larger dunes. For these dunes, the dune height is however 30-40% lower than the dune height predicted by Tjerry and Fredsøe. No conclusions can hence be drawn with respect to a natural combination of dune height and dune length by combining the results from the present work with the results by Tjerry/Fredsøe.

The flow resistance was calculated for the computed dunes for two dune lengths, L/D = 4 and 6. Two locations of the crest are tested since there is some uncertainty in the predicted crest height in the morphological calculations. The flow resistance calculations show that the prediction of the crest height is important in the calculation of the form drag and hence the total drag. The flow resistance was calculated for the steady dunes for two dune lengths, L/D=4 and 6. The results for the flow resistance in this thesis is interpreted as preliminary, since some grid dependency was found in the results. The overall conclusions, however, does not seem to be affected. The form drag calculation was found to be sensible to the height of the crest above the trough level. Effects of the numerical filter applied in the morphological calculation and of the horizontal grid-spacing are most pronounced in the area of the crest. The two extreme locations of the crest were hence tested in the calculations of the flow resistance: 1) as computed with the morphological model and 2) at the top of the dune. The results were compared to a model for the flow resistance presented by Engelund (1966, 1967). Furthermore, the form drag for the computed dunes were estimated from the Carnot formula. The calculated form drag were found to agree well with the results for the Carnot formula, when Carnot-coefficient found empirically by Engelund (1977) was applied. For dunes for the large Shields' parameters, the numerically calculated flow resistance exceeds the prediction by the Engelund resistance model. For the small dunes, the flow resistance computations with the two crest locations were found to be respectively higher (crest at the top of the dune) and lower (crest as computed in the morphological model) than predicted by the Engelund model. The best agreement is found with the crest located at the top. It should be noted, that further tests should be made to ensure grid independence.

Finally the grid dependency and influence of the morphological filter were investigated. The filter smooths the bedform, especially in the front area.
Chapter 4
UNSTEADY DUNES IN A UNIDIRECTIONAL FLOW

In the previous chapter dunes in equilibrium with the hydraulic conditions and the sediment properties were studied. When the hydraulic parameters change, for instance the discharge, the capacity for transporting sediment changes and therefore the properties of the dunes will change. The time scale for the change in the hydraulic parameters is important: if a change in the flow happens rapidly the dunes will not immediately reach equilibrium with this new flow situation, because it takes time for the bedforms to change to new geometrical properties. A certain time lag between the change in the flow and the new equilibrium condition for the bedform will exist. The morphological development of unsteady dunes with time due to a change in the capacity of carrying sediment is investigated in this chapter.

Such a change in the flow properties occurs often in nature, for instance during a flood situation or in a dry period, when the discharge of the river changes, and hence increases or decreases the sediment transport. As illustrated in Chapter 3, the bedforms carry up to 2/3 of the flow resistance in a river. The change in the dune properties are thus important in relation to for instance flood forecasting.

In the following two different flow situations are considered: 1) a sudden change (decrease or increase) in the flow condition and 2) a slowly varying change in the flow, i.e. for instance during a flood. The first flow situation serves to investigate and verify the rate of change of the dune height after the sudden change of the flow. For verification the results are compared to analytical work by Fredsoe, 1979. In section 4.2 the flow, sediment transport and erosion/deposition pattern along a dune just before and after an immediate change in the flow properties are described. The morphological development of the dunes is studied with respect to the two flow situations (section 4.3 and 4.4).

4.1 Methodology

When the hydraulic parameters change, the dune height responds to the new hydraulic situation before the dune length. The dune height immediately changes when the sediment transport field changes, while a change in the length requires the dunes to interact to either merge or split up, which must be expected to take place on a longer time scale. In the present study the length of the dune (or actually the calculation domain) is constant and hence only the response in the dune height and dune shape is studied. Within the calculation domain only one dune is present in the calculations, but due to the periodic boundary conditions the set-up compares to considering a train of similar dunes.
The methodology in the present chapter is very similar to the methodology in
the previous chapter. The only difference is that a change in the hydraulic parameters
is introduced in order to study the unsteady behaviour of the dunes.

As the initial bedform and corresponding flow field, the equilibrium dune from
run s3L4 in Chapter 3 (see Table 3.1) is chosen. This dune is mainly characterized
by a Shields’ parameter of 0.11 for an undisturbed bed and a dune length of 4 water
depths. This initial flow situation (in equilibrium with the bed) is denoted 1 and
the flow situation after the change in the discharge is denoted 2. The hydraulic
parameters for flow situation 2 is applied to the initial bed from Chapter 3, and this
is where the morphological calculations start (at the time $t^*=0$) in this chapter. The
initial situation can hence be interpreted as the situation, where the flow in a channel
has been constant long enough for the bedform to be in equilibrium with the flow.
The flow is then changed abruptly to flow situation 2 at $t^*=0$ according to a step
function, see Fig. 4.1. The capacity for carrying sediment hence changes.

The nondimensional input parameters for sediment calculation are $(k_N, Fr, Re)$. The increase/decrease in the discharge leads to an increase/decrease in the Shields’
parameter and hence sediment transport. The change in the discharge/Shields’
parameter is as in Chapter 3 in the numerical model applied by changing the Froude
number (which scales the Shields’ parameter in the model, see Section 2.4). The
roughness parameter, $k_N$, is maintained constant, $k_N = 0.001$.

The dimensional set of parameters is $(k_N V, D, g, \nu)$. For an unchanged grain
size (or $k_N$) of the bed, a constant mean water depth is hence applied in the calculations,
when $k_N/D$, is maintained constant and the Froude number is changed. The
change in the discharge is hence purely related to a change in the mean velocity. In
nature, a change in the discharge changes the velocity as well as the water depth.
The flow is, however, only a weak function of $k_N$ (Eq. 2.29), as long as the changes
in $k_N$ is ‘small’ (a change of $\pm$ 50% of its value causes only a small difference in
the variation of the nondimensional friction, $U^*/V$, along the bedform). The impor-
tance is hence the change in the Shields’ parameter, and not the exact combination
of $(k_N, Fr)$.

The response of the dune is calculated for four flow situations, see Table 4.1. The four flow situations correspond to the flow situations for four of the steady dunes
for $L/D=4$ in Chapter 3 (Table 3.1).

The last comment in this section is related to ‘numerical issues’. A variable
morphological time step has been found to improve the morphological calculations.
During the morphological calculations, the time step is hence varied according to the
immediate flow situation - more about this in Section 4.5.1

4.2 Flow and sediment transport immediately after a change in flow prop-
eries

A change of the flow field changes the skin friction along the bed and therefore
the sediment transport field as well as the erosion and deposition pattern along the
dunes. These changes are described in this section for a sudden increase and decrease
in the flow in order to understand the changes of the bedform in the morphological
calculations described later in this chapter.

In Fig. 4.2 and 4.3 the skin friction along a bedform is shown for respectively the situation where the dune is in equilibrium with the flow situation (as found in Chapter 3) and an increase (Fig. 4.2) and a decrease (Fig. 4.3) in the flow. The original (equilibrium) flow situation is denoted 1 and the flow situation after the change in the discharge is denoted 2. For the equilibrium bedform the Shields’ parameter on an undisturbed bed is \( \theta_{u} = 0.11 \) and is increased/decreased to \( \theta'_{u,1} = 0.11 \) and is increased/decreased to \( \theta'_{u,2} = 0.20 \) (run uu3, in Table 4.1) or \( \theta'_{u,2} = 0.08 \) (run uu2).

In flow situation 2 the skin friction on the bed respectively increases or decreases compared with the equilibrium flow situation. Since the bedform has not yet changed, the bed shear stress along the dune after the change in the discharge will be self-similar to the bed shear stress distribution before the change (as long as the Froude number is small, the undulations on the water surface can be neglected and hence the cross section is similar). which was also realized from the nondimensional description of the momentum equations in Chapter 2. This means that if the nondimensional skin friction is increased by a factor of \( \Delta \) the gradient in the skin friction along the dunes will also increase by a factor of \( \Delta = \frac{\theta'_{u,2}}{\theta'_{u,1}} \):

\[
\frac{\partial \theta'_{u,2}}{\partial x} = \Delta \frac{\partial \theta'_{u,1}}{\partial x} \tag{4.1}
\]

Taking the bed slope effect (gravity) and the critical Shields’ parameter into account, the effective Shields’ parameter available for sediment transport is not exactly proportional to the original skin friction. Therefore the bed load field in flow situation 2 is not exactly self-similar to the bed load field in flow situation 1 - but nearly.

<table>
<thead>
<tr>
<th>Run uu1</th>
<th>Run uu2</th>
<th>Run uu3</th>
<th>Run uu4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow situation 1</td>
<td>Flow situation 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta'_{u} = 0.11 ) ( \left( \frac{k}{g} = 0.001, Fr = 0.20 )</td>
<td>( \theta'_{u} = 0.062 ) ( \left( \frac{k}{g} = 0.001, Fr = 0.15 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta'_{u} = 0.11 ) ( \left( \frac{k}{g} = 0.001, Fr = 0.20 )</td>
<td>( \theta'_{u} = 0.080 ) ( \left( \frac{k}{g} = 0.001, Fr = 0.17 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta'_{u} = 0.11 ) ( \left( \frac{k}{g} = 0.001, Fr = 0.20 )</td>
<td>( \theta'_{u} = 0.20 ) ( \left( \frac{k}{g} = 0.001, Fr = 0.27 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta'_{u} = 0.11 ) ( \left( \frac{k}{g} = 0.001, Fr = 0.20 )</td>
<td>( \theta'_{u} = 0.40 ) ( \left( \frac{k}{g} = 0.001, Fr = 0.38 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1 Flow parameters for the four test cases before (1) and after (2) the change in the discharge.
The erosion/deposition pattern, $\frac{\partial h_p}{\partial t}$ where ($h_D = \frac{h}{L}$) immediately after the change in the discharge can be explained from the new bed load distribution. The following is noted in Fig. 4.2 and 4.3:

**Increasing bed load capacity:**

- An increase in the bed load capacity increases the erosion along the upstream slope, as well as increases the deposition at the front.

- In the equilibrium situation, the phase difference between the bed load transport field and the bedform is 0. After the increase in the discharge, the maximum bed load is located before the top. $\frac{\partial h_p}{\partial t}$ is hence positive at the top, i.e. deposition takes place and the dune height grows.

- The location on the dune, where the skin friction exceeds the critical ($\theta'_c = 0.05$) moves upstream, when the discharge increases. Sediment transport and hence erosion thus take place in the trough (where $\frac{\partial h_p}{\partial t} = 0$ before the change in the flow).

**Decreasing bed load capacity:**

- A decrease in the bed load capacity reduces the erosion along the upstream slope, as well as reduces the deposition at the front.

- The maximum bed load is located after the top, after the decrease in the discharge. $\frac{\partial h_p}{\partial t}$ is hence negative at the top, i.e. erosion takes place and the dune height decays.

- The location, where the skin friction exceeds the critical ($\theta'_c = 0.05$) moves downstream, when the discharge decreases. The point, where sediment transport begins downstream from the zone of separation is therefore moved a distance up upon the gentle slope on the dune.

4.3 **Morphology after an immediate change in the flow**

The morphological development of the initial bed, the equilibrium bedform found for run s3L4 ($\frac{E}{D} = 4$ and $\theta'_u = 0.11$) in Chapter 3, after a change in the flow parameters according to Table 4.1 is described in this section. Changes in the dune height are compared to findings by Fredsøe (1979). A model for prediction of the trough level change is presented.

4.3.1 **Change in dune height and shape**

The morphological development with time, from $t^* = 0$ to a new equilibrium for the bed takes place, is shown in Fig. 4.4 and Fig. 4.5 for respectively after a decrease in the discharge (run uu2) and after an increase in the discharge (run uu3). The initial changes are furthermore shown in Fig. 4.6 and 4.7 for the two situations with a decreasing sediment transport and in Fig. 4.8 and 4.9 for the two situations with
Figure 4.2 Friction, sediment transport and erosion/deposition pattern immediately before (index 1) and after (index 2) an abrupt increase in the discharge. Before the flow change the bedform is in equilibrium with the flow. $\theta_{u1}' = 0.11$. $\theta_{u2}' = 0.20$. 
Figure 4.3 Friction, sediment transport and erosion/deposition pattern immediately before (index 1) and after (index 2) an abrupt decrease in the discharge. $\theta'_{u1} = 0.11$. $\theta'_{u2} = 0.08$. 
and increasing sediment transport. The dune height development with time and the change in the level of the top and the trough level are shown in Fig. 4.10-4.12.

The morphological changes can be divided in the changes that take place at the top and the changes that take place in the trough:

**At the top**

The bed level at the top changes according to the phase difference between the top of the dune and the sediment transport field (see Fig. 4.11) as noted above in Section 4.2 (and in Section 1.2 about stability of dunes), where the flow and sediment transport field immediately after the change in the hydraulic situation are described. In the situations, where the water discharge decreases (run uu1 and uu2) and the maximum bed load is found after the top, erosion takes place at the top and the level of the top of the dune decreases as also seen in Fig. 4.11. Where the water discharge and sediment transport increases (run uu3 and uu4), the maximum bed load is found before the top, and sediment is deposited at the top causing the bed level at the top to increase.

**In the trough area**

The morphological changes in the trough area are dependent on whether discharge increases or decreases:

**Decreasing discharge (Run uu1 and uu2)** The deposition/erosion pattern can be divided into three phases with respect to the change of the trough level. The three phases are indicated with p1, p2 and p3 in Fig. 4.12.

**Phase 1:** In the beginning, just after the discharge decreases, the trough level is constant as no sediment is transported in the trough due to the low velocities and hence low skin friction in the separation zone. The front thus moves forward across the trough as seen in Fig. 4.6 and Fig. 4.7. The total dune height hence only decreases with the decrease in the bed level at the top of the dune (Fig. 4.10).

**Phase 2:** The second phase starts when the lower part of the front meets the location where the gentle slope of the original bedform starts (see Fig. 4.6). Downstream from the crest, the skin friction drops due to the expansion of the flow area and no bed load takes place before the location where the skin friction again exceeds the critical skin friction for sediment transport (compare upper and middle subfigures in Fig. 4.3). Since the skin friction decreases due to the reduction in the water discharge, the location of the skin friction exceeding the critical level for sediment transport, moves further downstream as described in Section 4.2 and is also shown in more details in Fig. 4.13. The sediment volume located between the point where the skin friction exceeds the critical level in flow situation 1 and the skin friction exceeds the critical level in case of flow situation 2 is so to say "left behind", since no forces exist to move the sediment in this area. Hence, the front moves up along this "old bedform" and the trough level therefore rises. The total dune height now decreases with the rate of the change of the bed level at the top as well as at the rate of change of the trough.
Figure 4.4  Morphological development after and abrupt decrease in the discharge and until a new equilibrium takes place (Run uu2). Initial bed is the uppermost figure and are in the following figures shown with thin curve.

Phase 3: The final trough level in the new flow situation is found in the third phase. The new trough level is now slowly found such that the separation length (or strictly speaking the location of the point where the critical skin friction is exceeded), the migration velocity and the erosion along the upstream slope is in equilibrium.

Increasing discharge (Run uu3 and uu4)  In this situation only the third phase (compared with the above description for the decreasing sediment transport rate) exists. Since the skin friction now increases after the water discharge increases, the location of the point, where the critical skin friction for sediment transport is exceeded, moves upstream (see Fig. 4.13). This means that in the former flat trough, sediment now starts being eroded, and hence the trough deepens (Fig. 4.8 and 4.9). Immediately after the increase in the water discharge, the total dune height is therefore grows with the increase of the bed level at the top plus the bed level deepening of the trough. The final trough level (and top level) is found under the same conditions as in the above described case such that equilibrium takes place.

The initial morphological development of the dune in the case of the increasing water discharge is seen to be influenced by an overlying perturbation of a shorter wave
Figure 4.5  Morphological development after an abrupt increase in the flow and untill a new equilibrium takes place (Run uu3). Initial bed is the uppermost figure and are in the following figures shown with thin curve.

Figure 4.6  Initial morphological development in case of decreasing water discharge. Run uu1: $\theta'_u = 0.11 \rightarrow 0.062$. 
Figure 4.7  Initial morphological development in case of decreasing water discharge. Run uu2: $\theta'_u = 0.11 \rightarrow 0.08$.

Figure 4.8  Initial morphological development in case of increasing water discharge. Run uu3: $\theta'_u = 0.11 \rightarrow 0.20$.

Figure 4.9  Initial morphological development in case of increasing water discharge. Run uu4: $\theta'_u = 0.11 \rightarrow 0.40$. 
length than the bedform (Fig. 4.8 and 4.9). The origin of this perturbation is in the
trough, and can be understood from the flow and sediment transport field as shown in
Fig. 4.13. Right after the reattachment point the skin friction rises quickly due to the
acceleration of the flow close to the bed caused by a favorable pressure gradient. A
large gradient in the skin friction is seen. Downstream from the reattachment point,
the boundary layer grows up, and a relaxation in the growth of the skin friction is
seen. The concave bedform at this location induces an additional pressure at the bed,
thus reducing the favorable pressure gradient, and this process adds to the growth of
the boundary layer.

For the sediment transport and erosion/deposition pattern this means that: a) before
the increase in the water discharge the skin friction does not exceed the critical
level until after the relaxation in the spatial gradient in the skin friction b) after the
increase in the water discharge, the skin friction initially exceeds the critical level
already where the spatial gradient in the skin friction distribution is large. A large
erosion hence takes place immediately downstream from the reattachment point, but
- as the rate of change in the skin friction decreases further downstream - the rate
of erosion also reduces. This creates a perturbation in the bed, which immediately
grows and migrates.

It is uncertain whether this perturbation in nature is similar to the modelled
perturbation, if it exists at all. Two details in the present model argues against that:
1) In nature the separation zone is dynamic. The location on the bedform where
the skin friction exceeds the critical level for sediment transport therefore varies with
time (on a time scale much smaller than the morphological development of the dune).
A large eddy model, where this dynamic effect is modelled would be able to model
this. 2) The numerical filter smooths out the details of a small spatial scale like this
perturbation.

4.3.2 The Fredsøe model for dune height change
Fredsøe (1979) developed a model which describes the initial change in the dune
height with time after a sudden change in the water discharge. The results from this
Figure 4.11 Change in the level of the top, $h_{\text{max}}$, of the dune after a sudden change in the flow. The dashed curves are the dune heights predicted by the Fredsøe model. Respectively increasing discharge (upper figures) and decreasing discharge (lower figures).
Figure 4.12 Change in trough level of the dune after a sudden change in the flow. The dashed curves are dune height predicted by the model for the trough. Respectively increasing discharge (upper figures) and decreasing discharge (lower figures). p1, p2 and p3 in the lower figures indicates phase 1, phase 2 and phase 3 as defined in the text.
model are compared to the calculations presented above. The same assumptions as in
the present model are applied: a train of regular two-dimensional dunes of the same
wave length and height are studied and only bed load is taken into account.

