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Application of asymptotic speed deficit concept to existing engineering wake model

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WAKE DECAY FOR THE INFINITE WIND TURBINE ARRAY

Application of asymptotic speed deficit concept to existing engineering wake model

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Application of asymptotic speed deficit concept to existing engineering wake model

Outline

• Background
• Asymptotic speed deficit from boundary layer considerations
• “WAsP Park” model details
• Asymptotic speed deficit of the “WAsP Park” model
• Adjustment of WAsP Park model
• Comparative wind farm predictions
• Conclusions
Very large wind farms:
  • Standard wake models seems to underpredict wake effects.

Recent investigations by Sten Frandsen [1, 2]:
  • The reason is the lack of accounting for the effect a large wind farm may have on the atmospheric boundary layer, e.g. by modifying the vertical wind profile.
  
• In some way the effect of an extended wind farm resembles that of a change in surface roughness: increased equivalent roughness length.

Idea:
  • While more detailed models are underway [3], modify the existing WAsP Park engineering wind farm wake model to take this boundary-layer effect into account.


Wake Decay for the Infinite Wind Turbine Array [3]
When should a wind farm be considered as large/infinite?

(Hand drawing illustrating the initial idea)
Asymptotic speed deficit from boundary layer considerations (2)

**BL-Limited infinite wind farm**

Geostrophic wind speed \( U(z|z > H) = G \)

Roughness \( z_0 \)

Friction velocity \( u_0 \)

Hub height shear \( t = \rho C_t U_h^2 \)

Jump in friction velocity at hub-height due to rotor thrust: \( \rho (u_{\text{eff}})^2 = \rho (u_i)^2 + t \)

Approximation: homogeneously distributed thrust \( c_t \)

\[
c_t = \frac{\pi}{8} \frac{C_t}{(s_r s_f)} \quad t = \rho C_t U_h^2
\]

\( s_r \) and \( s_f \): dimensionless* WTG-distances (along- and across-wind) *by \( D_{\text{rotor}} \)

\( Z < h \): profile according to ground surface friction velocity \( u_{i_1} \) / roughness \( z_0 \).

\( Z > h \): profile according to increased friction velocity \( u_{i_1}^{\text{eff}} = u_{i_0} \) / roughness \( z_0^{\text{eff}} = z_{00} \).

Equivalent, effective surface roughness: \( z_0^{\text{eff}} = h_H \cdot \exp \left( -\kappa / \sqrt{c_i + \left( \kappa / \ln(h_H / z_0) \right)^2} \right) \)
Asymptotic speed deficit from boundary layer considerations (3)

Approximate geostrophic drag-law

\[ G \approx \frac{u_*}{\kappa} \left( \ln \left( \frac{G}{f z_0} \right) - A_* \right) \]

General hub-height wind speed:

\[ U(h) = \frac{G}{1 + \left( \ln \left( \frac{G}{h \cdot f} \right) - A_* \right) i} \]

Free flow: \( i_0 = \frac{1}{\ln \frac{h}{z_0}} \)

Flow over wind farm:

\[ i_{Tot} = \sqrt{i_0^2 + i_{add}^2}, \quad i_{add} = \frac{\sqrt{c_i}}{\kappa} \]

Relative speed deficit \( \varepsilon \):

\[ 1 - \varepsilon = \frac{1 + \ln \left( \frac{G}{h \cdot f} \right) i_0}{1 + \ln \left( \frac{G}{h \cdot f'} \right) i_{Tot}} \]

Wake Decay for the Infinite Wind Turbine Array [6]
Asymptotic speed deficit from boundary layer considerations (3)

Comparison with wind farm (Horns Rev):
\[ s_r \approx s_f \approx 7, \ h=80\text{m}, \ D_R = 60\text{ m} \]

Wake deficit about 50% of the BL-limiting value.
Horns Rev wind farm NOT “infinite”.

---

**Horns Rev**

<table>
<thead>
<tr>
<th>Distance for severe wake interference ( k_{\text{wake}} = 0.075 )</th>
<th>Actual extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5 km</td>
<td>5 km</td>
</tr>
</tbody>
</table>

**Power density (W/m²) [4]**

<table>
<thead>
<tr>
<th>Horns Rev 2MW turb’s (observed)</th>
<th>Entire North Sea 5 MW turb’s (Frandsen – BL-limited)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Wake Evolution and speed deficit [5,6]

Speed deficit from single wake:

$$\delta V_{01\text{type}} = U_0 \left(1 - \sqrt{1 - C_t}\right) \left(\frac{D_0}{D_0 + 2kX_{01}}\right)^2 \frac{A_{\text{type\,overlap}}}{A_i^{(R)}}, \quad \text{(type)} = "\text{dir.}" , \ "\text{ref.}"$$

Resulting speed deficit at a downwind turbine:

$$\delta V_{\text{turb}}^2 = \sum_{i \in \text{upw\,turb}'} \left(\delta V_{i,\text{turb}}^{\text{dir.}}\right)^2 + \left(\delta V_{i,\text{turb}}^{\text{ref.}}\right)^2$$


Wake Decay for the Infinite Wind Turbine Array [8]
Asymptotic speed deficit of the “WAsP Park” model

**Speed deficit for a turbine in an infinite wind farm**

Speed deficit the same for all turbines, thus also the turbine thrusts.

