Springing Response due to Bidirectional Wave Excitation

Jelena Vidic-Perunovic
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Preface

This thesis is submitted as a partial fulfilment of the requirements for the Danish Ph.D. degree. The work was carried out at the Section of Coastal, Maritime and Structural Engineering, Department of Mechanical Engineering, The Technical University of Denmark, during the period March 2002 to May 2005. Professor Jørgen Juncher Jensen and Associate Professor Harry Bredford Bingham supervised the study.

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Special thanks to my supervisor Professor Jørgen Juncher Jensen for giving me an honourable opportunity to be one of his students. His guidance through the study, and all the valuable and inspiring advices were highly appreciated.

Thanks to dr. Magnus Lindgren from Det Norske Veritas (DNV) who initiated Workshops on Springing in March 2002 and kept their continuity with optimism. Thanks to my colleagues Gaute Storhaug, dr. Xuekang Gu, dr. Gabriel Holtsmark, Øyvind Lund Johansen, Jens Bloch Helmers, Michiel van Tongeren and the others, who took part in the workshops with great interest. Thanks to dr. Elzbieta Bittner Gregersen for valuable discussions.

The theory was validated by using full-scale measurement data provided by Det Norske Veritas (DNV). Many thanks to dr. Bo Cerup Simonsen for his support and help during my stay at the Department of Hydrodynamics and Structures, Det Norske Veritas (DNV), Norway, in early spring 2004.

Many thanks to dr. Jack Spencer and dr. Yung Sup Shin at the Department of Rule and Development at American Bureau of Shipping (ABS), Houston, USA who made my visit in spring 2005 possible.

Thanks to Tara and Zoran, my dear family, for being loving and supportive.
Many thanks to all my colleagues at the Section for creating a friendly, positive and dynamic atmosphere.

Jelena Vidić-Perunovic
Lyngby, Jun 24, 2005
Executive Summary

Springing is a two-node high frequency resonant vibration of the hull induced by unsteady wave pressure field on the hull. The excitation force may be rather complex - any wave activity (or their combination) in the Ocean matching the two-node natural hull vibration frequency. With some ship designs the hull natural frequency may get low enough that the corresponding level of excitation energy becomes large. Springing vibration negatively influences the fatigue life of the ship but, paradoxically, it still doesn't get much attention of the technical society.

Usually, non-linear hydroelastic theories deal with the unidirectional wave excitation. This is quite standard. The problem is how to include more than one directional wave systems described by a wave spectrum with arbitrary heading. The main objective of the present work has been to account for the additional second-order springing excitation coming from interacting directional waves.

The modification has been implemented in the Second Order Strip Theory (SOST) computer programme developed by Jensen and Pedersen, 1981; Jensen and Dogliani, 1996, based on the relative motion strip theory (Gerritsma and Beukelmann, 1964). The quadratic strip theory is an efficient tool for load calculation that has been validated and proven to give satisfactory results in the range of rigid body wave-induced loads.

The present results from the linear analysis show very good agreement with other computer programmes for wave-induced loads calculation. Compared to the results of the full-scale measurements they agree quite satisfactory, too. Some differences inevitably appear between the different codes since they are not consistent in calculation of hydrodynamic coefficients, diffraction potential etc.

On the contrary, the results from different non-linear (second order) high frequency springing analyses with unidirectional wave excitation are much more scattered. Some of the reasons are different level of wave excitation accounted in the different
theories, inclusion of additional hydrodynamic phenomena e.g. slamming in the time-domain procedures, the structural damping coefficient uncertainty or some purely numerical details in the programme execution. Comparison with full-scale measurements clearly shows that in some cases all the presented computer programmes strongly underestimate the level of springing stresses in the hull. Not only a discrepancy with full-scale measurements exists, but worse is that no tendency in the measurement trend is captured. An important source of high frequency springing excitation is undoubtedly missing.

The full-scale measurements that are presented in the thesis and have been used for the validation are unique because, to the author's knowledge, this is the first time that the wave data were collected simultaneously with stress records on the deck of the ship. This is highly appreciated because one can use the precise input and not only the most probable sea state statistics. The actual picture of the sea waves leads to the conclusion that the sea surface is rarely unidirectional and that two main wave directions usually can distinguish. This could be explained by existence of wind waves and swell in the ocean, or sudden change in wind direction that could create a wave system from the new direction while the energy is still not dissipated in the old waves.

The new excitation coming from second order interaction between the two considered wave systems is included. The improvement of the agreement between the calculated and measured high frequency springing response is high while there was almost no change in rigid body response.

This thesis should, hopefully, contribute to the better understanding of the springing response and it's excitation, recommend the way to include this new and very important excitation source and, finally, it should highlight the need to consider springing loads already in pre-design stage at least for some types of ships and loading conditions.
Synopsis

Springing betegner en to-knudet højfrekvent resonanssvingning i skroget, der fremkaldes af et kvasi-statisk bølge-induceret tryk på skroget. Belastningen er temmelig kompleks – en hvilken som helst holst bølgeaktivitet (eller kombination heraf) i havet, med en frekvens svarende til den to-knudede skrogsvingningsfrekvens kan inducere denne svingning. For nogle skibe kan denne frekvens være så lav, at det tilsvarende bølgeenerginiveau er stort nok til at inducere betydelige dynamiske spændinger med deraf følgende større sandsynlighed for træthedsbrud i skibes bærende struktur. Paradoksalt nok har dette emne ikke megen opmærksomhed i tekniske kredse, selvom der for nylig er dokumenteret meget alvorlige eksempler på sådanne skader.


Betragtes alene den lineære del af teorien, så en god overensstemmelse med andre computerprogrammer for beregning af lineære bølgefremkaldte belastninger. Ved sammenligning med resultaterne fra målingerne i fuld skala stemmer de beregnede resultater også tilfredsstillende overens. Visse forskelle vil altid opstå, fordi der er forskelle i beregningen af hydrodynamiske koefficienter, diffraktionspotentiale etc.

Derimod er resultaterne fra forskellige ikke-lineære (anden grads) højfrekvente springingsanalyser med ensrættet bølgedannelse meget forskellige. Nogle af grundene er forskelle i den bølgedannelse, der medtages, herunder transiente bølgeslag på forskibet, og usikkerheden vedrørende den konstruktionsmæssige dæmpningskoefficient.


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En sammenligning med målinger i fuld skala viser klart, at i nogle tilfælde undervurderer alle de diskuterede beregningsprocedurer stærkt niveauet af den springinginducerede spænding i skroget. Der er ikke blot en uoverensstemmelse med målingerne i fuld skala, men hvad værre er, er der heller ingen tendens i målingerne. Der mangler således utvivlsomt en vigtig kilde for generering af springing.

Målingerne i fuld skala, der er benyttet i denne afhandling og brugt til valideringen er enestående, fordi så vidt forfatteren ved, er dette første gang bølgedata er blevet indsamlet samtidigt med spændingsregistreringer på dækket af skibet. Dette er meget værdifuldt, fordi det præcise bølgeinput, og ikke blot den mest sandsynlige havoverfladetilstandsstatistik, kendes. Det aktuelle billede af de målte havbølger leder til den konklusion, at havoverfladen sjældent er ensrettet, og at to hovedbølgeretninger sædværdifuld kan skelnes. Dette kan forklares ved eksistensen af både vindbølger og donninger i havet eller ved pludseligt skift i vindretningen, som kunne skabe et bølgesystem fra den nye retning, mens energien stadig ikke døet ud i de gamle bølger.

De nye effekter, som kommer fra den medtagne anden ordens vekselvirkning mellem to betragtede bølgesystemer, er dokumenteret og valideret i nærværende arbejde. Overensstemmelsen mellem de beregnede og målte springingsrespons er betydeligt bedre end uden disse effekter medtaget, mens der næsten ingen ændring er i det direkte bølgeinducerede respons.

Denne afhandling kan forhåbentlig bidrage til en bedre forståelse af springing i skibe og dens dannelse, og sætte fokus på behovet for at betragte springingsbelastninger allerede i designstadiet.
Kratak Pregled

Springingom nazivamo dvočvornu rezonantnu vibraciju brodskog trupa u visokoj frekvenciji, indukovanu promenljivim poljem pritiska od talasa koji deluju na brod. Pobudna sila može biti prilično složena – bilo kakva aktivnost morskih talasa (ili njihova kombinacija) čija se frekvencija podudara sa sopstvenom frekvencijom vibracije trupa. Kod nekih tipova brodova sopstvena frekvencija trupa može biti dovoljno niska da odgovarajući nivo pobudne energije postane visok. Springing vibracija negativno utiče na vek trajanja usled zamora materijala i, paradoksalno, još uvek ne privlači dovoljno pažnje od strane stručnjaka u ovom polju tehnike.

Nelinearne hidroelastične teorije obično podrazumevaju pobudu od talasa koji putuju svi u istom pravcu i smeru. To je standardna pretpostavka. Pitanje je kako uključiti više sistema talasa iz različitih pravaca (sve talasne komponente u okviru talasnog sistema iz jednog pravca putujući u istom smeru) pod proizvoljnim međusobnim uglom napredovanja. Glavni zadatak ovog rada je da se uzme u obzir dodatni izvor pobude drugog reda relevantne za springing koji vodi poreklo od međusobne interakcije dva sistema talasa iz različitih pravaca.


Rezultati dobijeni linearnom analizom su u saglasnosti sa ostalim kompjuterskim programima za izračunavanje opterećenja od talasa. Zadovoljavajuća saglasnost postoji i u poređenju sa rezultatima merenja u pravoj veličini. Ipak, neizbežne razlike se javljaju između različitih kompjuterskih programa zbog nekonzistentnosti u načinu izračunavanja hidrodinamičkih koeficijenata, difrakcionog potencijala, itd.

Rezultati različitih nelinearnih springing analiza drugog reda, koji podrazumevaju talasnu pobudu iz istog pravca i smera, su naprotiv veoma razuđeni. Mogući razlozi za...
to su nivoi pobude uzeti u obzir u različitim teorijama, uključenje dodatnih hidrodinamičkih efekata u procedure u vremenskom domenu, kao npr. sleming pramčanog dela, neizvesnosti u vezi sa prigušenjem u samoj brodskoj strukturi ili, pak, neki sasvim numerički detalji pri izvršavanju kompjuterskog programa. Poređenja sa merenjima u pravoj veličini jasno pokazuju da u nekim slučajevima svi prikazani kompjuterski programi značajno podcenjuju veličinu napona usled springinga u brodskom trupu. Ne samo da postoji nepodudarnost rezultata, već ne postoji nikakav trend sa merenjima. Bez svake sumnje, nedostaje vaza izvor pobude springinga u visokoj frekvenciji.

Merenja u pravoj veličini, predstavljena u disertaciji i korišćena za validaciju teorije, su jedinstvena jer, prema autorkinom znanju, ovo je prvi pokušaj da se podaci o pobudnim talasima prikupe simultano sa merenim vremenskim serijama napona na palubu broda. Ovo je veoma korisno, jer se talasi mogu opisati znatno preciznije nego koristeći statističke karakteristike stanja mora npr. sa najvećom verovatnočom pojavljivanja. Merena slika površine mora pokazuje da su talasi retko jednosmerni i u istom pravcu, te da se obično mogu razlikovati dva glavna pravca (svaki od njih sa istosmernim talasima). Ovo se objašnjava postojanjem talasa usled dejstva vetra i talasa usled zaostalog mora, ili brzom promenom pravca duvanja vetra usled čega je kreiran novi talasni sistem dok energija starog jos nije ugušena.

Uključena je nova pobuda usled interakcije drugog dela između dva postomatana sistema talasa. Saglasnost između izračunatog i izmerenog visokofrekventnog opterećenja usled springinga je značajno poboljšano, dok je opterećenje krutog tela skoro nepromenjeno.

Ova disertacija bi, po mogućtvu, trebala da doprise boljem razumevanju springinga i mehanizma njegove pobude, da ukaže na način da se uzme u obzir ovaj novi i veoma uticajan izvor pobude i, konačno, da naglaši potrebu da se opterećenja usled springinga razmatraju već u fazi predprojekta, barem za neke tipove brodova i uslove natovarenosti.
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Nomenclature

Coordinate System

\( OXYZ \) Global coordinate system fixed at the mean water level position

\( Oxz \) Local coordinate system fixed to the ship (translating with the mean forward speed \( U \) in the +x direction) and with the origin at the stern. The vertical coordinate \( z \) equals zero at the mean water level. \( z \) is positive upwards.

Roman Symbols

\( U \) Mean forward speed of the ship
\( \dot{z} \) Vertical relative displacement
\( t \) Time
\( F_{HH}(x,t) \) Vertical total hydrodynamic force on a ship section
\( m \) Sectional added mass
\( N \) Sectional hydrodynamic damping
\( L \) Length between perpendiculars
\( B \) Breadth moulded
\( B(x) \) Sectional breadth
\( T \) Sectional draught
\( A \) Sectional submerged area
\( w \) Vertical displacement of a ship section
\( p \) Wave pressure
\( F_{HH} \) Hydrodynamic force
<table>
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<th>Symbol</th>
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<tbody>
<tr>
<td>$F_R$</td>
<td>Froude-Krylov force</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration of gravity</td>
</tr>
<tr>
<td>$a$</td>
<td>Wave amplitude</td>
</tr>
<tr>
<td>$k$</td>
<td>Wave number</td>
</tr>
<tr>
<td>$\vec{V}$</td>
<td>Fluid velocity vector</td>
</tr>
<tr>
<td>$M$</td>
<td>Bending moment</td>
</tr>
<tr>
<td>$E$</td>
<td>Youngs modulus of elasticity</td>
</tr>
<tr>
<td>$E[]$</td>
<td>Expectation function</td>
</tr>
<tr>
<td>$R_{\sigma}(\tau)$</td>
<td>Covariance function</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus</td>
</tr>
<tr>
<td>$F$</td>
<td>Shear force</td>
</tr>
<tr>
<td>$I$</td>
<td>Sectional moment of inertia</td>
</tr>
<tr>
<td>$m_r(x)$</td>
<td>Longitudinal distribution of the mass of the ship</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius of gyration</td>
</tr>
<tr>
<td>$m_r r^2(x)$</td>
<td>Mass moment of inertia round the y axes</td>
</tr>
<tr>
<td>$u_*$</td>
<td>Wind speed</td>
</tr>
<tr>
<td>$S$</td>
<td>Wave spectrum</td>
</tr>
<tr>
<td>$S_r$</td>
<td>Response spectrum</td>
</tr>
<tr>
<td>$S^e$</td>
<td>Encounter wave spectrum</td>
</tr>
<tr>
<td>$S^y$</td>
<td>Target wave spectrum</td>
</tr>
<tr>
<td>$S^L$</td>
<td>Linear part of the wave spectrum</td>
</tr>
<tr>
<td>$S^{NL}$</td>
<td>Non-linear part of the wave spectrum</td>
</tr>
<tr>
<td>$M_{r^a}$</td>
<td>Vertical bending moment linear coefficients</td>
</tr>
<tr>
<td>$M_{r^a}$</td>
<td>Vertical bending moment second order coefficients</td>
</tr>
<tr>
<td>$V_r$</td>
<td>Variance of the wave spectral density</td>
</tr>
<tr>
<td>$s$</td>
<td>Standard deviation of the response</td>
</tr>
<tr>
<td>$H_s$</td>
<td>Significant wave height</td>
</tr>
<tr>
<td>$T_z$</td>
<td>Zero up-crossing period</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Wave peak period</td>
</tr>
<tr>
<td>$a_1,a_2,d,\bar{\beta},C$</td>
<td>Lewis transformation coefficients</td>
</tr>
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Nomenclature

Greek Symbols

\( \beta_i \)  Heading of the \( i \)th wave system relative to the ship
\( \beta \) Dominant wave heading relative to the ship
\( \eta \) Wave elevation
\( \kappa \) Smith correction factor
\( \phi \) Velocity potential
\( \chi \) Independent stochastic variable
\( \Lambda_{d} \) Standard deviation coefficients of the second order contribution
\( \lambda_{k} \) Standard deviation coefficients of the first order contribution
\( \theta \) Wave phase
\( \omega \) Angular frequency of wave
\( \omega_e \) Angular frequency of encounter wave
\( \omega_p \) Wave peak frequency
\( \omega_s \) Springing resonant frequency
\( \bar{\omega}_r \) Vertical \( r \)th mode natural frequency of the dry hull
\( \rho \) Density of water
\( \eta_m \) Normal damping coefficient
\( \eta_s \) Tangential damping coefficient
\( \alpha_e, \nu_r \) Eigenvectors for the \( r \)th vibration mode
\( \alpha_e, \beta_e \) Phillips constants
\( \alpha_d \) Damping coefficient
\( \alpha \) Sectional area coefficient
\( \bar{\beta} \) Breadth to draught coefficient
\( \delta \) Logarithmic decrement damping coefficient of the hull
\( \delta_i \) Kronecker delta function
\( \xi \) Damping to critical damping ratio coefficient
\( \varepsilon \) Spectral bandwidth
\( \hat{\varepsilon} \) Nonlinearity parameter
\( \hat{\gamma} \) Peakedness factor
\( \chi(k) \) Wave number moduli spectrum
\( \varphi \) Angular displacement
\( \hat{\varphi} \) Polar coordinate, angle relative to the wind direction
ν  Spectral bandwidth

**Abreviations**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ISSC</td>
<td>International Ship and Offshore Structures Congress</td>
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<tr>
<td>VWBM</td>
<td>Vertical wave bending moment</td>
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<td>DNV</td>
<td>Det Norske Veritas</td>
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<td>SOST</td>
<td>Second Order Strip Theory</td>
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<tr>
<td>WASIM</td>
<td>Software for calculation of Wave Loads on Vessels with Forward Speed</td>
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<tr>
<td>VERES</td>
<td>Software for calculation of Vessel Response</td>
</tr>
<tr>
<td>SINO</td>
<td>Simulation of Nonlinear Responses</td>
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<td>LAMP</td>
<td>Large Amplitude Motion Programme</td>
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<td>JONSWAP</td>
<td>Joint North Sea Wave Project</td>
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<tr>
<td>HF</td>
<td>High frequency</td>
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<tr>
<td>WF</td>
<td>Wave frequency</td>
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<tr>
<td>P</td>
<td>Port side</td>
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<td>SB</td>
<td>Starboard side</td>
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<tr>
<td>WAVEX</td>
<td>Marine Radar Wave Extractor</td>
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<tr>
<td>WAFO</td>
<td>Wave Analysis for Fatigue and Oceanography</td>
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<tr>
<td>LC</td>
<td>Long crested (waves)</td>
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<tr>
<td>SC</td>
<td>Short crested (waves)</td>
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Chapter 1

Introduction

1.1 Overview and Background

Usually, a ship can be treated as a rigid body when the wave-induced sectional forces are calculated. This is implicitly assumed in the current rules of the classification societies provided the midship sectional moment of inertia of the hull girder exceeds a prescribed value. However, with the current ship's tendency towards increasing size, lower depth to length ratios and the use of high tensile steel, ships become more flexible so that under certain sea conditions the wave-induced vibrations of the hull become important. The contribution from the vibration-induced bending moment to the direct wave-induced loads can dramatically reduce the fatigue life of the hull girder. According to e.g. Storhaug et al., 2003, cracking has appeared in the deck plates due to vibration after very short time in operation, sometimes also in combination with corrosion, high stresses concentrations in certain areas or badly designed structural details. This implies that the fatigue damage has increased by an order of magnitude compared to a rigid body analysis.

When in the 1960's many ships exceeding 200m in length entered into service, the concern about springing, i.e. the two-node vertical hull girder vibration was increased. Significant springing vibrations were noticed in full-form ships and full-scale measurements of stresses were performed on several ships. Comparisons between full-scale measurements of the stresses and numerical calculations using different linear and non-linear procedures can be found for instance in Goodman, 1970 (the tanker SS Myrina), Stiansen at al., 1978 (three Great Lake bulk carriers), Gran, 1976 (the tanker Esso Bonn). Lack of agreement between the measured and calculated
responses was mainly attributed to uncertainties in the structural damping and the high-frequency tail of the wave spectrum.

Today, the master of the a ship, guided by modern monitoring decision support systems, is able to avoid severe storms and moderate sea states become the most probable sailing condition due to weather routing based on weather forecast. Ships are thus generally operating in low sea states and related low periods and that way they can keep up a higher speed. All these conditions imply, however, relatively susceptibility to high-frequency springing responses.

Extensive reviews on springing can be found in the proceedings of the International Ship and Offshore Structures Congress (ISSC). However, only recently very detailed full-scale springing measurements for a large ocean-going bulk carrier have been compiled and published, including information on the wave environment recorded by a wave radar system. Storhaug et al., 2003 compared parts of these data with theoretical load predictions using several linear and non-linear hydrodynamic procedures. Some agreement is found but, generally, the springing predictions are lower than seen in the measurements. For the linear calculations the omission of non-linear terms is believed to be the reason for the discrepancy, whereas for the non-linear calculations the assumption of a single long-crested wave system might omit important cross-coupling terms. The latter assumption is partly removed in the present thesis considering the second order excitation from a bi-directional wave system.

While many researchers point out in their studies the importance of accounting for wave induced high frequency loads, a dynamic load analysis based on more direct approach than the prescriptive rule base design still stays an optional choice in the regulations from the Classification Societies. In many cases economy and strict deadlines don't allow for a more sophisticated and time consuming direct calculations that include hydro-elasticity. Springing stays ignored also in the Common Structural Rules (ABS, DNV and LR Joint Rules) that will be effective in July 2005.

1.2 Objectives

Whenever the frequency of the excitation force on the ship matches the two node natural frequency of the hull, the loads rapidly increase due to generally a very low damping in the hull. The complexity and variety of linear and higher order forces that may excite springing is large. Attempts to better understand the physics of the wave induced springing vibration of the hull and to account for some of these excitation mechanisms are presented in this thesis. Finally, some conclusions are drawn about hull forms and sea conditions, which can lead to the higher springing vibration.

Assuming a situation with two uni-directional wave spectra describing the sea surface activity, a springing response at the 2-node vibration frequency $\omega_s$ is excited by:
(1) Waves with encounter frequency equal to $\omega_s$, which is accounted by the linear theory.

(2) Waves with encounter frequencies $\omega_i$ and $\omega_j$ where $\omega_i + \omega_j = \omega_s$, which is treated by the second order super-harmonic terms.

(3) In case of existence of another wave system (e.g. swell coming from a different direction or some local secondary wind sea) presented by the wave spectrum $S_2$, the two cases (1) and (2) above would equally apply to the wave components of the second wave spectrum ($\omega_m$ and $\omega_p$), see Fig. 1.

(4) Waves with encounter frequencies $\omega_i$ and $\omega_m$, where $\omega_i + \omega_m = \omega_s$ and this case is treated by second order cross coupling terms due to second order interaction between components of the two incident wave spectra $S_1$ and $S_2$. This cross-coupling is the main topic in this thesis.