A change in the discharge from $Q_1$ to $Q_2$ is assumed in the model by Fredsøe.
The bedforms are in equilibrium with the flow conditions at the discharge $Q_1$ and
are therefore migrating downstream with the migration velocity $a_1$ without changing
their shape. The local bed load is hence (according the derivation in Section 3.2):

$$q_1 = a_1(1 - n)h$$

(4.2)

- $h$ is here the local bed level measured from the trough. When the discharge is
changed to $Q_2$, the sediment transport changes. Just after the change of the dis-
charge, when the bedforms still have not changed the geometrical properties, the new
migration velocity can be found from the new bed load, $q_2$, and geometrical con-
derations. As the front slope remains constant, the bed load transported over the crest
is deposited along the front slope, causing a migration velocity of:

$$a_2 = \left( \frac{q_2}{(1 - n)h} \right)_{top}$$

(4.3)

The change in the dune height is in the Fredsøe model assumed to take place
only at the top. The rate of change of the dune height with time is then found from
Eq. 3.2 in Chapter 3):

$$\left( \frac{dh}{dt} \right)_{top} = \left( \frac{\partial h}{\partial t} \right)_{top} + a_2 \left( \frac{\partial h}{\partial x} \right)_{top}$$

(4.4)

or by use of Eq. 4.3, and the continuity equation for sediment 3.3:

$$\left( \frac{dh}{dt} \right)_{top} = \frac{1}{1 - n} \left( \frac{\partial q_2}{\partial x} \right)_{top} + \frac{a_2}{h_{top}(1 - n)} \frac{\partial h}{\partial x}$$

(4.5)

Rewriting the gradient in sediment transport by $\frac{\partial q}{\partial x} = \frac{\partial \theta}{\partial x} \frac{\partial q}{\partial \theta}$, the non-dimensional version of this equation is:

$$\frac{1}{\sqrt{(s - 1)gd^3}} \left( \frac{dh}{dt} \right)_{top} = -\frac{1}{1 - n} \frac{d\Phi_2}{d\theta} \left( \frac{d\theta}{dx} \right)_{top} + \frac{a_2}{h_{top}(1 - n)} \frac{\partial h}{\partial x}$$

(4.6)

An estimate of the sediment field at flow situation 2 to establish the longitudi-
nal variation of the sediment transport at the top is hence required. Now, since only
bed load is taken into account, the local rate of sediment transport can be related to
the instantaneous local value of the skin friction, and can be found from Eq. 4.1.
As the shear stress distribution at flow situation 1 is in equilibrium with the flow, a relation for the sediment transport field at flow situation 2 is further derived in the paper by Fredsøe from the equations above and an expression for the rate of change of the bed level at the dune top.

\[
\frac{dH}{dt} = \sqrt{(s - 1)gd^3} \cdot \left[ 1 - \Delta \left( \frac{1}{\phi\frac{d\phi}{d\varphi}} \right) \right] \phi_2
\]  

(4.7)

- where all parameters are taken at the top. From the Meyer-Peter bed load formula in expression for \( \frac{d\phi_c}{d\varphi} \) is easily found as:

\[
\frac{d\phi_c}{d\varphi} = 12 \cdot \int \theta' - \theta_c
\]  

(4.8)

Since the change in the trough is assumed negligible in the paper, the rate of change of the bed in Eq. 4.7 is regarded as the change in the total dune height. Eq. 4.7 and Eq. 4.8 hence give a formula for the rate of change of the dune height initially after a change in the water discharge under the assumption that the trough level is unchanged. The formula was compared to experiments performed by Gee (1973) by using also Engelunds resistance formula, and good/reasonable agreement was found.

4.3.3 Initial time scale: Comparison with the Fredsøe model

The rates of change of the dune height for the computed dunes are compared to the Fredsøe model described above. The initial growth/decay of the dune height is hence calculated for the four flow situations in Table 4.1 using Eq. 4.7 and 4.8. The prescribed dune height using the Fredsøe model is shown in Fig. 4.10 along with the development of the dune height as calculated by the numerical model. Since the Fredsøe model only prescribes the change that takes place at the top of the dunes, the model is also used to describe the top level of the dune after the new flow situation. This is shown in Fig. 4.11.

The Fredsøe model is seen to predict the initial development of the total dune height very well for the two situations where the water discharge is reduced, whereas the model underestimates the increase in the total dune height in the two situations, where the water discharge is increased. Comparing the actual development of the dunes as described in Section 4.3 with the assumption of initially no change in the trough level taken by Fredsøe (1979) this comes as no surprise:

- In the case of a decreasing water discharge the trough level was found to initially stay at the same level, as assumed by Fredsøe, and only the change of the top level is initially causing the reduction of the total dune height. In Fig. 4.11 the development in the top level of the dune is seen to compare well with the rate of change of the total dune height found by the Fredsøe model.

- In the case of an increasing water discharge, the trough deepens immediately after the abrupt change in the flow. In addition to the increasing level at the top level, this development in the trough level hence also causes the total dune
height to increase. While the total dune height immediately after the increased water discharge is not well described with the Fredsøe model, a comparison of the time development of the dune level at the top and the Fredsøe model shows that the model predicts the increase in the top level initially very well.

4.3.4 Initial time scale: A model for the trough

The initial development of the trough was found to influence the rate of change of the dune height after a change in the discharge. The mechanisms described qualitatively in Section 4.3.1 are quantified in the following by simple physical considerations. The aim is to describe the initial change of the trough level by using the flow and sediment transport fields immediately after the change in the flow conditions. This also serves to verify the morphological calculation by the model. The equation that describes the change in the bed is the general version of Eq. 4.4:

\[
\frac{dh_D}{dt^*} = \frac{\partial h_D}{\partial t^*} + a^* \frac{\partial h_D}{\partial x}
\]  

(4.9)

According to the observations in Section 4.3.1 a distinction is made between the situation of an increasing and a decreasing discharge.

Increasing sediment transport rate

The deepening of the trough immediately after the flow change was found to be caused by the movement of the location on the dune where the skin friction exceeds the critical level for sediment transport. Details of the skin friction and sediment transport fields are given in Fig. 4.13. Before the flow change, \( \theta' \) equals \( \theta_c' \) where the gentle upstream slope begins. This point is denoted \( x_{c1} \). In flow situation 2 this location has moved a distance of \( \Delta x \) upstream to \( x_{c2} \). An estimate of the initial erosion rate in the trough can hence be found by estimating the gradient in the sediment transport over \( \Delta x \) from the sediment transport field in flow situation 2, as seen in the figure:

\[
\frac{dh_D}{dt^*} \bigg|_{top, t^*=0} = -\frac{\Delta \Phi}{\Delta x} = -\frac{\Delta \Phi}{x_{c1} - x_{c2}}
\]

For the two situations with an increasing discharge this gives \( \frac{dh_D}{dt^*} \bigg|_{top, t^*=0} = 0.83 \) for uu3 and \( \frac{dh_D}{dt^*} \bigg|_{top, t^*=0} = 2.40 \) for uu4. This rate of change of the trough level compares well with the initial change in the trough in the morphological computations as shown in Fig. 4.12.

Decreasing sediment transport rate

The situation is slightly more complicated in this case. The trough level changes and the time in each of the three phases as defined Section 4.3.1 is estimated:
Phase 1: The front migrates along the flat trough, and the level of the trough stays the same:

\[
\frac{dh}{dt^*}|_{\text{trough,ph1}} = 0
\]  

(4.10)

The duration of phase 1 can be estimated from the migration velocity and the length of the flat part of the trough:

\[
\Delta t^*_{\text{ph1}} = \frac{\Delta x_{\text{trough}}}{a_2^*}
\]  

(4.11)

where the migration velocity is estimated from Eq. 4.3. Phase 1 therefore takes place in the time interval \( t^* = [0 - \Delta t^*_{\text{ph1}}] \). The initial values for \( \Phi_{b,\text{top,2}} \) and the dune height, \( H/D \), are used, and the migration velocity is hence assumed to be constant.

Phase 2: The front migrates up along the upstream part of the gentle slope, where no erosion has taken place since \( \theta_2' < \theta_c' \) along this part of the dune. The bed slope, \( \frac{dh}{dx} \), is hence unchanged for this part of the bed. The change in the trough level as the front migrates up this slope is found from Eq. 4.9, where \( \frac{dh}{dx} = 0 \):

\[
\frac{dh}{dt^*}|_{\text{trough,ph2}} = a_2^* \frac{\partial h}{\partial x}
\]  

(4.12)

By estimating the bed slope (denoted \( \frac{dh}{dx}|_{\text{toe}} \)) between the location of \( \theta_1' = \theta_c' \) and \( \theta_2' = \theta_c' \), an estimate for the trough level increase is found. Phase 2 ends, when the front reaches the location where \( \theta_2' = \theta_c' \). The duration of Phase 2 is thus estimated as the time it takes the front to migrate the distance between \( x(\theta_1' = \theta_c') \) and \( x(\theta_2' = \theta_c') \):

\[
\Delta t^*_{\text{ph2}} = \frac{x(\theta_2' = \theta_c') - x(\theta_1' = \theta_c')}{a_2^*}
\]  

(4.13)

Phase 2 thus takes place in the time interval \( t^* = \Delta t^*_{\text{ph1}} - (\Delta t^*_{\text{ph1}} + \Delta t^*_{\text{ph2}}) \).

Phase 3: Downstream from the point of \( \theta_2' = \theta_c' \) \( (x_{c,2}) \) erosion take place, starting at \( t^* = 0 \). The development in the bed from this point should be calculated by integrating Eq. 4.9 with respect to time from \( t^* = [0 - (\Delta t^*_{\text{ph1}} + \Delta t^*_{\text{ph2}})] \). The trough level change in phase 3 can then be found from Eq. 4.12. The integration is done in a ”quick” way as described below:

At \( x_{c,2} \) and \( t^* = 0 \) the rate of erosion is \( -\frac{\partial \Phi_b}{\partial x}(x_{c,2}) \), which is found from the calculated transport field to \( t^*=0 \) for flow situation 2. This rate of erosion is used as an estimate also ”some distance” downstream from \( x_{c,2} \), which is the same as assuming the increase in \( \Phi_{b,2} \) is linear. As the front of the dune moves downstream, the location of \( x_{c,2} \) also moves, i.e. the point where the skin friction exceeds the critical level for sediment transport moves downstream with the front. For a fixed \( x \)-value there is hence only a certain time, \( \Delta t^*(x) \), in which erosion takes place. The erosion for a given \( x \) (where \( x>x_{c,2} \)) and \( t^* \) can be estimated as:
\[ \Delta h_D(x, t^*) = \frac{\partial h_D}{\partial t^*} \cdot \Delta t^*(x) \approx -\frac{\partial \Phi_b}{\partial x}(x_{c2}) \cdot \Delta t^*(x) \]  

(4.14)

where \( \Delta t^*(x) \) is found using the assumption that \( x_{c2} \) moves forward with the migration velocity \( a_2^* \):

\[ \Delta t^*(x) = \frac{x - x_{c2}}{a_2^*} \]  

(4.15)

\( \Delta h_D(x) \) is found at the time \( t^* = \Delta t_{p1}^* + \Delta t_{p2}^* \), which is when Phase 3 begins. This erosion depth is then subtracted from the original bed. Next Eq. 4.12 can be used to determine the trough level from this time and forward.

The change of the trough level with time using this method of estimation is shown in Fig. 4.12 along with the trough level change as calculated by Dune2D for run uu1 and uu2. The applied numerical values of the constants for the two situations are given in Table 4.2. Results for \( \frac{\partial h_D}{\partial t^*} \) and the duration of each phase are given in Table 4.3.

A few comments related to these results are given in the following: The duration of Phase 1 and 2 as well as the rate of change of the trough level are seen to be estimated very well. In Phase 3, however, the method only catches the development of run uu2. The predicted bedform downstream from \( x_{c2} \) calculated by Eq. 4.14 is shown in Fig. 4.8 and 4.6 respectively. In run uu1 the trough level and hence the change in trough level is predicted well (\( \frac{\partial h_D}{\partial t^*} \) was found to be nearly 0, and hence \( \frac{\partial h_D}{\partial t^*} \approx 0 \)). The trough level for uu1 is not predicted well - the erosion \( \Delta h_D \) (Eq.4.14) is underestimated. This indicates that the assumptions in phase 3 are violated, especially in run uu1.

The assumption that the location where \( \theta'_2 > \theta'_c(x_{c2}) \) is moving forward with the migration velocity was found problematic. In Fig. 4.6 as well as in 4.7, the erosion is seen to start before the front has reached the location where \( \theta'_{2} > \theta'_c \) at \( t^* = 0 \) (\( x_{c2} \)). This was in the beginning believed to be a result of the numerical filter, but studying the skin friction distribution with time after the change in the flow properties, it was revealed that \( x_{c2} \) initially does not move forward with the rate of the migration velocity. The location of \( x_{c2} \) is in fact initially more or less constant or even moving slightly upstream. This seems to be a boundary layer effect: As the front moves forward across the trough, the shape of the trough does not have a flat part just after the front, but increases immediately downstream from the minimum trough level, especially in run uu1. At the point of reattachment, the streamlines hence converge due to the this slope of the bed. This causes an additional shear in the velocity profile and the skin friction increases. The effect of this is that \( x_{c2} \) does not move downstream as fast as assumed and the time interval for erosion is therefore underestimated by Eq. 4.15.

4.3.5 Time scale for the new equilibrium situation

The time lag for the dunes to obtain new geometrical properties and shape such that they are in equilibrium with the new flow situation is discussed in this section. The
Figure 4.13 Details of changes in the bed shear stress bed and sediment transport in the trough area of the bed in case of an abrupt increasing (upper figures, run uu3) and decreasing discharge (lower figures, run uu2).

| Phase | $a_2^*$ | $\Delta x_{trough}$ | $\frac{dn}{dx}|_{toc}$ | $\frac{\partial \Phi_b}{\partial x}(x_{c1})$ | $x_{c1} - x_{c2}$ |
|-------|---------|---------------------|----------------------|---------------------------------|------------------|
| uu1   | 0.55    | 0.25                | 0.05                 | 0.021                           | 0.95             |
| uu2   | 0.78    | 0.25                | 0.03                 | 0.052                           | 0.55             |

Table 4.2 Parameters needed to calculate the change in the trough level

<table>
<thead>
<tr>
<th>Phase</th>
<th>$\frac{dn}{dt}^*$</th>
<th>$\Delta t^*$</th>
<th>$\frac{dn}{dt}^*$</th>
<th>$\Delta t^*$</th>
<th>$\frac{dn}{dt}^*$</th>
<th>$\Delta t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>uu1</td>
<td>0</td>
<td>0.45</td>
<td>0.017</td>
<td>1.72</td>
<td>$\approx 0$</td>
<td>-</td>
</tr>
<tr>
<td>uu2</td>
<td>0</td>
<td>0.32</td>
<td>0.024</td>
<td>0.70</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.3 Rate of change of the trough level and duration of the three phases
time lag can be expressed in either the actual time or as the distance they migrate downstream before they have obtained the new equilibrium situation.

The morphological non-dimensional time lag, $T_{95}^*$, is shown for the computed dunes in Fig. 4.14 (left). $T_{95}^*$ is defined from the dune height change between flow situation 1 and 2. The total change in the dune height is $|H_1 - H_2|$. $T_{95}^*$ is found from the data in Fig. 4.10 as the time it takes for the dunes to change their height by 95% of the total change in dune height between flow situation 1 and 2. Or in other words - when $t^* > T_{95}^*$ the dunes only have to change the remaining 5% of the total dune height change to be in equilibrium with flow situation 2:

$$T_{95}^* : 0.95 < \frac{|H_1 - H(T_{95}^*)|}{|H_1 - H_2|} < 1.05 \quad (4.16)$$

where the notation * indicates nondimensionalizing of the time with the sediments properties, see Eq. 2.33. A more precise method of defining the time lag could be thought of. However as the variation in the dune height - especially for the situations with the increasing discharge - is influenced by the overlying perturbation, which is described in the end of Section 4.3.1, a more complicated method is not believed to reveal more information for the present calculations.

The corresponding distance, $L_{95}^*$, the dunes have migrated downstream in the period $T_{95}^*$, is given in Fig. 4.14 (right) made nondimensional with the dune length. The distance is defined as the distance between the location of the top of the dune at $t^* = 0$ and the top of the dune at $t^* = T_{95}^*$:

$$L_{95}^* = x_{top} (T_{95}^*) - x_{top} (0) \quad (4.17)$$

**Time Lag**

With respect to the time scale it can be seen from Eq. 4.7 that the initial development in the dune height (or actually the level of the top) depends on:

- the relative change in the Shields’ parameter, $\Delta = \frac{\theta' - \theta_1'}{\theta_1'}$
- the Shields’ parameter itself, and
- the dune length

Although it is shown above that the mechanisms taking place in the trough are also important for the development of the dunes, the same parameters are important for the time scale to equilibrium.

Keeping the dependence on the dune height on the Shields’ parameter (Fig. 3.10) for the steady dunes in mind, it can be seen that depending on the sand dune (or Shields’ parameter) in flow situation 1, a relative increase or decrease in the Shields’ parameter of for instance 10% causes a much larger change in the dune height if $\theta_1' \approx 0.1$ than if $\theta_1' \approx 0.3$. 

The time lag to the new equilibrium situation is hence expected to be scaled roughly by the volume of sand (depending on \( \theta_1', \Delta, \frac{\Delta}{\theta_1'} \)) to be relocated and the sediment transport rate (\( \theta_2' \)).

A few comments should be added to the time lag for the two flow situations with the increasing discharge. Assuming that the dune height grows in a more smooth manner (which might be the case if a LES model was used instead as described in Section 4.3.1), there is a possibility that the time lag to equilibrium is smaller than the time lag given in Fig. 4.14 (possibly as much as a factor 2 less).

**Migration length**

The distance the dunes migrate downstream before the new equilibrium situation, see Fig. 4.14 (right), is found to be approximately 0.5-1 times their own length. In the two situations with the increasing discharge (uu3 and uu4) the migration distance is larger than in the two flow situations with the decreasing discharge (uu1 and uu2).

When the dunes migrate forward after the abrupt change in the flow, translation as well as deformation of the dunes take place. A difference is noted in the deformation process of the dunes after a decrease respectively increase in the discharge:

- **Decrease in the discharge (eg. Fig. 4.6):** The change from the larger bedform in flow situation 1 to the smaller bedform 2 takes place by a) deformations in the top area of the dune according to the local sediment transport field (Section 4.3.1) and by b) the forward migration of the dune, where a part of the change in the trough area takes place by ”leaving the old bedform behind” (Section 4.3.1).

- **Increase in the discharge (eg. Fig. 4.8):** The change in the bedform at the top as well as at the trough takes place by deformations caused by the local sediment transport.

Fredsoe (1979) found the migration lengths to equilibrium to be larger than found here. He evaluated the length scale from the rate of change of the dune level at the top and the change in the dune height. Fredsoe found the migration lengths to be in the order of 2.5-5.8 dune lengths by applying the methodology outlined in Section 4.3.3 to experimental data by Gee (1973). Whereas Fredsoe’s method takes into account only the processes at the top, the present method takes into account also the processes taking place in the trough. Comparing the rate of change of the top level with the rate of change at the trough level (Fig. 4.12 and 4.11), it is clearly seen that change in the trough level is important for the change in total dune height. By including the processes in the trough level, the process of obtaining equilibrium is hence speeded up.

On the other hand, the separation zone in the present calculations is estimated too short. With a larger separation zone the (flat) trough will be longer. The migration length to obtain equilibrium is hence expected to be underestimated by the model: In the case of a decreasing discharge, the dunes must migrate a longer distance across the flat trough before the trough level starts to increase (see Fig. 4.7). If the discharge increases the location downstream the from the crest, where the erosion
begins to deepen the trough (see Fig. 4.8) will also move further downstream.

Finally the length of the dunes might have an influence on the time lag to equilibrium. The present calculations are performed for a constant dune length of 4 water depths. When the dunes also have to change their length, the time lag to equilibrium is expected to be longer.

$L_{50}^*$ is also influenced by the uncertainty in the estimate of the time lag; with a smaller time lag the migration length will be shorter than the one given in Fig. 4.14 (right) for the two flow situations with the increasing discharge.