Infinite (convergent!!) sum:

\[
(\delta V)^2 = \left(U_{\text{upwind}}\varepsilon_0\right)^2 \sum_{j=1}^{\infty} N(s_j)\varepsilon_w(x_j)^2; \quad \varepsilon_w(x) = \left(\frac{D_R}{D_R + 2kx} \right)^2; \quad \varepsilon_0 = \left(1 - \sqrt{1 - C_t}\right)
\]

- \(x_j\): Distance to upwind turbine row \(j\). \(N(x_j)\): number of turbines row \(j\) throwing wake on the rotor in focus.
- \(U_{\text{upwind}}\): Wind speed immediately upwind of a turbine

The infinite sum may be approximated by an infinite integral - a simple function \(G\):

\[
\frac{\delta V}{U_{\text{upwind}}\varepsilon_0} = G_{\text{park}}(k; s_r, s_f, h/D_R, C_t)
\]

Since \(U_{\text{upwind}} = U_w = U_{\text{free}} - \delta V\):

\[
\frac{\delta V}{U_{\text{Free}}} = \varepsilon_w = \frac{\varepsilon_{w,\text{app}}}{1 + \varepsilon_{w,\text{app}}}; \quad \varepsilon_{w,\text{app}} = \varepsilon_0 G_{\text{park}}(\text{layout}; k)
\]

Wake Decay for the Infinite Wind Turbine Array [9]
Adjustment to match the BL-based asymptotic speed deficit

For “deep” positions the wake expansion coefficient $k$ of the Park Model is modified to approach the BL-based asymptotic speed deficit value $k_{\text{inf}}$:

$$\delta V_{\text{inf, park}}(k_{\text{inf}}; [s_r, s_f, h, C_t]) = \delta V_{\text{BL-based}}(s_r, s_f, h, C_t)$$

The $k$-change applies when a wake overlaps with a downwind rotor (to both wakes involved), using a relaxation factor $F_{\text{relax}}$:

$$k_{j+1}^{\text{adj}} = k_{j}^{\text{adj}} + (k_{\text{inf}}^{\text{adj}} - k_{j}^{\text{adj}}) \frac{A_{\text{overlap}}}{A_w} F_{\text{relax}}$$

The change of the wake expansion coefficient is indicated.

Model-paramters used in the following (based on Horns Rev data):

- $k_{\text{initial}} := 0.075$ (recommended value for onshore!)
- $F_{\text{relax}} = 0.2$
Comparative wind farm predictions: Horns Rev (1)

Turbines: 2MW, $D_R = 80m$, $H_{hub} = 60m$
Layout: $s_r = s_f = 7$

Wake Decay for the Infinite Wind Turbine Array [11]
Comparative wind farm predictions: Horns Rev (2)

Wind direction: 270° +/- 3°
Wind speed: 8.5 m/s +/- 0.5 m/s

Wind direction: 270° +/- 3°
Wind speed: 12.0 m/s +/- 0.5 m/s

Wake Decay for the Infinite Wind Turbine Array [12]
Comparative wind farm predictions: Horns Rev (3)

Wind direction:
222° ± 3°

Wind speed:
8.5 m/s ± 0.5 m/s

Wind direction:
222° ± 3°

Wind speed:
12.0 m/s ± 0.5 m/s

Wake Decay for the Infinite Wind Turbine Array [13]
Comparative wind farm predictions: Nysted (1)

Turbines: 2.33 MW, \(D_R = 82\text{m}, H_{hub} = 69\text{m}\)
Layout: \(s_r = 10.6, s_f = 5.9\)

Wake Decay for the Infinite Wind Turbine Array [14]
Comparative wind farm predictions: Nysted(2)

Wind direction: 278° +/- 2.5°
Wind speed: 10.0 m/s +/- 0.5 m/s

Wind direction: 263° +/- 2.5°
Wind speed: 10.2 m/s +/- 0.5 m/s

Wake Decay for the Infinite Wind Turbine Array [15]
Conclusions

• The adjustment of the wake expansion coefficient towards a value matching the BL-limited asymptotic speed deficit seems a valuable engineering approach.
• A value for the wake expansion coefficient close to that normally used for onshore – locations seems reasonable in this approach also for off-shore wind farms.
• The model (relaxation factor) needs to be fine-tuned in order not to produce over estimations.
• The model needs to be tested on situations with wake effects between neighboring wind farms.