![Wave spectral density and Response spectrum](image)

Fig. 1: Wave excitation leading to springing

Wind-driven waves and swell usually differ in energy level as well as in the heading angle relative to the vessel. Normally, they are combined to give a double-peaked total wave energy spectrum. Very early some authors stressed the importance of such double-peaked excitation spectra in the springing analysis (Van Gunsteren, 1978), as
they deviate considerably from the standard analytical wave spectra (Pierson-Moskowitz, JONSWAP). Double-peaked wave spectra that take into account both wind-driven and swell parts of the sea have been formulated e.g. the Torsethaugen wave spectrum (Torsethaugen, 1996). However, they only describe the energy distribution as a function of frequency but do not account for the wave direction.

With new measurement equipment, notably the wave radar, it is now possible on board the vessel to obtain rather detailed, albeit not always very accurate, information on the wave environment. This data typically includes wave statistics but also the wave spectral density as a function of direction and frequency. A recent example (Storhaug et al., 2003) concerning data collected in the North Atlantic Ocean clearly shows the existence of more than one wave system, each with different heading angle.

In this thesis, an extension has been made to the Second Order Strip Theory (SOST) (Jensen and Pedersen, 1979, 1981 and Jensen and Dogliani, 1996) to include bi-directional wave excitation. The effect on the ship's response to the second order interaction between two long-crested wave systems is derived. Due to the non-linear free surface boundary conditions for the fluid, non-linear cross-coupling terms between two long-crested wave systems arise, to be added to second order terms from each of the two wave systems alone. The result is frequency sum terms giving an increase of the high-frequency response compared to the response in second order unidirectional waves. Thus, the springing-induced response will be significantly influenced by the additional second order cross-coupling terms. The direct wave-induced response (i.e. the rigid body response) will remain mainly unaffected by the cross-coupling terms and given as the sum of the two statistically independent directional responses, as the cross-coupling response terms are very small in the wave frequency range. In the further text, the fundamental rigid body response will be denoted *wave-induced response* and the high-frequency response of the flexible hull *springing*, both induced by the wave pressure acting on the hull.

Deep water waves are assumed with a specified linear wave spectrum. A finite water depth would have no influence on the ship's springing response due to the small wavelengths responsible for the springing excitation. Moreover, a difference in the tail of the wave spectrum in shallow water compared to deep water would mainly influence the linearly excited springing response, which is significantly smaller than the second order springing response. However, it would be straightforward to include finite water depth using a linear theory (Vidic-Perunovic and Jensen, 2004) as a starting point.

It will be shown that the springing responses obtained after inclusion of the bi-directional excitation forces become a much better agreement with the available full-scale springing measurements.
1.3 Structure of the Thesis

After the introduction in Chapter 1 the thesis is arranged as follows. Chapter 2 presents the derivation of the second order non-linear hydrodynamic forces on the hull in case of bi-directional wave excitation with the arbitrary wave heading. The second order cross interaction terms have been included in the excitation force. In connection to the structural response of the ship, structural damping is discussed. Also, a slight modification to the short terms response statistics is outlined, due to interaction between the two wave systems.

In Chapter 3 the Smith correction factor has been derived for the Lewis hull sectional form with the aim to provide a better approximation in the high frequency domain relevant for springing response. Additionally, the Smith correction for different analytical hull sectional forms is presented and the results are discussed.

Chapter 4 deals with the high frequency excitation wave spectrum. An analytical form of the wave spectrum is suggested with variable wave frequency decay rate. This may be a convenient way to describe a spectrum containing more energy in the tail than conventional, the JONSWAP or Pierson-Moskowitz spectrum form.

Springing is usually a low amplitude high frequency vibration of limited importance. However, in Chapter 5 examples of the measured large amplitude springing stresses are given, that have caused severe damages onboard a large ocean going bulk carrier. The problem has been analysed using unidirectional second order theory to predict springing loads. Comparing the results with different hydroelastic calculations has varified the unidirectional theory but, comparison with measurement data has shown that an additional springing excitation source is lacking in the analysis.

Therefore, in Chapter 6 a bi-directional analysis has been made for the same bulk carrier. The wave characteristics obtained by measured directional wave data are used as input. The springing loads are further compared with a large amount of response measurements. The agreement between calculations and measurements is greatly improved by the inclusion of the cross-interaction terms between the two excitation wave systems.

In Chapter 7 calculations for a containership are presented and compared with available measurements.

Finally, in Chapter 8 the thesis is concluded and some recommendations for the future work are given.
Chapter 2

Hydrodynamic Forces

2.1 Introduction

In this chapter second order expressions are derived for the incident wave potential, the wave elevation and the dynamic pressure for bidirectional wave excitation. The general formulas for two wave systems intersecting at an arbitrary angle are given. The incident wave potential is presented due to bichromatic and bidirectional waves. Second order non-linear free surface boundary conditions (kinematic and dynamic) are solved and the second order wave potential has been obtained. In order to obtain the total dynamic force on the hull, the expressions for the Froude-Krylov force and the hydrodynamic force have been derived including the modification due to bi-directional waves. The new second order terms arise due to cross-coupling of waves from different directions. The springing response depends strongly on non-linear (sum frequency) terms, especially as large contributions to the high frequency response from the superharmonic interaction terms are expected in the opposing wave conditions, which is thoroughly discussed in this chapter.

The equations of motion of the hull can be solved now presenting the hull to behave as a single Timoshenko beam with free ends, as the wave excitation force, hull stiffness, mass distribution and added mass of the water and damping are known.

The structural damping is then discussed, which is the dominating source of damping in the domain of springing frequencies since the hydrodynamic damping is typically very small. The uncertainties related to determination of the hull structural damping coefficient in engineering practice are shown to be significant.
Modifications of the short-term response is given due to additional excitation wave system. The new terms in the spectral density functions appear due to the interaction between the two unidirectional wave trains.

2.2 Literature Overview

After severe springing was experienced in Great Lake bulk carriers, efforts were made to analyse the problem and to calculate springing responses. First attempts were made using linear strip theories such as Van Gunsteren, 1970, Goodman, 1971, Gran, 1973. The inability to predict the level of springing was explained by uncertainty in the hull internal damping and inaccuracy of (or complete lack of information about) the measured excitation wave spectrum.

Bishop and Price, 1979, introduced a generalized approach where the ship hull is modelled as a beam, and the rigid body motions and the deflection are defined as motions in the ‘generalized’ modes.

Later, the excitation forces were formulated using the slender body theory where the wave length was assumed to be small, comparable with the ship breadth. The theory is explained using a potential flow approach in Troesch, 1979, Skjordal and Faltinsen, 1980, or Beck and Troesch, 1980, where the diffraction potential was solved directly (instead of applying the Haskind relations) by use of matched asymptotic expansions and the problem was divided into a far field and a near field solution.

The Second Order Strip Theory was proposed in Jensen and Pedersen, 1979, and Jensen and Pedersen, 1981, based on the relative motion strip theory concept. Based on this Second Order Strip Theory, Jensen and Dogliani, 1996, accounted for the proper short-term analysis and investigated the response in stationary and non-stationary seas as well as the contribution of the non-linear springing loads to the fatigue.

The influence of non-linearities was experimentally confirmed in Slocum, 1983, and Slocum and Troesch, 1983. Non-linear effects are theoretically accounted for in e.g. Troesch, 1984. Domnisoru and Domnisoru, 2000, who conducted experiments using an elastic ship model and found large contribution from vibratory responses excited by regular waves. Non-linear effects in oblique seas due to second order diffraction pressures are included in He at al., 2004, based on slender body linear diffraction theory by Troesch, 1979. Here only the first order incident wave potential is solved and accounted for by the linear velocity squared term in the Bernoulli equation while the second order potential is neglected in the analysis. On the contrary, in the present study second order the incident wave potential is accounted for in the wave excitation force and extended to include bidirectional waves.
Finally, three-dimensional methods are more advanced though robust and computationally time consuming. The two main approaches are using the wave Green function (example of implementation of the method is LAMP, Lin et al., 1994) or Rankine sources (example of implementation of the method is WASIM). Using a wave Green function satisfies the free surface boundary conditions and only the hull surface needs to be discretized. Using Rankine sources both hull surface and free surface should be discretized since the singularities do not satisfy free surface boundary conditions (Kring, 1994, Vada, 1994).

2.3 Non-linear Strip Theory

2.3.1 Second Order Strip Theory

The Second Order Strip Theory of Jensen and Pedersen, 1979 was originally developed for ships sailing in unidirectional random seas, and it represents a simple generalisation of the linear strip theory formulation. The quadratic solutions are established by a perturbational procedure performed about the mean still water level.

The first order solution coincides with the linear theory approach being either

- The potential flow strip theory (Salvesen, Tuck and Faltinsen, 1970) or
- The relative motion strip theory (Gerritsma and Beukelman, 1964)

The non-linear second order terms are determined by including:

- Second-order incident waves
- Variation of the sectional breadth (non-vertical sides) with draught
- Variation in hydrodynamic forces during the vertical motions of the hull

The free surface is modelled by second order Stokes waves and the hydrodynamic coefficients and the breadth of the ship are perturbed about the mean waterline, which results in non-linear hydrodynamic and restoring forces. The quadratic theory is formulated in the frequency domain and the statistical results are therefore easily obtained. The procedure is computationally fast.
2.3.2 Coordinate System

The global coordinate system $OXYZ$ is fixed in the mean water level position and the local coordinates $oxyz$ as fixed to the ship with the origin at the stern. The two systems coincide when the ship is at rest. Both coordinate systems are Cartesian, right-handed, with the vertical axes positive upwards. The mean forward speed $U$ of the ship is in the positive $x$ direction. The ship is sailing in bidirectional waves with heading angles $\beta_1$ and $\beta_2$ relatively to the ship’s heading. $\beta = 180 \text{deg}$ denotes head sea.

2.3.3 Non-linear Hydrodynamic Force

The external forces on the hull are determined according to expressions proposed by Gerritsma and Beukelman, 1964, modified to account for hydrodynamic coefficients and breadth dependent on the sectional immersion. The total hydrodynamic force per unit length acting on the hull in the position $x$ consists of the time derivation of the momentum of the added mass of the surrounding water, a damping term and a restoring term, all dependent on the relative motion (Jensen and Pedersen, 1979):

\[
F(x,t) = -\left[ \frac{D}{Dt} \left\{ m(\bar{z},x) \frac{D\bar{z}}{Dt} \right\} + N(\bar{z},x) \frac{D\bar{z}}{Dt} + \int_{-T}^{T} B(z,x) \left. \frac{\partial p}{\partial z} \right|_{z+w} \, dz \right] \tag{2.1}
\]

Fig. 2.1: Coordinate system.
The relative motion concept is used in this formula. Hence, the difference \( \tilde{z} \) between the absolute displacement of the hull \( w(x,t) \) and the surface elevation of the sea \( \eta(x,t) \), as modified by the Smith correction factor \( \kappa \) to take into account diffraction effects:

\[
\tilde{z} = w - \kappa \eta
\]

The total derivative with respect to time \( t \) is defined by

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{dx}{dt} \frac{\partial}{\partial x} = \frac{\partial}{\partial t} - U \frac{\partial}{\partial x}
\]

where \( U \) is the forward speed of the vessel. The breadth at the water line \( B \), the added mass \( m \) and the damping coefficient \( N \) are evaluated around \( \tilde{z} = 0 \) by taking into account the constant and the linear term in a Taylor series expansion at each quantity about \( \tilde{z} = 0 \):

\[
B(\tilde{z}, x) = B(0, x) + \tilde{z} \left. \frac{\partial B}{\partial \tilde{z}} \right|_{\tilde{z}=0} = B_0(x) + \tilde{z}(x,t)B_1(x)
\]

\[
m(\tilde{z}, x) = m(0, x) + \tilde{z} \left. \frac{\partial m}{\partial \tilde{z}} \right|_{\tilde{z}=0} = m_0(x) + \tilde{z}(x,t)m_1(x)
\]

\[
N(\tilde{z}, x) = N(0, x) + \tilde{z} \left. \frac{\partial N}{\partial \tilde{z}} \right|_{\tilde{z}=0} = N_0(x) + \tilde{z}(x,t)N_1(x)
\]

\section*{2.4 Second Order Wave Potential and Wave Elevation}

The flow due to ocean waves is treated as incompressible and irrotational. Hence, beside the Laplace equation within the fluid and the infinite water depth bottom boundary condition, the wave potential \( \phi = \phi(X, Y, Z, t) \) has to satisfy the following boundary conditions:

- The kinematic free surface boundary condition:

\[
\frac{D \eta}{Dt} = \frac{\partial \phi}{\partial Z}
\]

The total derivative with respect to time \( t \) is needed with respect to particle propagation, since the \( XYZ \)-coordinate system is fixed globally with \( XY \) in the still water plane and \( Z \) pointing upwards.

- The dynamic free surface boundary condition:
\[
\frac{\partial \phi}{\partial t} + \frac{1}{2} \vec{V}^2 + g\eta = 0 \tag{2.4}
\]

where the fluid velocity vector \( \vec{V} = (V_x, V_y, V_z) \), the acceleration of gravity is \( g \) and the vertical water surface elevation is denoted by \( \eta = \eta(X,Y,t) \).

By taking the total derivative of the dynamic free surface boundary condition,

\[
\left( \frac{\partial}{\partial t} + \vec{V} \nabla \right) \left( \frac{\partial \phi}{\partial t} + \frac{1}{2} \vec{V}^2 + g\eta \right) = 0 \tag{2.5}
\]

the following expression is obtained:

\[
\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial}{\partial t} \left( \frac{\vec{V}^2}{2} + g \frac{\partial \eta}{\partial t} + \vec{V} \nabla \frac{\partial \phi}{\partial t} + \frac{1}{2} \vec{V} \nabla \vec{V}^2 \right) + \vec{V} \nabla g\eta = 0 \tag{2.6}
\]

By using \( \frac{\partial}{\partial t} \vec{V} = \frac{1}{2} \frac{\partial}{\partial t} \vec{V}^2 \) and Eq. (2.3) the condition for the potential at the free surface level \( Z = \eta \) becomes (e.g. Mei, 1994):

\[
\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial Z} + \frac{\partial}{\partial t} \left( \frac{\vec{V}^2}{2} + \frac{1}{2} \vec{V} \nabla \vec{V}^2 \right) = 0 \tag{2.7}
\]

The wave potential and the wave elevation are approximated to the second order with

\[
\phi(X,Y,Z,t) = \phi^{(1)}(X,Y,Z,t) + \phi^{(2)}(X,Y,Z,t);
\]

\[
\eta(X,Y,t) = \eta^{(1)}(X,Y,t) + \eta^{(2)}(X,Y,t)
\]

while the terms appearing in the free surface boundary conditions are expanded in a MacLaurin series around the still water level to

\[
\frac{\partial^2 \phi}{\partial t^2} \bigg|_{Z=\eta} = \frac{\partial^2 \phi}{\partial t^2} \bigg|_{Z=0} + \eta^{(1)} \frac{\partial}{\partial Z} \frac{\partial^2 \phi}{\partial t^2} \bigg|_{Z=0} \tag{2.8}
\]

\[
\frac{\partial \phi}{\partial Z} \bigg|_{Z=\eta} = \frac{\partial \phi}{\partial Z} \bigg|_{Z=0} + \eta^{(1)} \frac{\partial^2 \phi}{\partial Z^2} \bigg|_{Z=0} \tag{2.9}
\]
By inserting Eqs.(2.8) - (2.9) into Eq.(2.4) and Eq.(2.7), the following equations are obtained for $Z=0$:

\[
\frac{\partial \phi^{(2)}}{\partial t} + \eta^{(1)} \frac{\partial^{2} \phi^{(1)}}{\partial t \partial Z} + \frac{1}{2} \left| \nabla \phi^{(1)} \right|^2 + g \eta^{(2)} = 0
\]  
(2.10)

\[
\frac{\partial^{2} \phi^{(2)}}{\partial t^2} + \eta^{(1)} \frac{\partial}{\partial Z} \frac{\partial^{2} \phi^{(1)}}{\partial t^2} + g \frac{\partial \phi^{(1)}}{\partial Z} + g \eta^{(1)} \frac{\partial^{2} \phi^{(1)}}{\partial Z^2} + \frac{\partial}{\partial t} \left| \nabla \phi^{(1)} \right|^2 = 0
\]  
(2.11)

where the velocity vector $\vec{V}$ is replaced by $\nabla \phi^{(1)}$. From Eqs. (2.10) and (2.11) it then follows, Longuet-Higgins, 1963:

\[
\left( \frac{\partial^2}{\partial t^2} + g \frac{\partial}{\partial Z} \right) \phi^{(2)} = -\frac{\partial}{\partial t} \left| \nabla \phi^{(1)} \right|^2 - \eta^{(1)} \frac{\partial}{\partial Z} \left( \frac{\partial^2}{\partial t^2} + g \frac{\partial}{\partial Z} \right) \phi^{(1)}
\]  
(2.12)

\[
\eta^{(2)} = -\frac{1}{g} \left( \frac{\partial \phi^{(2)}}{\partial t} + \frac{1}{2} \left| \nabla \phi^{(1)} \right|^2 + \eta^{(1)} \frac{\partial^2 \phi^{(1)}}{\partial t \partial Z} \right)
\]  
(2.13)

By substituting into Eq. (2.12) for the first order velocity potential

\[
\phi^{(1)} = \sum_{i} \frac{a_{i} \omega_{i}}{k_{i}} e^{k_{i}Z} \sin \psi_{i} + \sum_{m} \frac{a_{m} \omega_{m}}{k_{m}} e^{k_{m}Z} \sin \psi_{m}
\]  
(2.14)

the solution for the bidirectional case becomes
\[ \phi^{(2)} = \frac{1}{2} \sum_{i,j=1}^{n} a_i a_j \max(-\omega_i, \omega_j) e^{jk \cdot \mathbf{r}} \sin(\psi_i - \psi_j) + \frac{1}{2} \sum_{m,p=1}^{n} a_m a_p \max(-\omega_m, \omega_p) e^{jk \cdot \mathbf{r}} \sin(\psi_m - \psi_p) \]

\[
+ \sum_{i,m=1}^{n} a_i a_m \omega_i \omega_m \left[ \frac{(1 + \cos(\beta_i - \beta_m))}{(\omega_i - \omega_m)} e^{2k_r} \sin(\psi_i - \psi_m) - \frac{g_k}{(\omega_i - \omega_m)} e^{2k_z} \sin(\psi_i + \psi_m) \right] \\
\]

\[
+ \frac{(\cos(\beta_i - \beta_m) - 1)}{(\omega_i + \omega_m)} e^{2k_z} \sin(\psi_i + \psi_m) \left[ \frac{g_k}{(\omega_i + \omega_m)} \right]
\]

(2.15)

where the cross-coupling wave numbers \( k_i \) are defined as

\[ k_i = \sqrt{k_i^2 + k_m^2 \pm 2k_i k_m \cos(\beta_i - \beta_m)} \]

and \( \beta_1, \beta_2 \) are the heading angles of the first and second wave systems relatively to the ship.

Furthermore,

\[ \max(-\omega_\alpha, \omega_\varepsilon) \equiv \begin{cases} -\omega_\alpha & \text{if } \omega_\alpha > \omega_\varepsilon \\ \omega_\varepsilon & \text{if } \omega_\alpha < \omega_\varepsilon \end{cases} \]

and

\[ \psi_\alpha = k_\alpha \cos \beta_\alpha x - \omega_\alpha t + \theta_\alpha \]

where the counters \( i, j \) denote the first wave system with the heading angle \( \beta_1 \) and the counters \( m, p \) stand for the second wave system approaching the ship at the angle \( \beta_2 \).

In all these expressions \( k_i \) is the wave number and \( \omega_j \) is the wave frequency \( \left( \omega_j^2 = gk_j \right) \), \( a_i \) is the wave amplitude and \( \theta_i \) is the phase angle.

The first order wave elevation associated with Eq. (2.14) is
\[ \eta^{(1)} = \sum_{i=1}^{n} a_i \cos \psi_i + \sum_{m=1}^{n} a_m \cos \psi_m \quad (2.16) \]

whereas the second order wave elevation follows from Eq. (2.13):

\[
\eta^{(2)} = \frac{1}{4} \sum_{i,j=1}^{n} a_j \left( k_i + k_j \right) \cos \left( \psi_i + \psi_j \right) - \left| k_i - k_j \right| \cos \left( \psi_i - \psi_j \right) \\
+ \frac{1}{4} \sum_{m,p=1}^{n} a_p \left( k_m + k_p \right) \cos \left( \psi_m + \psi_p \right) - \left| k_m - k_p \right| \cos \left( \psi_m - \psi_p \right) \\
+ \sum_{i,m=1}^{n} a_i a_m \left\{ \left( \sqrt{k_i} k_m \left( 1 + \cos \left( \beta_i - \beta_m \right) \right) \left( \frac{1}{1 - r^2} - \frac{1}{2} \right) + \frac{1}{2} (k_i + k_m) \right) \cos \left( \psi_i - \psi_m \right) \\
+ \left( \sqrt{k_i} k_m \cos \left( \beta_i - \beta_m \right) - 1 \right) \left( \frac{1}{1 - r^2} - \frac{1}{2} \right) + \frac{1}{2} (k_i + k_m) \right\} \cos \left( \psi_i + \psi_m \right) \right\} \\
\quad (2.17) \]

with

\[ r_s = \frac{k_i}{\left( \sqrt{k_i} \pm \sqrt{k_m} \right)^2} \quad (2.18) \]

**2.5 Second Order Pressure**

The dynamic pressure is derived from the Bernoulli equation:

\[
p^{(1)} = -\rho \frac{\partial \phi^{(1)}}{\partial t} \quad \text{and} \quad p^{(2)} = -\rho \left\{ \frac{\partial \phi^{(2)}}{\partial t} + \frac{1}{2} \left( \nabla \phi^{(1)} \right)^2 \right\}
\]

where \( \rho \) is the density of sea water. The first order pressure becomes
\[ p^{(1)} = \rho g \left( \sum_{i=1}^{n} a_i e^{i k_z z} \cos \psi_i + \sum_{m=1}^{n} a_m e^{i k_z z} \cos \psi_m \right) \]  

(2.19)

and the second order pressure takes the form

\[ p^{(2)} = -\frac{1}{2} \rho \sum_{i,j=1}^{n} a_i a_j \left\{ [\omega_i - \omega_j] \max\{\omega_i, \omega_j\} e^{i k_z z} \cos(\psi_i - \psi_j) \right. \]
\[ + \omega_i \omega_j e^{(k_z + k_x) z} \cos(\psi_i - \psi_j) \} \]
\[ - \frac{1}{2} \rho \sum_{m,p=1}^{n} a_m a_p \left\{ [\omega_m - \omega_p] \max\{\omega_m, \omega_p\} e^{i k_z z} \cos(\psi_m - \psi_p) \right. \]
\[ + \omega_m \omega_p e^{(k_z + k_x) z} \cos(\psi_m - \psi_p) \} \]
\[ - \rho \sum_{i,m=1}^{n} a_i a_m \omega_i \omega_m \left\{ - \frac{1 + \cos(\beta_1 - \beta_2)}{1 - r_i} e^{2k_z z} \cos(\psi_i - \psi_m) \right. \]
\[ - \frac{\cos(\beta_1 - \beta_2) - 1}{1 - r_i} e^{2k_z z} \cos(\psi_i + \psi_m) \]
\[ + \frac{1}{2} (1 + \cos(\beta_1 - \beta_2)) e^{(k_z + k_x) z} \cos(\psi_i - \psi_m) \]
\[ - \frac{1}{2} (1 - \cos(\beta_1 - \beta_2)) e^{(k_z + k_x) z} \cos(\psi_i - \psi_m) \} \}

(2.20)

Note that for wave components with opposite directions \((\cos(\beta_1 - \beta_2) = -1)\) and the same wave number \((k_i = k_m)\), the cross coupling wave number \(k_+ = 0\). For such combinations of wind and swell wave components, the sum frequency cross-coupling term in Eq. (2.20) does not decay with distance \(Z\) below the mean water level and may hence yield a significant contribution to the springing response (Vidic-Perunovic and Jensen, 2004). The corresponding cross-coupling frequency difference term is seen to disappear.
2.6 Wave Excitation Force

The sectional Froude-Krylov force due to the incident wave pressure $p$ is given as the sum of linear and quadratic parts, Jensen and Pedersen (1979):

$$F_R = -\int_{-T}^{T} B(z,x) \frac{\partial p}{\partial z} \left|_{z=w} \right. \, dz \approx F_R^{(1)} + F_R^{(2)}$$

(2.21)

where $B(z,x)$ is the sectional breadth variation of the submerged part of the section at $x=x$ and $T(x)$ is the sectional draught. Substitution of Eqs. (17) - (18) into Eq. (19) yields

$$F_R^{(1)} = -\rho g B_0 \left\{ w_1 - \sum_{i=1}^{n} a_i \kappa_i \cos \psi_i - \sum_{m=1}^{n} a_m \kappa_m \cos \psi_m \right\}$$

(2.22)

and
Here $w_1$, $w_2$ denote the first and second order deflection of the hull, equal to the sum of linear and second order responses, respectively, due to each of the two long-crested wave systems. Furthermore, $B_o(x) = B(0, x)$ is the sectional breadth at the still water line and $B_i(x) = \frac{dB(z,x)}{dz}$ at $z = 0$, see Eq. (2.2).