4.4 Morphology during a flood wave

The morphology during a simplified flood wave is calculated in the following. Flood waves, where the discharge in the rivers increases during a limited period, often occur in rivers after a heavy storm. The difference from the situations described in the previous sections is that now the discharge is unsteady. Since the flow during a flood wave varies slowly, i.e. on a time scale comparable to the morphological response time, the similarities with the previous considerations in this chapter are obvious. When the discharge in the river increases, the friction on the bed increases. This is seen from the relations of flow resistance (see Chapter 3, Section 3.5). When the skin friction increases, the morphology responds to the changed flow and sediment conditions in a similar way as the dunes after a sudden change in discharge. In the calculations in the previous sections, the discharge is constant after the abrupt change in the flow condition and a new equilibrium situation for the flow-bedform interaction is found. During the flood wave studied in the following, the discharge is constantly changing. Due to the time lag in the response of the dunes to the flow, the dune height and shape corresponding to e.g. the maximum flow is thus not necessarily
reached.

4.4.1 Methodology

A simplified flood wave is studied, as seen in the upper part of Fig. 4.15. The initial bedform is again the steady state case for run s3L4 in Chapter 3, which is characterized by \( \beta = 4, \theta'_{u} = 0.11 \). The initial situation is therefore a stream/river/channel, where the flow has been constant long enough for the bed to obtain equilibrium at the time, when the flood wave begins. A flood wave of a duration \( T_p^* \) is studied.

The instationary change in the friction and hence sediment transport during a flood wave is modelled by applying a variation, \( f(t) = \frac{V'(t)}{V} \) to the mean velocity, \( V \), as outlined in Section 2.4. \( V'(t) \) is the instantaneous mean depth averaged velocity and \( V \) is the mean depth averaged velocity in the initial situation. The flood wave is hence modelled like a slowly varying current. The velocity is increased linearly for \( \frac{t}{T_p^*} = [0;0.5] \) and then decreased in a similar way in the time interval \( \frac{t}{T_p^*} = [0.5;1.0] \). The maximum amplification of the mean water velocity in the calculated flood wave is \( \frac{V'(t)}{V} = 1.35 \).

The remaining nondimensional parameters are kept constant: \( k_N/D=0.001 \), Froude= \( \frac{V}{\sqrt{gD}} = 0.20 \), Re=2.5e6. It should be noted, that these are now in the instationary flow situation, formulated in the flow parameters for the initial situation, \( V \) and \( D \). The change in the Shields’ parameter on an undisturbed channel bed corresponding to the change in the flow during the flood wave is in the range \( \theta'_{u} = [0.11, 0.20] \).

During flood wave in nature, does not only the mean flow velocity change. The water depth also changes and \( k_N/D \) hence changes. However, with regard to the roughness parameter, \( k_N/D \), the same line of arguments apply as outlined in Section 4.1; the relative influence of \( k_N/D \) is much smaller that the influence of the Shields’ parameter.

The duration of the studied flood wave is (nondimensionalized with sediment properties) \( T_p^* = 0.43 \). The time scale for the flood wave should be commented on in relation to the findings in the previous section. For the relevant range of the Shields’ parameter, Fig. 4.14 shows that for a situation with an abrupt change in the Shields’ parameter from \( \theta'_{u} = 0.11 \) to \( \theta'_{u} = 0.20 \) (corresponding to a change from \( \theta_{top} = 0.14 \) to \( \theta'_{u} = 0.26 \)), the time scale for a steady situation to take place is \( T_{95} = 0.26 \), i.e. close to half of the duration of the flood wave. However, in this situation, the sediment transport rate after the change in the discharge corresponds approximately to the highest Shields’ parameter. During the flood wave the Shields’ parameter and hence the sediment transport rate is lower in the beginning and then slowly increase. The morphological changes of the dune will hence take a longer time.

The morphological time step is varied according to the immediate flow situation - see Section 4.5.1.

4.4.2 Change in dune height and bedform

Since we have already seen that the total flow resistance is highly dependent on the height of the dune (Section 3.5) the key observations in this section are coupled to the dune height. The increase in the dune height is seen in the center part of Fig. 4.15. The dune height during the flood wave is compared to the dune height, which
Morphology during a flood wave

![Graph showing velocity, dune height, and level variation over time]

Figure 4.15 Variation in the mean velocity (top) during the modelled flood wave with a total duration of $T_F^*$ and corresponding variation in $\theta_u'$ (upper middle subfigure). The dune height variation (lower middle subfigure - fully drawn curve) is compared to the equilibrium dune height in a steady flow with the same mean velocity (dashed curve). Variation of the top and trough level is shown in the bottom figure.

corresponds to the equilibrium dune height for a steady flow situation with the same mean velocity (or more precisely: the same Shields’ parameter for an undisturbed bed, $\theta_u'$). An approximate dune height variation with $\theta_u'$ for a dune length of $L/D=4$, is found by modifying the expression obtained by Fredsoe (1982) (for an equilibrium length of the dune), see Fig. 4.17, such that a fit to the dune heights found in Chapter 3 is obtained. The expression fits the relevant area, $\theta_u' = 0.11 - 0.20$, within +/- 3%. From Fig. 4.15 the following is noticed:

- The time scale for the decrease in the dune height is seen to be about $2\frac{1}{2}$ times the time scale for increase in the dune height. This difference in time scale for respectively an increasing bed load capacity situation and a situation of a decreasing bed load capacity corresponds well with the difference in time scale found in the previous chapter.

- A phase lag is observed between the maximum mean velocity and the maximum dune height. The dune height keeps increasing even after the mean velocity
Figure 4.16  Hysteresis effect in the dune height during the flood wave due to the time lag between the flow and the morphology. The change with time is counter clockwise starting at t=0 for $\theta' = 0.11$.

Figure 4.17  Approximated expression for steady dune height for a dune length of $L/D=4$ as a function of of the Shields' parameter on a undisturbed bed. +: dune heights in steady flow as found in Chapter 3.
starts to decrease. This is explained by the observation, that the dune height at the time of the peak mean velocity is lower than the corresponding dune height for the steady flow. The dune is therefore still lower than the equilibrium dune height for the flow condition at this time and will hence keep increasing in height.

- The dune height increases to a maximum height, which is 3% larger than the maximum height for a corresponding steady flow situation. It should be added that although the amplification is not large, the result is interesting, since this behavior can only be caused by the unsteadiness.

The lagged response in the morphology after a change in the flow is furthermore illustrated in Fig. 4.16. The dune height variation with the nondimensional bed shear stress during the flood is related by a hysteresis loop. Such an effect is also identified in measurements of the dune response during a flood by Julien, 2000, and is also mentioned by Dalrymple and Rhodes, 1995.

To understand the details and the time development of the dune height, the changes in the dune height should be held up against the morphological development. In Fig. 4.18 the development in the dune is shown in respectively the time intervals of increasing, decreasing and steady flow velocity.

- During the period of increase in the water velocity, the dune height is lower than the dune height that corresponds to the same Shields’ parameter for an undisturbed bed. Hence, the flow changes faster than the bed is able to change in order to be in equilibrium with the flow field at all times.

- The increase in the dune height is seen to be caused by an increase in the top level as well as a decrease in the trough level. The rate of change in the trough level is faster than the rate of change in the top level. This was also observed after an abrupt change in the discharge (see Fig. 4.11 and Fig. 4.12).

- The migration velocity of the dune increases with the increasing discharge, due to the increasing bed load capacity.

- After the decrease in the discharge begins, the level of the top still increases, in fact the rate of change of the top increases during and also right after the peak in the discharge. The growth at the top of the dune is determined by a balance in the gradient in the sediment transport at the top and the "migration"-term \( \frac{\partial h}{\partial x} \), as prescribed by the continuity equation:

\[
\frac{dh}{dt} = -\frac{1}{1 - n \frac{\partial q}{\partial x}} + a \frac{\partial h}{\partial x} \tag{4.18}
\]

The gradient in the sediment transport at the top is determined by the rate of sediment transport (or Shields’ parameter) and the phase difference between
Numerical issues

the bedform and the sediment transport (ultimately governed by the shear stress distribution and thus the instantaneous bedform shape). After the discharge decreases, the sediment transport and hence also the migration velocity decrease.

- The level of the trough practically stays the same after the decrease in the discharge begins. Again the balance is governed by the continuity equation (Eq. 4.18). The erosion/deposition in the trough is caused by the mechanisms outlined in Section 4.3.1. Thus, the following is important: a) The location, where friction exceeds the critical level for sediment transport b) the gradient in the sediment transport downstream this point and c) the migration velocity. In this case the dune height (especially at the crest) immediately after the decrease of the discharge increases and the location of \( \theta'_c \) moves downstream. Due to the decreasing sediment transport, the migration velocity decreases - this effect reduces the previous effect.

\[ t/T_F = 1.0 \rightarrow \text{Steady discharge} \]

- After the discharge is steady again, the dune height decays by reducing the top level and increasing the trough level as described in Section 4.3.1 until the bedform is again steady.

- The migration velocity is seen to be more or less constant. Since the discharge is now constant, the friction on the bed is more or less constant and the migration velocity only varies with the change in the dune height.

4.5 Numerical issues

4.5.1 Variable morphological time step

The time step (which is formulated in flow parameters in the model) was changed during the calculations. This improved, the morphological calculation by ensuring, that the effects of the filter were "constant" during the calculation. If the time step was kept the same during the calculation (i.e. at a time step small enough for stability also at the highest sediment transport rates) the effect of the filter was reflected in the morphology for the lowest transport rates.

The morphological Courant-criterium was in Chapter 2 found to be:

\[ \Delta t < \frac{\partial h}{\partial x} \frac{\partial q_h}{\partial x} \Delta x \]  (4.19)

In a morphological calculation with an unsteady flow situation it was found that a variable time step according to a constant morphological Courant number was preferred over a constant time step. The nondimensional time step in the model is defined using the flow variables, \( \Delta t \cdot V \). On nondimensional grounds Eq. 4.19 can be written as:
Figure 4.18 Variation in dune shape and migration during and immediately after flood wave. In the figure the time step between each bedform is the same.
\[
\frac{\Delta t \cdot V}{D} < \frac{\partial (h/D)}{\partial (x/D)} \frac{Fr}{(s-1) \left( \frac{d}{T} \right)^3} \frac{\Delta x/D}{\delta T/D} \quad (4.20)
\]

In a morphological calculation with an unsteady flow situation, where we want to increase/decrease the morphological time step with the morphological Courant-number approximately constant, this reduces to:

\[
\frac{\Delta t \cdot V}{D} < K \frac{Fr}{\Phi_b} \quad (4.21)
\]

- where we have used that the spatial gradient in the sediment transport is approximately proportional to the transport itself, \(\frac{\partial \Phi_b}{\partial (x/D)} \sim \Phi_b\). For the time step at two different flow situations in the in the morphological calculations in this Chapter, where the bed load rate varies with time, the time step should be scaled such that:

\[
\frac{\left( \frac{\Delta t \cdot V}{D} \right)_1}{\left( \frac{\Delta t \cdot V}{D} \right)_2} = \left( \frac{Fr}{\Phi_b} \right)_1 \left( \frac{Fr}{\Phi_b} \right)_2^{-1} \quad (4.22)
\]

For stability reasons this was not entirely possible. Instead a curve was fitted to corresponding values of \(\left( \frac{\Delta t \cdot V}{D} \right)\) and \(\left( \frac{Fr}{\Phi_b} \right)\) as found in the steady flow calculations. In the steady flow calculations the time step was found by trial and error, but increased as much as possible to speed up the calculations as well as avoid unnecessary filtering effects. The following is the time step variation with the parameter \(\frac{Fr}{\Phi_b}\):

\[
\frac{\Delta t \cdot V}{D} = -0.9081 \left( \frac{Fr}{\Phi_b} \right)^2 + 8.347 \left( \frac{Fr}{\Phi_b} \right) - 0.1653 \quad (4.23)
\]

The linear term in Eq. 4.23 is the dominating one, while the quadratic term acts to decrease the time step for lower sediment transport rates, which is a problem if a pure linear relationship is used as suggested by Eq. 4.22.

### 4.6 Summery

Unsteady dunes in a unidirectional flow were studied in this chapter. The mechanisms for the dune height change after a change in the sediment transport were focused upon for two reasons 1) to understand the growth and the decay of the dunes when the discharge changes 2) to verify the morphological calculations.

The response of the total dune height to the flow conditions was divided into the changes of the top and of the trough level of the dunes. The change in the level at the top of the dune after a sudden change in the flow condition, was found to compare very well with the model by Fredsøe (1979), which describes the initial change after a sudden change in the flow.
Summary

The processes at the trough were found to be governed by the change in the location of the skin friction, $\theta'$, exceeding the critical skin friction for sediment transport, $\theta^*$. This location moves relative to the bedform, when the flow conditions change. When the Shields’ parameter increases the location moves downstream and vice versa.

For the increasing flow conditions this process deepens the trough immediately after the increase in the flow. This deepening was found to have an order of magnitude which is comparable to the change of the top-level, and is hence important for the total growth of the dune height. For a decrease in the flow, the change in the trough level takes part in three phases. A method for estimating of the change of the trough level with time in each of the three phases based on the calculated flow and sediment transport immediately after the change in the flow conditions was presented. The results were found to compare well with the numerical calculations.

The development of dunes by a simplified flood wave was calculated. This example added the effects of an unsteady flow (slowly varying) to the previously mentioned observations. The growth of the dune height (the change of the level of the dune top as well as the change in the trough level) was found to follow the same physical mechanisms as immediately after an abrupt change in the flow. A phase lag between the maximum dune height and the maximum flow discharge was observed due to the time lag between the flow and morphology.
Chapter 5
UNSTEADY DUNES DUE TO REVERSAL OF THE FLOW DIRECTION

In this chapter the effects of a change in the current direction on the dunes are studied. Dunes in nature opposed to a reversing current are usually connected to tidal flows. The present chapter therefore merely serves as a background for Chapter 6, in which dunes in a periodically reversing flow situation is studied for a time scale of the flow reversal similar to tidal periods (approx. 12 or 24 hours). In the present chapter the physical mechanisms, which change the dune shape after flow reversal are described.

The methodology is similar to the approach taken in Chapter 4 (Unsteady dunes in a unidirectional flow) - the steady dunes found in Chapter 3 are used as initial bedforms and are applied to a similar flow situation, however the direction of the current is reversed. As in the previous chapters only bed load is considered in the sediment transport field.

The flow and sediment transport fields immediately before and after flow reversal are compared in Section 5.2 in order to understand the morphological changes of the dunes occurring right after flow reversal. The morphological changes of the sand dunes due to the flow reversal are described in Section 5.3. Finally amplification of the dune height due to the flow reversal and the time scale for a full reversal of the bedform are found.

5.1 Methodology
The morphological development of bedforms after flow reversal is studied in this section. The two main objectives are

1. the time scale for the reversal of the bedform
2. the change of the bedform shape with time

The bedforms studied are the steady dunes found in Chapter 3 for a dune length $L_D = 4$ and Shields’ parameters in the range $\theta'_u = 0.062 - 0.40$. With regard to a periodically reversing flow (with a relatively long period like the tidal flow), these bedforms are interesting to study in the context of the two bullet points above due to the following: (1) is important in a tidal flow situation, where the flow reverses every (approximately) 6 or 12 hours. The time scale for reversal of the steady dunes indicates whether a full reversal of the bedform, i.e. a steady state situation is reached during each tidal half-period. (2) is important for the understanding of the evolution of the bed, and for example for the variation of the flow resistance with time.
The flow and sediment transport field at flow reversal

<table>
<thead>
<tr>
<th>Run</th>
<th>Initial bed (Chapter 3)</th>
<th>L/D</th>
<th>$\frac{u}{f}$</th>
<th>$\theta'_u$</th>
<th>Fr</th>
</tr>
</thead>
<tbody>
<tr>
<td>r1L4</td>
<td>s1L4</td>
<td>4</td>
<td>0.001</td>
<td>0.062</td>
<td>0.15</td>
</tr>
<tr>
<td>r2L4</td>
<td>s2L4</td>
<td>4</td>
<td>0.001</td>
<td>0.08</td>
<td>0.17</td>
</tr>
<tr>
<td>r3L4</td>
<td>s3L4</td>
<td>4</td>
<td>0.001</td>
<td>0.11</td>
<td>0.20</td>
</tr>
<tr>
<td>r4L4</td>
<td>s4L4</td>
<td>4</td>
<td>0.001</td>
<td>0.20</td>
<td>0.27</td>
</tr>
<tr>
<td>r6L4</td>
<td>s6L4</td>
<td>4</td>
<td>0.001</td>
<td>0.40</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Table 5.1 Morphological calculations with a reversal of the flow direction. Re=2.5e6 in all calculations.

Figure 5.1 Flow is changed according to a stepfunction - the direction of the flow is reversed at $t^* = 0$.

The methodology in the present chapter is similar to the approach in the two previous chapters. The bedforms are applied to a flow field, which is similar with regard to the nondimensional values defining the flow, only the flow direction is reversed. The flow situation is hence changed according to a step-function, see Fig. 5.1, where the discharge is changing from $Q$ to $-Q$ at $t^* = 0$. The nondimensional flow parameters are already explained Chapter 3. The test matrix is therefore as given in Table 5.1, and the flow parameters are seen to be similar to the flow parameters in Table 3.1.

The morphological development of the dune is calculated by the numerical model after the flow direction is reversed and until the (reversed) bedform is again in equilibrium with the flow field and the dune is steady.

5.2 The flow and sediment transport field at flow reversal

The flow and sediment transport variation along dunes immediately before and after the abrupt flow reversal are described in the following. The aim is to obtain a background for understanding the morphological changes of the dunes in the situation of reversal of the flow. The actual morphological changes after the flow reversal are described in Section 5.3.

In Fig. 5.2 and Fig. 5.3 the skin friction and the corresponding sediment transport field is shown immediately before and after the flow reversal in the upper and middle figures for two different Shields’ parameters. In Fig. 5.2 the bedform is from run s4L4 characterized by a dune length $\frac{L}{D} = 4$ and $\theta'_u = 0.20$ and in Fig. 5.3 the bedform is from runs s2L4, and characterized by $\frac{L}{D} = 4$ and $\theta'_u = 0.08$. The flow
parameters are the same as given in Table 3.1 for L/D=4 in Chapter 3. In the lower figures the erosion and deposition pattern is given:

5.2.1 Immediately before the flow reversal

The Shields’ parameter, bed load and erosion/deposition along the dune immediately before the flow reversal is shown in the left hand side figures in Fig. 5.2 and Fig. 5.3. The flow and erosion/deposition pattern of dunes in equilibrium with the flow has previously been described in details - see Section 3.3. Here we just note that erosion takes places along the upstream gentle slope and a zone of depositions is found at the steep downstream slope, causing the dune to migrate downstream.

5.2.2 Immediately after the flow reversal

The Shields’ parameter, bed load and erosion/deposition along the dune immediately after the flow reversal is shown in the right hand side figures in Fig. 5.2 and Fig. 5.3 (note that in Fig. 5.2 the two lowest figures are just showing the same, but with different scaling on the y-axis)

   At the dune front, which now - after the flow reversal - faces the upstream direction, a strong acceleration takes place due to the stronger contraction and hence stronger convergence of streamlines. The Shields’ parameter thus increases along the front, which causes an increase in the bed load and hence erosion of sediment takes place from the front as seen in the lower right figures. On the top of the dune, just downstream of the crest, the Shields’ parameter decreases due to the relaxation in the near wall flow (the boundary layer thickness increases and the curvature acceleration decreases). The bed load capacity thus decreases rapidly and a deposition zone is hence found immediately at the top. Along the downstream slope the flow decelerates due to the weak expansion of the flow area causing an increased flow resistance. The Shields’ parameter hence decreases, which causes the bed load to decrease and a weak deposition is therefor seen along the downstream slope.

