Wake decay constant for the infinite wind turbine array
Application of asymptotic speed deficit concept to existing engineering wake model

Rathmann, Ole Steen; Frandsen, Sten Tronæs; Nielsen, Morten

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Application of asymptotic speed deficit concept to existing engineering wake model

Ole Rathmann, Sten Frandsen and Morten Nielsen
Risoe-DTU, National Laboratory for Sustainable Energy

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WAKE DECAY FOR THE INFINITE WIND TURBINE ARRAY

Application of asymptotic speed deficit concept to existing engineering wake model

Outline
• Background
• Asymptotic speed deficit from boundary layer considerations
• “WAsP Park” model details
• Asymptotic speed deficit of the “WAsP Park” model
• Adjustment of WAsP Park model
• Comparative wind farm predictions
• Conclusions
Background

Very large wind farms:

• Standard wake models seems to underpredict wake effects.

Recent investigations by Sten Frandsen [1, 2]:

• The reason is the lack of accounting for the effect a large wind farm may have on the atmospheric boundary layer, e.g. by modifying the vertical wind profile.

• In some way the effect of an extended wind farm resembles that of a change in surface roughness: increased equivalent roughness length.

Idea:

• While more detailed models are underway [3], modify the existing WAsP Park engineering wind farm wake model to take this boundary-layer effect into account.

Asymptotic speed deficit from boundary layer considerations (1)

When should a wind farm be considered as large/infinite?

(Hand drawing illustrating the initial idea)
Asymptotic speed deficit from boundary layer considerations (2)

BL-Limited infinite wind farm

Geostrophic wind speed \( U(z > H) = G \)

\[ \ln(z) \]

\( H \)

\( G \)

\( U_h \)

\( \rho C_t \frac{U_h^2}{2} \)

Friction velocity \( u_0 \)

Roughness \( z_0 \)

Hub height shear \( t = \rho C_t U_h^2 \)

Jump in friction velocity at hub-height due to rotor thrust: \( \rho (u_{\text{eff}})^2 = \rho (u_*)^2 + t \)

Approximation: homogeneously distributed thrust \( c_t \)

\( c_t = \frac{\pi}{8} \frac{C_t}{s_r s_f} \), \( t = \rho C_t U_h^2 \)

\( s_r \) and \( s_f \): dimensionless* WTG-distances (along- and across-wind) \*by \( D_{\text{rotor}} \)

\( Z < h \): profile according to ground surface friction velocity \( u_* \) / roughness \( z_0 \).

\( Z > h \): profile according to increased friction velocity \( u_{\text{eff}} (= u_*^0) \) / roughness \( z_{0\text{eff}} (=z_{00}) \).

Equivalent, effective surface roughness:

\( z_{0\text{eff}}^H = h_H \cdot \exp \left( -\frac{\kappa}{c_i} + \left( \frac{\kappa}{\ln(h_H/z_0)} \right)^2 \right) \)

Wake Decay for the Infinite Wind Turbine Array [5]
Asymptotic speed deficit from boundary layer considerations (3)

Approximate geostrophic drag-law

\[ G \approx \frac{u_*}{\kappa} \left( \ln \left( \frac{G}{f z_0} \right) - A_* \right) \]

General hub-height wind speed:

\[ U(h) = \frac{G}{1 + \left( \ln \frac{G}{h f} - A_* \right) i} \]

Free flow: \[ i_0 = \frac{1}{\ln \frac{h}{z_0}} \]

Flow over wind farm: \[ i_{\text{Tot}} = \sqrt{i_0^2 + i_{\text{add}}^2}, \quad i_{\text{add}} = \frac{\sqrt{c_i}}{\kappa} \]

Relative speed deficit \( \varepsilon \):

\[ 1 - \varepsilon = \frac{1 + \ln \left( \frac{G}{h f} \right) i_0}{1 + \ln \left( \frac{G}{h f} \right) i_{\text{Tot}}} \]
Asymptotic speed deficit from boundary layer considerations (3)

Comparison with wind farm (Horns Rev):
$s_r \approx s_f \approx 7$, $h=80\text{ m}$, $D_R = 60\text{ m}$

Wake deficit about 50% of the BL-limiting value.
Horns Rev wind farm NOT “infinite”.

<table>
<thead>
<tr>
<th>Horns Rev</th>
<th>Power density (W/m$^2$) [4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance for severe wake interference ($k_{\text{wake}}=0.075$)</td>
<td>Actual extension</td>
</tr>
<tr>
<td>7.5 km</td>
<td>5 km</td>
</tr>
</tbody>
</table>

Wake Evolution and speed deficit [5,6]

\[ \delta V_{01}^{(type)} = U_0 \left( 1 - \sqrt{1 - C_t} \right) \left( \frac{D_0}{D_0 + 2kX_{01}} \right)^2 \frac{A_{(type\,overlap)}}{A_i^{(R)}} \], \quad (type) = "dir." , "ref."