The Smith correction $\kappa$ is in general defined by (Jensen, 2001):

$$\kappa(k) = 1 - \frac{k}{B_o(x)} \int_0^x B(z,x) e^{ik} dz$$  \hspace{1cm} (2.24)
In the numerical calculations the sectional shapes have been modelled by Lewis forms and an approximate form for the corresponding Smith correction factor $\kappa$ is given and explained in Chapter 3. The maximum second order force will occur for two very close wave numbers $k_i$ and $k_m$.

### 2.7 Hydrodynamic Force

The sectional hydrodynamic force $F_{h}$ is approximated as follows, Jensen and Pedersen (1979):

$$F_{h}(x,t) = -\frac{D}{Dt}\left(m \frac{D\tilde{z}}{Dt}\right) - N \frac{D\tilde{w}}{Dt}$$

(2.25)

where the sectional added mass and damping are denoted by $m(x)$ and $N(x)$, respectively. By assuming a linear variation of the sectional added mass $m(x,t)$ and the hydrodynamic damping $N(x,t)$ as well as the water line breadth $B$ with immersion $\tilde{z}(x,t)$ (mean values $m_0$, $N_0$, slope $m_1$ and $N_1$, see Eq. (2.2)), the hydrodynamic force can be written as a sum of a linear and a second order term. The first order hydrodynamic force becomes

$$F_{h}^{(1)} = -m_0 \frac{D^2\tilde{w}_1}{Dt^2} - n_0 \frac{D\tilde{w}_1}{Dt} - \sum_{i=1}^{n} a_i \kappa_i \omega_i \left\{ m_0 \omega_i \cos \psi_i - n_0 \sin \psi_i \right\}$$

$$- \sum_{m=1}^{n} a_m \kappa_m \omega_m \left\{ m_0 \omega_m \cos \psi_m - n_0 \sin \psi_m \right\}$$

(2.26)

where $n = N - U \frac{\partial m}{\partial x}$ and $U$ is the forward speed of the ship. The second order hydrodynamic force takes the form:
The second order hydrodynamic force given by Eq. (2.27) will have a smaller contribution to the high-frequency response than the second order Froude-Krylov force given by Eq. (2.23) since it does not include the exponential pressure term.
included in the Smith correction factor but depends only on hydrodynamic coefficients, added mass and damping.

In case of unidirectional waves the second wave system and the cross interaction terms should simply be omitted from the expressions presented in this chapter.

### 2.8 Ship Structural Response

The lower modes of vibration can be accurately determined by modelling the hull as a single beam. The quadratic theory takes into account the flexibility of the ship by modelling the hull as a free-free Timoshenko beam and modal analysis is used to calculate the hull deflections. The vertical symmetric responses are considered, heave, pitch, vertical bending moment and shear force. In Fig. 2.1 the measured two-node springing vibration mode for the bulk carrier considered in Storhaug et al., 2003, is given.

![Fig. 2.2: Two-node vibration of the hull](image)

### 2.8.1 Constitutive Equations for the Flexible Beam

By modification of the equations to account for the visco-elastic stress-strain relationship suggested by Kumai, 1958, the constitutive equations for the Timoshenko beam become
where \( w(x,t) \) and \( \phi(x,t) \) are the total deflection and angular displacement (the slope), respectively, and \( E \) is the Young's modulus of elasticity, \( G \) the shear modulus, \( I \) is the cross-sectional inertia, \( A \) is the sectional area and \( \mu \) is the constant dependent on the cross-sectional shape due to non-uniform distribution of shear stresses over the sectional area geometry. The visco-elastic properties are introduced throughout the material constants \( \eta_m \) and \( \eta_s \).

The equilibrium of moments yields

\[
\frac{\partial M}{\partial x} - m_s r^2 \frac{\partial^3 \varphi}{\partial t^2} = -F 
\]

and the equilibrium of forces gives:

\[
\frac{\partial F}{\partial x} = m_s \frac{\partial^2 w}{\partial t^2} - F 
\]

where \( m_s(x) \) is the ship mass per unit length, \( m_{sr}r^2(x) \) the mass moment of inertia round the \( y \) axes and \( F(x,t) \) is the applied external force per unit length, as described in Section 2.2.

The following equations of motion in the vertical plane of a non-prismatic Timoshenko beam are obtained by inserting Eqs.(2.28)-(2.29) into Eqs.(2.30)-(2.31):

\[
\frac{\partial}{\partial x} \left[ EI \left( 1 + \eta_m \frac{\partial}{\partial t} \right) \frac{\partial \varphi}{\partial x} \right] + \mu GA \left( 1 + \eta_s \frac{\partial}{\partial t} \right) \left( \frac{\partial w}{\partial x} - \varphi \right) - m_s \frac{\partial^2 \varphi}{\partial t^2} = 0
\]

and

\[
\frac{\partial}{\partial x} \left[ \mu GA \left( 1 + \eta_s \frac{\partial}{\partial t} \right) \left( \frac{\partial w}{\partial x} - \varphi \right) \right] - m_s \frac{\partial^2 w}{\partial t^2} = -F(x,t)
\]
For simplicity it is normally assumed that the damping parameters \( \eta_M, \eta_S \) are equal and independent of the cross section of the beam, \( \eta_M = \eta_S = \eta \). This is a reasonable assumption since only the overall damping in the ship structure can easily be measured but not the damping per section.

The solution to Eqs. (2.32) and (2.33) is found by modal superposition. Any function satisfying the boundary conditions can be written as an infinite linear combination of eigenvectors of this problem. Thus, the angular displacement and the total deflection of the beam may be written as a linear series in the form:

\[
\varphi(x,t) = \sum_{i=0}^{N} u_i(t) \alpha_i(x) \quad (2.34)
\]

\[
w(x,t) = \sum_{i=0}^{N} u_i(t) v_i(x) \quad (2.35)
\]

where the time-dependent coefficients \( u_i(t) \) are to be determined and the eigenvectors \( \alpha_i(x), v_i(x) \) are non-zero solutions to the free, undamped vibration case.

The eigenvectors are found from the eigenvalue problem:

\[
\frac{\partial}{\partial x} \left( EI \frac{\partial \alpha_i}{\partial x} \right) + \mu GA \left( \frac{\partial v_i}{\partial x} - \alpha_i \right) = \bar{\omega}_i^2 m_i r^2 \alpha_i \quad (2.38)
\]

\[
\frac{\partial}{\partial x} \left[ \mu GA \left( \frac{\partial v_i}{\partial x} - \alpha_i \right) \right] = \bar{\omega}_i^2 m_i v_i \quad (2.39)
\]

where \( \alpha_i(x) \) and \( v_i(x) \) correspond to a certain natural frequency \( \bar{\omega}_i \) and where \( \{v_0, \alpha_0\}, \{v_1, \alpha_1\}, \{v_2, \alpha_2\} \) represent heave, pitch and the two-node vibration mode, respectively, etc. The boundary conditions are:

\[
\left. \left( EI \frac{\partial \alpha}{\partial x} \right) \right|_{x=0,L} = 0
\]

and

\[
\left. \left[ \mu GA \left( \frac{\partial v}{\partial x} - \alpha \right) \right] \right|_{x=0,L} = 0
\]

The eigenfunctions \( (v_i, \alpha_i) \) and \( (v_j, \alpha_j) \) are orthogonalised and normalised so that
\[
\int_0^L (m_x r^2 \alpha_x \alpha_j + m_y v_y v_j) dx = \delta_{ij} \tag{2.40}
\]

and

\[
\int_0^L [\alpha_x (EI' \alpha') + \alpha_x \mu GA (v_j' - \alpha_j) + v_j (\mu GA (v_j' - \alpha_j))'] dx = \delta_{ij} \tag{2.41}
\]

where \( \delta_{ij} \) is the Kronecker symbol:

\[
\delta_{ij} = \begin{cases} 
1, & i = j \\
0, & i \neq j 
\end{cases}
\]

and where \((\cdot)' \equiv \frac{d}{dx}\).

From Eqs. (2.32) – (2.34) and Eqs. (2.40) – (2.41) the following equations of motion are obtained:

\[
\frac{\partial^2 u_j}{\partial t^2} + \eta \omega_j^2 \frac{\partial u_j}{\partial t} + \omega_j^2 u_j = \int_0^L v_j F(x, t, \sum_{i=0}^N u_i, v_i) dx \quad \text{where} \quad j = 0, 1, 2, \ldots, N \tag{2.42}
\]

where \( \omega_j \) is the eigenvalue corresponding to the \( j \)th eigenfunction.

The solutions to Eq. (2.42) are found (similarly as for the exciting forces) as a sum of the first and second order solutions:

\[
u_j(t) = u_j^{(1)}(t) + u_j^{(2)}(t) \tag{2.43}
\]

First and second order deflections are determined from Eq. (2.42), replacing \( F(x, t) \) with the expressions for Froude-Krylov and hydrodynamic force. Thus, the vertical displacement \( w(x, t) \), the rotation angle \( \phi(x, t) \) and hence the wave-induced bending moment \( M(x, t) = M^{(1)}(x, t) + M^{(2)}(x, t) \) may be expressed as functions of the incoming wave elevation. Explicit vertical bending moment coefficients are given in Jensen and Pedersen, 1979.
2.9 Structural Damping

2.9.1 Damping Mechanisms

Determination of the damping in complex structures has always presented a problem because of the large variety of different physical mechanisms that contribute to damping. It is also very difficult to model damping due to lack of knowledge regarding most of the damping sources.

The hull oscillatory energy is dissipated through several processes, the most important of which may be described by:

- Wave radiation away from the ship (as the result of ship motion) - the hydrodynamic damping
- Material hysteresis, especially in weldings
- Dry friction within cargo – Coulomb damping

The relative magnitudes of these mechanisms change with frequency.

Structural damping becomes the governing part of the total damping in the high-frequency region typical for springing response. The hydrodynamic damping converges to zero - the motions of the ship are small and there is very little wave radiation for frequencies around the two-node natural hull vibration frequency.

During the ship's operation, the total damping is increased by some hydrodynamic effects e.g. by vibration energy loss due to flow separation and generation of vortices at the submerged transom stern sections, vorticity around the bilge keel due to vertical motion etc.

According to Söding, 1975, the following kinds of damping are considered to be especially influential:

“Damping forces from an assumed flow separation at the bilge keels during vertical movements of the ship sections. These forces which, presumably, are similar in nature to the roll damping moments caused by the bilge keels may contribute substantially to the vibration damping.”

“Flow separation at transom stern, if the transom stern is submerged, at least for a certain percentage of time, by the action of waves. In many cases, the vorticies generated by the oscillating transom may dissipate the major part of the vibration energy.”
In Fig. 2.3 the classification of damping is given, which enter the equations of motion of the hull.

![Damping division diagram]

**2.9.2 Internal Damping**

The structural damping will mainly be due to material hysteresis in weldings of the steel plates, as they experience stress cycling. The magnitudes are typically in the wide range of 0.04 to 0.6 per cent of the critical damping of the structure, according to e.g. Betts, Bishop and Price, 1976. Further information on the damping coefficient for steel plates may be found e.g. in Stevenson, 1980, who compiled many measurement values and recommendations depending on the stress level, or So et al., 1990, who described the technique of measurement of material damping and gave results for different materials (steel, aluminium and GRP).

Dry friction within the cargo would increase the overall damping for ships in the full load condition. According to different measurements, cargo damping is supposed to be as important as structural damping in ships (Jensen and Madsen, 1977; Betts, Bishop and Price, 1976).

Coloumb damping (or dry friction) in the hull plates was important with riveted structures (Johnson, 1950-51), and is hence not relevant today.
2.9 Structural Damping

Some other effects may be influential like welding residual stresses as discussed in Betts, Bishop and Price, 1976.

### 2.9.3 Empirical Recommendations

The discrepancies between different predictions and measurements given in the literature are relatively large, and it is difficult to decide which value to use. The determination of the structural damping parameter is one of the uncertainties connected with ship response calculations.

Different recommendations for the structural damping coefficients are available from the literature depending on the ship type, vibration mode, length of the ship and loading condition, the way connections are made etc. Some of the expressions are listed below:

- Kumai, (1958) \[ \delta_i = \frac{3.5}{L} \]
- Tomita, (1964) \[ \delta_i = 2.4 \cdot 10^{-3} \omega_i \]
- Hirowatari (1963) \[ \delta_i = 1.065 \cdot 10^{-2} \omega_i^{1/2} \]
- Johnson, Ayling and Couchman, (1962) \[ \delta_i = C_i \omega_i, \quad C_i = 2 \cdot 10^{-3} \]
- Aertssen and de Lembre, (1971) \[ \delta_i = 7.3 \cdot 10^{-3} \omega_i \]
- Johnson and Ayling, (1962) \[ \frac{\alpha_{di}}{\alpha_{d0}} = \left( \frac{\omega_i}{\omega_0} \right)^{1.75} \]
- Van Gunsteren, (1973) \[ \alpha_{di} = \frac{\omega_i}{200} \] (2.46)

The damping coefficient \( \alpha_d \) relates to the structural damping ratio as \( \xi = \frac{\alpha_d}{\omega_i} \).

By use of the expressions Eq. (2.46) to estimate hull damping for the specific ship, the magnitude of the damping coefficients varies widely and does not explicitly take into account the actual ship design and loading condition.

Usually, the structural (internal) damping is determined by full-scale measurements for the specific ship. The logarithmic decrement \( \delta \), defined as the natural logarithm of the ratio between two successive maxima in the measured response, can be obtained by giving the structure an impulse load (e.g. impact by hammer or slamming) and recording the decrease of the vibration. The difficulty here is how to
filter out the damping dominating the two-node vibration only, since the structural response contains contributions from different vibration modes. Another procedure is outlined below.

### 2.9.4 Determination of the Hull Damping from the Ship Response Record

The structural damping parameter may be determined from an analysis of the field measurements of ship vibration. The measured quantity may be the stress on midship from which the dynamics of the ship can be identified. Linear approach is considered in the following (Rüdinger et al., 2001 and Rüdinger, 2004).

**Linear analysis.** If the system is assumed to be linear and the excitation is a Gaussian process, exact analytical procedures can be used to determine the probabilistic characteristics of the response. These procedures will be used here to determine the dynamics of the two-node vibration.

For a linear system the equation of motion governing the two-node vibration mode can be written as follows, see Eq. (2.42):

\[
\frac{d^2\sigma}{dt^2} + 2\xi\omega_s \frac{d\sigma}{dt} + \omega_s^2 \sigma = f(t)
\]

(2.47)

where \( \sigma(t) \) is the stress, \( \omega_s \) is the springing frequency, \( \xi \) is the damping ratio and \( f(t) \) is the excitation generated by waves. The equation of motion is normalised with respect to the effective mass corresponding to this mode. The modal deformation is proportional to the stress for a linear model and can therefore be measured in terms of the stress \( \sigma(t) \). The spectrum \( S_\sigma(\omega) \) is defined as:

\[
S_\sigma(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_\sigma(\tau) e^{-i\omega \tau} d\tau
\]

\[
R_\sigma(\tau) = E[\sigma(t)\sigma(t+\tau)]
\]

(2.48)

where \( R_\sigma(\tau) \) is the stationary covariance function and \( E[\ ] \) is the mean value operator. The spectrum of the system Eq. (2.47) is then given by

\[
S_\sigma(\omega) = \frac{1}{(\omega_s^2 - \omega^2)^2 + 4\xi^2 \omega_s^2 \omega^2} S_f(\omega)
\]

(2.49)
where $S_f(\omega)$ is the spectrum of the Gaussian excitation process $f(t)$. Only the resonant part of the response is considered. In this range the spectral density of the excitation function is assumed to decay as a power function, i.e. as

$$S_f(\omega) \approx \frac{A}{\omega^n} \text{ for say, } 2.5 < \omega < 4.5 \text{ rad/s}$$  \hspace{1cm} (2.50)

where $n$ is the power and $A$ is the intensity factor. In this range the spectrum is given by Eq. (2.49) and Eq. (2.50) and depends on the four parameters $\omega_s$, $\xi$, $A$ and $n$. These parameters are estimated by fitting the expression to estimates of the spectrum obtained from the records.

In Fig. 2.4 damping coefficients are presented, which are determined by use of the stress records from a bulk carrier.

![Fig. 2.4: Estimated damping coefficient according to the linear analysis. The cases are defined later in Chapter 5.](image-url)
2.10 Short-Term Statistics

The stochastic process can be divided into periods in which the wave signal could be considered as stationary i.e. the statistical properties are independent of the absolute value of time. Such a stationary period usually lasts only for few hours (usually up to three hours) and it is denoted to be 'short-term'. The sea is here assumed to be long-crested (all wave components are travelling in the same direction, parallel to each other).

In addition to the linear response second order theory includes the second order response operators, i.e. the subharmonic and superharmonic terms related to the frequency difference and sum frequency, respectively. The response of the wave–induced vertical bending moment can be written in compressed form:

\[
\frac{M(x,0)}{\lambda} = \sum_{k=1}^{4n} \hat{\Lambda}_{k} x_k + \hat{\epsilon} \sum_{k=1}^{4n} \sum_{l=1}^{4n} \Lambda_{kl} x_k x_l
\]

(2.51)

where the dimension of the first and second order coefficients is increased from \(2n\) (originally considered in Jensen and Pedersen, 1979) to \(4n\) used to account for the two wave spectra. The independent stochastic variables are defined as

\[
\chi_k = \frac{\vartheta_k}{\sqrt{V_k}}
\]

(2.52)

The wave amplitude and the phase lag of each wave component are chosen so that \(\vartheta_k = a_k \cos \theta_k\) and \(\vartheta_{k+n} = a_k \sin \theta_k\) are jointly normally distributed, with the mean value equal to zero, and with the variance of the wave spectral density given by

\[
V_k = V_{k+n} = S(\omega_k) = \frac{1}{2} a_k^2.
\]

The bending moment \(M(x,0)\) at the time \(t = 0\) is given as the sum of linear and second order contribution:

\[
M(x,0) = \sum_{r=1}^{n} \left( \vartheta_r M_r^c(x) - \vartheta_{r+n} M_r^s(x) \right) + \sum_{r,t=1}^{n} \left\{ \left( M_r^{c+} (x) + M_r^{c-} (x) \right) \varrho_{r,t} \varrho_{r+n,t} - \left( M_r^{s+} (x) - M_r^{s-} (x) \right) \varrho_{r,s} \varrho_{r+n,s} \right\}
\]

(2.53)

where the bending moment coefficients \(M_{r}^{\alpha}\) are calculated exactly from the expressions given in Jensen and Pedersen, 1979, but by use of the second order total
hydrodynamic force as derived in the present chapter, accounting for the influence of the additional wave system and the cross interaction terms.

\( \lambda \) is the standard deviation of the first order contribution to the wave-induced bending moment given by

\[
\lambda = \left( \sum_{r=1}^{n} \left( M_r^c \right)^2 + \left( M_r^s \right)^2 \right)^{1/2} \tag{2.54}
\]

The parameter \( \varepsilon \) is a small non-linearity parameter determined from the following condition:

\[
\sum_{k=1}^{4n} \sum_{l=1}^{4n} \Lambda_{kl} \Lambda_{kl} = (\varepsilon \lambda)^2 \tag{2.55}
\]

The first order coefficients \( \lambda_k \) and the second order coefficients \( \Lambda_{kl} \) now read

\[
\lambda_k = \begin{bmatrix} \lambda_{k1} \\ \lambda_{k2} \end{bmatrix} \tag{2.56}
\]

and

\[
\Lambda_{kl} = \begin{bmatrix} \Lambda_{l/11} & \Lambda_{l/12} \\ \Lambda_{l/21} & \Lambda_{l/22} \end{bmatrix} \tag{2.57}
\]

where the non-diagonal terms represent the cross interaction terms due to second order wave-wave interaction.