Comparing the erosion and deposition pattern at two different Shields’ parameters (lower figures in Fig. 5.2 and Fig. 5.3) the same general features are found - only the bed load is less and the rate of change of the bedform is smaller for the lower Shields’ parameter. There is a slight difference in the deposition/erosion pattern. In the case of the high Shields’ parameter the major deposition zone is immediately downstream the crest. In the situation with the low Shields’ parameter a relaxation in the flow is also seen after the crest, but due to a further contraction of the flow area after the crest, the maximum sediment transport is not found until further towards the top. A more gentle deposition pattern is hence found.

   The effect of curvature acceleration is also noticed here. Especially in the situation with the high Shields’ parameter (Fig. 5.2) a deposition zone is found in the trough (as in Section 4.3.1). The reason for this is the change in curvature of the bed leading to curvature acceleration: the concave bedform, where the gentle slope meets the trough causes a higher pressure at the bed and hence induces a favorable pressure gradient in the direction of the trough. The Shields’ parameter as well as the bed load are increased. The sediment is deposited, when the curvature acceleration decreases in the trough, where the curvature of the bed is low.
Figure 5.2  Friction, sediment transport and erosion/deposition immediately before and after flow reversal. $\theta'_u = 0.20$. $k_N/D = 0.001$. $L/D = 4$. $H/D = 0.16$. 
Figure 5.3 Friction, sediment transport and erosion/deposition immediately before and after flow reversal. \( \theta''_u = 0.08 \), \( k_N/D = 0.001 \), \( L/D = 4 \), \( H/D = 0.09 \).

5.3 Morphology after flow reversal

5.3.1 Initial morphological development

An example of the initial change of the bedform after flow reversal is shown in Fig. 5.4 for the highest Shields’ parameter calculated, \( \theta''_u = 0.40 \). The morphological development of the dune corresponds to the erosion and deposition pattern found by studying the flow field in Section 5.2: Erosion takes place at the dune front and sediment is deposited on the top of the dune. This deposition creates a minor dune on top of the "old" dune, which causes the top of the dune to reverse. This overlying dune has a front which, as time passes, migrates downstream the slope of the old dune as the bedform reverses more and more. From Fig. 5.4 it is seen that the migration velocity of the front decreases with time as the height of the downstream slope increases. The migration of the front takes place by deposition of the bed load being transported over the crest. This corresponds to the migration of steady dunes as described in Section 3.2, where the migration velocity can be found from \( a = \frac{\theta''_u}{H} \). If the dune height, \( H \), in this equation is exchanged with the height of the downstream slope, the migration velocity, \( a \), is decreasing as the height is increasing as seen in Fig. 5.4.

The flow field is changed from the initial flow field as described in Section 5.2, when the top of the dune is reversed. The skin friction over the dune is shown in Fig. 5.5. Downstream from the crest of the "overlying dune", the flow separates (if the slope of the front is steep enough), just like the separation downstream the front of a steady dune. After the reattachment, a new boundary layer will build up. A relaxation of the flow takes place and the skin friction starts to increase. However,
since the flow area is expanding, the flow resistance increases downstream from the reattachment point and skin friction starts to decrease further downstream.

In Fig. 5.6 the initial reversal of a bedform at a lower sediment transport rate, $\theta'_u = 0.08$, is shown. In general the picture is the same: the top of the dune reverses and the new downstream front migrates down the gentle slope of the old bedform. Two differences is seen: a) small instabilities exist on the top of the dune, these are due to the time step probably being on "the limit" (almost too large for the morphological scheme) - the wiggles disappear as the migration velocity decreases.(see more details in Section 5.6 about numerical issues in relation to this Chapter) b) the overlying bedform created on top of the old one is more rounded and c) downstream from the reversing top small perturbations are generated. These small perturbations actually influence the dune height variation quite a lot (as we will see later), so they are of some interest. They might be the result of the effect described above (with a new boundary layer growing up), in combination with curvature acceleration effects, caused by the curvature of the bedform. The perturbations are not seen in a stronger flow as seen in Fig. 5.4, where the flow is separating and the opposing pressure gradient is larger due to the larger expansion of the flow area. It can be speculated whether this increased retardation of the flow, cause the perturbations not to show up. Further investigations should be made to draw any conclusions.

The development of the dune are very much like seen in observations after a flow reverse in a tidal cycle, see for instance Langhorne, 1982.

5.3.2 Towards new equilibrium situation

As time passes the bedform reverses fully and the overlying bedform develops to be the new bedform. In Fig. 5.7 and 5.8 the full reversal of the two dunes are shown from the initial steady bedform until the bedform is steady in the opposite direction. The bedform development for the high Shields’ parameter shows no surprises. The development of the bedform for the small Shields’ parameter is more complicated. The above mentioned perturbations (and the wiggles) disappear again, but first they migrates downstream into the trough and influences the gentle slope on the downstream dune. "Influences" here means that the perturbations migrate up the gentle slope (overlying perturbations migrate faster than the bedform itself), but also the flow fields will be influence by the existence of these perturbations. A more indirect influence is hence also exist.

5.4 Dune height amplification

During the reversal of the dune, the dune height is amplified compared to the steady dune height. The maximum amplification of the dune height is shown in Fig. 5.10. In a tidal flow situation the maximum dune height is thus expected to be larger than the steady state dune height, but this is further discussed in the next chapter. The overlying dune at the top is the reason for the amplification of the dune height. During the time it takes for this dune to pass over the top, the dune height is larger than the steady dune height.
Figure 5.4  Initial reversal of bedform after a flow reversal. Solid bold line indicates the bedform just before flow reversal. $\theta'_u = 0.4 \frac{H_{steady}}{D} = 0.18. \frac{L}{D} = 4$. Run r6L4. Flow direction is from left to right.

Figure 5.5  The skin friction distribution after the bedform has partially reversed. $\theta'_u = 0.40. t^* = 0.014. k_N/D = 0.001. L/D = 4$. Run r6L4. Flow direction is from left to right.
Figure 5.6  Initial reversal of bedform after a flow reversal. Solid bold line indicates the bedform just before flow reversal.  $\theta_u' = 0.08$.  $\frac{H}{D_{steady}} = 0.09$.  $\frac{L}{D} = 4$. Run r2L4. Flow direction is from left to right.

Figure 5.7  Full reversal of bedform after a flow reversal. The bold line indicates the bedform just before the flow reversal $\theta_u' = 0.40$.  $\frac{H}{D_{steady}} = 0.18$.  $\frac{L}{D} = 4$. Run r64L. Flow direction is from left to right.
Figure 5.8  Full reversal of bedform after a flow reversal. The bold line indicates the bedform just before the flow reversal $\theta_u' = 0.08$. $\frac{H}{L_{steady}} = 0.09$. $\frac{L}{D} = 4$. Run r2L4. Flow direction is from left to right.
Figure 5.9  Dune height development with time after the flow reverse.

Figure 5.10  Maximum amplification of dune height during the reversal of the dune, and time scale for an equilibrium situation to take place.
5.5 Time scale

The change of the dune height with time is shown in Fig. 5.9 for five Shields’ parameters. The time scale for full reversal of the bedform is estimated, and shown in Fig. 5.10. The definition of the morphological time scale for equilibrium is similar to the one used in Chapter 4. The dune height reaches during the reversal of the bedform a maximum dune height, $H_{\text{max}}$, and when the bedform is fully reversed it obtains the dune height for the steady dune, $H_{\text{steady}}$. Thus, the difference $H_{\text{max}} - H_{\text{steady}}$ is the maximum dune height variation. The time scale for the flow reversal, $T_{95}^*$, is defined as the time it takes until the dune height only remains to change by 5% of the difference $H_{\text{max}} - H_{\text{steady}}$ before the dune is again steady:

$$T_{95}^* : 0.95 < \frac{|H_{\text{max}} - H(T_{95}^*)|}{|H_{\text{max}} - H_{\text{steady}}|} < 1.05$$  \hspace{1cm} (5.1)

-where the nondimensional morphological time is as previously defined: $t^* = \frac{\sqrt{g(s-1)d^3}}{D^2}t$.

The dune height development for especially for the small Shields’ parameters reveal that some reasoning must be applied to determine the time scale. Let us take as an example the change of the dune height for $\theta'_{\text{u}} = 0.08$. After the peak in the dune height at approx. $t^* = 0.25$, the dune height has a depression at approx. $t^* = 0.5$ and furthermore decreases to below the original dune height before the steady state dune height is found. The reason for this behavior should be investigated to find the physical (or numerical) explanation. Thus, studying the development of the dune in Fig. 5.12, the reason for the first depression in the dune height is seen to be the perturbations explained above, which travel through the trough. This naturally decreases the dune height. Since these perturbations are as previously explained judged to be physical, the depression of the dune height is accepted as being physical as well. The following increase in the dune height is found to decrease with the effect of the morphological filter (further explanation in Section 5.6), but seems to be part of the general decrease of the dune height at the top. The decrease of the dune height to below the steady dune height, on the other hand, to disappears with a decreasing influence of the morphological filter, and is hence judged to be nonphysical. Taking this into consideration, the time scale is evaluated to $t^* = 1.0$. The dune height development for $\theta'_{\text{u}} = 0.062$ and $\theta'_{\text{u}} = 0.11$ is following the same pattern. The time scale is hence judged to be of some uncertainty - the uncertainty range for the calculations for the small values of $\theta'_{\text{u}}$ may be as high as $\pm 20\%$.

The time scale decreases with an increasing Shields’ parameter as seen in Fig. 5.10. A numerical example gives an idea of the order of magnitude. If $\theta'_{\text{u}} = 0.11$ the dune height in Chapter 3 was found to $\frac{H}{D} \approx 0.13$ for $\frac{L}{D} \approx 0.13$. The time scale for full reversal of the bedform is found to $T_{95}^* = 0.49$ (Fig. 5.10). If the water depth is 3 meters, the dune height is $H = 0.39m$ and the dune length is $L = 12m$. For a grain size of 1 mm the reversal of the dune takes approximately 9.8 hours. If the water depths is 1 m, the dune height and length is now $H = 0.13m$ and $L = 4m$. Keeping the same grain diameter, the time for reversal is now only approx. 1.1 hour. Due to the smaller dune size, the reversal takes place much faster (since the sediment transport rate is kept constant in the example).
The calculations of the time scale for reversal of the dune is performed only for a dune length of 4 water depths. Increasing the dune length (or rather domain length) the dune height also increases as we found in 3. The volume of the sand body of the dune is then larger and therefore a larger time scale should be expected, since a larger volume must be moved.

5.6 Numerical issues

In Chapter 3 the influence of the morphological filter was studied by investigating the influence of the filter on the equilibrium condition for steady dunes. In this section the equilibrium condition is not the main result, but the unsteady dune shape in itself. The influence of the numerical filter is instead studied by a) testing the effect of ”more” and ”less” filter and b) by ensuring that the local rate of change of the bedform is always dominated by the physical terms and not by the numerical filter.

Morphological filtering is performed after each update of the bed, i.e. in each morphological time step. If the time step is decreased (and the spacial discretization of the bed is the same), the filter effect is larger (the rate of change per. time step is smaller but the filter effect is the same). Two calculations with $\theta' = 0.08$ is shown in Fig. 5.11 with two different morphological time steps. The initial dune height change with time is the same, but with a smaller time step (”more” filter) the depression in the dune height before the equilibrium state is reached, is much more pronounced. The morphological development in the case with the lower time step is shown in Fig. 5.12 Compared with the same calculation just with the higher morphological time step (and hence less filter) as in Fig. 5.8, the morphological development in the two cases shows that this dune height change origins from the migration of the small perturbations, which is generated initially. With a higher morphological time step they disappear much faster than with a smaller time step. The instabilities on the top of the dune in Fig. 5.8 does not show up in Fig. 5.12.

The modification of the local rate of change of the bed caused by the filter within three different morphological time steps is shown in Fig. 5.13. The notation is the same as used in Fig. 3.24 in Chapter 3: $\frac{\partial h}{\partial t}|_{sf}$ is the rate of change of the bed due to the divergence in the bed load field as calculated from the skin friction, $\frac{\partial h}{\partial t}|_{a}$ is the rate of change caused by the avalanche function and $\frac{\partial h}{\partial t}|_{nf}$ is the effect on the bed change cause by the numerical filter.

The morphological filter smoothes the rate of change of the bed, but the effect is only significant, where the bed is strongly curved at the fronts.

5.7 Summery

Steady dunes in equilibrium with a given discharge (found in Chapter 3) were applied to a similar flow field, only the flow direction was abruptly reversed. The reversal of dunes after the reversal in the flow direction was modelled.

The morphological modelling of the dunes agree with observations in regard to the behaviour of dunes immediately after a change in the flow direction in a tidal flow. Initially, erosion takes place from the former front, and deposits on the top of the bedform. The top of the dune reverses this way, and a front facing the (new)
Figure 5.11  Effect of numerical filter on dune height development with time. $\theta'_w = 0.08$.

Figure 5.12  Morphological calculation precisely as in run r2L4, just with a lower morphological time step ($\Delta t_D = 5$) than in Fig. 5.8 ($\Delta t_D = 10$), and hence a larger effect of the morphological filter. Flow direction: left to right.
Figure 5.13  Numerical filter smoothes the local rate change of the bed. $\theta'_a = 0.11$. Flow direction: right to left.
downstream direction is created and migrates to become the new (fully reversed) dune. The amplification of the max dune height and the time scale for full reversal of the dune is found. The dune height amplification after flow reversal is 8-32 % with the largest amplifications found for the highest $\theta_{\alpha}$. The nondimensional morphological time scale for reversal, $T^*_{95}$, decreases with the Shields parameter.
Chapter 6
DUNES IN A TIDAL FLOW

Unsteady dunes in a tidal flow is studied in this section. Only the "equilibrium" situation is studied, i.e. the situation where the bedforms are fully periodic with the periodically reversing flow. These equilibrium tidal dunes are calculated using the numerical model in a number of different tidal flow situations.

For dunes in equilibrium with the flow, the time scale for a new steady situation to take place, was found in In Chapter 5. This time scale is in the following used to understand the geometry of the tidal dunes. In a tidal flow, the flow direction is changed not only once, as in Chapter 5, but periodically on a time scale corresponding to a tidal flow situation.

The flow description is in this chapter still a step-function, such that the velocity within each half-tidal cycle is constant. The geometrical properties for the calculated dunes (shape and dune height, dune length is kept constant) are analyzed in Section 6.2 and 6.3.

As already mentioned in Chapter 1 the sand body influenced by the reversal of the flow direction is dependent on the period, T, the size of the dune and the sediment transport in each half period. It is interesting to differentiate between the sand volume which is active during the tidal cycle (also called the active layer) and the sand volume, which is undisturbed, for several reasons. This is the subject in Section 6.4. In the case that a pipe line is planned to be buried in the sea bed, it should be located in the volume of sand, which is not moving in order to be protected against the hydrodynamic forces produced by the wave and currents. Naturally, only very small migration distances of the sand dunes, can be accepted, if a pipeline is to be buried in a part of the sand volume. Another reason to take interest in the active layer could be for biological/ecological/environmental reasons.

The last but not the least interesting matter in this chapter is the stabilizing mechanisms for tidal dunes (Section 6.6). Stability of the dunes are analyzed from the point of view of applying a phase-averaged respectively phase-resolved approach for the sediment transport and morphology.

6.1 Methodology

6.1.1 Flow description and impact on morphology

The bedforms are calculated using the numerical model Dune2D. Tidal currents in bedform analysis are often calculated by either the shallow water equations, or by a 2DV model simpler than the one applied in the present work. Some 2DV model
assume hydrostatic pressure and calculate the vertical velocity from a continuity equation. The present method, as described in Chapter 2, includes the full solution of the Navier Stokes equations with non-hydrostatic pressure.

The only simplification in the present analysis of tidal flow is with respect to the time variation in the mean flow during the tidal cycle: The sinusoidally time-varying forcing is substituted by a constant forcing during the two tidal periods, see Fig. 6.1. The above simplification has limited impact on the dune morphology, as further described below. At least three main physical characteristics of the tidal flow, which is left out by the step-function representation of velocity, should be commented on regarding the importance for the morphological development of the dunes:

1. the instantaneous flow acceleration
2. the time variation in the mean velocity and hence in the sediment transport during the tidal cycle
3. the spring/neck cycles, which cause the velocity amplitude in the tidal flow to have with a periodic variation of approximately 14 days

The key thing to notice is that for the evaluation of the bedform change during a tidal cycle, the time variation of the flow is less important. The importance is the rate of the sediment field along the bedform integrated over the tidal cycle (or half cycle). The advantage of the present stepfunction approach is connected to the numerical calculations. With a nearly constant sediment transport, the morphological time-stepping and effect of the filtering scheme is much easier to control. A time dependent variation of the flow through the tidal period is not included in the present study. An approach like the one taken in Section 4.4, where the morphological changes of the bed during a flood wave was modelled could be applied.

The absence of the spring and neck cycles in the present work is believed to be a much more important limitation.
The acceleration in the momentum equations

The pressure gradient in the tidal wave is varying very slowly, and the near wall velocity is hence in phase with the outer flow. The tidal wave is in that respect different from the short periodic surface waves, and when considering sediment transport and morphology the notation tidal current is more proper. In boundary layers for short periodic waves, a lag exists between the near bed velocity and the outer flow velocity. This lag is due to differences in inertia in the two water masses, and the fast changes in pressure gradient caused by the surface waves. This lag causes accordingly a lag between the shear stress, and hence the sediment transport, and the outer current velocity. The acceleration of the flow is therefore important with respect to the bottom shear stress and hence the sediment transport in short periodic surface waves. In a tidal current the bottom shear stress is practically in phase with the current speed and such a lag is hence very small (due to the long periodic variation of the pressure gradient). The sediment transport, especially bed load, is therefore nearly in phase with the current velocity, and the acceleration of the flow not important for the sediment transport.

The time-variation in the velocity

Under (short periodic surface) waves the boundary layer is very thin and sometimes laminar during part of the period. In a tidal flow the boundary layer has time to grow and extends over the entire water depth. Furthermore, the flow is fully turbulent. For a given roughness, a fully turbulent flow (i.e., Re-independent) along a fixed bed is self-similar with respect to a variation in the mean velocity. This was also realized in the non-dimensional description of the flow equations in Chapter 2. Similarity in the near-wall velocity along the bed leads to similarity in the skin friction distribution. For two flow situations, 1 and 2, with different mean velocities, where the above is fulfilled, we have:

\[ \theta'_1 = K_1 \cdot \theta'_2 \implies \frac{\partial \theta'_1}{\partial x} = K_1 \cdot \frac{\partial \theta'_2}{\partial x} \]  

(6.1)

-where K is a constant. From the Meyer-Peter and Müller bed load formula, the sediment transport is given as:

\[ \Phi_b = 8 \left( \theta' - \theta'_c - \mu \frac{\partial h}{\partial x} \right)^{3/2} \]  

(6.2)

Ignoring for a moment the dependency on \( \theta'_c \) and neglecting the gravity effect due to the bed slope, the longitudinal gradient in the sediment transport is:

\[ \frac{\partial \Phi_b}{\partial x} = 12 \sqrt{\theta} \frac{\partial \theta'}{\partial x} \]  

(6.3)
- or when combining 6.1 with 6.3, we obtain that the gradient in the sediment transport in flow situation 1 and 2, are similar.

\[
\frac{\partial \Phi_{b,1}}{\partial x} = K_2 \cdot \frac{\partial \Phi_{b,2}}{\partial x}
\]

This is an important result: The morphological changes along the bed, \( \frac{\partial h}{\partial t} = -\frac{1}{\kappa} \frac{\partial \Phi}{\partial x} \) are self-similar in the two flow situations. The development of the bed will be the same no matter how the velocity of the tidal flow varies with time, as long as the total sediment transport integrated over one tidal period (or half period) is the same.

