Lower Bound for the Radiation $Q$ of Electrically Small Magnetic Dipole Antennas With Solid Magnetodielectric Core

Kim, Oleksiy S.; Breinbjerg, Olav

Published in:
I E E E Transactions on Antennas and Propagation

Link to article, DOI:
10.1109/TAP.2010.2096394

Publication date:
2011

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
Lower Bound for the Radiation $Q$ of Electrically Small Magnetic Dipole Antennas With Solid Magnetodielectric Core

Oleksiy S. Kim and Olav Breinbjerg

Abstract—A new lower bound for the radiation $Q$ of electrically small spherical magnetic dipole antennas with solid magnetodielectric core is derived in closed form using the exact theory. The new bound approaches the Chu lower bound from above as the antenna electrical size decreases. For $ka < 0.863$, the new bound is lower than the bounds for spherical magnetic as well as electric dipole antennas composed of impressed electric currents in free space.

Index Terms—Chu lower bound, electrically small antennas, fundamental limitations, magnetic dipole, quality factor.

I. INTRODUCTION

An ideally electrically small magnetic dipole antenna composed of an impressed electric current on the surface of an imaginary sphere exhibits a radiation $Q$ three times the Chu lower bound [1], [2]. When the surface current encloses a magnetic core the internal stored magnetic energy reduces, and the Chu lower bound [3] can be reached for a vanishingly small antenna. However, the behavior of a finite-size magnetic dipole antenna is more complicated due to the internal resonances in the magnetic core. Exact analytical expressions derived in [4] show that the minimum achievable $Q$ is a function of the antenna electrical size. This function represents the lower bound for the radiation $Q$ of a magnetic dipole antenna with a magnetodielectric core. In this communication, taking account of the rigorous theory presented in [4], we derive an analytical expression for this lower bound and define its range of validity.

II. LOWER BOUND FOR THE QUALITY FACTOR

We consider an impressed electric current on the surface of a spherical material core with relative permeability $\mu_s$ and permittivity $\varepsilon_r$. In a spherical coordinate system $(r \theta \phi)$ the surface current density radiating the $TE_{m0}$ spherical mode (magnetic dipole mode) is

$$\mathbf{J} = \hat{a}_\phi J_0(kr) \sin \theta$$  \hspace{1cm} (1)

where $\hat{a}_\phi$ is the azimuthal unit vector and $J_0$ is the amplitude (A/m). Closed-form expressions for the stored magnetic $W_H$ and electric $W_E$ energies as well as for the radiated power $P_{\text{rad}}$ are derived in [4]. From those, the radiation quality factor $Q$ can be expressed in compact form, subject to $W_H \geq W_E$,

$$Q(ka, \mu_s, \varepsilon_r) = \left\{ 1 + \frac{1}{\mu_s} L(k,a) \right\} Q_{\text{Chu}}(ka)$$  \hspace{1cm} (2)

where $k$ is free-space wave number, $a$ is the radius of the antenna, $k_s = 1/\sqrt{\mu_0 \varepsilon_0}$ is the wave number in the core material, the function $L(k,a)$

$$L(k,a) = \frac{1 + (k,a) J_0(k,a) Y_0(k,a)}{2 J_1(k,a)} - 1$$  \hspace{1cm} (3)

Manuscript received February 17, 2010; revised June 24, 2010; accepted August 31, 2010. Date of publication December 03, 2010; date of current version February 02, 2011. This work was supported by the Danish Research Council for Technology and Production Sciences within the TopAnt project.

The authors are with the Department of Electrical Engineering, Electromagnetic Systems, Technical University of Denmark, DK-2800 Kgs. Lyngby, Denmark (e-mail: osk@elektro.dtu.dk; ob@elektro.dtu.dk).