The linear standard deviation for each encounter frequency and referring to each wave system is

\[
\lambda_{k_i} = \begin{cases} \left( M_k^c \right)^2 V_{ki} / \lambda, & k \leq n \\ -\left( M_{k-n}^s \right)^2 V_{(k-n)i} / \lambda, & k > n \end{cases} \tag{2.58}
\]

The second order coefficients become
\[ \Lambda_{ij} = \left\{ \begin{array}{ll} 
\left( M_{{kl}^{(s)}} \right)_{ij} + \left( M_{{kl}^{(c)}} \right)_{ij} \sqrt{V_{kVVMM}^l}, & k,l \leq n \\
\left( -M_{k,j-n}^{(s)} \right)_{ij} + \left( M_{k,j-n}^{(c)} \right)_{ij} \sqrt{V_{nVVMM}^l}, & k \leq n, l > n \\
-\left( M_{k-n,l}^{(s)} \right)_{ij} + \left( M_{k-n,l}^{(c)} \right)_{ij} \sqrt{V_{(k-n)VMM}^l}, & k > n, l \leq n \\
-\left( M_{k-n,l-n}^{(s)} \right)_{ij} + \left( M_{k-n,l-n}^{(c)} \right)_{ij} \sqrt{V_{(k-n)VMM}^l}, & k, l > n 
\end{array} \right. \]

(2.59)

where the counters assume the values \( i,j = 1,2 \). When \( i = j \), coefficients are presented by diagonal terms, and when \( i \neq j \) by non-diagonal terms in the matrix Eq. (2.57).

When the focus is only on the modification due to the additional wave spectrum the first order spectral density function is unchanged, except for inclusion of the second wave system, and it is given in the usual form:

\[ S_{M_{ij}^{(0)}}(\omega_r) = \left[ \left( M_r^{(s)} \right)^2 + \left( M_r^{(c)} \right)^2 \right] S_r^{(e)}(\omega_r) \]

(2.60)

The second order spectral density function, focusing e.g. on super-harmonic terms, is given as

\[ S_{M_{ij}^{(2)}}(\omega_r) = \sum_{t=r-t}^{\min(n,r-1)} 4 \left[ \left( M_{st}^{(s)} \right)^2 + \left( M_{st}^{(c)} \right)^2 \right] S_t^{(e)}(\omega_t) S_r^{(e)}(\omega_r) \Delta \omega \]

(2.61)

The subharmonic terms in the response spectrum are determined as

\[ S_{M_{ij}^{(-2)}}(\omega_r) = \sum_{t=r+t}^{r-t} 4 f_r \left[ \left( M_{st}^{(s)} \right)^2 + \left( M_{st}^{(c)} \right)^2 \right] S_t^{(e)}(\omega_t) S_r^{(e)}(\omega_r) \Delta \omega \]

(2.62)

where \( f_r = 1 \) if \( r = 0 \) and \( f_r = 2 \) if \( r \neq 0 \), \( \omega_t = t \Delta \omega \), and where counters assume values \( i,j = 1,2 \). In the case where \( i \neq j \) the influence of both wave spectra is accounted for by the cross coupling terms, so that e.g. \( S_1^{(e)}(\omega_r) \) represents the first and \( S_2^{(e)}(\omega_r) \) the second unidirectional wave spectrum. A detailed description of the first
and second order coefficients $\lambda_j$ and $\Lambda_{jk}$ and the non-linearity parameter $\hat{\varepsilon}$ in Eq. (2.55) is given in Jensen and Pedersen, 1979, and the spectral density functions for unidirectional analysis are given in Jensen and Pedersen, 1981.

The variance of the total response $X$ as the sum of the linear and second order parts is given as

$$E[X^2] = E[(X^{(1)})^2] + 2E[X^{(1)}]E[X^{(2)}]$$

(2.63)

neglecting fourth order non-linear terms.

From Eq. (2.60) it follows that the total standard deviation $s$ of the response becomes

$$s^2 = \left(\frac{\hat{\varepsilon}}{s^{(1)}}\right)^2 + \left(\frac{s^{(2)}}{\hat{\varepsilon}}\right)^2 = \hat{\lambda}^2 \left(1 + 2\hat{\varepsilon}^2\right)$$

(2.64)

where the variance of the second order part further equals

$$2\hat{\varepsilon}^2 \hat{\lambda}^2 = \int_0^\infty \left(S^{+}_{M^{(2)}_{ij}}(\omega) + S^{-}_{M^{(2)}_{ij}}(\omega)\right) d\omega$$

(2.65)

The deep-water wave spectrum used here can be either the Pierson-Moskowitz, the JONSWAP wave spectrum, or in general any numerical wave spectrum.

**Pierson-Moskowitz** – A unidirectional spectrum, applicable to fully developed conditions and dependent only on the characteristic wind speed. Pierson and Moskowitz, 1964, examined wave spectra obtained from North Atlantic weather ships. They proposed a limiting form based on the one-dimensional spectrum. It is given as

$$S(\omega) = \frac{\alpha_s g^2}{(2\pi)^4} \frac{1}{\omega^5} \exp\left(-\frac{\hat{B}}{\omega^4}\right)$$

(2.66)

where $\alpha_s = 0.0081$ denotes the Phillips constant and $\hat{B} = 0.74\left(g / 2\pi u_\ast\right)^4$, $u_\ast$ being the wind speed. The form of the spectrum implemented here is also denoted the ISSC spectrum and it is given in terms of two statistical parameters $H_s$ and $T_z$ as

$$S(\omega) = 4\pi^3 H_s^2 T_z^{-4} \omega^{-5} \exp\left(-16\pi^3 (\omega T_z)^{-4}\right)$$

(2.67)

* First and second order coefficients are multiplied by $\hat{\varepsilon}\hat{\lambda}$ in the present analysis.
JONSWAP - The Joint North Sea Wave Project (Hasselmann et al., 1973) was an extensive wave measurement programme, an international attempt to investigate fetch limited wave evolution. It constitutes a modification to the Pierson-Moskowitz spectrum to account for the effect of fetch restrictions and provides for a much more sharply peaked spectrum. It is given as

\[
S(\omega) = \frac{\alpha_* g^2}{(2\pi)^4 \omega^8} \exp\left(-\frac{5}{4} \left(\frac{\omega}{\omega_0}\right)^4\right) \tilde{\gamma}^{\alpha_0} 
\]

(2.68)

where

\[
a_* = \exp\left(-\frac{(\omega - \omega_0)^2}{2\sigma_*^2 \omega_0^2}\right)
\]

\[
\sigma_* = \begin{cases} 
0.07, & \omega \leq \omega_0 \\
0.09, & \omega > \omega_0
\end{cases}
\]

\[\omega_0\] is the peak frequency and \(\tilde{\gamma}\) is the ratio of the maximum spectral density to that of the corresponding Pierson-Moskowitz spectrum. This was found to have a mean value of about \(\tilde{\gamma} = 3.3\), corresponding to a much more sharply peaked JONSWAP spectrum than the Pierson-Moskowitz prediction.
Chapter 3

The Smith Correction Factor

3.1 General definition

The Froude-Krylov force is evaluated by dynamic pressure integration over the surface of the hull in the same way as it was done to obtain the hydrostatic buoyancy force.

The vertical Froude-Krylov excitation force is directly proportional to the Smith correction factor. The Smith correction factor accounts for the exponential decay of the dynamic pressure under the wave, with respect to increasing distance from the still water level to the bottom of the hull, due to orbital motion of the water particles under the wave.

The Smith correction factor depends on the wave frequency and shorter waves, of course, experiencing more modification. Thus, accurate expressions are needed for the Smith correction at frequencies relevant to springing.
3.2 The Smith Correction Factor for the Lewis Sectional Form

In the numerical calculations the sectional shapes have been modelled by use of Lewis forms. It is not strictly necessary but will facilitate the input to the calculation procedure.

A transformation of the unit circle to a more ship-like cross section was proposed by Lewis, 1929:

\[ y = d \left( (1 + a_i) \cos \theta + a_3 \cos 3\theta \right) \] (3.1)

\[ z = -d \left( (1 - a_i) \sin \theta - a_3 \sin 3\theta \right) \] (3.2)

The coefficients \( a_i \) and \( a_3 \) are expressed by the draught to beam ratio \( \beta = \frac{2T}{B} \) and the sectional area coefficient \( \alpha = \frac{A}{BT} \), where \( B, T \) and \( A \) are the waterline breadth, the draught and the hull sectional submerged area, respectively.

From \( \bar{\beta} = \frac{1 - a_i + a_3}{1 + a_i + a_3} \) and \( A = \int_0^{\pi/2} y(\theta) \frac{dz}{d\theta} d\theta \), the unknown coefficients may be expressed as

\[ a_i = C(1 - \bar{\beta}) \quad ; \quad a_3 = C(1 + \bar{\beta}) - 1 \] (3.3)

where

\[ C = \left[ \frac{3}{2} (1 + \bar{\beta}) - \frac{1}{4} \left( 1 + \bar{\beta} \right)^2 + 2 \bar{\beta} \left( 1 - \frac{4}{\pi} \alpha \right) \right]^{-1} \] (3.4)

and the scale parameter

\[ d = \frac{B}{4C} \] (3.5)

From Eq. (3.1) the following relations are obtained:

\[ B(x, z) = d \left( (1 + a_i) \cos \theta + a_3 \cos 3\theta \right) \] (3.6)

\[ B_0(x) = d \left( 1 + a_i + a_3 \right) \] (3.7)
3.2 The Smith Correction Factor for the Lewis Sectional Form

Hence, Eq. (2.24) yields (the definition of the Smith factor):

\[
\kappa(k) = 1 - kd \int_0^{\pi/2} e^{-kd(1 - \alpha_1 \sin \theta - \alpha_2 \sin 3\theta)} \left(1 + \alpha_3 \cos \theta + \alpha_3 \cos 3\theta\right) \left(1 + \alpha_1 + \alpha_3\right)
\]
\[
\times \left[\left(1 - \alpha_1 \cos \theta - 3\alpha_3 \cos 3\theta\right) d\theta\right]
\]
(3.8)

Results obtained by numerical integration are presented in Figs. 3.1 - 3.5, for different sectional area coefficients \( \alpha \) and draught to beam ratios \( \beta \).

In order to approximate the Smith correction factor by a simple function the solution is assumed in the form

\[
\kappa(k) = \exp\left[-akT + (a\alpha + b\alpha^2 + c\alpha^3)(kT)^2\right]
\]
(3.9)

Coefficients \(a, b, c\) are found using the least square method to minimize the difference between results of the numerical integration and the new approximation. This yields the following approximate Smith correction factor form:

\[
\kappa(k) = \exp\left[-akT + (0.02\alpha + 0.1\alpha^2 - 0.1\alpha^3)(kT)^2\right]
\]
(3.10)

This expression yields a very close approximation to Eq. (2.24), also in the wave number range important to springing excitation. In the wave number range typical of the fundamental response (rigid body response) Eq. (2.24) and Eq. (3.10) would give almost the same results.

The results of the comparison between the exact solution (evaluated by numerical integration) and the approximate form are given in Figs. 3.1 - 3.5, and the Smith correction factors for different analytical sectional forms are given in Appendix 1. More details are found in Vidic-Perunovic and Jensen, 2002.

The results are given for different breadth to draught ratios \( \beta \) and sectional area coefficients \( \alpha \) and are also compared to the Smith correction factor for the box (rectangular) section: \( \kappa = e^{-kT} \). The agreement between the exact and the approximate solutions is shown to be satisfactory. For low \( kT \) values \( (kT \leq 1) \) there is almost no difference in the results but for higher wave numbers large deviations are seen. The results for the Smith correction factor \( \kappa \) are dependent on the sectional area coefficient \( \alpha \) but very little dependent on the breadth to draught coefficient \( \beta \) (as seen in Figs. 3.1 - 3.5), therefore \( \beta \) is not included in the approximation given by Eq. (3.9).
Figs. 3.1-3.3: Approximative solution for the Smith correction compared to the exact solution (given by three $\beta$ coefficients) and to the box section.
3.2 The Smith Correction Factor for the Lewis Sectional Form

Figs. 3.4-3.5: Approximative solution for the Smith correction compared to the exact solution (given by three $\beta$ coefficients) and to the box section.
Fig. 3.6: The Smith correction factor $\kappa$ approximated for the Lewis sectional form presented for different sectional area coefficients $\alpha$.

The following notation is used in Eq. (2.23) in Chapter 2:

$$\kappa_j = \kappa(k_j); \kappa_{|i\pm|} = \kappa(|k_i \pm k_m|); \kappa_s = \kappa(k_s)$$

(3.11)

Results obtained by use of the approximate expression are presented in Fig. 3.6 for different sectional area coefficients as a function of the non-dimensional wave number $kT$. The Smith correction factor increases when the sectional form is taken into account and the lowest prediction is obtained for the rectangular form, i.e. when $\alpha = 1$. The vertical component of the normal excitation force is larger for the more pronounced slope on the side. Naturally, $\kappa$ has larger values for the sections with finer form. This means that by neglecting the sectional form and assuming the box-like section, large errors may occur in the Smith correction factor, especially at the higher frequencies.

For forms with a small sectional area and shallow draught at the forward sections, the $kT$ values might be less than unity even at springing frequencies and in this range the Smith correction factor for the rectangular section and Lewis form almost coincide. On the contrary, for forms with deeper draught at the front bow part, springing is expected to appear at higher $kT$ numbers ($kT$ around 10).
A comparison of the Smith correction factor for a number of sectional forms that have the same sectional area coefficient is given in Fig. 3.7. The result for the trapezoidal section is the largest, whereas the prediction for the rectangular (box) section gives the lowest value of the Smith correction. The vertical component of the wave excitation force increases with the side slope of the section. The Lewis sectional form intersects the waterline perpendicularly and the predictions for $\kappa$ are therefore lower than for the parabolic and trapezoidal sections. In the case of the rectangular shape (box), only the horizontal force component is present on the side.

Fig. 3.7: *The Smith correction factor for sectional forms with the same sectional area coefficient $\alpha = 0.7$.*

The expressions for the Smith correction factor for different sectional forms are derived in Appendix A.

In the previous expressions deep water is assumed. Finite water depth is not included in the present analysis. However, the general expression for the Smith correction factor accounting for the water depth $d$ reads

$$\kappa = \kappa(x,k,d,T) = 1 - \frac{k}{B_0(x)} \int_{-d}^{0} \sinh(k(z+d)) \frac{B(z,x)}{\cosh(kd)} dz$$  \hspace{1cm} (3.12)

More details are given in Vidic-Perunovic and Jensen, 2003.
Chapter 4

The High-Frequency Excitation Spectrum

4.1 Introduction

Stochastic ocean waves are considered to be approximately Gaussian distributed stationary processes in short-term periods ranging from one to several hours. Such processes are completely described by the spectral density function $S(\omega)$.

Accurate description of the high-frequency tail of the excitation wave spectrum is needed for prediction of wave loads on wave-affected structures, especially for high-frequency wave loads such as springing.

An increasing number of references support the idea of more energy content in the high-frequency part of the spectrum than predicted by conventional JONSWAP or Pierson-Moskowitz spectra. The finding is based on observations, theoretical considerations and non-linear wave realisation.

In this chapter the emphasis is placed on understanding the physics and form of the wave excitation spectrum in the tail after the spectral peak. A method is described for extrapolating the wave spectrum as input to the non-linear hydrodynamic codes for dynamic load calculation, when full information about the tail is missing or e.g. the accuracy of the measured data in the high-frequency tail is poor.

Finally, a more general analytical form of the wave spectrum is presented that accounts for different rates of decay in different frequency ranges. The new form is then compared to the conventional Pierson-Moskowitz spectrum and the Gamma
spectrum with specified frequency power decay \( n = -4 \), taking care of energy conservation.

### 4.2 The Form of the Spectrum

#### 4.2.1 Theoretical Background

Analytical efforts have been made in order to understand the evolution of the wind wave spectrum and the energy distribution inside it.

The 'equilibrium law' was used for the first time in the classical paper by Phillips, 1958, determined by the physical parameters governing the free surface continuity. He proposed the physical model where the wind input energy was balanced by the energy dissipation through wave breaking. This imposed the upper frequency limit of applicability, independent of the wind speed. By use of the dimensional analysis of the equilibrium range model, the surface elevation spectral density function over a certain range of wave frequencies was derived as

\[
S(\omega) = \alpha g^2 \omega^{-5} \tag{4.1}
\]

where \( g \) is the acceleration of gravity and \( \alpha \) is the equilibrium range (Phillips) constant.

Pierson and Moskowitz, 1964, analysed a large amount of wave data measured from two North Atlantic weather ships the *Weather Explorer* and the *Weather Reporter* and proposed a form of the spectrum in agreement with the original Phillips \( \omega^{-5} \) form, fitted around the peak of the spectrum so that

\[
S(\omega) = \left( \frac{\alpha g^2}{\omega_b^5} \right) e^{-\beta \left( \frac{\omega_b}{\omega} \right)^4} \tag{4.2}
\]

where \( \omega_b = g/u_* \), \( u_* \) being the wind speed reported from the weather ships, the Phillips constant \( \alpha_* = 0.0081 \) and \( \beta_* = 0.74 \) determined from measurements data fit. The spectrum is defined for frequencies from zero to infinity and the area below the spectrum is equal to the variance of the wave record. A more convenient formulation is in terms of the parameters \((H_s, T_1)\) related to the spectral moments. Thus, two parameters rather than one parameter \((u_*)\) are used to characterise the sea state.

The introduced equilibrium range was subject to different physical speculations.
Longuet-Higgins, 1969, estimated the actual amount of energy loss through wave breaking and used it to determine the constant $\alpha$.

Kitaigorodskii, 1983, and Phillips, 1985, extended the equilibrium range theory to account for the wind strength in the higher-frequency range and they obtained the high-frequency form of the spectrum

$$S(\omega) = \beta \cdot g^{-0.5} \omega^{-4}$$  \hspace{1cm} (4.3)

Both authors considered the range of the wave numbers where the wind input energy, the energy transferred by the non-linear wave-wave interactions and the amount of energy dissipated by different processes (through breaking, water viscosity etc.) are balanced, each author arguing for different importance levels and locations of the energy components for the short gravity wave region.

The deep-water wave number equilibrium range spectrum, Phillips, 1985, is given as

$$\Psi(k, \phi) = C f(\phi) u_* g^{-1/2} k^{-7/2}$$  \hspace{1cm} (4.4)

where the wave number vector is described by use of the polar coordinates $k$ and $\phi$, $C$ is a constant and $u_*$ is the wind speed at the elevation of 10m. The same form has been found by Kitaigorodskii, 1983, on a somewhat different dynamical basis.

In accordance with the relations in Kitaigorodskii et al., 1975, for the wave number moduli spectrum

$$\chi(k) = \int_{|k|} \Psi(k) d\vec{k} = \int_{-\pi}^{\pi} \Psi(k, \phi) k d\phi$$  \hspace{1cm} (4.5)

the wave number vector being defined as $\vec{k} = (k \cos \phi, k \sin \phi)$ and, eventually $\Psi(k, \phi) = \Psi(k, f(\phi))$.

The spreading function $f(\phi)$ describes the wave energy angular spreading at any angle $\phi$ relatively to the wind direction and satisfies the following condition:

$$\int_{-\pi}^{\pi} f(\phi) d\phi = 1$$  \hspace{1cm} (4.6)

Thus, by use of Eqs. (4.4)-(4.6), the one-dimensional spectrum regardless of the direction of the wave component is derived as

$$\chi(k) = C u_* g^{-1/2} k^{-5/2}$$  \hspace{1cm} (4.7)
The relation between the frequency spectrum $S(\omega)$ and the wave number moduli spectrum $\chi(k)$ is as follows (simply from energy conservation):

$$S(\omega) = \frac{\chi(k)}{\frac{\partial \chi}{\partial \omega}}$$

By use of $\omega^2 = kg$ the frequency wave spectrum is thus proportional to $\omega^{-4}$ in the equilibrium range, i.e.

$$S(\omega) = \alpha_* u_g \omega^{-4}$$

where $\alpha_* = 2C$ is a constant to be determined empirically (Phillips, 1985).

The applicability of the latter form should in theory be limited to an upper wave number or frequency at which other small-scale physical effects become important. (Such effects, as for instance capillarity, do not dominate the range of equilibrium). In Kitaigorodskii, 1983, this frequency limit is given as the non-dimensional frequency $\omega u_g^{-1} = 4$, after which the 'old' Phillips scaling is valid. The lower boundary of the equilibrium range should theoretically be the frequency where the non-linear wave interactions become influential, which is after the peak frequency.

Banner and Young, 1994, have shown that the variation of the energy portions of wind input or dissipation in the total energy balance of the equilibrium range, the decay rate of the spectrum and the Phillips constant $\alpha_*$ will be affected.

### 4.2.2 Measurements

Full-scale measurements performed under different geophysical conditions suggest more energy in the tail of the spectrum. Some of the studies, though, result in variable frequency exponent and wave data largely scattered in the high frequency region. Forristall, 1981, used the wavestaff measurements made in the Gulf of Mexico, and the analysis showed that the decay rate of the spectrum in the frequency range between $\omega_* \leq \omega \leq 2.5\omega_*$, where $\omega_*$ is the mean frequency, corresponds to $\omega^{-4}$.

Kahma, 1981, studied the wave spectrum by means of wave buoys located in the Bothnian Sea. The Phillips constant $\alpha_*$ was determined according to Longuet-Higgins, 1969, and the decay rate of the spectrum was found to be close to $\omega^{-4}$, for the dimensionless frequency $(\omega u_g^{-1})$ being less than 4.
Battjes et al., 1987, have reanalysed the wave data collected during JONSWAP and found out that an $\omega^{-4}$ dependency fits better in the high frequency spectrum.

Liu, 1989, tried to estimate the slope of the spectrum equilibrium range with a large amount of data recorded in the Great Lakes, and he recommended $\omega^{-4}$ as the best fit. The frequency exponent $n$ determined from the measurements is given in percentage in Fig. 4.1.

![Probability distribution of the exponent $n$ in the relationship $F(f) \propto f^{-n}$ for the high-frequency portion of the spectrum][1]. Figure taken from Young, (1999).

Prevosto et al., 1996, analysed the deep-water sets of wave data collected around the Portuguese coast, the island of Crete, for the North Sea and the Norwegian Sea using both wave buoys and wave radars. The large range of $H_s$ and $T_s$ was analysed and exponents above the spectral peak were obtained by regression. The tail will behave more steeply for steeper waves. All data suggests power decay of $\omega^{-4}$ and none of the data supports the Phillips $\omega^{-5}$ theory. In terms of the wave number spectrum, less variation in the exponents in the tail of the wave number spectrum is noticed than in the frequency spectrum and the model $k^{-2.5}$ may be applicable.

Rodriguez and Soares, 1999, analysed measurements recorded in a location off the Portuguese coast and the tail behaviour showed significant variability. They concluded that the frequency exponent varies between $n = -3$ and $n = -6$, and in order to reduce the uncertainty and scatter in the wave data analysis, the same techniques in the method of spectral averaging, the same data length etc. should be used.
Violante-Carvalho et al., 2002, analysed the deep-water wave database collected around the coast of Rio de Janeiro by the Brazilian Oil Company. The swell was removed from the measured spectrum since the area is swell dominated and the wind sea growth was studied. Their conclusion was that there is no predominant value for the high frequency exponent, actually the data was scattered between $n = -4$ and $n = -6$. They suggest that a spectral form with the variable high-frequency exponent $S(\omega) \propto \omega^{-n}$ might be a better option and that the reason for the large scatter of the data may also be the inaccuracy of the measurements in the high frequency region.