To arrive at this result, the slope effect on the effective Shields’ parameter and the critical skin friction for sediment transport is ignored. In situations, where the skin friction and hence the sediment transport is large, this is a good assumption. In situations, where neither the effect of gravity nor the critical shear stress can be neglected, full similarity between the effective skin friction at two different mean velocities will not exist, but is should be noted that:

1. due to the strong non-linearity between the velocity and the sediment transport \((\Phi \sim V^3)\), the majority of the sediment transport, and hence morphological changes, takes place when the tidal current (and skin friction) is largest

2. for small differences between \(\theta'_1\) and \(\theta'_2\) (\(K_1=O(1)\) in Eq. 6.1) the effective skin friction distributions in the two flow situations are still nearly proportional, and

3. the location of \(\theta'_{\text{eff}} = \theta'_c\) relative to the bedform will move during the tidal period.

With regard to item 3) it was revealed in Chapter 4 that this has an impact on the morphology in the trough area for dunes in an unsteady unidirectional current. However, the location of \(\theta'_{\text{eff}} = \theta'_c\) is not always in the trough area for reversing dunes, so the impact in the case of tidal dunes can not be compared with the dunes in a unidirectional current. With a time varying flow velocity, however, the bedform might obtain a more smooth shape than when the step function approach is applied for the flow velocity - for the same reason as if a dynamic description of the separation zone was included. Taking 1) and 2) into account, suggests that the morphological impact of 3) on the dune shape is not large.

**Spring/neap cycles**

The spring and neap cycles lead the velocity amplitude in the tidal flow to increase/decrease with a harmonic variation on a period of approximately 14 days and result according to Stride (1982) in differences in the tidal range between spring and neap tide of 37% (with local variations of the ocean) (Stride (1982)). The difference in the rate of sediment transport during spring/neap cycles are hence significant. If the time scale for the reversal of a tidal bedform (depending strongly on their size) are in the same order of magnitude as the spring/neap variation, the response of
the bedform will naturally be influenced by this variation. This limitation in the
morphological calculations is believed to be much more important than neglecting
the instantaneous flow acceleration term as well as the time-variation of the flow within
the tidal cycle. As a starting point for numerical calculation of tidal sand dunes, the
present method is however believed to be well suited.

6.1.2 Morphological calculations - test-matrix

Nondimensional parameters

The morphological development of an initial bedform is calculated until the situation
where the bedform shape and dimensions are periodic with the periodically varying
flow. This situation is denoted the "equilibrium situation". The initial bedform is
either a sinusoidally perturbed bed or the steady state bedform for the same Shields’
parameter. The length of the domain, L, is in all the tidal calculations 4 water depths.
The dune length (= the domain length) is hence also kept constant during the tidal
period. One test case has been made for the dependency on the initial bed, and as
in the case of the steady dunes, the final dune shows no variation to the choice of
the initial bedform. Due to the long calculation time, "short cuts" are taken where
possible. For the last calculations performed it was found that the quickest way to
obtain the final result was to start out with a coarse grid (40 grid points pr. dune
length) and, when equilibrium was found, change to a finer grid and continue the
calculation until a new equilibrium takes place.

For the tidal flow, the tidal period must be parameterized in the model. The
frequency of the tidal wave in this step-function representation of the tidal flow, can
be nondimensionalized by the mean velocity, V, and the mean water depth, D:

\[ \omega_D = \frac{2\pi}{T_{tide} \cdot \frac{V}{D}} \]  

(6.4)

In addition to this time scale for the tidal flow, the remaining nondimensional para-

ters are the roughness parameter, \( \frac{k_N}{D} \), the Reynolds number, Re, and the Froude

number, Fr, exactly as in the previous chapters. Instead of using \( \omega_D \), which is defined

in flow parameters, the time scale for the tidal period can also be given in parameters
relevant for the sediment transport. The tidal period is hence given as, \( T_{tide}^* \),

which replaces the nondimensional frequency, \( \omega_D \), of the tidal flow. In terms of the
morphological development of the bedform, this parameter is more relevant:

\[ T_{tide}^* = T_{tide} \cdot \left( \frac{(s - 1) gd^5}{D^2} \right) \]  

(6.5)

The nondimensional set of parameters is hence \( \left( \frac{k_N}{D} , Fr, Re, T_{tide}^* \right) \). The dimen-
sional set of parameters is \( \left( k_N, D, V, g, \nu, T_{tide} \right) \). Three of those dimensional parameters have natural values: \( g \), \( \nu \) and \( T_{tide} \). The nondimensional parameters should hence be chosen according to these three values. As also described in Chapter 2, this can be done (for instance) by expressing the nondimensional time scale in terms of the
other nondimensional values. Thus, \( T^*_{\text{tide}} \) can by combining Eq. 2.28 and Eq. 2.34 in Chapter 2 be expressed as:

\[
T^*_{\text{tide}} = \frac{T_{\text{tide}}}{\text{Re}^{1/3}} \left( \frac{F_r \cdot \sqrt{g}}{F_r \cdot \nu^{1/3}} \right)^{4/3} \sqrt{(s - 1) g \left( \frac{k_N/D}{2.5} \right)^3} \tag{6.6}
\]

A test matrix with the parameters mentioned above for calculations performed in the present chapter is shown in Table 6.1. The parameters are chosen such that dimensional values are: \( T = 12 \) hours for the tidal period, the gravity parameter is \( g = 9.82 \) m/s\(^2\), and the molecular viscosity \( \nu = 1e^{-6} \) m\(^2\)/s, except for run T8 (\( T = 24 \) hours) and run T9 (\( T = 6 \) hours, for comparison). The order of magnitude of the dimensional parameters are: \( d = O(1 \) mm\), \( D = O(2.5 \) m\), \( V = O(1 \) m/s\) and the size of the bedforms are in this case found later in this chapter to be \( H = O(0.20 \) m\) and \( L = O(10 \) m\). All calculations are therefore representatives of relatively small tidal dunes.

In Table 6.1 alternative combinations of the Reynolds number, \( \text{Re} \), and the tidal period, \( T_{\text{tide}} \), is given for Run 8 and 9. For fully turbulent flow situations, flow is independent on the Re-parameter. From Eq. 6.6 it hence becomes clear that the same morphological calculation (given the nondimensional parameters \( \frac{k_N}{F_r}, F_r, \text{Re}, T^*_{\text{tide}} \)) can be used to study morphological development in different combinations of \( (T_{\text{tide}}, \text{Re}) \), if the value of the Reynolds number is changed, when the nondimensional set of parameters are recalculated into dimensional parameter. Since the tidal period only takes values of approximately 12 or 24 hours, Eq. 6.6 states, that if the tidal period is decreased by a factor of 2 as in run T8, the Re-number is decreased by a factor of 8.

### The \( R_{\text{tide}} \)-parameter

A new additional parameter is included in Table 6.1. It is the parameter, \( R_{\text{tide}} \), which by the author is defined as the time of a tidal half-period in relation to the time scale for flow reversal, \( T^*_{g5} \), of a steady fully developed dune for the same \( \theta'_u \), as found in Chapter 3, Section 3.4.4:

\[
R_{\text{tide}} (\theta'_u) = \frac{\frac{1}{2}T^*_{\text{tide}}}{T^*_{g5}} \tag{6.7}
\]

The value of \( R_{\text{tide}} \) is used in the analysis of the tidal dunes. The physical meaning of this parameter should hence be briefly discussed, but will in the light of the calculations maybe be more meaningful.

\( R_{\text{tide}} \) turns out later in this chapter for a given \( \theta'_u \) to be an indicator for the ratio between:

1. The total volume of the sediment transport during the tidal cycle, and
2. The volume of the sand dune (defined later, roughly: the volume of the sand body above the trough level)
Methodology

In the following it is shown, that the definition of $R_{tide}$ in Eq. 6.7 also lead to $R_{tide}$ being the approximate ratio between the volume of sediment transported within a tidal half cycle (i.e. in each direction during the tidal flow) and the volume of sediment transported for reversing a steady bedform for the same nondimensional Shields’ parameter, $\theta_u^\prime$.

For given value of $\theta_u^\prime$, a small value of $R_{tide}$ according to Eq. 6.7 indicates, that the tidal period is small compared to the time for reversal of the steady dune corresponding to $\theta_u^\prime$. $\theta_u^\prime$ is the Shields’ parameter on a undisturbed bed, i.e. without bedforms. However, as also mentioned in Chapter 2, $\theta_u^\prime$ is in the calculations with a constant discharge (constant $V$ and $D$) and a constant roughness, $k_n/D$, an approximate value for the spatially averaged friction along the bed, also when the bed is covered by dunes, and the shape of the dunes change with time.

Hence, in two morphological calculations with the same value of $\theta_u^\prime$, it is a good assumption, that the average Shields’ parameter along the bedform practically takes a value of $\theta_u^\prime$. The spatially averaged nondimensional bed load, $\Phi_b(t)$, is therefore also nearly the same, and is not varying very much with time, since the discharge and roughness is constant with time in the numerical calculations here. During a tidal period the bed load corresponding to $\theta_u^\prime$, $\Phi_{b,u}$, will hence be approximately the same as the time and space averaged sediment transport, $\Phi_{b,tide}$, along a dune:

$$\\Phi_{b,tide} = \frac{1}{T} \int_0^{T_{tide}} \frac{1}{L_D} \int_0^{L_D} \Phi_b (x, t) dx dt \approx \Phi_{b,u} \quad (6.8)$$

-where $\Phi$ indicates space-averaging of the bed load along one dune length and $\Phi$ indicates time-averaging over one tidal cycle.

The nondimensional volume of sediment transported within one tidal half cycle can be found as:

$$V_{sed,tide} = \frac{1}{2} \int_0^{T_{tide}} \frac{1}{L/D} \int_0^{L_D} \Phi_b (x_D, t^*) dx_D dt^* \quad (6.9)$$

- and can by using Eq. 6.8 be approximated as:

$$V_{sed,tide} \approx \Phi_{b,u} \cdot \frac{1}{2} T_{tide} \quad (6.10)$$

The nondimensional volume, $V_{sed}$, is in relation to dimensional parameters, a volume of sediment made nondimensional with $D^2$:

$$V_{sed} = \frac{\text{Vol. of sediment transport} \ [\text{m}^3/\text{m}]}{D^2 \ [\text{m}]} \quad (6.11)$$

With the same line of arguments, the volume of sediment transported to reverse the bedform for a given $\theta_u^\prime$ in Chapter 5 can be approximated as:
<table>
<thead>
<tr>
<th>Run</th>
<th>$\theta'_u$</th>
<th>$\theta'_\Phi$</th>
<th>$T^*_\text{tide}$</th>
<th>$R_{\text{tide}}$</th>
<th>$\frac{k_n}{D}$</th>
<th>$\omega_D \cdot 10^{-7}$</th>
<th>Fr</th>
<th>Re</th>
<th>$T_{\text{tide}}$ [hours]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>0.08</td>
<td>0.0827</td>
<td>0.83</td>
<td>0.38</td>
<td>0.001</td>
<td>1.33</td>
<td>0.17</td>
<td>2.5e6</td>
<td>12</td>
</tr>
<tr>
<td>1b</td>
<td>0.08</td>
<td>0.0828</td>
<td>0.83</td>
<td>0.38</td>
<td>-</td>
<td>1.33</td>
<td>0.17</td>
<td>2.5e6</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.11</td>
<td>0.109</td>
<td>0.88</td>
<td>0.89</td>
<td>-</td>
<td>0.86</td>
<td>0.20</td>
<td>2.5e6</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
<td>0.186</td>
<td>0.61</td>
<td>1.39</td>
<td>-</td>
<td>3.90</td>
<td>0.27</td>
<td>10e6</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.20</td>
<td>0.193</td>
<td>0.97</td>
<td>2.20</td>
<td>-</td>
<td>1.55</td>
<td>0.27</td>
<td>2.5e6</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>0.40</td>
<td>0.373</td>
<td>1.09</td>
<td>6.42</td>
<td>-</td>
<td>0.15</td>
<td>0.40</td>
<td>2.5e6</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>0.11</td>
<td>0.112</td>
<td>1.75</td>
<td>1.78</td>
<td>-</td>
<td>0.22</td>
<td>0.20</td>
<td>2.5e6 (0.31e6)</td>
<td>24 (12)</td>
</tr>
<tr>
<td>9</td>
<td>0.11</td>
<td>0.117</td>
<td>0.44</td>
<td>0.44</td>
<td>-</td>
<td>3.45</td>
<td>0.20</td>
<td>2.5e6 (20e6)</td>
<td>6 (12)</td>
</tr>
</tbody>
</table>

Table 6.1 Characteristic parameters and input parameters to morphological calculations with periodic reversing flow. $T$ is tidal period provided the natural choice of the values of gravity $g=9.82$ m/s$^2$ and $\nu=1e-6$ s$^{-1}$. Only difference between 1a and 1b is the initial bedform: T1a (sine, H/D=0.05) and T1b steady bedform for $\theta'_u=0.08$

\[
V_{\text{sed.reverse}}(\theta'_u) \approx \Phi_{b,u} \cdot \frac{1}{2} T^*_95
\]  \hspace{1cm} (6.12)

Combining Eq. 6.7 with Eq. 6.10 and Eq. 6.12 it is seen that $R_{\text{tide}}$ expresses approximately the ratio between the volume of sediment transport during a tidal period and the volume for reversal of the steady dune for a given $\theta'_u$:

\[
R_{\text{tide}}(\theta'_u) \approx \frac{\frac{1}{2} V_{\text{sed.tide}}}{V_{\text{sed.reverse}}}
\]  \hspace{1cm} (6.13)

Using the definition of the non-dimensional morphological time, nondimensional tidal period, $T^*_{\text{tide}}$, can be given as:

\[
T^*_{\text{tide}} = \frac{\sqrt{g(s-1)d^3}}{D^2} T = \frac{\sqrt{g(s-1)d^3} \overline{q_{b,tide}} \cdot T}{\overline{q_{b,tide}} \cdot D} = \frac{1}{\Phi_{b,tide}} \frac{V_{\text{ol.sed.tide}}}{D^2}
\]  \hspace{1cm} (6.14)

The last term can be understood in relation to the geometrical properties of the dune. Since the dunes in the present work (roughly) scale with the water depth for a given Shields’ parameter ($\frac{H}{D}(\theta')$ and $\frac{k_n}{D}(\theta')$), the volume of the bedform in dimensional values increases with increasing water depth. $D^2$ can hence be seen as a scale for the (dimensional, m$^3$/m) volume (sand body) of the dune. For a constant $\theta'_u$ and therefore a constant $\Phi_{b,tide}$, a small value of $T^*_{\text{tide}}$ and hence a small value of $R_{\text{tide}}$, expresses the sediment transport during a tidal cycle is small relative to the bedform size.

6.2 Tidal bedform shape

An example of the time development of the dune from the initial situation to equilibrium is shown in Fig. 6.2. The initial bedform is sinusoidal with a height of H/D=0.10.
Tidal bedform shape

Basically the development of the dune follows the same pattern as described in Section 5.3. Immediately after the flow reversal, erosion takes place at the steepest slope, which is now facing the upstream direction. The sediment deposits at the top and along the gentle downstream side, generating a steep front facing the downstream direction. Thus, the upper part of the bedform reverses first. Contrary to the flow situation tested in Chapter 5, where the flow direction is only changed initially, the bedform do not necessarily obtain equilibrium with the tidal velocity (the velocity in the step function) within each half-period.

The bedform grows/decays and deforms until a point, where the equilibrium is reached and the bedform is changing periodically with the flow.

The time development in the dune height is shown in Fig. 6.3 for all the calculations. The following is noted:

- In the equilibrium state the dune height varies periodically. Immediately after flow reversal the dune height increases quickly, and - after obtaining a maximum - slowly decreases to the same dune height as just before the flow reversal.

- In run T6, T7, T8 a constant dune height is seen in each period before flow reversal. This shows that a steady bedform is reached within each tidal half-period. From Table 6.1 it is seen that for these three runs \( R_{tide} \) is above 1, which as mentioned above indicates that the bedform is fully reversed. In run T5, \( R_{tide} \) is also above one, but a clear sign of a steady dune is not seen from Fig. 6.3.

- In several of the runs (T1a, T5, T8, and T9) a sudden change in the dune height variation is seen, eg. for \( t \approx 7.5 \cdot 12 \) hours in run T9. This change is where the grid is refined from \( N=40 \) grid points along the dune to \( N=80 \) points. A further description of the grid-dependency is given in Section 6.7.

Examples of the morphological change of the dune in the equilibrium situation is shown in Fig.’s. 6.4 - 6.8. Fig. 6.4 and Fig. 6.5 (run T1a) respectively Fig. 6.6 and Fig. 6.7 (run T2) shows a similar morphological development. Within a tidal cycle, the bedform reverses and has at \( t=T \) returned to its initial location \( t=0 \). The bedform shape is also reversed to the initial shape at \( t=T \). The instabilities at the top of the dune was treated in Section 5.6.

A difference between run T1a/T2 and run T7, however, is noted (Figs. 6.4, 6.6 and 6.8). In T7 (Fig. 6.8) the dune reverses and also migrates more than one dune length downstream. The equilibrium bed in T1a and T2 (Figs. 6.4 and 6.6), on the other hand, merely reverses the upper part of the dune before the flow reverses again and hence do not actually migrate within each half-tidal period. Contrary to the bedform in T7 a part of the bedform is therefore left undisturbed by the tidal flow in run T1a and T2, as seen in Fig. 6.5 and 6.7. For these two runs, the \( R_{tide} \) parameter is less than 1, which indicates that the bedform is not fully reversed within a tidal cycle. The undisturbed volume is denoted the ‘inactive volume’ and is defined by the line of \( h_{min}(x, t \in 0 - T) \). A second volume, the ‘convolution volume’, can be defined as \( h_{max}(x, t \subseteq 0 - T) \), i.e. the maximum bed height during the tidal cycle at
Geometrical properties

a given location. From Fig. 6.5 and 6.7 this volume can also easily be defined. This
leads us to the time-averaged shape of the bedform, which is simply found as:

$$h_{\text{mean}}(x) = \frac{1}{T} \int_0^T h(x, t) dt$$ \hspace{1cm} (6.15)

The three bedforms/volumes are shown in the definition sketch in Fig. 6.9, and are
shown for each of the calculations in Table 6.1 in Fig. 6.10.

The three volumes characterizes the main, and maybe most important, fea-
tures, of the tidal dunes. As mentioned in the introduction to this chapter, knowledge
on the inactive part of the bed is relevant in case of for instance pipe line burial in
the sea bed. The convolution volume describes the maximum possible dune height,
which might be important to know for navigation or dredging reasons. Together with
the mean bedform, the convolution and inactive volume describes the activity of the
bed: if the three volumes are more or less identical, the bedform is nearly inactive.
The least active bed in Fig. 6.10 is run T1, while run T7 is the most active. The
active and inactive volumes will be further discussed and quantified in Section 6.4.
The dune height is the subject in the next section.