Resulting speed deficit at a downwind turbine:

\[ \delta V_{turb}^2 = \sum_{i \in upw\,turb's} \left( (\delta V_{i,\,turb}^{(dir.)})^2 + (\delta V_{i,\,turb}^{(ref.)})^2 \right) \]

Asymptotic speed deficit of the “WAsP Park” model

**Speed deficit for a turbine in an infinite wind farm**

Speed deficit the same for all turbines, thus also the turbine thrusts.

Infinite (convergent!!) sum:

\[
(\delta V)^2 = \left( U_{\text{upwind}} \varepsilon_0 \right)^2 \sum_{j=1}^{\infty} N(s_j) \varepsilon_w(x_j)^2; \quad \varepsilon_w(x) = \left( \frac{D_R}{D_R + 2kx} \right)^2; \quad \varepsilon_0 = \left( 1 - \sqrt{1 - C_t} \right)
\]

- \(x_j\): Distance to upwind turbine row \(j\).
- \(N(x_j)\): number of turbines row \(j\) throwing wake on the rotor in focus.
- \(U_{\text{upwind}}\): Wind speed immediately upwind of a turbine.

The infinite sum may be approximated by an infinite integral - a simple function \(G\):

\[
\frac{\delta V}{U_{\text{upwind}} \varepsilon_0} = G_{\text{park}} \left( k; s_r, s_f, h/D_R, C_t \right)
\]

Since \(U_{\text{upwind}} = U_w = U_{\text{free}} - \delta V\):

\[
\frac{\delta V}{U_{\text{Free}}} = \varepsilon_w = \frac{\varepsilon_w^{\text{app}}}{1 + \varepsilon_w^{\text{app}}}; \quad \varepsilon_w^{\text{app}} = \varepsilon_0 G_{\text{park}}(\text{layout}; k)
\]

Wake Decay for the Infinite Wind Turbine Array [9]
Adjustment of the “WAsP Park” model

Adjustment to match the BL-based asymptotic speed deficit

For “deep” positions the wake expansion coefficient $k$ of the Park Model is modified to approach the BL-based asymptotic speed deficit value $k_{\text{inf}}$:

$$\delta V_{\text{infin.park}}(k_{\text{inf}};[s_r,s_f,h,C_t]) = \delta V_{\text{BL-based}}(s_r,s_f,h,C_t)$$

The $k$-change applies when a wake overlaps with a downwind rotor (to both wakes involved), using a relaxation factor $F_{\text{relax}}$:

$$k_{adj}^{j+1} = k_{adj}^j + (k_{\text{inf}} - k_{adj}^j) \frac{A_{\text{overlap}}}{A_w} F_{\text{relax}}$$

The change of the wake expansion coefficient is indicated.

Model-parameters used in the following (based on Horns Rev data):

- $k_{\text{initial}} := 0.075$ (recommended value for onshore!)
- $F_{\text{relax}} = 0.2$

Wake Decay for the Infinite Wind Turbine Array [10]
Comparative wind farm predictions: Horns Rev (1)

Turbines: 2MW, $D_R = 80m$, $H_{hub} = 60m$
Layout: $s_r = s_f = 7$

Wake Decay for the Infinite Wind Turbine Array [11]
Comparative wind farm predictions: Horns Rev (2)

Wind direction: 270° +/- 3°
Wind speed: 8.5 m/s +/- 0.5 m/s

Wake Decay for the Infinite Wind Turbine Array [12]
Comparative wind farm predictions: Horns Rev (3)

Wind direction:
- 222° +/- 3°

Wind speed:
- 8.5 m/s +/- 0.5 m/s

Wind direction:
- 222° +/- 3°

Wind speed:
- 12.0 m/s +/- 0.5 m/s
Comparative wind farm predictions: Nysted (1)

Turbines: 2.33 MW, $D_R = 82\text{m}$, $H_{\text{hub}} = 69\text{m}$
Layout: $s_r = 10.6$, $s_f = 5.9$
Comparative wind farm predictions: Nysted(2)

Wind direction: 278° +/- 2.5°
Wind speed: 10.0 m/s +/- 0.5 m/s

Wind direction: 263° +/- 2.5°
Wind speed: 10.2 m/s +/- 0.5 m/s

Wake Decay for the Infinite Wind Turbine Array [15]
Conclusions

• The adjustment of the wake expansion coefficient towards a value matching the BL-limited asymptotic speed deficit seems a valuable engineering approach.

• A value for the wake expansion coefficient close to that normally used for onshore – locations seems reasonable in this approach also for off-shore wind farms.

• The model (relaxation factor) needs to be fine-tuned in order not to produce over estimations.

• The model needs to be tested on situations with wake effects between neighboring wind farms.

Wake Decay for the Infinite Wind Turbine Array [16]