### 4.2.3 Numerical Simulations

We may consider the total wave spectrum as the target spectrum $S^T(\omega)$ and it should therefore correspond to the measured wave data. It consists of the linear part $S^L(\omega)$ and the non-linear part $S^{NL}(\omega)$, where the non-linearities are included up to second order, both super- and subharmonic terms.

\[
S^L(\omega) + S^{NL}(\omega) = S^T(\omega)
\]  \hspace{1cm} (4.10)

The non-linear second order wave spectrum is simulated by using the WAFO program (Wave Analysis for Fatigue and Oceanography), a collection of Matlab m-files developed at Lund University (WAFO, 2000). WAFO is efficient software that contains routines for random second order wave simulation and statistical analysis as well as load and fatigue calculation.

From the target spectrum given in Fig. 4.2 by the solid line, second order components are extracted and removed and the spectrum is, therefore, linearised. The linear part of the spectrum is plotted by the dashed line (small dashes). In this way, the correct input spectrum is obtained for non-linear second order load calculation programs. This has been done in WAFO by a fix-point iteration on the spectral density using a non-linear simulation procedure. The second order wave spectrum is also simulated by using the input target spectrum (Fig. 4.2, large dashes). The dash-dotted line represents the linearisation of the second order spectrum, plotted by the upper dashed line, and it should coincide with the target spectrum. The unidirectional long-crested sea is used here for time and frequency domain simulations and infinite water depth is assumed.

The target wave spectrum in Fig. 4.2 is JONSWAP with the characteristics significant wave height $H_s = 5m$, peak period $T_p = 1.1s$ and factor of peakedness $\gamma = 1.1$.

As seen in Fig. 4.3, the simulated second order spectrum in the higher-frequency range exceeds the energy level contained by the conventional JONSWAP spectrum described by the $n = -5$ frequency exponent. Actually, it is close to the $n = -4$ decay rate given by the dash-dotted line. The effect on the truncated energy in the tail will be insignificant by setting the different cut-off frequency in the wave spectrum.
Based on the modified non-linear Schrödinger equation (Truelsen, 2000) Dysthe et al., 2003, performed three-dimensional fully non-linear numerical simulations of the gravity wave spectra in the horizontal physical plane \((x_k, y_k)\), where the intensity of the wave number \(k\) is 
\[ k = \sqrt{k_x^2 + k_y^2}. \]

The angularly integrated spectrum showed the evolution towards the power-law behaviour \(n = -4\) of the frequency exponent on the high-frequency side of the spectrum regardless of the spectral width. On the low – frequency side some steepening in the spectrum was noticed.

It is clear that the calculation of the wave spectrum using a higher order non-linear wave model would produce a thicker tail. Mori and Yasuda, 2002, used a fourth order non-linear wave model and compared the simulated wave spectra with the second order simulations for different spectral bandwidths. Including higher order non-linear wave interactions, the high-frequency tail of the spectrum will become more even, the second high-frequency peak will vanish and the tail will decay more slowly due to the increased energy level, according to their calculations.
Fig. 4.2: Wave spectral density in logarithmic form normalised by its maximal peak value. The following lines are plotted: the input spectrum - full line, the linearised input spectrum - lower dashed line, (small dashes), the second order spectrum simulated using input spectrum (full line) as linear input - upper dashed line.

Fig. 4.3: The rate of decay of the wave spectral density.
4.3 Measurements from WAVEX

The wave input to the load analysis of the bulk carrier in Chapter 5 is obtained from WAVEX (MIROS, 1998), the system attached to the standard navigation ship radar, able to measure directional waves. In Figs. 4.4-4.9 measured spectra are shown for the period of July to December year 1999 without separation in directions, though two main directions were noticed in the measurement data. The measured sea state parameters are given in Table 4.1, the significant wave height $H_s$ and the peak period $T_p$ of the total measured spectrum and $\beta$ denoting the dominant heading angle (the heading that contains the most energy) and is given in the WAVEX scale, meaning that 0 and 360deg stand for the head waves.

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>$H_s$ [m]</th>
<th>$T_p$ [s]</th>
<th>$\beta$ [deg]</th>
</tr>
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<tr>
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<td>2.62</td>
<td>6.37</td>
<td>315</td>
</tr>
<tr>
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<td>1.97</td>
<td>6.72</td>
<td>15</td>
</tr>
<tr>
<td>08/09/1999</td>
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<td>3.86</td>
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<td>345</td>
</tr>
<tr>
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<td>2.62</td>
<td>7.17</td>
<td>10</td>
</tr>
<tr>
<td>25/11/1999</td>
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<td>9.78</td>
<td>295</td>
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<td>19/12/1999</td>
<td>11:00</td>
<td>4.17</td>
<td>8.97</td>
<td>55</td>
</tr>
</tbody>
</table>

Table 4.1: Statistical characteristics of the sea state. WAVEX scale used meaning that 180deg denotes the following waves.

Spectral densities are normalised by the maximum peak value and given in the logarithmic form as a function of the wave frequency (the solid line). The lower dashed line (small dashes) represents the linear part of the total measured wave spectrum after the quadratic part of the spectrum has been removed, which should be entered in the second order load calculation. The upper dashed line (larger dashes) is obtained by second order simulation using the total measured spectrum (solid line) as input spectrum. The dash-dotted line almost coincides with the measured (target) spectrum.

The decay rates in the frequency domain $\omega^n$, where $n = -6, -5, -4$, are shown in the figures. It is difficult to generalise the frequency power of the measured spectra above the spectral peak. Due to uncertainty in the radar measuring system response for high frequencies close to the Nyquist frequency, the collected wave data is scattered. In the high-frequency region the slope of the measured wave spectrum corresponds neither to conventional $n = -5$ nor to $n = -4$, actually the measured spectrum seems to
decay much faster than the JONSWAP or even the $\omega^{-6}$ spectrum. For wave frequencies higher than, say, twice the $\omega_p = 2\pi / T_p$ the spectral densities increase with frequency contradictory to analytical predictions using dimensional analysis. This is probably due to frequency limited measuring instrumentation and not due to some physical reasons. Comparisons between the results by WAVEX and a reference instrument, i.e. the wave radar officially approved by Det Norske Veritas show that even much below the limiting Nyquist frequency, for the low signal to noise ratio it is difficult to estimate the shape of the wave spectrum (Gangeskar and Grønlie, 2000). This scatter is characteristic of low significant wave heights and higher frequencies. While there are deviations in $H_s$ and $T_s$, the wave direction is measured more accurately, except for very low wave energy when it is difficult to determine the energy peak direction.

Another disadvantage in connection with the measured wave data collected by the WAVEX system is the low cut-off frequency, which is certainly in negation with the present idea of using the data to study the high-frequency shape of the spectrum.

In conclusion, the measured wave spectrum has to be linearised in order to avoid unrealistic energy increase in the high-frequency tail after the spectrum has been subjected to non-linear response calculations. In the present analysis, the measured wave spectra for the North Atlantic Ocean obtained by WAVEX were approximated by using the classical JONSWAP $\omega^{-5}$ formulation in the high-frequency range. Second order correction of the linear wave spectrum formulated like Pierson-Moskowitz or JONSWAP spectrum will produce a frequency decay rate close to $n = -4$ in the high frequency tail, which is reasonable, on the dynamical grounds explained in references related to Subsection 4.2.1.
Figs. 4.4-4.6: Total measured wave spectral density.
Figs. 4.7-4.9: Total measured wave spectral density.
4.4 Analytical Solution for the Combined Wave Spectrum

A spectral form can be suggested that may better correspond to the real cases, according to the authors mentioned in 4.2.1. The spectrum would consist of e.g. two Gamma spectra, the first proportional to $\omega^{-5}$ and the second one regarding intermediate range decaying with $\omega^{-4}$. The formulation of the Gamma spectrum and the corresponding spectral moments, for the generalised parameters of the high-frequency power and the low-frequency steepness, could be found e.g. in Friis Hansen, 1994. Such representation is valid in the entire frequency region. However, different expressions should be applied to account for the combination of spectral forms and their integrations performed over separate frequency intervals.

The analytical form can be found for the combined wave spectrum consisting of the part $S_1(\omega)$ below the frequency $\omega_0 = x$ (decay rate $\omega^{-5}$) and $S_2(\omega)$ above this frequency (decay rate $\omega^{-4}$) referring to the equilibrium range. Such a spectrum can be described as a function of its statistical parameters, most conveniently by using spectral moments:

$$m_n = m_n^1 + m_n^2 = \int_0^x \omega^n S_1(\omega) d\omega + \int_x^\infty \omega^n S_2(\omega) d\omega \quad (4.11)$$

By substituting into Eq. (4.11) the forms of the spectrum $S_1(\omega) = A_1 \omega^{-5} e^{-B_1 \omega^4}$ and $S_2(\omega) = A_2 \omega^{-4} e^{-B_2 \omega^4}$ for the equilibrium range, the expressions for the spectral moments may be written as

$$m_n^1 = \frac{1}{4} \frac{A_1}{B_1^{n/4}} \gamma \left(1 - \frac{n}{4}, x \right) \quad (4.12)$$

$$m_n^2 = \frac{1}{4} \frac{A_2}{B_2^{n+1/4}} \Gamma \left(1 - \frac{(n+1)}{4}, x \right) \quad (4.13)$$

The proposed corrected form in the region below the frequency $\omega_0 = x$ is given as

$$S_1(\omega) = \frac{4}{D_1} \frac{m_0}{\omega_0^{5}} \left(\frac{\omega}{\omega_0}\right)^{-5} \exp \left\{ - \left(\frac{\omega}{\omega_0}\right)^{-4} \frac{1}{D_2} \right\} \quad (4.14)$$

with the coefficients
\[ D_1 = D_2 = \frac{\gamma(1/2, x)^2}{\gamma(1, x)} \]

where the lower incomplete Gamma function is given by the expression
\[ \gamma(a, x) = a^{-1} x^a e^{-x} \Gamma(a + 1, a + x) \]
representing the confluent hypergeometric function of the first kind. The boundary \( x \) will here be chosen at a point at or after the spectral peak.

![Graph of D1 and D2](image)

**Fig. 4.10: Coefficients D1 and D2 as functions of the upper boundary**

Coefficients \( D_1 \) and \( D_2 \) converge to \( \pi \) for large values of \( x \). For the infinite upper boundary, the wave spectrum \( S_1(\omega) \) would coincide with the Pierson-Moskowitz spectrum.

The region above the boundary \( x \) may be described by

\[ S_2(\omega) = \frac{4}{D_3} \frac{m_0}{\omega_{22}} \left( \frac{\omega}{\omega_{22}} \right)^4 \exp \left( -\frac{\omega}{\omega_{22}} \right)^4 \frac{1}{D_4} \]  \hspace{1cm} (4.15)

where

\[ D_3 = \frac{\Gamma(1/4, x)^{1/2}}{\Gamma(3/4, x)^{1/2}} \quad \text{and} \quad D_4 = \frac{\Gamma(1/4, x)^2}{\Gamma(3/4, x)^2} \]

defined by using the upper incomplete Gamma function \( \Gamma(a, x) \) with the peak frequency as the lower boundary. If the lower boundary \( x \) is close to zero, the
4.4 Analytical Solution for the Combined Wave Spectrum

spectrum will coincide with the Gamma spectrum $S(\omega, \xi, \zeta) = A \omega^{-\xi} e^{-B\omega^\zeta}$ with the factors $\xi = 4$ and $\zeta = 4$ (e.g. Friis Hansen, 1994).

Some higher order statistical moments depend strongly on the high frequency behaviour of the wave spectrum. For the Pierson-Moskowitz spectrum (decaying $\omega^{-5}$) the fourth moment $m_4 = \int_0^\infty \omega^4 S(\omega) d\omega$ becomes infinite and in this case the bandwidth of the spectrum $\varepsilon = \sqrt{1 - \frac{m_2^2}{m_1^2 m_4}}$ is an insensitive parameter equal to unity. This is in disagreement with the assumption of the narrow-banded Gaussian process.

It follows from Fig. 4.11 that the third spectral moment of the Gamma spectrum decaying with $\omega^{-4}$ will be infinite. To avoid these problems to some extent, according to Longuet-Higgins, 1983, a new spectral bandwidth (denoted as $\nu$) may be calculated to be only dependent on the lower order spectral moments like

$$\nu = \sqrt{\frac{m_0 m_2}{m_1^2}} - 1.$$ 

Practically, a cut-off frequency should be applied and higher spectral moments would have finite values and the spectral bandwidth $\varepsilon$ would be less than unity. Still, the difference between the two calculated coefficients $\varepsilon$ and $\nu$ may be large.

Furthermore, there would be no point in analysing the $\omega^{-3}$ decay rate since the zero-upcrossing period would be infinite.

![Fig. 4.11: Spectral moments of the $\omega^{-4}$ decaying Gamma wave spectrum.](image)
Now, by solving for the combined wave spectrum, the energy is to be conserved and spectral moments below and above the chosen boundary frequency may be added up. Furthermore, the continuity of the spectrum should be ensured continuing in the value and slope of the spectrum in the connecting point \( \omega_0 = x \). Thus, the following four conditions have to be satisfied:

\[
\begin{align*}
1. & \quad m_0 = m_0 + m_0 = \frac{1}{16} H_i^2 \\
2. & \quad m_2 = m_2 + m_2 = \frac{1}{16} H_i^2 \left( \frac{2\pi}{T_2} \right)^2 \\
3. & \quad S_1(\omega_0) = S_2(\omega_0) \\
4. & \quad \frac{dS_1(\omega_0)}{d\omega} = \frac{dS_2(\omega_0)}{d\omega}
\end{align*}
\]

(4.16)

Fig. 4.12: The combined spectrum with user defined separation frequency.
where $m_2 = \bar{\omega}_2^2 m_0$ and $m_2 = \bar{\omega}_2^2 m_0$, and where the second spectral moment is determined by use of Eq.(4.11) when $n = 2$.

From these conditions (Eq. (4.16)), the four unknowns $m_0, m_0, \bar{\omega}_1, \bar{\omega}_2$ are found and the resultant spectrum $S(\omega) = S_1(\omega) + S_2(\omega)$ is plotted in Fig. 4.13 and compared to the Pierson-Moskowitz and Gamma spectrum for the same sea state.

The characteristics of the wave spectrum presented in Fig. 4.13 are $H_s = 4.5m$ and $T_s = 7.5s$ and the specified frequency $\omega_0 = 0.85 rad/s$.

**Fig. 4.13:** The wave spectrum proportional to $\omega^{-5}$ (to 0.85 rad/s) and $\omega^{-4}$ (above 0.85 rad/s). The combined spectrum is plotted by full line in the frequency range $0 < \omega \leq \omega_0$, where it decays with the frequency power $n = -5$, and by dashed line (large dashes) for frequencies $\omega > \omega_0$, where the decay rate is $n = -4$.

Pierson-Moskowitz spectrum in the entire frequency domain (dashed line - small dashes) and Gamma spectrum of the same energy plotted by dash-dotted line.
4.5 Concluding Remarks

Non-linear effects in the wave spectrum (in Fig. 4.2 and Figs. 4.4-4.9) become influential at some frequency after the peak, which is dependent on the spectral shape in the second order theory that accounts only for the non-linear energy transfer. Each case from the measurements has to be treated individually due to the variability in number of peaks, differently distributed energy in the spectrum, etc.

The new analytical form of the wave spectrum plotted in Fig. 4.13 obviously differs from the Gamma spectrum, which decays with $\omega^{-4}$ in the whole frequency range. Due to the preserved $\omega^{-5}$ decay for $\omega \leq \omega_0$, the peak frequency of the combined analytical spectrum appears to be much closer to the Pierson-Moskowitz spectrum peak frequency.

As input to the theories based on non-linear wave excitation, the linear wave spectrum may be approximated by the JONSWAP or Pierson-Moskowitz spectrum (in general Gamma spectrum with a $\omega^{-5}$ decay rate), and the realisation of the irregular second order sea waves will support the more accepted concept of a spectrum tail proportional to $\omega^{-4}$, due to energy built up in the tail.
Chapter 5

Load Predictions for a Bulk Carrier by Unidirectional Wave Analysis

In the present chapter the dynamic load on the hull is calculated by use of different hydroelastic procedures and compared to the measured response on board a large ocean-going bulk carrier. Stress and wave measurements are explained in more detail. The accuracy of the collected wave data is discussed. The sensitivity of the calculated high-frequency results to wave heading and structural damping coefficient is discussed. In the calculation procedures the wave spectrum is treated as unidirectional.

5.1 Introduction

Extensive full-scale measurements of the dynamic stress were performed on Great Lakes bulk carriers in the 1970s. Cleary et al., 1971 and Stiansen et al., 1978, presented the first well documented cases of springing. These ships had low two-node vibration frequencies due to low stiffness of the hull and large length, which in combination with the limited draught and low encountered sea states gave a dynamic behaviour with the ratio of springing to wave induced bending moment being much higher than for ocean going ships.

Beside measurements on Great Lakes bulk carriers very little information on full-scale measurements is available, most of which dates from late 1960s and 1970s, and all of them showed the largest springing in ballast condition and for head seas or close to
head seas (Bell, 1968; Goodman, 1970; Little, 1971; Gran, 1974; Gran, 1976). The measurements were concerned only with the ship response and the wave description was disregarded. Although the fatigue life of the ship is negatively influenced by high-frequency two-node vibration, no severe damages to ocean-going ships were reported that could be attributed solely to springing.

Recently, Det Norske Veritas (DNV) was motivated to perform full-scale measurements of the ship structural response on a large ocean-going bulk carrier after a high level of vibration stresses had been observed in ballast condition. Wave data were also collected on the route simultaneously with the stress records, which contributes to the completeness of the study and uniqueness of the measurements.

### 5.2 Springing Response in a Large Ore Carrier

A large ore carrier (with the main characteristics given in Table 5.1 and hull sections shown in Fig. 5.2) is considered by Storaug et al. (2003). The ship was designed for carrying iron from the St Lawrence Gulf area in Canada to Rotterdam in the Netherlands. On the route to the west the ship operated in ballast condition and to the east in fully loaded condition. The route is approximately given in Fig. 5.1.

![Fig. 5.0: The route across the North Atlantic Ocean.](image)

The ship was mainly built of high-tensile steel. The moment of inertia around the horizontal axes (given in Table 5.1) was slightly above the value required by e.g. DNV Classification Rules. On voyages in ballast condition the ship experienced severe springing vibrations. The ship was operating in head sea conditions on the...
route. Measurements of the response were performed on board the ship and simultaneously the sea surface was monitored and operational data was recorded.

The ore carrier was severely damaged due to fatigue cracking in longitudinal structural members. High levels of the vibratory to wave stress response ratio were observed in the deck and the first damage appeared in longitudinal structural members after less than one year of operation. The ship was repaired in the summer of 2000. Wave measurements and stress records were collected for the period from July 1999 to May 2000 before the repairs, both in full load and ballast condition. The stress level due to the springing-induced vibrations was especially high in the ballast condition when the ship was sailing mainly in head waves, sometimes exceeding the wave-induced loads (in some cases the measured ratio of vibration stress standard deviation to wave stress standard deviation is larger than unity).
Table 5.1: Ship's characteristics.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars</td>
<td>294m</td>
</tr>
<tr>
<td>Breadth</td>
<td>53m</td>
</tr>
<tr>
<td>Draught (ballast condition)</td>
<td>11-12m</td>
</tr>
<tr>
<td>Deadweight</td>
<td>220 000t</td>
</tr>
<tr>
<td>Measured natural hull vibration frequency</td>
<td>0.53Hz</td>
</tr>
<tr>
<td>Max. service speed</td>
<td>14.9kn</td>
</tr>
<tr>
<td>Moment of inertia of section (around horizontal axes)</td>
<td>690m$^4$</td>
</tr>
</tbody>
</table>

Fig. 5.2: Body plan.
5.3 Stress Measurements

Stress records were available from strain gauges positioned in the midship part of the upper deck, both on port and starboard sides. The output from the strain gauges was the stress time history. The measuring period was 30min. and the sampling frequency of the stress signal was 10Hz. The vibration was not engine- or propeller-induced, as it was concluded from tests where the engine and propeller rotation rate was varied in calm sea (Van Tongeren, 2002). The ship was also equipped with pressure transducers positioned on the bottom of the bow part of the hull (at 0.94L) to measure bottom slamming. No bottom slamming was actually registered and it is hence believed that the vibratory stresses are solely due to steady state wave excitation, i.e. springing, and that the level of transient whipping vibrations is very low. This conclusion is supported by the moderate sea states encountered. There was no measuring equipment in the stern area, so stern slamming cannot completely be ruled out. However, the lines aft are not specifically prone to stern slamming.

\[
S_R(\omega_e) = ([\text{MPa}]^2 \text{s/rad})
\]

Fig. 5.3: An example of the measured stress spectral density in the frequency domain for the ore carrier given in terms of \(\omega/d\omega\), where \(d\omega = 0.003719\).

Torsion was disregarded in the present analysis, and only vertical vibration considered as the torsional mode for the ore carrier was approximately twice the two-node vertical vibration frequency (Fig. 5.3). In Lund-Johansen, 2002, it was found for the same ship that the three-node vertical bending mode is located at \(\omega_{3-a} = 6.78 \text{ rad/s}\). No response peak was observed around this frequency. This was probably due to the position of the strain gauges. The measured stress time history is taken to be the average of the port and starboard side time signals.

The significant vibrations measured were continuous with a very slow decay rate due to the small damping in the hull. The structural damping, which dominates the
springing response, has been determined for this particular ship by using the measurements of the ship's response and assuming a white noise excitation spectrum in the high-frequency region (Storhaug et al., 2003). The damping coefficient was found, according to Subsection 2.8.5 in Chapter 2, to be approximately 0.5-0.6 per cent of the critical damping and showed no sign of the significantly non-linear behaviour. Fig. 5.4 shows a sequence of the measured stress time series for one of the voyages in ballast condition. At least two dominant frequencies are present in the response. It is quite visible that for this time sequence the springing- and wave-induced hull vibrations were almost of the same magnitude.

![Stress Record in Ballast Condition](image)

**Fig. 5.4: An example of the stress record in ballast condition.**

Stress measurements in full load condition show that the level of springing stresses was less severe as expected, since the wave pressure on the bottom decreases exponentially with increasing draught, but also because of the following sea conditions mainly encountered by the ship on the full load voyages.

### 5.4 Wave Measurements

Wave data was collected by use of the WAVEX WAVE MEASURING SYSTEM connected to the ship's radar. Wave data was stored in two-dimensional matrices covering the wave frequency range up to 0.3Hz and wave headings in the whole range of 0-360deg, where 360deg in the measurements denotes head sea and 180deg...
following sea condition. The heading angle $\beta$ in the WAVEX analysis is defined so that $\beta = 180\,\text{deg}$ corresponds to following sea (Fig. 5.5). Wave data was collected for periods of 10min. An average 2-dimensional wave spectrum was determined by the WAVEX system.