6.3 Geometrical properties

The dune height and length of the tidal dunes are the primary characteristics of the
dunes. Also, the slopes of the bedform might be relevant in relation to flow resistance,
for instance in a tidal inlet. The constant dune length (= the domain length) in the
calculations is most likely a good assumption. An increase in the dune length would
require the neighboring dunes to reduce their length, which does not seem plausible.
Even during a spring-neap cycle where the peak velocity in the tidal flow varies (not
included here) Dalrymple and Rhodes (1995) reports the dune length to commonly
remain nearly constant, the variation being less than 20% of the mean length).

Since only one dune length (L = 4 waterdepths) are tested so far, mainly the
dune height is the subject of this section. The dune heights are compared to those
in a unidirectional steady current, and to those in Chapter 5, where the flow was
reversed upon initially steady dunes.

Three different dune heights are defined for the tidal bedforms:

Mean dune height during tidal cycle:

$$\left( \frac{H}{D} \right)_{\text{mean}} = \frac{1}{T} \int_0^T \frac{H}{D}(t) dt$$ \hspace{1cm} (6.16)

Maximum dune height during tidal cycle:

$$\left( \frac{H}{D} \right)_{\text{max}} = \max \left( \frac{H}{D}(t) \right), t \in (0, T)$$ \hspace{1cm} (6.17)
Figure 6.2  Morphological development from initial (sinousoidal) bed to equilibrium with tidal flow. Run T5 (N=40). The bold line is the initial bedform.
Figure 6.3  Dune height variation with time for all calculations. The sudden changes in the dune height variation within the same plot indicate refining of the grid as mentioned in the text.
Figure 6.4  Morphological change during one tidal period. Solid lines at t=0, T/2 and T. Run T1a. $\theta_{u} = 0.08$. $R_{tide} = 0.38$. 
Figure 6.5  Morphological change during one tidal period. Solid lines at t=0, T/2 and T. Run 1a. \( \theta'_{a} = 0.08 \). \( R_{tide} = 0.38 \).

Dune height of time-averaged bed: 
\[
\left( \frac{H}{D} \right)_{ta} = \max (h_{mean}(x)) - \min (h_{mean}(x))
\]  

While \( \left( \frac{H}{D} \right)_{\max} \) alone gives the maximum dune height found during a tidal flow, knowledge on \( \left( \frac{H}{D} \right)_{\max} \) as well as \( \left( \frac{H}{D} \right)_{mean} \) is a measure for the variation of the dune height in the tidal flow in the same way as the three bedforms previously given for each calculation in Fig. 6.10.

For the comparison between tidal dunes and steady dunes a characteristic Shields’ parameter, denoted \( \theta'_{p} \), is found from the time and spatially averaged bed load along each dune. For \( \theta'_{p} \), the bed load, \( \Phi_{b}(\theta'_{p}) \), along a (undisturbed) channel is the same as the time and space averaged bed load over the dune. \( \theta'_{p} \) is calculated for the steady dunes in the unidirectional flow (\( \frac{H}{D} = 4 \)) and for the equilibrium (time varying) dunes during the tidal period.

\( \theta'_{p} \) is hence found by the following calculations. First the time and spatially averaged bed load is found for the tidal dunes:
\[
\bar{\Phi}_{b,tide} = \frac{1}{T} \int_{0}^{T} \frac{1}{L} \int_{0}^{L} \Phi_{b}(x, t) \, dx \, dt
\]

For a channel flow the bed load is given by the Meyer-Peter and Müller bed load formula:
\[
\Phi_{b,channel} = 8 (\theta'_{p} - \theta_{c})^{3/2}
\]

If the bed load is required to be the same in the two flow situations, \( \theta'_{p} \) can be found as:
Figure 6.6  Morphological change during one tidal period. Solid lines at \( t=0, T/2 \) and \( T \). Run T2. \( \theta_u' = 0.11 \). \( R_{side} = 0.84 \).
\[
\Phi_{b,\text{tide}} = \Phi_{b,\text{channel}} \\
\theta'_{\Phi} = \left(\frac{1}{8} \Phi_{b,\text{tide}}\right)^{2/3} + \theta'_{e}
\]

The values for \(\theta'_{\Phi}\) is included in Table 6.1 and are seen to be very close to the \(\theta'_{u}\) values as assumed in Section 6.1.2 in the derivation of Eq. 6.13. In the unidirectional flow case only the space averaging along one dune is required to find \(\theta'_{\Phi}\) by the same method.

6.3.1 Bedform height
In Fig. 6.11 the dune heights for the tidal dunes, \(\frac{H}{D}\text{max}\) and \(\frac{H}{D}\text{mean}\) are compared to the steady dune height for \(L/D = 4\) (from Chapter 3) and the amplified dune height, in the situation when the flow direction is reversed on the steady dunes (from Chapter 5). The dune heights are plotted as a function of \(\theta'_{\Phi}\) and the value of \(R_{tide}\) defined from Eq. ?? is shown next to each data point for the tidal dunes.

In general, \(R_{tide}\) is seen to be a relevant parameter for the tidal dune height. According to the definition in Eq. 6.7, \(R_{tide}>1\) means that the bedform reverses fully and obtains the properties of the equilibrium dune in the opposite flow direction within each half tidal period. The observations are hence divided into the situation where \(R_{tide}<1\) and \(R_{tide}>1\):

For \(R_{tide}>1\):
The maximum dune height, \(\frac{H}{D}\text{max}\), during a tidal period equals the amplified dune height found in Chapter 5. Since the bedform obtains an equilibrium with the velocity of the step-function in the simplified tidal-flow in each half-period, this also explains the behavior of \(\frac{H}{D}\text{max}\) for \(R_{tide}>1\).
Figure 6.8  Morphological change during one tidal period. Solid lines at $t=0$, $T/2$ and $T$. Run T7. $\theta^u = 0.40$. $R_{side} = 6.42$. 
The mean dune height in the tidal period, $\frac{H}{D}_{\text{mean}}$, approaches the steady dune height for an increasing $R_{\text{tide}}$ since for a tidal period much longer than the time for the dune reversal, the dune height equals the steady dune height for a longer time.

The latter lead to the conclusion that the step-function approach for modelling dunes in the situation where $R_{\text{tide}}>1$ is less appropriate: In a time-varying (harmonic) flow like the natural tidal flow, the situation with a steady migrating dune in the end of each half-period is not possible, simply because the flow varies at all times. If the tidal period was still very long compared to the time of flow reversal. The dune height would be slowly decaying. Since the dune height decay is slower than dune height increase, though, $R_{\text{tide}}$ values in the area of 1 should be fine.

For $R_{\text{tide}}<1$:

$\frac{H}{D}_{\text{max}}$ as well as $\frac{H}{D}_{\text{mean}}$ exceeds the steady dune height as well as the amplified dune height, for the situation when the steady dune reverses. Furthermore, it can be observed that for the calculations tested, the dune height increases for a decreasing value of $R_{\text{tide}}$.

The explanation should be found in the morphological development of the dunes as sketched in Section 6.2: When the flow reverses before the equilibrium dune height is obtained in the opposite flow direction, the dune height increases for each flow reversal by depositing sediment at the top until an equilibrium situation prevails. Since the dune height obtains its maximum quickly after the flow reversal (see Fig. 6.3), a small value of $R_{\text{tide}}$ indicates that the bedform height is further from the equilibrium condition for the steady current case. The steady dune height is the lowest possible for $R_{\text{tide}}<1$.

The dune heights for the time-averaged bedforms (Fig. 6.12) are following the same trends as described above for the maximum dune height. Since the length of the dunes is constant, the steepness of the dunes, $\frac{H}{T}$, increases with a decreasing value of $R_{\text{tide}}$. The steepness of the slopes on the dunes are important in relation to stability (in the sense of finding an equilibrium bedform with the flow) since the stabilizing mechanism is the effect of gravity on the sediment transport on the slopes of the bed. The stability of tidal bedforms is treated later in this chapter.
Figure 6.10  Time averaged bedform, convolution volume and inactive volume for all calculations.
Figure 6.11  Mean and max dune heights during tidal cycle for tidal dunes. The value of $R_{tide}$ is shown next to each data point. Dune heights for the steady dunes (Chapter 3) and the maximum dune height after flow reversal in Chapter 5 are included.

There is some indication in Fig. 6.11 and Fig. 6.12 that the tidal dune height follows the same trend for a variation of the Shields’ parameter as the steady dunes - but now under the constraint that the value of $R_{tide}$ is constant. Further tests are necessary to draw any conclusions.

6.4 Active layer

Contrary to dunes in a unidirectional current, sand waves exposed to a pure tidal flow (i.e. without a net current) do not have a net migration with time, since they return to the same position after each tidal cycle (at least in the simplified tidal flow situation where no spring/neap tide occur). In situations where the sediment transported within each half cycle is relatively small compared to the size of the dune, an inactive volume of the dune exists as previously described. Above this inactive volume the sediment is active. This is for instance seen in Fig. 6.5. On the other hand an inactive part of the bedform do not always exist. The bedform in Fig. 6.8 reverses and migrates several dune lengths within each half tidal period and leaves no inactive part of the bed above the trough level.

The bedforms can hence be divided in respectively an active layer and an inactive bed. In the case of large bedforms (length $O(100 \text{ m})$) as found in the North Sea,
and relatively little sediment transport, the inactive volumes can be large. Conceptually, pipe lines can safely be buried within this inactively layer, whereas locating a pipe line outside the inactive part of the bed will cause the pipe to be exposed to the hydraulic forces from current and waves. In praxis, currents, storm-conditions etc. also need to be included in the analysis to define the inactive parts of the bed. Since burying as well as damage/failure of a pipe lines due to fatigue is costly, there is a demand for knowing "how active" the bed is, how large the inactive volume is, and which part of the bed is active/inactive.

The three volumes of the bedform, the inactive, the mean and the convolution volume as defined in Section 6.3 together with the total volume of sediment transport within each tidal period can be used as qualitative measures of how active the bed is. Notation of the volumes are shown in Fig. 6.9.

A fourth volume, $V_{sed}$, is given in Table 6.2. $V_{sed}$ is calculated from Eq. 6.9 and is the total volume of sediment transported within each half tidal cycle:

$$V_{sed} = \frac{1}{L} \int_0^L \int_0^{T/2} \frac{q_b}{1 - n D^2} dt dx$$  \hspace{1cm} (6.24)

Table 6.2 summarizes the volumes for the numerical calculations. In Fig. 6.13 and 6.14 the relation between the volumes are shown. For a fixed Shields’ parameter (i.e. constant sediment transport rate) a dependence on the $R_{tide}$ parameter is noticed. The inactive part of the bedform increases, when $R_{tide}$ is decreasing. In other words, when the nondimensional tidal period is decreasing compared with the non-dimensional time for the reversal of the steady dune for a comparable Shields’ parameter (Fig. 6.13). The reason is being revealed in Fig. 6.14, which shows that the total volume of sediment transport within each tidal period, in the cases of a low $R_{tide}$ value, is small compared to the volume of the bedforms.
Table 6.2 Inactive, mean and convolution volumes. Volume of sediment transported in each halph period.

<table>
<thead>
<tr>
<th>Run</th>
<th>$\theta'_a$</th>
<th>$\theta'_b$</th>
<th>$T_{tide}$</th>
<th>$R_{tide}$</th>
<th>$V_{inact}/V_{mean}$</th>
<th>$V_{mean}/V_{conv}$</th>
<th>$V_{conv}/V_{tide}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1a</td>
<td>0.08</td>
<td>0.0827</td>
<td>0.83</td>
<td>0.38</td>
<td>0.12</td>
<td>0.20</td>
<td>0.34</td>
</tr>
<tr>
<td>T1b</td>
<td>0.08</td>
<td>0.0828</td>
<td>0.83</td>
<td>0.38</td>
<td>0.12</td>
<td>0.21</td>
<td>0.34</td>
</tr>
<tr>
<td>T2</td>
<td>0.11</td>
<td>0.109</td>
<td>0.88</td>
<td>0.89</td>
<td>0.11</td>
<td>0.28</td>
<td>0.57</td>
</tr>
<tr>
<td>T5</td>
<td>0.20</td>
<td>0.189</td>
<td>0.61</td>
<td>1.39</td>
<td>0.08</td>
<td>0.38</td>
<td>0.76</td>
</tr>
<tr>
<td>T6</td>
<td>0.20</td>
<td>0.193</td>
<td>0.97</td>
<td>2.20</td>
<td>0.02</td>
<td>0.38</td>
<td>0.80</td>
</tr>
<tr>
<td>T7</td>
<td>0.40</td>
<td>0.373</td>
<td>1.09</td>
<td>6.42</td>
<td>0.02</td>
<td>0.45</td>
<td>0.99</td>
</tr>
<tr>
<td>T8</td>
<td>0.11</td>
<td>0.112</td>
<td>1.78</td>
<td>1.78</td>
<td>0.04</td>
<td>0.29</td>
<td>0.55</td>
</tr>
<tr>
<td>T9</td>
<td>0.11</td>
<td>0.117</td>
<td>0.44</td>
<td>0.44</td>
<td>0.18</td>
<td>0.31</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Figure 6.13 The ratio of the inactive volume to respective the volume of the time-averaged bedform and to the convolution volume. The value of the $R_{tide}$ parameter is given next to each datapoint.
Figure 6.14 The ratio of the volume of sediment transported within each tidal half-cycle, \( V_{\text{Vol, sed}} \), to the volume of the mean bed, \( V_{\text{Vol, mean}} \), (left) and to the convolution volume, \( V_{\text{Vol, conv}} \), (right). The value of the \( R_{\text{tide}} \) parameter is given next to each data point.

6.5 Discussion

The tidal bedforms are in the above subchapters discussed mainly with regard to the parameter \( R_{\text{tide}} \) and the Shields’ parameter. The value of \( R_{\text{tide}} \) is defined as the non-dimensional tidal period in comparison with the time scale for the reversal of a steady dune for the same Shields’ parameter. \( R_{\text{tide}} \) is in Section 6.4 found as an indicator of the ratio between the total volume of sediment transported within a tidal period to the size (volume) of the bedforms.

The time scale for a full reversal of the steady dunes are found in Chapter 5 by simply applying a change in the flow direction on steady dunes as found in Chapter 3. The time scale for a full reversal of the steady dunes is in Chapter 5 judged to be within an uncertainty range of \(+/- 20\%\) - the uncertainty is highest for the small Shields’ parameters. The value of \( R_{\text{tide}} \) given in the present chapter should be seen in the light of this uncertainty.

The dune heights for the calculated dunes shows a dependence on the Shields’ parameter and \( R_{\text{tide}} \). The results indicates weakly a dependence on the non-dimensional bottom friction which follows the same trend as the dune height for steady dunes for a given value of \( R_{\text{tide}} \). More calculations are necessary to obtain a more firm relation between the Shields’ parameter and the tidal dune height. The value of \( R_{\text{tide}} \) has shown to be a relevant parameter for the analysis of the dune height, even though the value of \( R_{\text{tide}} \) is purely connected to a time scale obtained for steady dunes together with the tidal period. If the tidal flow reverses on a time scale shorter than the time for full reversal of the corresponding steady dune (the value of \( \theta_{\text{b}} \) being the same), the dune height increases with a decreasing \( R_{\text{tide}} \). Furthermore it was found that the tidal dune height exceeds the steady dune height for the same time-averaged bed load rate.

The fact that the dune length is constant and always 4 water depths is a
limitation in the present work. This is expected to influence the equilibrium dune height, as is the case with the steady dunes. The same analysis as performed with steady dunes - varying the dune length (domain length in the numerical calculations) - would be interesting to do with the tidal flow as well.

In the present calculations the effect of surface waves are not included. Surface waves are expected to reduce the dune height. Especially if the water depth is relatively small, the effect of surface waves, for instance during a storm will increase the bottom friction and erode from the upper part of the dune. The importance of surface waves would be more crucial if suspended material were included in the analysis. In that case, waves would increase the turbulence level in the water column and keep the sediment in suspension.

The calculations performed here, are all representatives of relatively small tidal dunes. More precisely, they represent the more active dunes, however these are also in nature the smaller size dunes since the sediment transport rates are rarely large enough to move the large scale tidal dunes (dune length O(100 m)) as much as seen in the present chapter. The larger and less active dunes (Ride small and $\theta'$ small) are hence one of the "open ends". The lack of calculations of those dunes, is due to the fact, that morphological calculations with areas without any or very little sediment transport were found to be more problematic (see Section 6.7). If the bottom friction is only slightly above the critical for sediment movement, transport will only take place on the top of the dunes. The trough of the dunes will hence be inactive, and with the present approach, where a filter is applied on the morphology, extreme care must be taken of the numerical processes in these inactive areas. An "obvious solution" of just applying the filtering procedure where the bed is active causes (again under the present methodology) problems in the areas of changing from active to non-active bed.

6.6 Stability of tidal bedforms

The mechanisms leading to growth and stability of the tidal bedforms have briefly been mentioned in Chapter 1. This will be further discussed in the following in relation to the results from the numerical calculations. As mentioned in the review, many researcher study the mechanisms leading to growth of the tidal sand waves while only few are concerned with the sand waves in equilibrium. To the authors knowledge an understanding of the mechanisms leading to equilibrium of tidal sand waves is not fully understood. The following strongly suggests that not only the time-dependency of the flow and sediment transport field, the also of the bedform is necessary to describe (find) the equilibrium state of the bedform. In other words a phase-resolved method like the present is required. The tidally averaged sediment transport field (not even using a fully non-linear calculation method like the present) is not capable of determining the correct bedform height nor the correct shape.

The equilibrium of the sand waves is studied by considering:

1. The sediment transport field along the time averaged bedform,

2. The time averaged sediment transport field along the time varying bedform.
Figure 6.15 Profiles of horizontal velocity over time-averaged bed form for run 2. Lower figure shows profiles of streaming found by superposing the velocity profiles for flow in each direction.

The two different approaches are following the point of view taken by researchers applying respectively phase-averaged morphology (also used in the classical stability analysis) (bullet 1) and researcher applying phase-resolved morphology (bullet 2).

6.6.1 Streaming
The flow is calculated along the time averaged bedforms for run T2. Fig. 6.15 shows velocity profiles for the horizontal component. On the upstream side the streamlines are converging due to the contraction of the flow area, and the boundary layer is thin and the gradient in the near wall flow is large. On the downstream side, where the flow area is expanding and the flow resistance is increased, the boundary layer is thicker and the near wall flow is retarded. Reversing the flow, the same is naturally found since the bedform is symmetric, only reversing the upstream and downstream flow direction. The time averaged profiles (in this case the mean of the flow in the two flow directions) shows the flow cells caused by streaming. Two flow cells as sketched in Fig. 6.16 can be seen: A thin lower flow cell very close to the bottom, which is directed towards the top of the dune in the near bed flow, and a larger flow cell in the upper part of the flow area.

6.6.2 Time-averaged bed load: Phase-averaged and phase-resolved morphology
The pattern of streaming as calculated from the phase-averaged bedform in the previous section, shows the expected variation in the near-bed flow, as also found alike described in the literature (eg. Hulscher 1993, 1996). The upwards directed streaming in the near-bed flow is believed to balance the effect of gravity, such that bedform height/shape stabilizes. For stability of the bedform, it is required that the sediment
transport in both halves of the tidal period is the same - a possible difference will lead
the bedform to either grow (net sediment transport directed toward the top) or decay
(net sediment transport directed towards the trough). The question to be answered
here is therefore whether the sediment transport field calculated along the time aver-
egated bedform will give this balance (phase-averaged approach) or the whether the
time-dependency of the bedform and hence of the sediment transport field is necessary
to explain the balance in the sediment transport in the two half-periods.