Digital Object Identifier 10.1109/TAP.2010.2096394
The first four values of \( u_n \) and the corresponding constants \( \lambda_n \) are given in Table I. From Table I as well as from Fig. 2, which illustrates the expression (9), it is clear, that the first value \( u_1 \) yields the lowest \( \lambda_n \), and, consequently, the lowest \( Q_{\text{ch}}(ka, \varepsilon_r) \). Since the \( Q_{\text{ch}}(ka, \varepsilon_r) \) also reduces as the permittivity \( \varepsilon_r \) decreases, the optimal core must be pure magnetic with relative permeability

\[
\mu_s(ka) = \frac{u_n^2}{(ka)^3}. \tag{10}
\]

Finally, the lower bound for magnetic dipole antennas can be written from (9) as

\[
Q_{\text{LB}}(ka) = Q_1(ka, \varepsilon_r = 1) = \frac{1}{(ka)^3} + \frac{1 + \lambda_1}{ka} + (ka)\lambda_1. \tag{11}
\]

The conclusion is that for a given antenna electrical size \( ka \) there is an optimum magnetic core permeability (10) that ensures the lowest possible \( Q \) as given in (11). This \( Q \) is thus the lower bound for magnetic dipole antennas with solid magnetodielectric core.

As illustrated in Fig. 3, the new bound (11) is higher than the Chu lower bound, but approaches it as \( ka \to 0 \). At the same time, it shows that with a properly selected magnetic core an electrically small magnetic dipole antenna exhibits a \( Q \) that is lower not only as compared to the same antenna with air core, but also as compared to an air-core electric dipole antenna\(^1\) of the same size. We note, however, that in a region \( 0.863 < ka < 1.325 \)—although, in this region an antenna can hardly be considered electrically small—an air-core electric dipole antenna performs slightly better.

\^1Here, and throughout the communication, we assume that air-core magnetic and electric dipole antennas are composed of electric current distributions on an imaginary spherical surface in free space, that is, no other materials or objects, neither inside, nor on the surface, nor outside of the antenna, are present.
III. RANGE OF VALIDITY

All expressions given in Section II are exact and valid as long as the stored electric energy does not exceed the stored magnetic energy. Since these take turns in being dominant as the resonances are crossed, to define the range of validity for our expressions, we first need to find values of \((k_0, \mu_r, \varepsilon_r)\), for which

\[
W_{14}(k_0, \mu_r, \varepsilon_r) = W_{E}(k_0, \mu_r, \varepsilon_r).
\]  

The first set of values corresponds to the total resonance

\[
W_{14}^{-}(k_0, \mu_r, \varepsilon_r) + W_{14}^{+}(k_0, \mu_r, \varepsilon_r) = W_{E}^{-}(k_0, \mu_r, \varepsilon_r) + W_{E}^{+}(k_0, \mu_r, \varepsilon_r)
\]  

where \(W_{14}^{-}\) and \(W_{14}^{+}\) are, respectively, the magnetic and electric energies stored inside and outside the antenna core. The solution of (13) is not straightforward and has to be found numerically.

The second set of values is associated with the internal \(TE_{10}\)-mode resonances in the spherical core of the antenna, when in the absence of the external fields the internal magnetic \(W_{14}^{-}\) and electric \(W_{14}^{+}\) stored energies are balanced

\[
W_{14}^{-}(k_0, \mu_r, \varepsilon_r) = W_{14}^{+}(k_0, \mu_r, \varepsilon_r).
\]  

In contrast to (13), the solution of (14) is known—\(k_0, \mu_r = v_n\), where \(v_n\) are zeros of the spherical Bessel function \(j_n(v)\).

Since the bound (11) implies a pure magnetic core \((\varepsilon_r = 1)\), we can plot the result in a \((k_0, \mu_r)\) space, as shown in Fig. 4. Except for the shaded region, between the straight line representing the internal resonance (dependence \(k_0, \mu_r \approx v_1\)) and a curve denoting the total resonance, the stored magnetic energy dominates over the stored electric energy. Thus, the optimum parameters (10), also shown in Fig. 4, corresponding to the lower bound (11) are located in the valid region \(W_{14} > W_{E}\), which extends from the electrically small regime, \(k_0 \ll 1\), up to \(k_0 \approx 2.6\), at which (10) yields a total resonance. Consequently, the lower bound (11) is applicable in the range of antenna electrical sizes \(0 < k_0 < 2.6\).

IV. CONCLUSION

The lower bound for the radiation \(Q\) of magnetic dipole antennas with solid magnetodielectric core is derived explicitly. The bound can be reached by a spherical magnetic dipole antenna with a pure magnetic core, whose permeability is properly selected according to the antenna electrical size. Such an antenna exhibits a lower \(Q\) than the air-core magnetic and electric dipole antennas of the same size in the range of \(k_0 < 0.663\) as well as for \(1.325 < k_0 < 2.6\). The later value sets the upper limit for the range of validity of the presented bound.

REFERENCES