In Fig. 5.5 the wave heading angle is presented according to the WAVEX scale to be used in the following figures.

![Wave heading angle according to the WAVEX scale.](image)

A comparison between the wave measurements obtained by WAVEX and an additional wave radar approved by DNV as reference instrument shows the deviation of the significant wave height and wave period. For lower signal to noise ratio, the deviation is larger in the spectrum shape and the measurement accuracy seems to be questionable. The largest deviations are actually found in the value for the peak period (Gangeskar and Grønlie, 2000), since it depends on the second spectral moment and inaccuracies may be larger than in the significant wave height.

Fig. 5.6 relates standard deviations of the wave and high frequency stresses to the significant wave height and wind speed, respectively. Measurement data for ballast condition and for head waves in the range $(180\,\text{deg} \pm 30\,\text{deg})$ are included. The significant wave height is apparently not a very good measure of the vibration stress level. The wind speed gives a better indication while the reduction in springing vibration for the high Beaufort number is explained by speed reduction in higher sea states. For the wave-induced stresses (left-hand side of Fig. 5.6) the variation of the wave-induced stress standard deviation with the significant wave height is almost linear.
A very important issue and great advantage in the present analysis is that the wave system is able to determine the wave directions accurately. By use of 3-D processing (MIROS, 1998) wave directions will be accurately determined in the 0-360° interval. The wave system captures radar signals backscattered from the sea surface. The system collects polar images of the sea surface that have to be transformed to Cartesian coordinates in order to use the optimum fast Fourier transforms (FFT) available for spectral processing. Time series of radar images collected in short time intervals (radar antenna rotation time) are then transformed to wave number and frequency domain. There are some limitations as regards the radar characteristics and the FFT procedure itself. Wave directionality can be resolved for encounter frequencies ($\omega_e$) below the Nyquist frequency ($\omega_N$), which is dependent on the antenna rotation time ($T_r$) of the marine radar:
\[ \omega_e \leq \omega_N = \frac{2\pi}{2T_s} \]  \hspace{1cm} (5.1)

Using the dispersion relationship and solving with respect to \( T_s \) yield

\[ T_s \leq \frac{1}{2 \left[ f - \frac{2\pi^2}{g} U \cos \beta \right]} \]  \hspace{1cm} (5.2)

where \( f \) is the wave frequency in [Hz] and \( U \) is the ship's forward speed.

Fig. 5.7: Upper wave frequency limit for accurately determined wave directions as a function of the ship's speed.

Fig. 5.7 follows directly from Eq. (5.2) by assuming that the rotation time of the radar antenna is \( T_s = 1.25 \) s, which is a fair assumption considering that the speed of the ship was never larger than 16 knots (MIROS, 1998) and the ship heading \( \beta = 180 \) deg, since the head wave condition would represent the most conservative case. As the ship’s speed increases, the upper wave frequency for which wave directionality can be accurately determined will reduce.

In all analysed cases presented later in the thesis the direction of the approaching waves is correctly resolved, according to Eq. (5.2), since the peaks of both the directional wave spectra are far below the upper wave frequency limit.
5.5 Comparison with Measurements

The differences between predictions for vertical wave induced bending moment are discussed in e.g. Guedes Soares C et al., 1996, or Watanabe and Guedes Soares, 1999, who compared a number of hydrodynamic theories based either on strip theory or the 3-D Rankine source method and most of them accounting for the hydroelasticity. Focusing on the high-frequency springing response, this divergence in the calculations has been dealt with in Storghaug et al., 2003, Gu et al., 2003, as well as in the presented thesis, by comparisons between different hydroelastic theories.

5.5.1 Different Hydrodynamic Programs

A comparative study of springing load prediction was undertaken as a part of the workshops organised by Det Norske Veritas, Department of Hydrodynamic and Structures, starting in March, 2002. Four different hydrodynamic codes are used for the load calculation: SOST (DTU), WASIM (DNV), SINO (CSSRC) and VERES (Marintek). An overview of the programs and a comparison between them for selected cases are given in the following text. The conclusions of the study can also be found in two joint papers by Storhaug et al., 2003, and Gu et al., 2003.

WASIM (DNV)

WASIM is a 3-D Rankine source program for simulations in the time domain with panels covering the hull and free surface including a numerical beach. Forward speed can be chosen arbitrarily and regular and irregular waves can come from any direction, and at the same time. Non-linearities consist of Froude-Krylov and restoring forces calculated in the instantaneous position up to the incident wave surface. Higher order incident waves and slamming are optional. The flexibility of the hull is represented by normal modes including hydroelastic coupling. A further description is provided in Kring, 1994, Vada, 1994, and Tongeren, 2002.

Linear time domain simulations are performed including only the wet measured two-node mode. The two-dimensional measured wave spectra are used. The hydrodynamic damping is neglected and the structural damping ratio is set to 0.5%. The panel size is about 3x3m and 305 mass points are used.

SINO (CSSRC)

SINO is a non-linear strip theory program for time domain simulations in unidirectional irregular sea, the Froude-Krylov, restoring, added mass and damping coefficients are obtained at instantaneous draughts based on the relative motion at the
centre line. Furthermore, bow flare slamming, bottom impact and green water forces are considered. A free-free Timoshenko beam is used and hydroelastic effects are neglected. The memory effect is also neglected and in irregular sea the added mass and damping for the peak period are chosen to be representative. Infinite added mass and input structural damping are used for the flexible modes. A further description is given in Gu, 2003.

The hull is divided into 101 sections. An evenly distributed effective shear area is assumed and adjusted to give the measured springing frequency. Three flexible modes and a damping ratio of 0.6% are used. In case of two main headings, the largest energy heading is chosen for the analysis.

VERES (Marintek)

VERES is a non-linear strip theory program for time domain simulations in unidirectional irregular sea. Non-linear modification of Froude-Krylov, restoring, momentum slamming and radiation and diffraction forces are included. The memory effect is covered by convolution integrals. The structure is represented by a free-free non-uniform beam including shear deformation. Dry normal modes are used as input to the hydroelastic analysis. A further description is provided by Wu et al., 2002.

The hull is divided into 100 sections. 305 mass points, four flexible modes and a damping ratio of 0.7% were used. In case of two main headings, both are run and the mean is taken. The estimated effective shear area distribution is used.

5.5.2 Selected Cases

Ten cases have been arbitrarily selected for comparison, represented by different moderate to rough sea states. An overview of the sea states is given in Table 5.2.

The frequency range was given in [Hz] and limited by the cut-off frequency in the measured data positioned at 0.3Hz, which is lower than the natural vibration frequency of the hull. Another problem was the uncertainty related to the tail of the wave spectrum. Therefore the measured wave spectrum is extrapolated by the JONSWAP spectrum, preserving the significant wave height $H_s$ and the peak period $T_p$ instead of $T_z$ since the last parameter was overestimated for the limited frequency range. (A further explanation of the used wave spectrum is given in Chapter 4). The main heading(s) $\beta$ (0 and 360deg representing head sea since the WAVEX scale is used) and comments regarding short crested (SC) or long crested (LC) sea are given in the table.
Whenever there were more than one main heading, SOST results were obtained as the sum of statistically independent responses for each direction. The unidirectional wave spectrum for each heading was determined from the measurement data and fitted with the JONSWAP spectrum, or the numerical spectrum was used in case of large deviation from the analytical spectrum form.

<table>
<thead>
<tr>
<th>Case</th>
<th>Hs[m]</th>
<th>Tp[s]</th>
<th>V[m/s]</th>
<th>β [deg]</th>
<th>Comment</th>
<th>Date and local time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.5</td>
<td>10.75</td>
<td>7.80</td>
<td>335</td>
<td>LC, 1 peak</td>
<td>99.09.06 19.30</td>
</tr>
<tr>
<td>2</td>
<td>4.2</td>
<td>10.75</td>
<td>7.40</td>
<td>335</td>
<td>LC, 1 peak</td>
<td>99.09.06 22.00</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
<td>9.78</td>
<td>7.30</td>
<td>5/35</td>
<td>SC, 2 peak</td>
<td>99.09.07 12.30</td>
</tr>
<tr>
<td>4</td>
<td>2.8</td>
<td>7.68</td>
<td>6.97</td>
<td>75/355</td>
<td>SC, 2 peak</td>
<td>99.09.07 18.00</td>
</tr>
<tr>
<td>5</td>
<td>3.0</td>
<td>7.68</td>
<td>7.16</td>
<td>75/345</td>
<td>LC, 2 peak</td>
<td>99.09.07 19.30</td>
</tr>
<tr>
<td>6</td>
<td>4.1</td>
<td>8.96</td>
<td>6.93</td>
<td>355</td>
<td>SC, 1 peak skewed</td>
<td>99.09.08 00.00</td>
</tr>
<tr>
<td>7</td>
<td>4.0</td>
<td>8.96</td>
<td>6.80</td>
<td>345</td>
<td>SC, 1 peak skewed</td>
<td>99.09.08 00.30</td>
</tr>
<tr>
<td>8</td>
<td>4.2</td>
<td>8.96</td>
<td>6.60</td>
<td>315</td>
<td>SC, 1 peak skewed</td>
<td>99.09.08 02.30</td>
</tr>
<tr>
<td>9</td>
<td>4.2</td>
<td>8.96</td>
<td>7.80</td>
<td>5/275</td>
<td>LC, 2 peak</td>
<td>99.09.09 07.30</td>
</tr>
<tr>
<td>10</td>
<td>6.2</td>
<td>14.34</td>
<td>3.00</td>
<td>5</td>
<td>SC, 1 peak</td>
<td>00.02.08 00.30</td>
</tr>
</tbody>
</table>

Table 5.2: Selected cases. WAVEX scale used.

The unidirectional wave spectrum \( S(\omega) \) was defined by integration of the two-dimensional measured wave spectrum \( S(\omega, \beta) \), where spectral densities were given as a function of frequency and ship's heading:

\[
S(\omega) = \frac{360}{\beta = 0} S(\omega, \beta) d\beta \quad (5.3)
\]

The heading circle was divided into 36 intervals covering 10 degrees each and the numerical results from WAVEX were given as an average for the interval 0-10deg; 10-20deg ... 350-360deg.

In order to calculate the significant wave height and zero-upcrossing period for every of 10 cases and to compare them with DNV measurement results, spectral moments are needed.

\[
m_0 = \int_0^\infty S(\omega) d\omega \quad (5.4)
\]
$$m_2 = \int_0^\infty \omega^2 S(\omega) d\omega$$  \hspace{1cm} (5.6)$$

$$\sqrt{m_0} = \frac{H_s}{4} \quad T_Z = 2\pi \sqrt{\frac{m_0}{m_2}}$$  \hspace{1cm} (5.7)$$

where $m_0$ is variance or standard deviation squared and $m_2$ is the second spectral moment of the wave spectrum.

The relation for the spectral peak period reads

$$T_p = 1.408 \cdot T_Z$$  \hspace{1cm} (5.8)$$

but the peak period could also be found from the observed peak in the measured wave spectrum.

---

**Fig. 5.8:** Transfer function of vertical bending moment at 15 knots in head sea.
Transfer functions for the vertical wave bending moment at the midship section obtained by using different hydroelastic codes are presented in Fig. 5.8. The wave frequency bending moment depends on the formulation of the Froude-Krylov force and the calculation methods for the hydrodynamic coefficients. SOST is expected to give slightly different prediction for the wave frequency bending moment, since it accounts for the wave diffraction only approximately through the Smith correction. The same would apply to the linear springing bending moment. The location of natural frequency depends on the hull flexibility and calculation of added mass. The magnitude of the springing peak differs if the natural frequency coincides with hump (enhancement of the peak) or with hollow (peak reduction).

**5.5.3 Frequency Transfer Function Sensitivity to Structural Damping**

The variation in the bending moment frequency transfer function is presented by applying different values of structural damping coefficients. Results are obtained by use of SOST, VERES and WASIM hydroelastic codes and presented in Fig. 5.9 and Fig. 5.10. The ship is assumed to sail at 15 knots in head sea. The magnitude of the structural damping coefficient is varied to be $\xi = 0.001$, $\xi = 0.002$ and $\xi = 0.005$ as shown in the graph. The high frequency peak is sensitive to the damping value, while there is no change in fundamental wave frequency response dominated by the much larger hydrodynamic damping.
5.5 Comparison with Measurements

Fig. 5.9: Vertical bending moment transfer function sensitivity to structural damping coefficient, at 15 knots in head sea.

Fig. 5.10: Focus on springing peaks from Fig. 5.8.
5.5.4 Frequency Transfer Function Heading Analysis

The transfer function for the vertical wave bending moment at the midship section is considered and the cases with head and quartering waves are analysed. In Fig. 5.11 the WAVEX scale is used, which means that head wave is approaching from 0deg and following wave is denoted by 180deg. The bulk carrier sailing at a speed of 15kn is taken as an example. The predictions using two different hydroelastic codes SOST and SINO are compared (the figures are taken from the joint paper of Gu et al., 2003).

![Graph showing the transfer function in head and quartering sea. WAVEX scale used here 360deg denoting the head sea.](image)

It has been noticed throughout the different analyses that the high-frequency vertical wave bending moment transfer function may increase for some head quartering waves. This behaviour is very important to the numerical predictions. The frequency transfer function oscillates with the wave number, and it will have its zeros whenever the ratio of the ship length to wave length is close to an odd integer. For quartering waves the effective wave length (Fig. 5.12) will be smaller than in the case of headwaves ($\lambda_{eff} = \lambda \cos \beta$, where $\beta$ is the heading angle), and in the encounter frequency domain the humps will become ‘wider’. The springing peak will be sensitive to the location of the natural frequency relative to the humps and it will be
smaller if bounded by cancellation points. This would cause purely numerical amplification of the springing peak.

The oscillatory behaviour will be smoothed out in the non-linear spectral density of the response curve (that is in agreement with the response spectral density of the measured stress). Due to the frequency dispersion relationship, the peak of the incident wave spectrum will move to lower encounter frequencies. If this shift causes a better match between the wave spectrum peak frequency multiplied by an integer number and the two-node natural hull vibration frequency, the non-linear springing response may rapidly increase while the linear response can only become smaller.

In the wave frequency domain, the transfer function will, however, always decrease away from head sea.

5.5.5 Short-Term Results

The standard deviation \( s \) of the ship response is found by integrating the spectral density of the response \( S_R \) over the encounter frequency domain:

\[
s^2 = \int_0^\infty S_R(\omega_e) d\omega_e \tag{5.9}
\]