The sediment transport distribution along the dune time averaged within each
half period of the tidal cycle is thus calculated by applying both methods:

\[
\Phi_{b,1}(x) = \frac{2}{T} \int_0^{T/2} \Phi_b(x,t) \, dt \quad \Phi_{b,2}(x) = \frac{2}{T} \int_{T/2}^T \Phi_b(x,t) \, dt \quad (6.25)
\]

In Fig. 6.17 the bed load in each tidal half cycle for respectively the phase-
averaged method (corresponding to the streaming found in the previous section) and
the phase-resolved approach (according to Eq. 6.25) is shown. With regard to the bed
load pattern in the upper subfigure in Fig. 6.17 (phase-averaged sediment transport)
the net sediment transport on both slopes of the bedform is clearly directed towards
the top, since the bed load in the half period in which the slope is facing the upstream
direction is larger than when the slope is facing the downstream direction. The
phase-averaged approach hence suggests a higher and steeper dune shape than the
calculated, such that an enlarged effect of gravity can balance the sediment transport
in the two tidal half periods.

The time averaged sediment transport for the phase-resolved morphology (mid-
dle subfigure in Fig. 6.17) shows a much better agreement between the balance of
the sediment transport and the bedform. The net transport in either direction is
close to zero. A slight deviation is found on either side of the trough. This location
is approximately at the toe of the front in its "extreme" position during each tidal
cycle, i.e. just where front is before the flow reversal. The most plausible explanation
for this is the filter, which smooths the bedform at this point where the curvature of
the bed is high. Since the rate of bed load at this point throughout the tidal period
is also low (or at least never high) the effect of the filter is most likely to show up at this point.

Besides revealing the physics of the tidal bedform in equilibrium, it should be noted that this equilibrium between the bed and the sediment transport from a point of view of morphological modelling also shows that the physics of the sediment transport and not the numerical filter govern the bedform throughout the tidal cycle.

6.7 Numerical issues and stability

Morphological calculations of tidal dunes (at least in situations without a net current) were found to be clearly more problematic than dunes in the current situation. If for instance the bottom friction is only slightly above the critical for sediment movement, transport will only take place on the top of the dunes. The trough of the dunes will hence be inactive, and with the present approach, where a filter is applied on the morphology, extreme care must be taken of the numerical processes in these inactive areas. Even a very small, but consistent, effect of the filter can result in a wrong and unstable bedform, if the filter is causing ”artificial erosion or deposition” in this area. An example of this is shown in Fig. 6.18. For a too small time step the bedform decays instead of finding a stable dune height. It is not exactly clear whether the reason is the ”wiggles” of 5-7 grid points also described in Chapter 3 or it is a consistent error due to the filter in the trough. In calculations with a current, the bedform continuously migrates across the trough and any small error is less likely to cause
stability problems (although as we saw in Chapter 3 the dune height and shape can still be influenced by an error).

An ”obvious solution” seems to be to apply the filtering procedure only where the bed is active causes (again under the present methodology) problems in the areas of changing from active to non-active bed. A different solution should be found, and this is merely expected to be a matter of testing different numerical solutions out for filtering the numerical instabilities.

Grid-dependency is shown in Fig. 6.19. A courser grid causes a more smooth bedform - mainly at the top and at the lower part of the front at the time when the front is steepest. The situation in Fig. 6.19 is run 1b. The grid-dependency explains the abrupt change in dune height in Fig. 6.3, when the grid is refined. The dune height change due to grid refinement appears more dramatic in Fig. 6.3 than what is the case in 6.19.

The area of the dune front, especially around the lower part of the front, is more smooth when the course grid (N=40) is used. The resolution of the bed in this area is too small. This probably explains, the deviation in the bed load in the two flow directions in Fig. 6.17 at the lower part of the bed. A slightly steeper bedform at this location of the bedform is required to have full equilibrium in both flow directions.

6.8 Open ends

The present study of the tidal dunes is by the author seen as step towards a combination of the qualitatively as well as quantitatively understanding of the physical processes in relation to 2D tidal dunes. Thus the study is not ”filling in the blanks” and naturally leaves a number of open ends. A list of some of the relevant questions to be answered are listed below:

![Graph showing D/H vs. t/T](image-url)
Figure 6.19  Grid dependency for calculation T1b. Solid line is calculation with N=80 and punctured line is for calculation with N=40.
• Dune length dependence - what is the determining mechanisms for setting the dune length and how is the interaction between the height/length of the dunes.

• Large inactive tidal dunes have not been modelled in this study.

• Influence of spring/neap cycle. Very active/small tidal dunes will respond fast to the spring/neap cycle and be more or less in equilibrium with the tidal velocity at each tidal period. The less active dunes lags much more behind the changing flow conditions. This time lag is for instance in the present work seen as the time to obtain equilibrium with the tidal flow - the calculations with less active dunes (for instance T1 and T2), required many tidal periods before equilibrium was found (see Fig. 6.3). The unsteady effect of the spring/neap cycles can hence have a large effects on these dunes.

• Other unsteady flow situations in combination with tidal flow, ex. storm situations. Some tidal dunes might be generated during ”extreme” flows, while the ”normal” flow condition leave them more or less undisturbed if the critical bed shear stress is not exceeded.

• Calculation of flow resistance due to form drag from tidal dunes. Especially for tidal inlets and in a river mouth the flow resistance over tidal dunes can be relevant, since the flow resistance acts to increase the water depth.

6.9 Summery

Tidal dunes in equilibrium with a periodically reversing flow were computed in a number of test-cases, all with a constant dune length of 4 water depths. All test cases represent relatively small/active tidal dunes. A simplified flow description (in time) was applied using a constant velocity in each half-period.

The dunes were analyzed with respect to the dune height and the active/inactive volumes of the bed. An attempt has been made to relate the geometrical properties of the tidal dunes to a parameter, \( R_{tide} \). \( R_{tide} \) is a measure for the ratio of the volume of the sediment transported within a half tidal cycle and the volume of the bedform. The dunes were found to be higher and have a larger inactive volume, when the value of \( R_{tide} \) was small, under the constraint that the time and spatially averaged Shields’ parameter is the same. More calculations are necessary to give a firm relation between the Shields’ parameter and the dune height. The results indicate, for a constant value of \( R_{tide} \), a behavior similar to the steady dunes case, where the dune height increases and approaches a constant with an increasing Shields’ parameter.

To explain the dune height and shape of the equilibrium dunes, it is demonstrated that a phase-resolved flow/morphology are required. The non-linearity of the bedform/flow interaction during the tidal period is hence important to give a full description of tidal dunes. Using a phase-averaged approach, a net sediment transported directed towards the top of the equilibrium dune were found and hence a larger dune height would be found using this approach. Including the phase-resolved interaction between the bedform/flow therefore dampens the bedforms.
Chapter 7
UNSTEADY DUNES IN PERIODICALLY REVERSING FLOW
WITH A NET CURRENT

The last flow situation to be studied in this thesis is a periodically reversing flow with
a superimposed net flow. The period of the reversing flow has the order of magnitude
of the tidal period (T ~ 12 hours), and thus the corresponding flow in nature is a
tidal flow with a net current. This type of flow is typically found in a tidal delta area,
where the tidal flow is superposed by a strong or weak current from a river flowing
into the sea. However, currents and tidal flows are also commonly found coexisting
elsewhere in the sea.

In the pure tidal flow situation as studied in the previous chapter, the sediment
transport within each tidal half period is the same, i.e. the transport in each flow
direction is the same. There is hence no net sediment transport and the dunes do
therefore not migrate, but they simply reverse to a smaller or higher degree in each
tidal period.

When a tide and current are superimposed, the relative strength between
the tide and the current determines the direction of the net sediment transport.
Compared to the pure tide situation, this changes the morphological development
of the dunes during the tidal period, and furthermore causes the dune to migrate.
The morphology during the tidal period and the migration of the dunes is studied in
the present Chapter. The tide-current flow situation is exemplified by two different
flow cases: A strong tide superimposed by a weak current, and a strong current
superimposed by a weak tide.

7.1 Methodology

The methodology is very similar to the one applied in the previous Chapter, and is
only briefly outlined here. In the calculations it is assumed that the angle between the
current and the tidal flow i zero, and the situation is hence strictly two-dimensional.
The flow is, as previously, described by a step-function, and the same line of arguments
as presented in Section 6.1.1 in Chapter 6 relates the morphological development
during a natural flow situation (a harmonically flow variation of some kind) with the
morphological response to a step function. Whereas the discharge in the modelling
of the pure tide calculations is the same in each tidal half-period, the discharge in the
case of a tide superimposed by a current is different in the two tidal half-periods.

In Fig. 7.1, two different tide-current situations are shown. The discharge
related to the pure tide is denoted $Q_{\text{tide}}$, and the discharge related to the current
### Table 7.1 Characteristic parameters in tide-current calculations. $k_N/D=0.001$. L/D=4.

<table>
<thead>
<tr>
<th>Run</th>
<th>$Q_{cur}/Q_{tide}$</th>
<th>Fr</th>
<th>Re</th>
<th>$T_2$</th>
<th>$\theta_u$</th>
<th>Fr</th>
<th>Re</th>
<th>$T_2$</th>
<th>$\theta_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>tc1</td>
<td>0.24</td>
<td>0.24</td>
<td>3.0e6</td>
<td>0.59</td>
<td>0.26</td>
<td>0.16</td>
<td>2.0e6</td>
<td>0.177</td>
<td>0.07</td>
</tr>
<tr>
<td>tc2</td>
<td>0.2</td>
<td>0.24</td>
<td>3.0e6</td>
<td>0.59</td>
<td>0.26</td>
<td>(-)</td>
<td>(-)</td>
<td>2.0e6</td>
<td>0.177</td>
</tr>
</tbody>
</table>

$Q_{cur}$. The relationship $Q_{cur}/Q_{tide}$ specifies the relative strength of the current to the tide, and is as seen in the figure very important for the character of the flow. In the flow situation in the left hand side (strong current and weak tide), the flow has the character of a unidirectional current with a periodic change of the strength of the discharge. In the flow situation in the right hand side figure (strong tide and weak current) on the other hand, the flow has the character of a tidal flow, where the discharge in the two flow directions is, however, not the same. No ”typical” values of this relationship exist - in nature combinations from week currents/strong tidal flow to strong current/week tidal flow is found.

The importance for the morphology is the sediment transport during the tidal flow, and hence the change in the Shields’ parameter. The change in the discharge is hence again performed by increasing/decreasing the Shields’ parameter by changing the mean velocity.

In the numerical model the nondimensional numbers are chosen by a similar method as in Chapter ?? (Unsteady unidirectional flow with change in discharge) and Chapter 6 (Tidal flow). The nondimensional set of parameters are ($k_N$, $Fr$, $Re$, $T_{tide}$). The roughness parameter, $k_N$, is held constant throughout the calculation and the Froude number, $Fr$, and the Reynolds number are chosen according to the velocity signal of the superimposed of the current and tide. A different set of of nondimensional parameters is hence applied in flow situation A in Fig. 7.1 for $t \in [0; T_2]$ than in flow situation B $t \in [T_2; T]$. The nondimensional parameters are given in 7.1. The dimensional set of parameters are ($k_N$, $V$, $D$, $T$, $g$, $\nu$). The nondimensional tidal period is set as in Chapter 6 under the constraint of a natural choice of the values of $g=9.82$ $m/s^2$ and $\nu=1.10^{-6}$ $m^2/s$ and a tidal period of $T=12$ hours. The nondimensional tidal period is therefore found from the dimensional value of the tidal period, $T=12$ hours, and $k_N Fr$ and $Re$ (Eq. 6.6), which leads to defining different values of $T_2$ for $T_2$ in each tidal half period (see Table 7.1). The morphological time step is also changed in the two tidal half periods, see Section 4.5.1.

The Shields’ parameter is changing between $\theta_u = 0.26$ and $\theta_u = 0.07$. The shift leads hence to a large change in the sediment transport in the two tidal half cycles. This is reflected in the morphology which is the subject in the next section.

#### 7.2 Strong current and weak reversing flow

##### 7.2.1 Dune height and shape during a tidal period

The morphological development of the bed in the ”strong current - weak tide” flow situation as shown to the left in Fig. 7.1 is calculated from an initially perturbed
bed (sinusoidal bedform with H/D=0.10). The change in the dune height with time is shown in Fig. 7.2. The grid is refined from N=40 point in the horizontal direction along the bedform to N=80 during the calculation to reduce the calculation time to reach the equilibrium situation, i.e. the situation when the bedform change is periodic with the tidal period. The refinement shows up in the dune height variation as a ”jump”. The dune height change during the tidal period in the equilibrium situation, can be explained by the morphological development of the bedform. This is shown in respectively Fig. 7.3 and 7.4.

In the present flow situation the current is so much stronger than the tide, that the flow is always in the direction of the current. As will be seen in the following, the morphological development of the dunes hence has many similarities with the dunes described in Chapter 4, where the flow situation was a sudden increase/decrease in the discharge. The main difference between the present chapter and the situations described in Chapter 4 is, however, that due to the periodic change of the discharge, the dune immediately before each change in the discharge is not in equilibrium with the flow. While the similarity in the flow field immediately before and after the flow change still applies (Eq. 4.1), the morphological development may not follow the same pattern as outlined in Chapter 4 following respectively an increase or decrease in the discharge.

\[ t = [nT, \frac{T}{2} + nT] : \text{Tidal flow direction opposed to current direction} \]

At \( t = nT \) the discharge reduces abruptly to a lower value since the tidal flow acts to reduce the current. The dune height decreases slowly as seen from Fig. 7.2. In Fig. 7.4 it is seen that this dune height change mainly takes place at the top, while the trough maintains the same level. Erosion takes place from the gentle slope of the dune, but only from the upper part, where the critical bed shear stress is exceeded. This corresponds to ”phase 1” for the trough for unsteady uniform flow as described in Section 4.3.1. The migration rate is very low due to the low discharge/sediment transport.

\[ t = \left[ \frac{T}{2} + nT, (n + 1)T \right] : \text{Tidal flow direction in current direction} \]
At $t/T = 0.5$ the discharge changes to a higher discharge. In Fig. 7.2 the dune height is seen to initially decrease, with a higher rate of change than in the previous half-period, before the dune starts to grow in height. The reduction is actually caused by the mechanism described in phase 2 for a decreasing discharge in Section 4.3.1: When the discharge increases the location, where the bed shear stress exceeds the critical moves upstream. This has two consequences. The bed shear stress does, after the separation zone area, where the skin friction is low, grow up to exceed the critical at a location downstream from the deepest part of the trough and thus on the gently sloping part of the next dune. The front of the dune hence moves up this gentle slope as seen in Fig. 7.4. The trough level is therefore rising, and the dune height reduces. This is note-worthy since the dune height initially decreases, even though the discharge increases. This is different from the situation of an increasing discharge when the initial bed is in equilibrium with the flow prior to the change of discharge as in Chapter 4. The location, where $\theta'_{\text{eff}} > \theta'_c$ in that case, will always be IN the trough and deepen the trough level initially.

Second, the top level and the shape of the dune undergoes changes, which are "wiggly". This is a combination of the erosion pattern from the last half-period, where only the upper part of the dune was eroded, and the movement of the location where $\theta'_{\text{eff}} > \theta'_c$. The same line of comments as noted in the end of Section 4.3.1 in connection with the migration of a superimposed perturbation applies here: Since the separation zone is dynamic in nature, the location on the bedform where $\theta'_{\text{eff}} > \theta'_c$ moves slightly back and forth. A larger erosion in that area will hence take place in nature than in the simulation, since the dynamic behavior is not included in Dune2D. The "wiggly" tendency is probably more a modelling phenomenon than a natural phenomenon. Again, a LES-model, which can simulate the dynamic behavior of the separation zone, would reveal more information than the present model.

Following this reduction in the trough level and the shape changes, the dune height increases due to an increase of the top level and erosion in the trough. This is following the mechanisms for a dune applied to an increase in the discharge as outlined in Section 4.3.

In Fig. 7.2 the dune height of a steady dune in a steady current corresponding to the strongest as well as to the weakest discharge is included using the expression found in Chapter 4, see Fig. 4.17. The dune height is seen to have an order of magnitude which is related mostly to the highest discharge. For the dune to be in equilibrium, the dune height change in the two tidal half-periods must be the same, i.e. $\Delta h(t \in [nT, nT + T]) = -\Delta h(t \in [\frac{T}{2} + nT, (n + 1) T])$. Taking the observations of the time scale for the dune height change in an increasing/decreasing discharge in Chapter 4 into account, intuitively the decrease of the dune height in the tidal half-period, when the discharge is low, would be less, than the increase in dune height in the tidal half-period when the discharge is high. This makes it hard to see how an equilibrium situation takes place.

Two comments should be added to this. The first is related to the migration velocity, which is actually the subject of the next subsection. Here it is however noted, that the situation is different from the situations studied in Chapter 4, where the initial bedform in all the calculations was the same and in equilibrium with the
flow field before the increase respectively decrease in the discharge. This is not the situation here; the dune height before the increase in the discharge is larger than the equilibrium dune height for $Q_B$, hence the migration velocity is lower right after the increase in the discharge (compare Eq. 3.8). As noted in Section 4.4 about morphology during a flood wave, the local rate of change of the bed $\frac{\partial h}{\partial t}$ as well as the migration term $\frac{\partial h}{\partial t}$ influences the change in the dune height.

The second comment is related to the phenomenon described above, where the dune height initially decreases after the discharge increases. This effect seems to balance the difference in the time scale for the change of the dune height for respectively an increasing/decreasing discharge as found in Chapter 4, and is hence important for the equilibrium height of the dune.

### 7.2.2 Migration velocity

The migration velocity defined as the migration velocity of the center of gravity of the dune is shown in Fig. 7.5. The clear difference in the migration velocity seen in Fig. 7.3 between the tidal half-cycles with respectively the high discharge, $Q_A$, and the low discharge, $Q_B$, is also clearly seen in the morphology in Fig. 7.3.

The migration velocity of the dune is compared to the migration velocity of a steady dune, corresponding to the average mean velocity (i.e. the velocity of the current) in Fig. 7.1 (left). This dune was already found in Chapter 3 (Run s3L4: $\theta'_u = .11$, $k_N/D = 0.001$, Fr=0.2, Re=2.5e6).

The migration can furthermore be expressed as the distance the dune travel within one tidal cycle. For the dune in the 'current and tide' situation this distance is found to $\frac{\Delta L_{tide}}{T} = 0.76$, while the steady dune migrates $\frac{\Delta L_{steady}}{T} = 0.66$ dune lengths in the same time. The 'tide-current dune' hence migrates 15% longer than the steady dune, even though the mean depth averaged velocity (or rather the mean value of the skin friction, $\theta'_u$) is the same.

Without further investigations the following general observations/physical considerations are believed to be responsible for this:

- The non-linearity in the increase of sediment transport with an increasing velocity: $\Phi_b \propto V^3$. Hence, even though the average mean velocity is the same, i.e. $V_{steady} = \frac{1}{2}(V_A + V_B)$, this effect clearly increases the net sediment transport rate during one 'current+tide' cycle compared to the current situation - given that we have the same bedform.
Figure 7.3  Morphology during a tidal period, when a strong current is present. Run tc1.
Figure 7.4  Morphology during a tidal period, when a strong current is coexisting with a weak tide. Net current is directed from left to right. Run tc1.

Figure 7.5  Migration velocity in the 'strong current - weak tide' equilibrium situation. The mean migration velocity and the migration velocity in the "pure-current" situation (run s3L4 in Chapter 3) is included. Run tc1.