An example of the stress spectral density presented for one of the analysed cases is given in Fig. 5.13. The calculated response using three different hydrodynamic theories mentioned in 5.5.1, VERES, SOST and SINO, is compared to the measured stress spectrum.
Fig. 5.13: Simulated and measured stress spectra for case 1.

```
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<th>HF Pσ</th>
<th>WF SBσ</th>
<th>HF SBσ</th>
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<td>17.48</td>
<td>6.69</td>
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```

Table 5.3: Standard deviation of wave (WF) and high frequency (HF) stress in [MPa] for port (Pσ) and starboard side (SBσ).
The stress measurements on port and starboard sides have shown that the vertical mode is the main contribution to the vibration, since the high-frequency stresses are almost the same for both sides (Table 5.3).

The time series of the stress were also evaluated in order to determine the level of high-frequency wave excitation. The resonant part of the response is considered, i.e. frequencies in the range $2.5 < \omega < 4.5$ rad/s. By representing the ship in springing vibration as a system with one degree of freedom with linear stiffness, the response spectrum was found according to Eq. (2.48) in Chapter 2 and related to the excitation spectrum in Eq. (2.49). The rate of decay in the tail of the wave excitation spectrum is estimated by using the approximate expression (Rüdinger in Storhaug et al., 2003):

$$S_j(\omega) = \frac{A_j}{\omega^n} \quad \text{for} \quad \omega > 2.5 \text{ rad/s}$$

where $n$ is the decay rate and $A_j$ is the intensity factor. The slope of the wave spectrum is found for each of the ten full-scale records (Table 5.4) and on average it is close to the power $n = -4$. This is in good agreement with the findings from Chapter 4.

<table>
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</tr>
<tr>
<td>10</td>
<td>5.06</td>
</tr>
</tbody>
</table>

Table 5.4: Decay rate of the wave excitation spectrum for ten cases.

A comparison of the wave frequency stresses for ten cases is given in Fig. 5.14. The different calculations are in good agreement with each other and the trend in the measurements is captured. For the first three cases all the programs overestimate the stresses, which may be caused by an error in the peak period $T_p$ of the measured wave spectrum, since a little reduction in the $T_p$ would cause a significant reduction in the wave frequency standard deviation of the response (Storhaug et al., 2003).

In Fig. 5.15, a comparison of high frequency stresses is presented. The simulation results are largely scattered versus measurements, neither of the programs is able to capture the trend or the magnitude of the measured response, which is underestimated by all programs except SINO. SINO includes bottom slamming (which was not registered during the measurement periods) and gives larger predictions, especially in higher sea states. In SOST, the non-linear response (superharmonic terms) dominates over the linear prediction since there is almost no energy in the tail of the wave spectrum around the two-node vibration frequency. It seems that an important source of springing excitation is missing in all codes.
Fig. 5.14: Comparison of wave frequency stresses.

Fig. 5.15: Comparison of high frequency stresses.
5.6 Summary

The unidirectional second order strip theory has been verified by comparisons with the other hydrodynamic codes in both wave and high-frequency domains. Comparison for ten arbitrarily selected full-scale measurement cases shows that numerical predictions using four different non-linear hydrodynamic codes for load calculation agree well with measurements for the wave frequency induced stresses. The agreement is much less for high frequency part, where the measured response is strongly underestimated in the calculations. Furthermore, the calculations performed by each of the four theories do not follow the trend of the measurements. The load predictions seem to show least agreement for cases where waves are not only unidirectional and deviate from the analytical JONSWAP spectrum. Obviously, there is a lack of the springing excitation, which is not accounted for by any of the presented hydrodynamic codes. The additional high-frequency excitation may be due to directional waves and the results of such analysis are presented in the next chapter.
Chapter 6

Calculations and Measurements for a Bulk Carrier - Bidirectional Wave Analysis

6.1 Introduction

In the present chapter response calculations are performed for the bulk carrier in ballast condition (described in Chapter 5) using measured wave input and the results are compared with the numerous stress measurements. In addition the response of the fully loaded ship has been considered.

The response is obtained by bidirectional SOST analysis based on expressions for the excitation force derived in Chapter 2. The influence of the new second order interaction terms is shown. The measured wave energy is in general distributed on different frequencies and headings. In case of bidirectional waves, the total measured wave spectrum has been divided into two unidirectional components as input to the bidirectional analysis. The effect of different heading of both unidirectional wave spectra is illustrated and discussed. When there was only one dominant wave heading the unidirectional SOST analysis has been used again.

The results obtained by bidirectional analysis are presented in comparison with the results obtained by unidirectional SOST and linear analysis and finally compared to the measured response. Significant improvement in springing load prediction has been achieved.
6.2 Ballast Condition

6.2.1 Calculated Response

First, however, the importance of the second order cross-coupling terms is illustrated. Fig. 6.1 shows the spectral density of the vertical bending moment amidships calculated for one of the cases analysed in the following. The full line shows the results for the complete second order analysis, which accounts for both wave systems and the interaction terms between them. The dashed line stands for a second order unidirectional analysis with the same energy content as the bidirectional waves together but without the cross-coupling terms.

![Spectral density of vertical bending moment](image)

Fig. 6.1: Spectral density of vertical bending moment by using linear unidirectional analysis (dotted line), unidirectional waves (dash-dotted line) and including two wave directions (full line).

The response in Fig. 6.1 is presented on a logarithmic scale. In the wave frequency region the linear and non-linear theoretical predictions using unidirectional waves are nearly the same. The calculation with two wave headings is different due to the influence of the wave directionality. For low frequencies the second order response will have a small contribution from frequency difference terms.
Linear theory gives a very low high-frequency prediction of the response, as the excitation spectrum contains almost no energy in the high-frequency region around the resonant area. The difference in the high-frequency peak value of spectral density of the vertical bending moment by use of unidirectional waves and bidirectional waves with cross-coupling terms included is of one order of magnitude on an absolute scale. The high-frequency response including cross-coupling interaction terms is broader and the peak is sharper and higher.

6.2.2 Effect of Heading Angle

To investigate the influence of the heading angle of both wave spectra on the magnitude of the calculated springing response, the spectral density of the vertical wave bending moment has been computed for different heading conditions. Two JONSWAP wave spectra are used in the calculations, the first one presenting the dominant head waves. The first and second wave spectrum parameters were fixed to $H_s^1 = 4m$, $T_z^1 = 7.5s$, $\gamma_1 = 1.5$ and $H_s^2 = 1.5m$, $T_z^2 = 8s$ and $\gamma_2 = 1.0$. Wave spectra as functions of the wave frequency are given in Fig. 6.2.

![Wave spectral density](image)

Fig. 6.2: Wave spectral density used for the calculations in Figs. 6.3-6.6.

The heading angles of the first wave spectra are fixed and varied for the second wave spectra in order to present the situation with opposing wave spectra, or when the heading angles are quite close. In Fig. 6.3 the first wave spectrum is fixed to $\beta_1 = 180\text{deg}$ (denoting head sea) and the second spectrum is varied from $\beta_2 = 0 - 150\text{deg}$ with a step of 30deg. In Fig. 6.5 the first wave spectrum is fixed to
\( \beta_1 = 135 \text{ deg} \) and the second spectrum is varied from \( \beta_2 = 315 - 105 \text{ deg} \) with the 30 deg step.

The second wave spectrum represents a very low and long period sea state and an absolutely small high frequency response is expected in these conditions. Nevertheless, it contributes through the second order cross interaction terms, especially in cases when it is directed oppositely to the first wave spectrum.

The response is given on a logarithmic scale in Fig. 6.3 and Fig. 6.5. The largest contribution from second order cross interaction terms is expected when \( \beta_1 - \beta_2 = 180 \text{ deg} \), i.e. in the case of opposing waves due to maximum Smith correction factor. This is the black full line in Fig. 6.4 and Fig. 6.6, and as the angle difference becomes smaller the total response using bidirectional analysis diminishes as explained in detail in Chapter 2. It is seen that in the case when the first wave spectrum represents strictly head sea (Fig. 6.3 and Fig. 6.4) the responses for smaller angle difference give larger springing prediction e.g. for \( \beta_1 - \beta_2 = 60 \text{ deg} \) the high frequency response is larger than for \( \beta_1 - \beta_2 = 120 \text{ deg} \). Even though the influence of the cross interaction terms is larger for distant angles, the unidirectional response for each heading is larger for headings closer to head sea, and therefore the magnitude of the second peak in the total response spectrum is increased.
6.2 Ballast Condition

Fig. 6.3: Spectral density of vertical wave bending moment $S_p(\omega_e)$ evaluated for different heading angles. Heading of the first wave spectrum fixed to $\beta_1 = 180$ deg.

Fig. 6.4: Focus on springing peak from Fig. 6.3. The largest contribution from opposing waves. (Heading of the first wave spectrum fixed to $\beta_1 = 180$ deg.).
Fig. 6.5: Spectral density of vertical wave bending moment $S_{R}(\omega_e)$ evaluated for different heading angles. Heading of the first wave spectrum fixed to $\beta_1 = 135$ deg.

Fig. 6.6: Focus on springing peak from Fig. 6.5. The largest contribution from opposing waves. (Heading of the first wave spectrum fixed to $\beta_1 = 135$ deg.)
6.2.3 Comparison with measurements

For a short-term analysis, the wave characteristics and the ship's response are assumed to be stationary over a period of time which is typically of the order of hours. Even if the wind is unchanged and the energy level and the wave period stay approximately the same, the speed and the heading angle of the ship to the waves may vary due to manoeuvring. In order to make reasonable comparisons with the theoretical predictions, the time periods have been selected from the vast number of recorded data as those with the least change in sea and ship operational conditions. Intentionally, the selection is made to cover cases with high-frequency to wave-frequency response stress standard deviation ratios equal to or larger than 1, and to consider cases with clearly bidirectional wave systems. Furthermore, ballast condition and head sea as the dominant wave direction are chosen.

For the months of July to December 1999 and February to April 2000 (the exact dates and time periods are given in Table 6.1; January 2000 is disregarded from the present analysis since the ship was operating only in full-load condition), a selection of five measurement series for each of the months has been made. For these cases, by use of the wave measurement matrices, contour plots of the wave spectral density as a function of wave frequency and wave heading angle are made. They are shown in Figs. 6.7-6.15 for nine arbitrarily selected cases, one from each month analysed. Two main wave directions are clearly visible, each possessing almost the same amount of wave energy. The exceptions are in November 1999 and April 2000 when the sea was mainly unidirectional. Double peaks and swell coming from the wind-driven wave direction have also been noticed.

Separation of the wave spectra is performed. The wave spectral density as a function of the heading angle $S(\beta)$ is determined by integration from the frequency range and plotted over the heading range. Two main headings are then determined for two distinctive maxima. For the separation points, headings containing a minimum of wave energy (usually close to zero) have been chosen. A unidirectional wave spectrum is then determined for each direction. These spectra are generally approximated by JONSWAP spectra with suitable peak enhancement coefficients. However, if the shape of the spectrum deviates too much from the analytical spectrum, the wave spectral density is numerically defined in the calculations on the assumption that the measured spectra could be applied as linear wave spectra. This is a reasonable assumption as the sea states are moderate. The measured wave spectrum and its separation into two main directions are shown in Figs. 6.16-6.24 for the nine cases analysed.

The measurements for the nine cases (with the main characteristics given in Table 6.1) are compared to the theoretical predictions: linear and second order analysis that accounts for unidirectional waves only (lin 1-dir, SOST 1-dir in Figs. 6.25-6.42) and second order analysis accounting for two different wave directions (SOST 2-dir in Figs. 6.25-6.42). The comparison for July 1999 (Fig. 6.25) shows clearly a linear...
dependency of wave frequency stress standard deviations with the significant wave height $H_s$ and a parabolic dependency in the high-frequency case (Fig. 6.28), implying that the wave frequency response is dominated by linear excitation whereas the springing excitation is mainly due to second order effects. In August 1999 (Fig. 6.8) and February 2000 (Fig. 6.13) the approaching wave systems are exactly opposite to each another. This represents theoretically the worst case where the largest springing excitation is expected. The standard deviation of the measured springing stress is 3.7 times larger than the standard deviation of the measured wave frequency stress in August 1999, while the same ratio comes to even 5.4 in February 2000! Such a high ratio between the high-frequency and the wave response is due to the relatively low wave period but of course also a result of the very low wave frequency loads in this low sea state. The sea state is much higher in September (Fig. 6.27 and Fig. 6.30) and so is the corresponding wave period. The high-frequency response is still close in magnitude to the wave frequency response, but there is no tendency of a parabolic increase in springing results for such large wave heights. For higher sea states (typically November, December for the North Atlantic Ocean) the growth in springing loads with increasing $H_s$ is much less than in wave-induced loads. Both the measured and calculated high-frequency stresses will decrease in the larger sea states due to the corresponding increase in the zero-upcrossing wave period $T_z$, which was also noticed in other sets of measurements not shown here.

For all nine sets of measurements, comparisons with theoretical predictions for the wave frequency part (WF) give remarkably good agreement with the second order bidirectional analysis (SOST 2-dir). By use of unidirectional analysis, keeping the energy preserved and taking the larger energy heading angle as the heading angle, the wave frequency responses become strongly overpredicted (linear 1-dir, SOST 1-dir). The reason is that by defining only one wave heading angle all the excitation wave energy is artificially moved to one wave direction - in these cases head- or bow-quartering waves that are responsible for the largest wave response. In reality, the energy was spread also to following or stern-quartering waves and this reduced the total response. The difference between two unidirectional predictions (linear 1-dir and SOST 1-dir) will be larger in cases where a low-frequency swell is present, as the second order unidirectional theory (SOST 1-dir) accounts for the frequency difference second order terms. As an example, according to the measured data for September 1999 (contour plot shown in Fig. 6.9 and separation of the total measured wave spectrum shown in Fig. 6.18) there is no low-frequency part in the wave spectrum, and hence the difference between linear and non-linear unidirectional calculations is small as seen in Fig. 6.27. Especially, a larger difference is seen for the months of July 1999, October 1999, February 2000 and March 2000, when the low-frequency spectrum is significant.
Comparison for the high-frequency (HF) part of the response shows that second order bidirectional analysis (SOST 2-dir) gives results closest to the measurements. The second order cross-coupling terms give a large contribution to the total high-frequency response. This contribution will increase with significant wave height. The largest effect is found for exactly opposing waves (as in August 1999, Fig. 6.29, December 1999, Fig. 6.36 and February 2000, Fig. 6.40) as expected and explained in the theoretical background of the paper. In September 1999 (Fig. 6.30), though the significant wave height is larger than in August 1999, the contribution from second order interaction terms is relatively smaller due to the wave headings being closer to each other. By comparison of high-frequency results from March 2000 and February 2000, the same reasoning applies in relation to the possible large contribution from opposing waves. The measured high-frequency response is much higher in February 2000 than in March the same year, though in March 2000 the waves deviate from strictly opposing by not more than only about 10deg. Linear unidirectional analyses...
(linear 1-dir) clearly give the lowest results and definitely underpredict the springing response.

Contour plots of the wave spectral density for one of the cases in November 1999 and April 2000 (Fig. 6.11 and Fig. 6.15) show one dominant wave heading, and the unidirectional wave spectra for the same cases (Fig. 6.20 and Fig. 6.24) agree very well with the theoretical JONSWAP spectrum of the same energy, except in the high-frequency part where the accuracy of the measured wave data is questionable. For November 1999, SOST unidirectional theory was sufficient for response calculations (Fig. 6.32 and Fig. 6.35). A much higher wave frequency stress level is measured in December 1999 (Fig. 6.33), while the high-frequency stresses for November and December are of similar magnitude (Figs. 6.35-6.36). The wave spectra characteristics for these two months do not differ significantly in energy and peak position, but the main heading is slightly different. Still the theory predicts much higher fundamental wave frequency responses in November 1999. It seems that second order unidirectional theory is able to predict measured high-frequency stresses for the same month. Somewhat larger high-frequency stresses are measured in April 2000 (Fig. 6.42) even though the heading, speed, wave period and significant wave height are approximately the same as in November 1999. The trend of the measurements is still captured reasonably well. The calculated high-frequency response is greatly sensitive to the wave period and sometimes the heading. A small change in these parameters would significantly influence the high-frequency stress standard deviation.

Calculations of the vertical wave bending moment spectral density in ballast condition are presented in Appendix B, for each point in Figs. 6.25 - 6.42. The response is plotted in Figs. B.1 – B.45.
Figs. 6.7-6.9: Contour plots of the wave spectral density for the months July, August and September year 1999 as a function of wave frequency and heading. Note: head sea is 0deg according to the WAVEX scale.
Figs. 6.10-6.12: Contour plots of the wave spectral density for the months October, November and December year 1999 as a function of wave frequency and heading. Note: head sea is 0deg according to the WAVEX scale.
Fig. 6.13-6.15: Contour plots of the wave spectral density for the months February, March and April year 2000 as a function of wave frequency and heading. Note: head sea is 0deg according to the WAVEX scale.
Fig. 6.16-6.18: Separation of the total measured wave spectrum for the months July, August and September year 1999 into two main heading directions. Note: head sea is 0deg according to the WAVEX scale.
Fig. 6.19-6.21: Separation of the total measured wave spectrum for the months October, November and December year 1999 into two main heading directions. Note: head sea is 0deg according to the WAVEX scale.
Figs. 6.22-6.24: Separation of the total measured wave spectrum for the months February, March and April year 2000 into two main heading directions. Note: head sea is 0deg according...
Figs. 6.25-6.27: Comparison of the measured and calculated stress standard deviations for wave frequency (WF) for the months July, August and September year 1999.
Figs. 6.28-6.30: Comparison of the measured and calculated stress standard deviations for high frequency (HF) for the months July, August and September year 1999.
Figs. 6.31-6.33: Comparison of the measured and calculated stress standard deviations for wave frequency (WF) for the months October, November and December year 1999.
Figs. 6.34-6.36: Comparison of the measured and calculated stress standard deviations for high frequency (HF) for the months October, November and December year 1999.
Figs. 6.37-6.39: Comparison of the measured and calculated stress standard deviations for wave frequency (WF) for the months February, March and April year 2000.
Figs. 6.40-6.42: Comparison of the measured and calculated stress standard deviations for high frequency (HF) for the months February, March and April year 2000.
6.3 Full Load Condition

The ship was designed to carry iron ore from the St Lawrence Gulf in Canada to Rotterdam in the Netherlands. In cargo condition the draught varied between 19.49m at aft perpendicular and 17.77m at fore perpendicular. The measured natural hull vibration frequency was 0.463Hz. During operation in full load condition the vibratory fatigue damage was estimated to be 20-34% of the fatigue damage in ballast condition (Storhaug et al., 2003). The reduction in high frequency stresses was partly due to the fact that the sea on the east-bound fully loaded voyage was mainly following.

The main reduction in the high-frequency loads was due to the general decrease in the wave pressure with the vertical coordinate, i.e. the draught of the ship. A faster decay in Smith correction factor with increase in draught definitely provides lower peaks in the load frequency transfer function, both fundamental and high-frequency springing peaks.

Spectral densities of the vertical wave bending moment in ballast and full load condition are compared in Fig. 6.43. The bidirectional sea is described by two JONSWAP unidirectional spectra with different heading angle $\beta$ and sea state parameters as given in Table 6.2.

<table>
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</table>

Table 6.2: Characteristics of JONSWAP wave spectra.

Wave spectra 1 and 2 are arbitrarily but intentionally chosen to represent wind-driven waves (close to head waves) and low frequency swell (close to beam sea), respectively. By use of only unidirectional SOST analysis, the only springing response is excited by wave spectrum 1 (spectral density of VWBM 1 plotted by the dashed line (small dashes) in Fig. 6.43). The influence of the second order cross interaction terms in bidirectional SOST analysis is rather large (spectral density of VWBM 12 plotted by the full line in Fig. 6.43) even though there is almost no response from wave spectrum 2 due to the low wave period (spectral density of VWBM 2 plotted by the dash-dotted line in Fig. 6.43).

The natural two-node vibration frequency of the hull decreases for the loaded ship since the stiffness is unchanged. The resonant frequency shifts towards the part of the excitation wave spectrum that contains more energy. However, in full load condition a reduction in springing response is expected, as seen in Fig. 6.43.
Fig. 6.43: Spectral density of vertical wave bending moment as a function of the encounter frequency compared for the same bidirectional sea state in ballast and full load condition. Full line represents the combined response including cross interaction terms and both directions. Dashed line (small dashes) shows the response excited by wave spectrum 1 described by use of the blue dashed line. Dash-dotted line shows the response excited by wave spectrum 2 described by use of the red dashed line.
The increase of internal hull damping in full load condition relatively to the ballast condition is not taken into account in the presented calculation (Fig. 6.43). The same damping coefficient is used as for the calculations in Chapter 5 although additional energy will be dissipated through the friction of the cargo. By applying a larger structural damping coefficient, the second peak in the response curve will be diminished while the fundamental response will stay unaffected.

The ratios of high frequency (HF) to wave frequency (WF) stress standard deviations during voyages in August 1999 are illustrated in Fig. 6.44 from the available measurement data. Not surprisingly, much higher ratio HF/WF response is registered in ballast than in full load condition. Still during voyages in full load condition, measured springing stresses are rather high and the ratio HF/WF response is sometimes close to unity.

![Graph showing stress standard deviation ratio HF/WF](image)

Fig. 6.44: High-frequency (HF) to wave-frequency (WF) response ratio in ballast and full load condition measured in August 1999.

The stationary sea state is defined from the measured wave data on 25 August 1999 covering a two-hour short-term period from 15:00 to 17:00h. The contour plot in Fig. 6.45 shows two pronounced wave directions and the situation with low frequency following waves (swell) and wave frequency head waves (wind-driven part).
In Fig. 6.46 the total measured wave spectrum is divided into two unidirectional wave systems showing that the dominant wave heading is low-frequency (swell) following sea containing the most energy. The total significant wave height varies in the range 3.409 to 3.606m. The range of heading is 5-45deg for the first and 185-225deg for the second unidirectional wave system.

Similarly to the observations in the comparisons made for ballast condition in Section 6.2, springing response is strongly underestimated by unidirectional SOST or linear analysis (Fig. 6.48). Since two dominant wave directions are usually registered in the wave measurement data with large high- to wave-frequency response ratios, such a significant level of springing stresses could be a result of bidirectional wave (inter)action.

Calculations of the vertical wave bending moment spectral density in full load condition are presented in Appendix B, for each point in Fig. 6.47 and Fig. 6.48. The response is plotted in Figs. B.46 – B.50.
Fig. 6.45: Contour plots of the wave spectral density for 25 August 1999 as a function of wave frequency and heading. WAVEX scale used in the figure, meaning that 180deg denotes following waves.

Fig. 6.46: Separation of the total measured wave spectrum for August 1999 into two main heading directions. WAVEX scale used in the figure, meaning that 180deg denotes following waves.
Fig. 6.47: Comparison of the measured and calculated stress standard deviations for the wave frequency (WF) for August year 1999.

Fig. 6.48: Comparison of the measured and calculated stress standard deviations for the high frequency (HF) for August year 1999.
6.4 Summary

The high level of springing measured for the bulk carrier has been investigated by taking into account hull geometrical and structural characteristics, the loading condition and the ship's operational conditions for the given route. Measurements of the ship response in ballast condition showed that the ratio of high to wave frequency stress is very often larger than unity. Thorough calculations are performed using second order bi-directional theory and the results are compared to the predictions by second order unidirectional and linear unidirectional theory, and finally with measurements. By assuming only unidirectional or linear wave excitation the high-frequency (springing) response of the ship was severely underestimated. However, including wave directionality in the second order wave excitation terms greatly improves the theoretical predictions.

The situation of opposing wave systems is especially interesting because it represents a case of a second order pressure term, which does not decay with distance below the still water line. In this case the second order wave pressure on the bottom of the ship might be significant even at the very low wavelengths responsible for springing. Springing is expected to be largest in this situation while it should be less for situations with smaller angles between the wave systems.

In general, however, a reduction of the springing vibration level is expected in full load condition due to increased draught of the ship and therefore lower wave-induced pressure at the bottom, with speed reduction and in high sea states due to the corresponding increase in the wave period.

The contour lines of the wave spectral density in Figs. 6.4 - 6.12 and Fig. 6.42 in most of the cases clearly show the existence of two main wave directions in real sea states, which the present theory accounts for.

According to the plots of excitation wave spectral densities, especially during voyages in ballast condition (Figs. 6.16 – 6.24), wave spectra are usually multi peaked and deviate from the conventional JONSWAP spectrum form. The spectrum is much closer to JONSWAP form in cases when the waves are unidirectional, as seen from the plots for November 1999 (Fig. 6.20) and April 2000 (Fig. 6.24).

The wave frequency stresses are slightly overpredicted by the present calculations, especially in the comparisons for the year 2000. However, the second order strip theory shows very good agreement with the other theories for wave load prediction in Chapter 5. Wave spreading, which is not included in the present theory, would probably reduce the wave frequency loads. Strip theory is the basis for all the calculations presented in this thesis, Chapter 5 and Chapter 6. For bulk carriers and other full form ships three-dimensional effects may reduce the wave-induced bending moment by 15-20% as compared to the strip theory calculations (Bach-Gansmo and Lotsberg, 1989, in Jensen and Mansour, 2002).
Much lower springing stresses have been recorded during voyages in full load condition versus the stress level in ballast condition. Still, the ratios of high-frequency to wave frequency response are in some cases close to unity, as seen in Fig. 6.44, and conventional unidirectional Second Order Strip Theory and linear theory are unable to predict such springing loads.

Another possible source of springing excitation is bow-generated waves that would cause the same second order effect in front of the ship as described in this thesis. However, the present procedure is based on a strip theory approach and is therefore not able to treat local hull geometrical effects like the three-dimensional shape of the bulbous bow.
Chapter 7

Calculations and Measurements for a Container Ship

With an increase in the hull dimensions, especially the length, the two-node vertical vibration frequency of the hull decreases. Springing may then become an important fatigue life issue considering large and ultra large container ships. Another important aspect is that standard scatter diagrams that are based on wave data collected over the past several decades do not take into account recent improvement in weather forecasting procedures available to the master of the ship. Due to the weather routing additional information will be given on how to avoid severe storms. From the wave data observed and collected in recent years for the North Atlantic Ocean it is shown that the probability distribution function becomes significantly lower for the higher sea states than tabulated in the standard references (Olsen et al., 2004), especially for containerships. These moderate sea states that become the most probable sailing conditions for the ship may be important to springing due to the shorter wave periods. In these conditions, both linear and non-linear unidirectional excited high-frequency response may be of the same order of magnitude. However, the influence of the interaction terms due to bidirectional excitation is independent of the wave period. The same applies if the ship is operating in i.e. the Mediterranean Sea, where the sea states are generally lower.

In this chapter a comparison is made between the responses measured on board a post-panamax container ship and calculations performed by use of the second order strip theory for the bidirectional case where waves coming from more than one direction are observed on the voyage.
7.1 The Lexa Mærsk Containership

The hull sections of the Lexa Mærsk are given in Fig. 7.1 and the main characteristics in Table 7.1. The ship was operating from Rotterdam in the Netherlands to Buenos Aires in Argentina in the period 17 August – 09 September 2004. For the analysis 23 non-equally distributed sections have been modelled from the ship’s geometry.

![Hull sections](image)

**Fig. 7.1: Hull sections.**

<table>
<thead>
<tr>
<th>Table 7.1: Main characteristics.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length over all</strong></td>
</tr>
<tr>
<td><strong>Length between perpendiculars</strong></td>
</tr>
<tr>
<td><strong>Breadth</strong></td>
</tr>
<tr>
<td><strong>Draught</strong></td>
</tr>
<tr>
<td><strong>TEU</strong></td>
</tr>
<tr>
<td><strong>Block coefficient</strong></td>
</tr>
<tr>
<td><strong>Measured natural hull vibration frequency</strong></td>
</tr>
<tr>
<td><strong>Moment of inertia (around horizontal axes)</strong></td>
</tr>
</tbody>
</table>
Fig. 7.2: Mass distribution for the Lexa Mærsk.

In full load condition the mass distribution is taken according to Fig. 7.2.

### 7.1.1 Sea States from Observation

On 22 August 2004 at 07:00 and 22 August 2004 at 13:00 the ship was sailing from Bremerhaven in Germany to Le Havre in France, through the North Sea and the English Channel. On 24 August 2004 at 07:00 the ship was sailing from Le Havre in France to Gibraltar, through the Atlantic Ocean.

The sea state information was collected by visual observation. The following characteristics were available from Laugen and Smidemann, 2004:

- Ship position
- Wind speed
- Wind direction
- Ship speed
- Ship heading
- Significant wave height of the first wave spectrum
- Zero-upcrossing period of the first wave spectrum
- Significant wave height of the second wave spectrum
- Heading of the second wave spectrum
The zero-upcrossing period of the second wave system was never recorded. This makes the comparison extremely difficult. The sea state was actually always very low. During most of the time the wind was blowing in the stern developing the following wind sea and the second wave system was usually head or quartering seas.

Table 7.2: Sea states during the operation. Note: 180 deg denotes head sea.

<table>
<thead>
<tr>
<th>Date</th>
<th>22 August 2004</th>
<th>22 August 2004</th>
<th>24 August 2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>07:00</td>
<td>13:00</td>
<td>20:00</td>
</tr>
<tr>
<td>Ship speed [kn]</td>
<td>21</td>
<td>21.9</td>
<td>20.9</td>
</tr>
<tr>
<td>Wind speed [m/s]</td>
<td>8.9</td>
<td>4.3</td>
<td>11.1</td>
</tr>
<tr>
<td><strong>Observation data</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{s1}$ [m]</td>
<td>0.8</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>$H_{s2}$ [m]</td>
<td>1.5</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$T_{z1}$ [s]</td>
<td>2.0</td>
<td>/</td>
<td>3.0</td>
</tr>
<tr>
<td>$T_{z2}$ [s]</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>$\beta_1$ [deg]</td>
<td>77</td>
<td>49</td>
<td>86</td>
</tr>
<tr>
<td>$\beta_2$ [deg]</td>
<td>-183</td>
<td>101</td>
<td>-127</td>
</tr>
<tr>
<td><strong>Values used in calculations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{s1}$ [m]</td>
<td>0.8</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>$H_{s2}$ [m]</td>
<td>1.1</td>
<td>1.0</td>
<td>1.3</td>
</tr>
<tr>
<td>$T_{z1}$ [s]</td>
<td>2.0</td>
<td>2.0</td>
<td>3.5</td>
</tr>
<tr>
<td>$T_{z2}$ [s]</td>
<td>5.2</td>
<td>4.2</td>
<td>15</td>
</tr>
<tr>
<td>$\beta_1$ [deg]</td>
<td>77</td>
<td>50</td>
<td>93</td>
</tr>
<tr>
<td>$\beta_2$ [deg]</td>
<td>180</td>
<td>100</td>
<td>-70</td>
</tr>
</tbody>
</table>

The accuracy of the visual observation method is not always the highest. However, it gives a good idea of the existence of secondary sea and its global heading. Many times the full information was missing in the observation data, especially on the wave height of the second wave system.
7.2 Ship Response

7.2.1 Time Histories of the Response

Longitudinal strains were measured by the strain gauges positioned in the midship area close to the deck. The response time series were collected for periods of 60 minutes starting at each hour. From the strain records, bending moment frequency response spectra were determined using the Fourier transform.

In most of the measurements both wave-induced and springing stresses were very small. This was probably due to a generally very low sea state and mainly following waves encountered during the operation.

In Figs. 7.3 – 7.5 three sequences of the measured vertical wave bending moments are presented for each one-hour time series covering the same randomly determined portion of time. A small continuous springing vibration is present in all the records with period around 1 – 1.5 seconds.

From the available stress records the examples chosen represent the cases where there was some visible two-node vibration and also where the sea state was better described which means that both headings were given for the bidirectional analysis.

Figs. 7.3: The sequence of the measured vertical wave bending moment time history amidships of the Lexa Mærsk container ship on 22 August 2004 at 07:00.
Figs. 7.4: The sequence of the measured vertical wave bending moment time history amidships of the Lexa Mærsk container ship on 22 August 2004 at 13:00.

Figs. 7.5: Two sequences of the measured vertical wave bending moment time history amidships of the Lexa Mærsk container ship on 24 August 2004 at 20:00.
7.2.2 Comparison of the Response Spectral Densities

The response spectral densities of the vertical wave bending moment for three cases given in Table 7.2 are calculated by taking account of the wave directionality. The structural damping coefficient in full load increases due to friction in the cargo and it is assumed to be 0.8 per cent of the critical damping in the calculations. The wave input is as shown in Table 7.2 with the wave spectrum assumed to be Pierson-Moskowitz type. The results are compared with the response spectrum of the measured response time histories. The results are presented in Fig. 7.6 – Fig. 7.11.

The measurements of the ship response give a good indication of the magnitude of the wave-induced and springing vibration during the operation. On the other hand, the wave data collected by observation is not complete and its accuracy is questionable.

In the first two cases (22 August 2004 at 07:00, Fig. 7.6, and 22 August 2004 at 13:00, Fig. 7.8) the wave-induced response by the first wave spectra is insignificant. This is probably due to a very low significant wave height and following sea condition. Most of the response comes from the second wave spectrum. Only in the third case (24 August 2004 at 20:00, Fig. 7.10) the wave-induced response component excited by the first wave spectrum is significant due to the larger significant wave height and slightly different heading. The influence of the swell is visible in Fig. 7.8 and Fig. 7.10.

Wave headings relative to the ship have a significant influence on the agreement between the full-scale measurements and the calculations. The headings are based on rough estimate. Both the ship response in regular waves and the position of the wave spectrum are quite sensitive to the heading.

The agreement in the wave-induced response between calculations and measurements is not quite good. The zero-upcrossing period of the second wave system may be assumed to be wrong. Such a low frequency response spectrum could only be produced by the following wave excitation where the encounter frequency would diminish.

There is almost no contribution to springing from the first wave spectrum as seen in Fig. 7.7 and Fig. 7.9 (response plotted by dashed lines in pink colour). The exception is the third case (Fig. 7.11) where there is some springing response due to larger significant wave height and beam seas. Much more influential are the head waves from the second wave carrying more energy. Probably due to the very long period, the springing contribution from the second wave system induced response is negligible in Fig. 7.11 (response plotted by dashed lines in green colour). Though the springing response is small, and the angle between the two headings is not very large, the influence of the cross-coupling terms between the two wave systems is clearly visible in Fig 7.7 and Fig. 7.10.
Fig. 7.6: Spectral density of vertical wave bending moment: measured (grey), smoothed measured (red), response from the first wave spectrum (pink-dashed line), response from the second wave spectrum (green-dashed line), total response (blue line).

Fig. 7.7: Focus on springing peak from Fig. 7.6.
7.2 Ship Response

Fig. 7.8: Spectral density of vertical wave bending moment: measured (grey), smoothed measured (red), response from the first wave spectrum (pink-dashed line), response from the second wave spectrum (green-dashed line), total response (blue line).

Fig. 7.9: Focus on springing peak from Fig. 7.8.
Fig. 7.10: Spectral density of vertical wave bending moment: measured (grey), smoothed measured (red), response from the first wave spectrum (pink-dashed line), response from the second wave spectrum (green-dashed line), total response (blue line).

Fig. 7.11: Focus on springing peak from Fig. 7.10.
7.3 Summary

In this chapter some full-scale measurements for the container ship Lexa Mærsk are presented and compared to the calculations. The results are the frequency response spectra of the vertical wave-induced bending moment. Comparison is made to the response spectrum obtained by using the bidirectional second order strip theory.

The wave characteristics cannot be determined exactly and the sea state observation is just a rough visual estimation and therefore not very reliable. The wave heading could only be estimated with large uncertainty, say, ±30deg, or even more. This is a drawback as both wave-induced and springing response are quite sensitive to the wave heading.

Generally, the orders of magnitude of both the wave-induced and the springing responses agree and springing is clearly visible, although not important, in all the cases.
Chapter 8

Conclusions and Recommendations

8.1 Conclusions

In this thesis, high-frequency bi-directional springing excitation mechanisms are accounted for on the basis of the strip theory. As a result, the following can be concluded:

- Second Order Strip Theory has been generalized to account for bidirectional wave excitation. The ocean environment is complex and the free surface is modelled more realistically using second order bidirectional waves, as can be seen from the contour plots of two-dimensional measured wave spectral densities.

- The second order cross-coupling terms, due to the interaction between two wave systems progressing at arbitrary directions are included in the total hydrodynamic force.

- An important parameter is the angle between the wave components from the two wave systems. The influence of the cross-coupling terms will be largest for strictly opposing waves. In this case the interaction contribution is confined to the high-frequency response (only superharmonic terms in the hydrodynamic force remain). The Froude-Krylov force has the largest impact for two wave components with very close wave numbers. In this case the Smith correction factor has a maximum value and there is no exponential decay in the dynamic pressure with the vertical coordinate i.e. the sectional
draught. This conclusion is valid for the whole frequency range. These cross-coupling terms in the wave excitation force will, however, decrease for smaller angles between the wave systems.

- The Smith correction factor approximated for the Lewis sectional form yields a larger level of excitation in higher frequencies relevant for springing, than predicted by the solution for the rectangular form.

- The Smith correction factor increases with larger slope on the side of the section.

- The concept of more energy content in the tail of the excitation wave spectrum has been accepted, represented by $\omega^{-4}$ wave frequency decay rate. In that case it was shown to be sufficient to use a conventional wave spectrum form proportional to $\omega^{-2}$ as input to the non-linear codes for load calculation.

- An analytical form of the wave spectrum is presented, which accounts for different wave frequency decay rates below and above the certain flexible point (e.g. some frequency after the peak frequency). Such a form might be more convenient to represent reality.

- From the comparisons between full-scale measurements and calculations by different unidirectional theories for the large ocean going bulk carrier in ballast condition, it can be concluded that the theoretical predictions agree well with each other both in wave and high frequency calculations. However, the measured high frequency response is strongly underpredicted in most cases by use of any unidirectional theory mentioned in the thesis. Furthermore, the trend in full-scale measurements of the response is not at all captured.

- From the comparisons between full-scale measurements and calculations for the large ocean going bulk carrier it is concluded that including the bidirectional wave excitation, gives an obvious improvement and much better agreement with measurements is achieved. By using unidirectional linear or Second Order Strip Theory in cases when there are more than one differently directed excitation wave systems, the response may be significantly underpredicted.

- Full-scale measurements for a container ship show generally a low level of springing vibration. The agreement between measurements and calculations using bidirectional second order strip theory is acceptable and the theory is able to predict the correct level of springing. It is difficult to perform any comparisons accurately since the accuracy of the wave measured by visual observation method is uncertain.
8.2 Further Recommendations

- More full-scale measurements both of the ship response and wave environment are needed, in order to analyse the level of springing in different sea states and for different ship forms.

- Efforts should be made to improve the technique for collecting wave data during voyages, to be able to determine accurately the form of the spectrum at high frequencies and to measure the directional spectrum since the cross interaction between the directional spectra represents an additional high-frequency excitation source.

- Experimental work can be done using ship models with variable form e.g. variable stern and bow parts in order to investigate phenomena like stern slamming or bow generated waves, that would contribute to the springing excitation.

- Recommendations for avoiding significant springing loads should be included in the rules of classification societies, especially for hull forms more susceptible to springing. This may not be possible to achieve due to the complexity and sensitivity of the high-frequency response. Considering the remarks from the beginning of the thesis, that springing loads are not considered specifically in the recommendations of classification societies, direct calculations for a ship may be required. In that case, Second Order Strip Theory would be an efficient tool for wave induced load prediction on the elastic hull. The calculations are performed fast and it gives a good indication of the level of springing in the hull.
Bibliography


Appendix A

Smith Correction Factor for Different Analytical Sectional Forms

In the following section exact expressions for the Smith correction factor is found for different analytical sectional forms.

Circular section

\[ z = -R \sin \theta \]
\[ B_0 = 2R \]
\[ B(z) = -2R \cos \theta \]
\[ dz = -R \cos \theta d\theta \]
After introducing these relations into the general formula for Smith effect we obtain:

\[ \kappa = \int_{0}^{\pi/2} \sin \theta \cdot e^{-kR \sin \theta} d\theta \]  
\[ (A.2) \]

Using formula for Taylor series expansion \( e^x = \sum_{n=0}^{\infty} \frac{(x)^n}{n!} \)

\[ \kappa = \sum_{n=0}^{\infty} R^n \frac{(-k)^n}{n!} \cdot a_{n+1} \]  
\[ (A.3) \]

where

\[ a_{n+1} = \int_{0}^{\pi/2} \sin^{n+1} \theta \cdot d\theta \]  
\[ (A.4) \]

The first four terms in the exact solution expansion become:

\[ \begin{array}{c|c}
 n=0 & 1 \\
 n=1 & -k\pi R / 4 \\
 n=2 & k^2 R^2 / 3 \\
 n=3 & -k^3 R^3 \pi / 32 \\
\end{array} \]  
\[ (A.5) \]

**Rectangular section**

![Rectangular section diagram]

The rectangular section is described by

\[ \frac{B(x, z)}{B_0(x)} = 1 \]  
\[ (A.6) \]

Inserting Eq. (A.6) in Smith correction factor following is obtained:
\[ \kappa = e^{-kt} \]  

**Wedge section**

![Diagram of a wedge section]

The wedge section is described by

\[ B(x, z) = B_0(x) \left( 1 + \frac{z}{T} \right) \]  

The Smith correction becomes

\[ \kappa = \frac{1}{kT} \left( 1 - e^{-kt} \right) \]  

**Trapezoidal section**

![Diagram of a trapezoidal section]

The trapezoidal section is described by

\[ B(x, z) = B_0 + \frac{z}{T} (B_0 - B_1) \]
The Smith correction becomes

$$\kappa = \frac{B_1}{B_0} e^{-kT} + \frac{1}{kT} \left( 1 - \frac{B_1}{B_0} \right) \left( 1 - e^{-kT} \right)$$  \hspace{1cm} (A.11)

**Parabolic section**

![Parabolic section diagram]

The section is described by following expressions:

$$B(x, z) = \left( \frac{z}{T} + 1 \right)^n \cdot B_0$$  \hspace{1cm} (A.12)

where $-T \leq z \leq 0$.

Inserting the relation Eq. (A.12) into the definition for the Smith correction factor yields

$$\kappa = 1 - e^{-kT} \sum_{m=0}^{\infty} \frac{(kT)^{m+1}}{m!(n+1+m)}$$  \hspace{1cm} (A.13)

Using the relation $n = \frac{1}{\alpha} - 1$, it can be written

$$\kappa = 1 - e^{-kT} \sum_{m=0}^{\infty} \frac{(kT)^{m+1}}{m!(m + \frac{1}{\alpha})}$$  \hspace{1cm} (A.14)

The formula is derived using I.S.Gradshteyn and I.M.Ryzhik, 1980.
Appendix B

Calculations for a Bulk Carrier

B.1 Ballast condition

For the bulk carrier described in Chapter 5 and Chapter 6 additional calculations of the spectral density of the response in ballast condition are presented here. Vertical wave bending moment spectral density is calculated using bidirectional second order analysis, unidirectional second order analysis and unidirectional linear analysis, and the three different calculations are plotted and compared in Figs. B.1 – B. 45. The response spectral densities are calculated for July 1999, August 1999, September 1999, October 1999, November 1999, December 1999, February 2000, March 2000 and April 2000. Five plots for each month (for a specific date) are presented covering the short-term period of about three hours. The results for stress standard deviation given in Chapter 6 are obtained using the calculated response presented here. The bidirectional wave input used for calculations Fig. B.1 – B.45 is described in Chapter 6. Significant wave heights related to the total wave input spectrum, (corresponding to Figs. 6.25 – 6.42 in Chapter 6), are given below the plots for each month.
App. B: Calculations for a Bulk Carrier

![Graph 1](19 July 1999 04:30  U=8.4 m/s)

- VWBM Spectral density [(Nm)^2/s/rad]
- \(\omega_e\) [rad/s]
- \(\beta_1 = 155\) deg
- \(\beta_1 = -35\) deg

![Graph 2](19 July 1999 05:30  U=8.4 m/s)

- VWBM Spectral density [(Nm)^2/s/rad]
- \(\omega_e\) [rad/s]
- \(\beta_1 = 155\) deg
- \(\beta_1 = -25\) deg

![Graph 3](19 July 1999 07:00  U=8.5 m/s)

- VWBM Spectral density [(Nm)^2/s/rad]
- \(\omega_e\) [rad/s]
- \(\beta_1 = 135\) deg
- \(\beta_1 = -15\) deg
Fig. B.1-B.5: Calculated vertical wave bending moment spectral density in July 1999. Comparison between the bi-directional SOST including cross-interaction terms (black continuous line), unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue line).

On the same plot first and second wave excitation spectra (blue dashed line-small dashes and pink dashed line-small dashes) are given like $S(\omega) \times 2 \times 10^{17}$ [m$^2$ s / rad].

Significant wave heights of the total wave input spectrum are $H_s = 2.7m$, 2.48m, 2.61m, 2.19m, 1.69m, respectively.
App. B: Calculations for a Bulk Carrier

13 August 1999 06:30  $U=8.3\,\text{m/s}$

- $\beta_1 = 165^\circ$
- $\beta_1 = -15^\circ$

13 August 1999 07:00  $U=8.3\,\text{m/s}$

- $\beta_1 = 165^\circ$
- $\beta_1 = -15^\circ$

13 August 1999 07:30  $U=8.3\,\text{m/s}$

- $\beta_1 = 165^\circ$
- $\beta_1 = -15^\circ$
Fig. B.6-B.10: Calculated vertical wave bending moment spectral density in August 1999. Comparison between the bi-directional SOST including cross-interaction terms (black continuous line), unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue line).

On the same plot first and second wave excitation spectra (blue dashed line-small dashes and pink dashed line-small dashes) are given like $S(\omega_e) \times 2 \times 10^{17} [m^2/s/\text{rad}]$.

Significant wave heights of the total wave input spectrum are $H_h = 1.97 m$, 2.26 m, 2.20 m, 2.08 m, 1.73 m, respectively.
Fig. B.11-B.16: Calculated vertical wave bending moment spectral density in September 1999. Comparison between the bi-directional SOST including cross-interaction terms (black continuous line), unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue line).

On the same plot first and second wave excitation spectra (blue dashed line-small dashes and pink dashed line-small dashes) are given like $S(\omega_e) \times 2 \times 10^{17}$ [$m^2/s/rad$].

Significant wave heights of the total wave input spectrum are, $H_s = 3.86m$, 4.87m, 4.80m, 4.71m, 4.75m, respectively.
App. B: Calculations for a Bulk Carrier

![Graphs showing VWBM Spectral density vs. \( \omega_e \) for different times and wind speeds.]

- 05 Oct 1999 02:00, \( U = 7.8 \text{ m/s} \):
  - \( \beta_1 = 180^\circ \text{deg} \)
  - \( \beta_1 = 5^\circ \text{deg} \)

- 05 Oct 1999 02:30, \( U = 7.7 \text{ m/s} \):
  - \( \beta_1 = 170^\circ \text{deg} \)
  - \( \beta_1 = 0^\circ \text{deg} \)

- 05 Oct 1999 03:00, \( U = 7.7 \text{ m/s} \):
  - \( \beta_1 = 180^\circ \text{deg} \)
  - \( \beta_1 = 10^\circ \text{deg} \)
Fig. B.16-B.20: Calculated vertical wave bending moment spectral density in October 1999. Comparison between the bi-directional SOST including cross-interaction terms (black continuous line), unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue line).

On the same plot first and second wave excitation spectra (blue dashed line-small dashes and pink dashed line-small dashes) are given like $S(\omega_e) \times 2 \times 10^{17} [m^2/s/\text{rad}]$.

Significant wave heights of the total wave input spectrum are, $H_s = 2.13\text{m}, 2.24\text{m}, 2.26\text{m}, 2.7\text{m}, 2.62\text{m}$, respectively.
VWBM Spectral density [(Nm)$^2$/rad]

- 25 Nov 1999 01:00  U=7.3 m/s  $\beta = 115$ deg
- 25 Nov 1999 01:30  U=7.1 m/s  $\beta = 135$ deg
- 25 Nov 1999 02:30  U=6.9 m/s  $\beta = 125$ deg
Fig. B.21-B.25: Calculated vertical wave bending moment spectral density in November 1999. Comparison between unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue line).

On the same plot unidirectional wave excitation spectrum (blue dashed line-small dashes) is given like $S(\omega_x) \times 2 \times 10^{17} \,[\text{m}^2 \text{s} / \text{rad}].$

Significant wave heights of the total wave input spectrum are $H_s = 4.16\text{m}, 4.54\text{m}, 4.69\text{m}, 4.44\text{m}, 4.40\text{m},$ respectively.
App. B: Calculations for a Bulk Carrier

19 Dec 1999 08:30  U=7.0 m/s

19 Dec 1999 09:00  U=6.9 m/s

19 Dec 1999 10:00  U=6.6 m/s
Fig. B.26-B.30: Calculated vertical wave bending moment spectral density in December 1999. Comparison between the bi-directional SOST including cross-interaction terms (black continuous line), unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue line).

On the same plot first and second wave excitation spectra (blue dashed line-small dashes and pink dashed line-small dashes) are given like $S(\omega) \times 2 \times 10^{17} [\text{m}^2 \text{s}^{-1} \text{rad}]$.

Significant wave heights of the total wave input spectrum are, $H_s = 4.17 \text{m}, 4.18 \text{m}, 4.20 \text{m}, 4.17 \text{m}, 4.12 \text{m}$, respectively.
04 February 2000 21:30  \( U=8.2\text{ m/s} \)

04 February 2000 22:00  \( U=8.2\text{ m/s} \)

04 February 2000 22:30  \( U=8.2\text{ m/s} \)

VWBM Spectral density \([\text{Nm}^2\text{s/rad}]\)

\( \beta_1 = 165\text{deg} \)

\( \beta_1 = -15\text{deg} \)

\( \beta_1 = 165\text{deg} \)

\( \beta_1 = -15\text{deg} \)
Fig. B.31-B.35: Calculated vertical wave bending moment spectral density in February 2000. Comparison between the bi-directional SOST including cross-interaction terms (black continuous line), unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue line).

On the same plot first and second wave excitation spectra (blue dashed line-small dashes and pink dashed line-small dashes) are given like $S(\omega) \times 2 \times 10^{17} [m^2/s/\text{rad}]$.

Significant wave heights of the total wave input spectrum are, $H_s = 1.93m, 1.77m, 1.77m, 1.73m, 1.59m$, respectively.
App. B: Calculations for a Bulk Carrier

06 March 2000 04:30  U=7.4 m/s

06 March 2000 05:00  U=7.5 m/s

06 March 2000 06:00  U=7.7 m/s
Fig. B.36-B.40: Calculated vertical wave bending moment spectral density in March 2000. Comparison between the bi-directional SOST including cross-interaction terms (black continuous line), unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue line).

On the same plot first and second wave excitation spectra (blue dashed line-small dashes and pink dashed line-small dashes) are given like $S(\omega) \times 2 \times 10^{17}$ [m$^2$ s/rad].

Significant wave heights of the total wave input spectrum are, $H_s = 1.86$ m, 1.87 m, 2.10 m, 2.15 m, 2.32 m, respectively.
04 April 2000 22:00  U=6.8 m/s

04 April 2000 22:30  U=6.8 m/s

04 April 2000 23:00  U=6.8 m/s
Fig. B.41-B.45: Calculated vertical wave bending moment spectral density in April 2000. Comparison between unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue line).

On the same plot unidirectional wave excitation spectrum (blue dashed line-small dashes) is given like $S(\omega) \times 2 \times 10^{17} [m^2/s/\text{rad}]$.

Significant wave heights of the total wave input spectrum are, $H_s = 4.47m$, 4.20m, 4.23m, 4.12m, 4.18m, respectively.
B.2 Full Load Condition

Calculations of the spectral density of the response for the bulk carrier in full load condition are presented here. The response spectral densities are calculated for August 1999. Results in Figs. B.46 – B.50 are obtained in the same way as explained in section B.1.

Related significant wave heights to the total wave input spectrum, corresponding to Figs. 6.47 – 6.48 in Chapter 6, are given below the plots.
B.2 Full Load Condition

\[
\begin{align*}
\text{VWBM Spectral density [Nm}^2\text{s/rad]} & \quad \text{25 August 1999 15:00} \quad U=7.8 \text{ m/s} \\
\beta_1 &= 45\text{deg} \quad \beta_1 &= -175\text{deg}
\end{align*}
\]

\[
\begin{align*}
\text{VWBM Spectral density [Nm}^2\text{s/rad]} & \quad \text{25 August 1999 15:30} \quad U=7.8 \text{ m/s} \\
\beta_1 &= 15\text{deg} \quad \beta_1 &= -175\text{deg}
\end{align*}
\]

\[
\begin{align*}
\text{VWBM Spectral density [Nm}^2\text{s/rad]} & \quad \text{25 August 1999 16:00} \quad U=7.8 \text{ m/s} \\
\beta_1 &= 150\text{deg} \quad \beta_1 &= -35\text{deg}
\end{align*}
\]
Fig. B.46-B.50: Calculated vertical wave bending moment spectral density in August 2000. Comparison between unidirectional SOST (black dashed line-small dashes) and unidirectional linear analysis (blue dashed line-large dashes).

On the same plot first and second wave excitation spectra (blue dashed line-small dashes and pink dashed line-small dashes) are given like $S(\omega) \times 2 \times 10^{17} [m^2/s/rad]$.

Significant wave heights of the total wave input spectrum are, $H_s = 3.49m$, $3.6m$, $3.4m$, $3.41m$, $3.61m$, respectively.
<table>
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<th>Year</th>
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