- The dune height in the 'current-tide' situation is larger than the corresponding dune height for the average current in a steady current situation. In fact, the dune height is oscillating around a value in the order of the dune height for the larger mean velocity (or Shields’ parameter) - Fig. 7.2. Since the dune height is thus also larger, the migration velocity is decreased by this higher dune height (Eq. 3.8).

The above arguments are very course - modification of the dune within each tidal half-period also influences the sediment budget. Furthermore, the skin friction distribution along the dunes changes and influences the sediment transport rate.

7.3 Weak current and strong reversing flow

7.3.1 Dune height and shape during a tidal period

When the current is weak and the reversing flow is strong, the morphological development of the dunes has not surprisingly a stronger similarity with the tidal flow than
with the unidirectional flow situation. The dune height change with time is shown in Fig. 7.6 from the initial sinusoidally perturbed bed with a height of 0.1 water depth. As in the previous calculation, a refinement of the grid has been performed during the calculation towards equilibrium, here at $t/T \approx 6$.

The morphological change during the tidal period is shown in Fig. 7.7 and Fig. 7.8, in both figures the net current is directed from left to right. The bedform change during the two tidal half-periods is very similar. The morphological response of the bedform after the flow reverses - whether the flow is strong (when the tidal velocity and the current is directed in the same direction) or weak (when tidal velocity is directed in the opposite direction of the current) - is as described in the previous Chapter. The upper part of the dune reverses and migrates down the slope of the bedform from the previous half-period. The different strength in the discharge is reflected by the volume of the dune reversed within half a tidal period. The latter means that in the case with the low discharge, only the top of the dune is reversed, while when the discharge is high, the dune fully reverses and migrates a distance downstream. A steady state situation, however, for the flow/bedform interaction is not reached before the flow reverses again.

A difference in the maximum dune height during the two tidal half-periods is seen in Fig. 7.6 and also in the morphology in Fig. 7.8. The difference takes place at the top - during the entire tidal period, the trough-level maintains the same trough-level. The dune height just before flow reversal is also different.

7.3.2 Migration velocity
The net current alone is too small to cause sediment transport by itself if the tidal flow was not present. Furthermore, the tidal flow alone does not cause any net migration by itself, but merely reverses the bedform within each tidal half-cycle. The combination of the tide and the current, though, causes the dune to migrate. The migration velocity for the dune (migration of center of gravity) is shown in Fig. 7.9. The dune migrates relatively much faster forward, when the tidal velocity and the current velocity is in the same direction, i.e. when $Q=Q_A$, than when the tidal flow is retarded by the current when $Q=Q_B$. A small net flow (20% of the tidal flow) hence is very important for the net migration of the bedform.

In the previous flow situation the migration was compared to the migration in the 'pure-current' situation to see whether/how much the co-existence of the tidal flow with the current increased the migration. In Fig. 7.9 a comparison is made to
Figure 7.7  Morphology during a tidal period. Strong tide and weak current. Run tc2.
the migration velocity for the tidal current in the case this is a unidirectional current, primarily to have an order of magnitude to compare the migration in the present situation with. Expressed in dune lengths the dune migrate $\Delta L_{\text{tc2}} = 0.56L$ in the 'strong tide-weak current' situation within one tidal period. The migration distance for a dune generated by $Q_{\text{tide}}$ as a current is $\Delta L = 0.66L$ (again this corresponds to the dune in run s3L4 as in the previous subchapter). The tidal dune hence migrates approx. 85% of the distance travelled by a steady dune corresponding to $Q_{\text{tide}}$.

7.4 Open ends
The two situations are primarily illustrating the tide-current interaction and is not nearly covering the issue of tidal sand dunes in a combined tide and current situation. As one example of a limitation of the present work, the dune length is again kept constant and is assumed to be 4 water depths. This is expected to effect the dune height as in the previous chapters. Many parameters are "in play" in tide-current morphology, but for a start an understanding of the mechanisms must be obtained.
7.5 Summery

Two different flow situations illustrating a combined current and tidal flow has been investigated. The equilibrium bedform, i.e. the situation, when the bedform is periodic with the tidal period, is studied with respect to the bedform and height change during the tidal period. The migration velocity is calculated.

The first flow situation is a strong current overlaid by a weak tide. The net current is unidirectional during the entire tidal period, but with alternating strength of the current due to the tide. The morphological development of the dune during the tidal period hence has strong similarities with dunes in an unsteady unidirectional flow. The same basic mechanisms are recognized as in Chapter 4: movement of the location, where the skin friction exceeds the critical for sediment transport is important for the trough level changes. The top level changes according to the phase difference between the top and the sediment transport. Furthermore, it is found that the dune height has an order of magnitude corresponding to the highest flow velocity.

The second flow situation is a weak current combined with a strong tide. The net flow has the main characteristics of a tidal flow but with a difference in the strength of the two currents in the two tidal half-cycles. The morphological development of the dune during the tidal period hence has strong similarities with dunes in a tidal flow. The dune reverses in the two half-cycles to a degree corresponding to the difference in the strength of the flow in the two half cycles. Furthermore, the difference in the current strength results in a significant migration in the direction of the current.
Chapter 8
CONCLUSION

This thesis summarizes a study on morphological modelling of two-dimensional steady and unsteady sand dunes. The modelling system consists of a fully non-linear flow model, a sediment transport model and a morphological model. Only bed load was considered in the sediment transport field. The focus in the study was on the physical mechanisms related to the stability of the dunes and to the mechanisms responsible for the change in the shape and the geometrical properties of the dunes in unsteady flow situations.

Dunes in unidirectional flows and reversing flows were studied. Steady dunes in equilibrium with a unidirectional flow were calculated for fixed dune lengths between 3 and 8 water depths. The main conclusions with regard to the dune properties from this part of the study were: 1) For a given dune length, the dune height variation with the nondimensional bed shear stress, the Shields’ parameter, was found to have a trend similar to the expression for the dune height found by a semi-analytical method by Fredsøe (1982). 2) For an increasing dune length, the dune height approaches the dune height predicted by the expression by Fredsøe. The inequilibrium in the flow at the top of the dune for the shorter dunes explain the reduced dune height for these dunes 3) The dune height seems to approach a maximum value for an increasing dune length. The maximum value is smaller than the dune height predicted by Fredsøe. 4) The steepness of the calculated dunes was found to compare well with the steepness calculations presented by Tjerry (1995) and Tjerry and Fredsøe (to be publ.). The best agreement was found for dune lengths L/D=4-6 for the smallest dunes (θ′< 0.15) and for dune lengths of L/D=4-5 for the larger dunes. The mechanisms explaining the stability of the dunes for a given height and shape are the same as already identified by Fredsøe (1982), Tjerry (1995) and Tjerry and Fredsøe (to be publ.). Stability is obtained when equilibrium between the following is obtained: phase lag due to turbulence relaxation mechanisms, phase lag due to curvature acceleration of the flow (over curved bedform), gravity effects on sediment transport on a sloping bed, flow acceleration due to contraction and expansion of the flow area.

The flow resistance was calculated for the steady dunes for two dune lengths, L/D=4 and 6. The form drag calculation was found to be sensitive to the height of the crest above the trough level. The two extreme locations of the crest were hence tested in the calculations of the flow resistance: 1) as computed with the morphological model and 2) at the top of the dune. The results were compared to a model for the flow resistance presented by Engelund (1966, 1967). Furthermore, the form drag
Conclusion

for the computed dunes was estimated from the Carnot formula. The calculated form drag was found to agree well with the results for the Carnot formula, when the Carnot-coefficient found empirically by Engelund (1977) was applied. For dunes for the large Shields’ parameters, the numerically calculated flow resistance exceeds the prediction by the Engelund resistance model. For the small dunes, the flow resistance computations with the two crest locations were found to be respectively higher (crest at the top of the dune) and lower (crest as computed in the morphological model) than predicted by the Engelund model. The best agreement, however, was found with the crest located at the top. It should be noted, that further tests should be made to ensure grid independency, and the results presented here are preliminary.

Unsteady dunes in a unidirectional flow were studied mainly by means of calculating the change in the bedform after an abrupt change in the discharge. The discharge was constant before and after the change in the flow. The rate of change of the dune heights was compared to results by Fredsøe (1979). Fredsøe’s model, which predicts the rate of change of the top level of the dunes, was found to compare indeed very well with the computed dunes. The self-similarity in the flow after an abrupt change in the flow and a changed migration rate hence explains the growth or decay at the top. The change in the trough level, however, was found to be equally important for the prediction of the total dune height after a change in the discharge. This change was found to be related to the change in the skin friction distribution along the dune and mainly to the movement of the location on the dune, where the critical level of the bed shear stress for sediment transport is exceeded. The changes in the trough level with time were divided in three phases. In each of those, the mechanisms for the trough level change were identified. Estimates of the trough level change in each phase were then calculated based on the computed sediment transport field immediately after the change in the flow. The estimates show a good agreement with the computed morphological results.

The study of unsteady dunes in a unidirectional current was furthermore exemplified by a simulation of the morphological response of a dune during a conceptual flood wave with time-varying change in the discharge. The changes in the dune height and shape were studied - the dune length was remained constant. The same mechanisms as found after an abrupt change in the flow were identified. The variation in the dune height showed a ‘hysteresis effect’ caused by the lagged response in the morphology to a change in the flow. This effect was also observed in measurements on dune-dynamics in a river during a flood (e.g. Julien, 2000, Dalrymple and Rhodes, 1995).

The response in the morphology of dunes after a change in the flow direction was studied in three different flow situations. First, the change in the bedform shape immediately after a reversal of the flow direction was studied by simply reversing the flow direction on already steady dunes in the opposite flow direction. The dune shape changes initially by eroding from the steepest slope on the dune, which faces in the downstream direction after the change in the flow direction, due to the contraction of the flow area in the flow direction and hence acceleration of the flow. The sediment transport is deposited such that a new dune front is generated on the top of the dune facing the downstream direction. The upper part of the dune hence reverses first,
and the new front migrates down the old bedform. This behavior agrees well with observations in nature (Langhorne, 1982). Furthermore, the time scale and migration length for a full reversal of the dunes for different values of the Shields’ parameters were found.

Dunes in equilibrium with a periodic change in the flow direction were calculated for a period, T, comparable to the tidal period. The calculated dunes had a fixed length of 4 water depths. The maximum dune heights were found to be slightly larger than the dunes in a steady current for the same rate of sediment transport. The increased dune height after reversal was responsible for this.

Mainly two physical properties were found to be important for the dune height: The first is the ratio between total volume of sediment transported during one tidal period to the volume of the bedform. The second is the Shields’ parameter. Further calculations should be performed to make any clear conclusions. However, the following were observed in the present calculations: There is an indication that the dune height increases with an increasing Shields’ parameter for a constant value of $R_{side}$. This follows the same trend as a steady current dune in the pure bed load situation. For a reduced relationship between the total volume of sediment transported within a tidal period and the size (volume) of the bedform, the dune height increases. The latter appears intuitively to be a paradox, but was found to be related to the reversal mechanism within each tidal half-period. When less sediment is transported within a tidal cycle, the sediment mainly deposits on the top of the bedform in each tidal half-cycle. A further increase in the sediment transported within a tidal period, causes the deposited material on the top of the dune to be transported further away from the top of the dune. It should be noted that the bedforms studied in the present thesis were all relatively active bedforms. This means that, the large tidal bedforms also observed in nature, where only the upper part reverses slightly during a tidal period are not covered in the present thesis.

The tidal bedforms were studied with respect to the inactive and active part of the bedforms. This is mainly relevant in connection with pipe line burial. The same properties are relevant for the active/inactive volumes as is the case for the dune height. The inactive part of the bedform increases when the volume of sediment transported during a tidal period relative to the bedform volume decreases.

Finally the stability mechanisms of the tidal bedforms were studied. It is demonstrated that the stability of the bedforms cannot be explained from the time variation in the flow and sediment field along the tidally time-averaged bedform alone. The sediment transport field is not in equilibrium with the bed, but causes a further increase in the dune height/steeplness of the time-averaged bedform. Time-variation in the bedform interaction with the flow and sediment transport field is required to explain the equilibrium between the time-averaged sediment transport field and the time-averaged bedform. It is noted that 1) the phase-resolved mechanisms are necessary to explain the stability of tidal bedforms, but 2) if a phase-resolved method is applied this will result in larger sand dunes, since the phase-resolved mechanisms were found to have a dampening effect.

The last flow situation studied was the case of a co-existing tide and current. Two calculations served to exemplify the mechanisms in a tide-current flow: a) a
Conclusion

strong current with a super-imposed weak tidal flow and b) a strong tidal flow with a superimposed weak current. In a) the flow was unidirectional through the entire tidal period. The superimposed current acted to reduce and increase the strength of the current in each of the tidal half periods. A strong similarity with the dunes in the unsteady unidirectional current was found. The same physical mechanisms were found to govern the morphological changes in the dune shape and height. The differences found were especially related to the fact that the dunes were not in equilibrium with the flow at the time of the change in the discharge. In b) the flow direction reversed at the initiation of each half period. The superimposed current increased or decreased the strength of the flow. The morphology of the dune had a strong similarity with the pure tide flow. In both situations the weak component, being either the superimposed tide or current, changes the morphology and especially the migration velocity of the dune drastically compared to the ‘pure current’ or ‘pure tide’ situation.

To conclude, the numerical calculations of steady as well as unsteady dunes were promising. Dunes in equilibrium with the steady flow as well as the unsteady flow are found to follow the expected mechanisms. By using the numerical model, further insight into the mechanics of dunes has been obtained in this study.

Mainly two limitations in the present work are obvious: The first is the influence of suspended material. The dune calculations and observations are only valid when bed load dominates in the total sediment transport rate. When the Shields’ parameters exceed 0.2-0.3, it is crucial to include the suspended material in the sediment transport. The second is the mechanisms involved in the interaction between several dunes, leading to the length/height relationship for instance in unsteady flows.
Chapter 9
REFERENCES


References


References


References


Chapter 10
LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Migration velocity of sand dune</td>
</tr>
<tr>
<td>$c_b$</td>
<td>Bed concentration for suspended material</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Local skin friction coeﬃcient</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Skin friction coeﬃcient</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Local form drag coeﬃcient</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Form drag coeﬃcient</td>
</tr>
<tr>
<td>Cr</td>
<td>Morphological Courant Number</td>
</tr>
<tr>
<td>d</td>
<td>Grain diameter</td>
</tr>
<tr>
<td>D</td>
<td>Mean water depth, measured from mean bed level</td>
</tr>
<tr>
<td>D'</td>
<td>Boundary layer thickness</td>
</tr>
<tr>
<td>f</td>
<td>Normalized skin friction distribution</td>
</tr>
<tr>
<td>f</td>
<td>Time varying velocity signal for numerical model</td>
</tr>
<tr>
<td>Fr</td>
<td>Froude number, $F_r = \frac{V}{\sqrt{gh}}$</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration of gravity, 9.82 m/s$^2$</td>
</tr>
<tr>
<td>h</td>
<td>Local bed elevation</td>
</tr>
<tr>
<td>$h_{raw}$</td>
<td>Unﬁltered bed elevation</td>
</tr>
<tr>
<td>$h_D$</td>
<td>$\frac{h}{D}$</td>
</tr>
<tr>
<td>H</td>
<td>Dune height, from crest to trough</td>
</tr>
<tr>
<td>$\Delta H''$</td>
<td>Expansion loss in Carnot formula</td>
</tr>
<tr>
<td>k</td>
<td>Turbulent kinetic energy</td>
</tr>
<tr>
<td>I</td>
<td>Energy slope</td>
</tr>
<tr>
<td>$k_N$</td>
<td>Equivalent Nikuradse roughness</td>
</tr>
<tr>
<td>$k_{N}^+$</td>
<td>Equivalent Nikuradse roughness in wall values, $\frac{U_{*} k_N}{\nu}$</td>
</tr>
<tr>
<td>L</td>
<td>Dune length</td>
</tr>
<tr>
<td>$L_D$</td>
<td>$\frac{L}{D}$</td>
</tr>
<tr>
<td>L$_1$</td>
<td>Dune length, trough to top</td>
</tr>
<tr>
<td>L$_{95}$</td>
<td>Migration length to equilibrium</td>
</tr>
<tr>
<td>n</td>
<td>Pore volume, 0.4</td>
</tr>
<tr>
<td>L</td>
<td>Dune length</td>
</tr>
<tr>
<td>$L_D$</td>
<td>$\frac{L}{D}$</td>
</tr>
<tr>
<td>L$_1$</td>
<td>Dune length, trough to top</td>
</tr>
<tr>
<td>L$_{95}$</td>
<td>Migration length to equilibrium</td>
</tr>
<tr>
<td>n</td>
<td>Pore volume, 0.4</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

n  Number of tidal period
N  Number of horizontal gridpoints per dune
q  Sediment transport
Q  Discharge
p  Pressure
q_b  Rate of bed load
R_{tide}  Parameter for the tidal period
Re  Reynolds number, \( \frac{V \cdot D}{\nu} \)
s  Specific density of bed material, 2.65
S  Strain rate
t  Time
\Delta t  Morphological time step, nondim. with flow parameters
\Delta t_D  Morphological time step
t_D  Time in numerical model, nondim. with flow parameters
t^*  Time, nondimensionalised with sediment properties
T  Tidal period
T_D  Time scale in numerical model, nondim. with flow parameters
T_F  Duration of flood wave
T_{95}  Time lag
T_{tide}  Tidal period, nondimensionalized with sediment properties
u  horizontal velocity component
U'  Friction velocity
v  Vertical velocity component
V  Mean velocity
V'  Time varying mean velocity
V(x)  Local depth averaged velocity
V_{conv}  Convolution bedvolume in tidal flow
V_{inact}  Inactive bedvolume in tidal flow
V_{mean}  Mean volume in tidal flow
x  Horizontal coordinate
x_D  \( \frac{x}{D} \)
\Delta x  Horizontal grid spacing
w_s  Fall velocity
Z  Rouse parameter
y  Vertical coordinate
y^+  Vertical coordinate in wall units, \( \frac{y \cdot \kappa \cdot \nu}{\nu} \)
\alpha  Carnot coefficient
\epsilon  Dissipation of turbulent kinetic energy
\theta  Nondimensional total flow resistance
\theta'  Shields’ parameter, nondimensional bed shear stress
\theta''  Nondimensional form drag
\theta_c  Critical Shields’ parameter
\theta'_{eff}  Effective Shields’ parameter
\theta'_{top}  Shields’ parameter at the top of a sand dune
**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_u$</td>
<td>Shields’ parameter for a undisturbed channel bed</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Gravity parameter in sediment transport on sloping bed, 0.10</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Molecular viscosity, $1 \times 10^{-6} \text{m}^2/\text{s}$</td>
</tr>
<tr>
<td>$\nu_T$</td>
<td>Eddy viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density for water</td>
</tr>
<tr>
<td>$\rho_{sand}$</td>
<td>Density for the sediment, $2650 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>$\tau_b$</td>
<td>Bed shear stress</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Nondimensional sediment transport</td>
</tr>
<tr>
<td>$\Phi_b$</td>
<td>Nondimensional bed load</td>
</tr>
<tr>
<td>$\Phi_{b, u}$</td>
<td>Nondimensional bed load for undisturbed bed</td>
</tr>
<tr>
<td>$\Phi_s$</td>
<td>Nondimensional suspended load</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Specific dissipation rate</td>
</tr>
<tr>
<td>$\omega_D$</td>
<td>Wave frequency for tidal wave</td>
</tr>
</tbody>
</table>